Reliability of Module Based Software System

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Abstract

This paper consider the problem of determining the reliability of a software system which can be decomposed in a number of modules. We have derived the expression of the reliability of a system using the Markovian model for the transfer of control between modules in order. We have given the expression of reliability by considering both benign and catastrophic failure. The expression of reliability presented in this work is applicable for some control software which are designed to detect its own internal errors.

1 Introduction

Now a days large scale software systems are used in every walk of life. The price of software are much higher than the cost of hardware when we consider a huge computer intensive system. Moreover the penalty cost incurred by a false outcome of a system is enormous. To address such a challenge posed by this technological trend, during the last three decades extensive research has focused on the area of software reliability. The consideration of software reliability is increasing because of the growing emphasis on software that is reusable (as opposed to software that is written for a terminal mission), where it is essential to demonstrate that the system will perform reliably for a variety of end-user applications.

A software system is defined here as a " collection of programs and system files such that the system files are accessed and altered only by the programs in the collection ". Each element in this collection will be called a module - for instance, a module might be a program, a subprogram, or a file. The performance (and hence the reliability) of the system clearly depends on that of each individual module and the relationship between these modules and the system; in this regard a software system is quite similar to any other system. However, the actual relationship between system and module reliabilities is quite unique and depends on the specific definition of software reliability as well as on

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the structure of the overall system. In this paper we focus on software systems that can be decomposed into a finite number of modules.

In testing a software one may test the system as a whole, but in practice, different organizational entities are assigned responsibility of developing different modules. So it will be more beneficial in the context of both cost and time test the individual modules instead of testing them together. In order to do this, some mathematical models, often referred to as Software Reliability Growth Models (SRGM) are used to enable the software reliability practitioners to estimate the expected future reliability of a software under development and accordingly allocate time, money, human resources to a project. Often these reliability growth models express software system reliability in terms of the individual module reliabilities which is favorable under both time and cost constraints.

Cheung (2), first expressed the system reliability in terms of the component reliabilities. Poore et al. (1) suggest allocating the targeted system reliability goal among the components and then testing the individual components to verify whether the component reliabilities meet the allocated goals at a specified level of confidence, where as Easterling, Mazumdar, Spencer and Diegert (6), has discussed this method may lead to estimates of overly conservative sample size requirements for component testing. Yang et. al. has implemented the idea of using testability to estimate software reliability. They have also provided the basic steps involve estimating testability, evaluating how well software was written, and assessing the relationship between testing and usage by assuming the modules are independently functioning. They have also compared their results with those obtained by using two reliability growth models. Rajgopal et. al. has used a Markovian model for the transfer of control between modules in order to develop the system reliability expression in terms of the module reliabilities in case of a dependent setup. They have also discussed a procedure for determining the minimum number of tests required of each module such that the probability of certifying a system whose reliability falls below a specified value R_0 is less than a specified small fraction β. Bondavalli et. al. has considered the concept of benign failure and catastrophic failure for determining the software reliability for a iterative program.

In this paper we have expressed the system reliability in terms of testability of a particular module following Yang et. al. for dependent modules and have introduced the concepts of benign and catastrophic failure following Bondavalli et. al. in case of a system where it can be decomposed in a finite number of dependently functional modules. The section 2 discuss the notations and preliminaries, section 3 gives the expression of the probability of correct output for a specific input. Recent research [26] has shown a strong correlation between reliability and coverage criteria (Lott et al. (2005), Khun et. al. (2002), Yilmaz et. al. (2004) etc.), although it is very difficult to quantify this relation. Dalal et al. [6] and many more has examined this relationship between unit-test statement coverage and system-test faults later attributed to those units.

Present work has been organized in 4 sections the section 2 gives the notation and preliminaries of software reliability in terms testability of a module. In the 3rd sections we have derived the probability of correct output of a particular system corresponding to a particular input considering both the case presence and absence of benign failure. In Section 4 we present a brief discussions about the procedure mentioned here.

2 Notations and Preliminaries

There is no rigorous definition of 'Quality'. But it can be weakly defined as the fitness of purpose of any product to its users. Similarly software quality is defined as the conformance to explicitly stated functions and performance requirements, explicitly documented development standards and implicit characteristics that are expected of all professionally crafted software(Cai Kai-Yuan Cai (3)). Alternatively, the quality of a software may be characterized by some quality factors of a software - reliability, efficiency, correctness, usability, testability etc.

