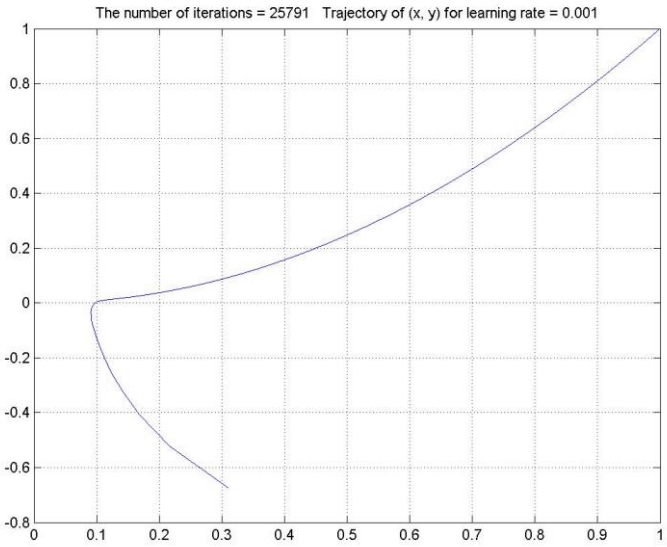
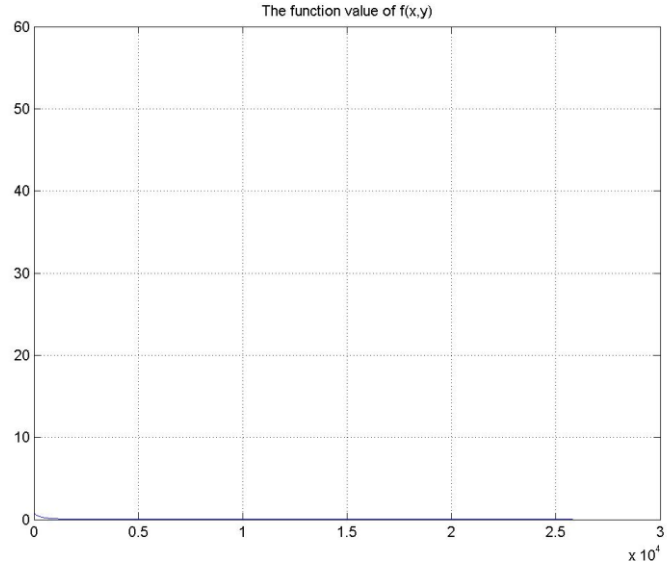
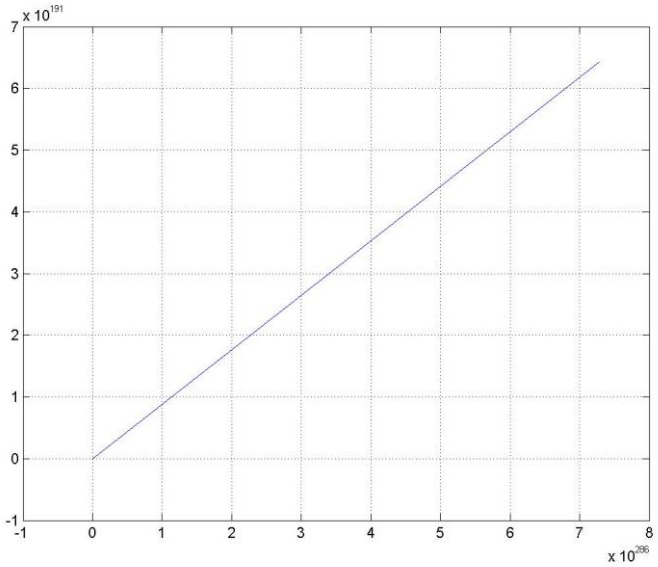
Q1.

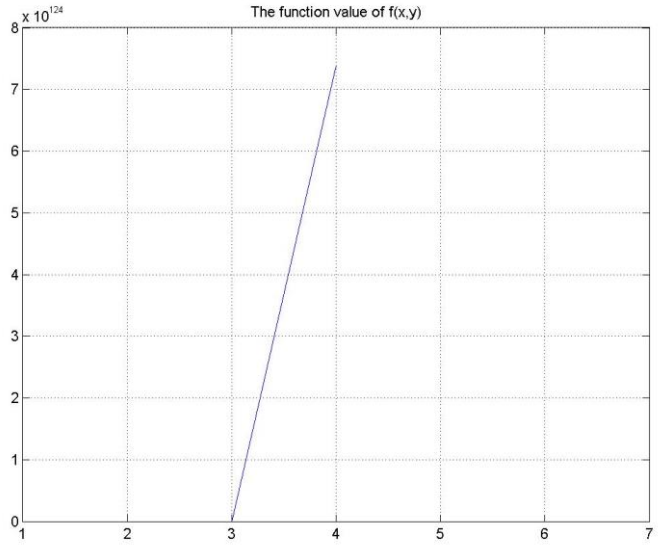
b) Steepest (Gradient) descent method with learning rate = 0.001.
The figure below shows the number of iterations when f(x, y) converges to 0(1e-10) and the trajectory of (x, y) in the 2-dimensional space.