Reliability of a software system may be viewed as the expected value of probability of failure-free operation of a program for a randomly chosen set of input variables. The term failure in the context of software reliability implies a result other than what was expected from the software for a set of inputs. Following Voas et. al. (1995) we define the testability of a particular system as the probability of failure of the system for a particular input when it is assumed that there is at least one fault in the system. Suppose we have a software system which can be decomposed in N modules. Thus the testability of a particular module, say ith ($\forall i = 1(1)N$) module, is given by

$$
\begin{aligned}
p_i \quad = \quad & \text{Prob[that the } i\text{th module will give incorrect output } | \text{ there is at least one fault,} \\
& \text{probability distribution of input]}
\end{aligned} \tag{1}
$$

The expression for the probability that the ith module will contain error if the module has tested n_i times successfully, is given by the following (Yang et. al. (1998))

$$
\alpha_i(t) = \frac{\alpha_i(0)(1 - p_i)^{n_i}}{\alpha_i(0)(1 - p_i)^{n_i} + 1 - \alpha_i(0)} \tag{2}
$$

where $\alpha_i(0)$ is the probability of failure of the system before testing. Let $\pi_t(x)$ is the probability of a system giving correct output corresponding to a particular set of input x. The expression of $\pi_t(x)$ by assuming the independent setup is given by (Yang et. al. (1998))

$$
\pi_t(x) = \prod_{i \in S}(1 - q_i \alpha_i(t)) \tag{3}
$$

where q_i is the revealibility of the i th module and $S(x)$ is the set of those modules which will be executed by the input x. The reliability of a software system is given by

$$
R_t = \int_{x \in X} \pi_t(x)\phi(x)dx \tag{4}
$$

where X is the set of all possible inputs and $\phi(x)$ is the probability distribution of x.

3 Detailed Expression of $\pi_t(x)$ for Dependent Setup

A software system is necessarily an iterative. In each iteration a particular module accepts a value and produce an output. The outcomes of an individual iteration may

be: i) success, i.e., the delivery of a correct result, ii) a benign failure of the program, i.e., an output that is not correct but does not, by itself, cause the entire mission of the controlled system to fail, or iii) a catastrophic failure, i.e., an output that causes the immediate failure of the entire mission. The characterization of failures in benign and catastrophic is discussed with example by Bondavalli. et. al. (). In this section we derive the expression of $\pi_t(x)$ first of all only considering the catastrophic failure and then in the subsequent subsection considering the benign and catastrophic failure simultaneously.

3.1   Expression of $\pi_t(x)$: No Benign Failure in the System

Consider the above software system with N modules. Let p_{ij} be the probability that the control from the ith module will be transferred to the jth module with correct execution ($\forall i = 1(1)N, \forall j = 1(1)N$). Let S be a state of successful completion of the system. As S is achievable from any one of the module so we define p_{iS} ($\forall i = 1(1)N$) as the probability of successful completion of the mission from the ith module. Here we must have $p_{iS} + \sum_{j=1}^{n} p_{ij} = 1$.

As we have a faulty system, that is, we have a system where there is at least one fault or if the faults can be classified into categories then there are at most one fault of each category. So we introduce another state F, i.e., unsuccessful completion of the mission. As any module may be faulty so the state F also can be achieved from any of the module. We define p_{iF} as the probability of unsuccessful completion of the module i ($\forall i = 1(1)N$). The transition probability matrix takes the following form for the above setup.

$$Q = \begin{pmatrix} p_{11}(1 - \alpha_1^x(t)) & p_{12}(1 - \alpha_1^x(t)) & \dots & p_{1N}(1 - \alpha_1^x(t)) & p_{1S}(1 - \alpha_1^x(t)) & \alpha_1^x(t) \\ p_{21}(1 - \alpha_2^x(t)) & p_{22}(1 - \alpha_2^x(t)) & \dots & p_{2N}(1 - \alpha_2^x(t)) & p_{2S}(1 - \alpha_2^x(t)) & \alpha_2^x(t) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ p_{N1}(1 - \alpha_N^x(t)) & p_{N2}(1 - \alpha_N^x(t)) & \dots & p_{NN}(1 - \alpha_N^x(t)) & p_{NS}(1 - \alpha_N^x(t)) & \alpha_N^x(t) \\ 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

where $\alpha_i^x(t)$ is the probability of faulty completion of the ith module for the input x. The expression of $\alpha_i^x(t)$ is given by

$$\alpha_i^x(t) = q_i \alpha_i(t) \quad (6)$$

If we assume that the first block is the control block then the probability of correct completion of the mission for the given input x is given by (Parzen (1962))

$$\pi_t(x) = \sum_{i=1}^{N} (I_N - \hat{Q})_{1i}^{-1} p_{iS}(1 - \alpha_i^x(t)) \quad (7)$$

where $\hat{Q}$ is the sub-matrix of Q deleting its last two columns and rows.

3.2 Expression of $\pi_t(x)$: Benign Failure and Catastrophic Failure are in the System

From the software viewpoint solely, and without referring to any specific application, we assume here that all detected failures (default safe values of the control outputs from the computer) do not prevent the mission to continue and are in this sense benign, whereas undetected failures are conservatively assumed to have a "catastrophic" effect on the controlled system. Obviously, if knowledge of the consequences of software failures on the system was available for a specific system, the proper splitting of software failures into benign and catastrophic could be precisely made. We make the following assumption to model the system.

Suppose SS is a state where the total system, that is all the N modules, runs without any fault of either kind. Let B_i be the state where the system is running in benign failure of ith level, that is after i iterations the system will enter in the state SS. As the previous subsection S and F denotes the successful completion of the mission and completion of the mission with a failure respectively. The mission will fail if their is a catastrophic failure in the system. Let us also assume that if there is a benign failure of length greater than a threshold value, say n_c, then the system will enter in a catastrophic failure region. Although this assumption will take the model a little away from reality, a model should be good enough to handle a benign failure of any arbitrary random length, but this assumption will make the calculation of reliability expression easier which will increase its practical application. At this point note that the state S, that is the successful completion of the program, can be achieved only from the state SS, where as the state F can be achieved from any of the state SS or B_i's ($\forall i = 1(1)N$), but we assume here the control will be transferred from the state B_i to B_{i-1} only to reduce the number of parameters in the model.

The transition probability matrix will be as follows

$$Q = \begin{pmatrix} Q_{00} & Q_{01}^b & Q_{02}^b & \cdots & Q_{0(n_c-2)}^b & Q_{0(n_c-1)}^b & Q_{0n_c}^b & S^0 & F^0 \\ Q_{10}^b & O & O & \cdots & O & O & O & \bar{0} & \bar{0} \\ O & Q_{21}^b & O & \cdots & O & O & O & \bar{0} & \bar{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ O & O & O & \cdots & Q_{(n_c-1)(n_c-2)}^b & O & O & \bar{0} & \bar{0} \\ O & O & O & \cdots & O & Q_{n_c(n_c-1)}^b & O & \bar{0} & \bar{0} \\ \bar{0}' & \bar{0}' & \bar{0}' & \cdots & \bar{0}' & \bar{0}' & \bar{0}' & 1 & 0 \\ \bar{0}' & \bar{0}' & \bar{0}' & \cdots & \bar{0}' & \bar{0}' & \bar{0}' & 0 & 1 \end{pmatrix} \tag{8}$$

Here the matrix Q_{00} is a $N \times N$ matrix which describes that the flow is running without entering in benign failure or catastrophic failure. The matrix Q_{0k}^b is also a $N \times N$ matrix giving the transition probabilities of the flow of control from stable state to the kth level benign failure ($\forall k = 1(1)n_c$). Similarly, the matrix Q_{kl}^b which is also $N \times N$ denotes the transition probabilities of the control entering from the kth level benign failure to l th level ($\forall k = 1(1)n_c \forall l = 1(1)n_c$). From the kth level benign failure we can only achieve the $k-1$th level benign failure so $Q_{kl}^b = O$ ($\forall l \neq k-1$). Where O is the null matrix of order $N \times N$. S^0 is a $N \times 1$ vector of the transition probabilities of successful completion of the mission from the stable state. As the mission can terminate successfully only from

the stable state so the rest of the entries in this column are all zero. $\bar{0}$ denotes a null vector of length N and $\bar{0}'$ denotes transpose of $\bar{0}$. Finally, F^0 is a column vector of length N giving probabilities of reaching the state of catastrophic failure from the stable state.