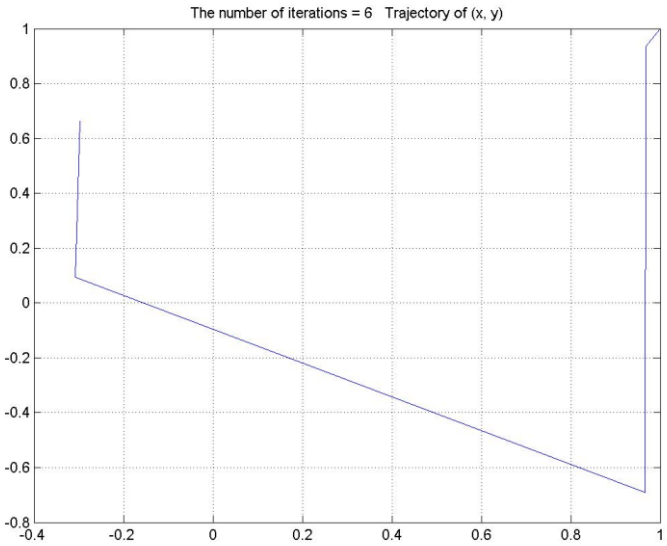
Steepest (Gradient) descent method with learning rate = 1.0.
The figure below shows the number of iterations when f(x, y) converges to 0(1e-10) and the trajectory of (x, y) in the 2-dimensional space.

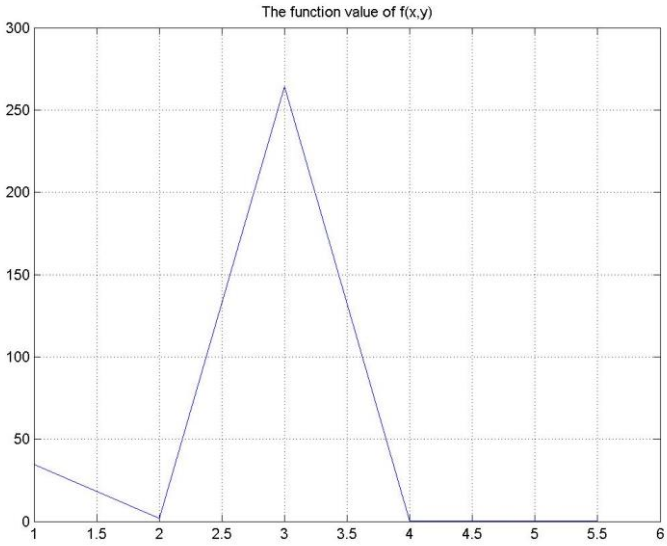




Since the learning rate is too large, the result could not converge.

c) Newton's method.
The figure below shows the number of iterations when f(x, y) converges to 0(1e-10) and the trajectory of (x, y) in the 2-dimensional space.





The following is the MATLAB code of steepest descent method:

```
%%INPUT
clear all; close all;
i = 1;
x(i) = rand(1)*2-1;
y(i) = rand(1)*2-1;
f(i) = (1-x(i))^2 + 100*(y(i)-x(i)^2)^2;
learning_rate = 0.001;

%%CACULATION
while f(i) > 1e-10
fx(i) = 2*x(i)-2+400*(x(i)^3-x(i)*y(i));
fy(i) = 200*(y(i)-x(i)^2);
i = i + 1;
x(i) = x(i-1) - learning_rate*fx(i-1);
y(i) = y(i-1) - learning_rate*fy(i-1);
f(i) = (1-x(i))^2 + 100*(y(i)-x(i)^2)^2;
end

%%PLOT
plot(x,y,'-');
grid on;
discription1 = sprintf('The number of iterations = %d  Trajectory of
(x, y) for learning rate = 0.001',i);
title(discription1);
saveas(gcf,'Q1_2_1.jpg');

iteration=1:i;
figure;
plot (iteration,f(iteration),'-');
grid on;
discription2 = sprintf('The function value of f(x,y)');
title(discription2);
saveas(gcf,'Q1_2_2.jpg');
```

The following is the MATLAB code of Newton's method:

```
%%INPUT
clear all; close all;
i = 1;
x(i) = rand(1)*2-1;
y(i) = rand(1)*2-1;
f(i) = (1-x(i))^2 + 100*(y(i)-x(i)^2)^2;
learning_rate = 0.001;

%%CACULATION
while f(i) > 1e-10
fx(i) = 2*x(i)-2+400*(x(i)^3-x(i)*y(i));
fy(i) = 200*(y(i)-x(i)^2);
H{i} = [1200*x(i)^2-400*y(i)+2 -400*x(i); -400*x(i) 200];
i = i + 1;
tmp = [x(i-1);y(i-1)] - inv(H{i-1})*[fx(i-1);fy(i-1)];
x(i) = tmp(1);
y(i) = tmp(2);
f(i) = (1-x(i))^2 + 100*(y(i)-x(i)^2)^2;
end

%%PLOT
plot(x,y,'-');
grid on;
discription1 = sprintf('The number of iterations = %d  Trajectory of
(x, y)',i);
title(discription1);
saveas(gcf,'Q1_3_1.jpg');

iteration=1:i;
figure;
plot (iteration,f(iteration),'-');
grid on;
discription2 = sprintf('The function value of f(x,y)');
title(discription2);
saveas(gcf,'Q1_3_2.jpg');
```