To give the structure of sub-matrices Q_{00}, let us define p_{ij}^{SS} be the probability of the control to enter from the ith module to jth module in the state SS. So the matrix Q_{00} is given by

$$Q_{00} = \begin{pmatrix} p_{11}^{SS} & p_{12}^{SS} & \cdots & p_{1N}^{SS} \\ p_{21}^{SS} & p_{22}^{SS} & \cdots & p_{2N}^{SS} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1}^{SS} & p_{N2}^{SS} & \cdots & p_{NN}^{SS} \end{pmatrix} \tag{9}$$

Let us also define p_{ij}^{SB} be the probability that the control will be transferred from the module i to the module j from the state SS to any of benign failure. Let also p_k^B the probability that the control will enter in B_k, thus the probability that the control will enter in the jth module from the ith module in the state B_k is given by $p_{ij}^{SB} p_k^B$. So the matrix Q_{0k}^b will take the following form

$$Q_{0k}^b = \begin{pmatrix} p_{11}^{SB} p_k^B & p_{12}^{SB} p_k^B & \cdots & p_{1N}^{SB} p_k^B \\ p_{21}^{SB} p_k^B & p_{22}^{SB} p_k^B & \cdots & p_{2N}^{SB} p_k^B \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1}^{SB} p_k^B & p_{N2}^{SB} p_k^B & \cdots & p_{NN}^{SB} p_k^B \end{pmatrix} \tag{10}$$

If p_{iS} and p_{iF} is respectively the successful completion of the mission and achieving catastrophic failure from the ith module. Then we must have

$$\sum_{j=1}^{N} p_{ij}^{SS} + \sum_{k=1}^{n_c} p_k^B \sum_{j=1}^{N} p_{ij}^{SB} + p_{iS} + p_{iF} = 1 \qquad \forall i = 1(1)N \tag{11}$$

The matrix Q_{kk-1}^b takes the following form

$$Q_{kk-1}^b = \begin{pmatrix} p_{11}^{bb} & p_{12}^{bb} & \cdots & p_{1N}^{bb} \\ p_{21}^{bb} & p_{22}^{bb} & \cdots & p_{2N}^{bb} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1}^{bb} & p_{N2}^{bb} & \cdots & p_{NN}^{bb} \end{pmatrix} \tag{12}$$

Here we have

$$\sum_{j=1}^{N} p_{ij}^{bb} = 1 \qquad \forall i = 1(1)N \tag{13}$$

Finally, the matrix Q_{10}^b is the matrix of transition probabilities, say p_{ij}^{bS}, that the flow of control will be transferred from the ith to the jth module and from the B_1 to SS. Here also

$$\sum_{j=1}^{N} p_{ij}^{bS} = 1 \qquad \forall i = 1(1)N \tag{14}$$

By assuming as before the first module as the control module the expression of $\pi_t(x)$ is given

$$\pi_t(x) = \sum_{i=1}^{N} (I_{Nn_c} - \hat{Q})_{1i}^{-1} p_{iS} \tag{15}$$

where $\hat{Q}$ is once again the sub-matrix of Q deleting its last two columns and rows.

4   Conclusions

In this work we have given an expression of the reliability of a software system which can be divided in a finite number of modules. The transition probabilities we have considered can be easily estimated using maximum likelihood method of estimation.

Consider the setup without benign failure, suppose ith block is tested n_i times, out of which x_j^i times the control is transferred to the jth state $(\forall i = 1(1)N \& \forall j = 1(1)N, S, F)$. The maximum likelihood estimates of p_{ij} is $x_j^i/(\sum_{i=1}^{N} x_j^i + x_S^i)$ and that of $\alpha_i^x(t)$ is x_F^i/n_i. Hence estimate of $\pi_t(x)$ can be obtained and let it be denoted by $\hat{\pi}_t(x)$. Finally the estimate of reliability of a system can be given by

$$\hat{R}_t = \frac{1}{|W|} \sum_{x \in W} \hat{\pi}_t(x) \tag{16}$$

where W is the set of all inputs which are used for testing. This is an extension of some previous work and the model what we have considered are more realistic for some control software which are designed to detect its own internal errors and then issue a safe output and reset itself to a known state from which the program is likely to proceed correctly.

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