

## ■ Initial State

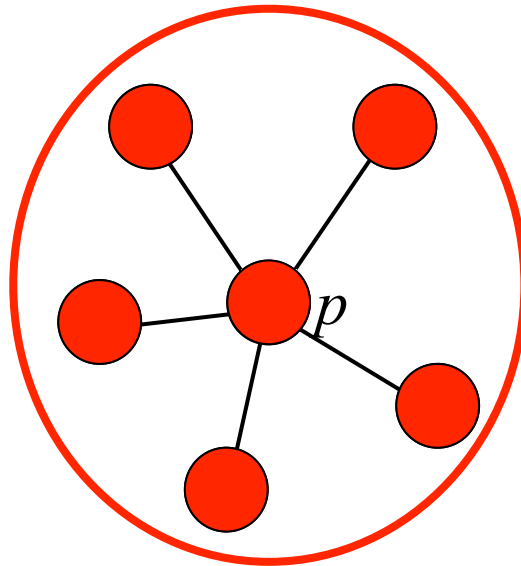
- ① Incorrect messages in channels
- ② Incorrect values in variables

*Starting from **any initial configuration**, the system eventually reaches a configuration from which its behavior is correct.*

*Dijkstra 1974*

# State Model

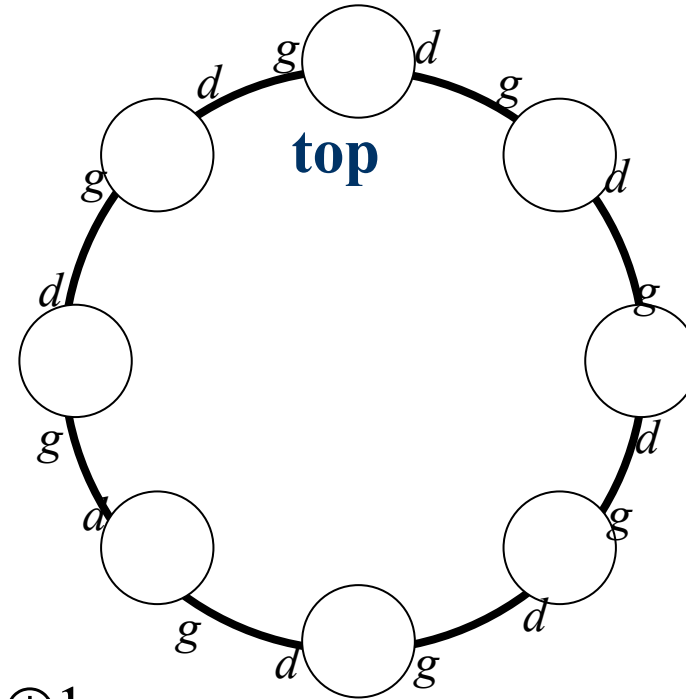
<Gard> → <Action>



# Anneau à jeton

## Spécification

- Sûreté : *Au plus un jeton dans le système*
- Vivacité : *Chaque processeur obtient le jeton infiniment souvent*



**$p = top$**

TR1 :  $(v_p = v_g) \rightarrow v_p := v_g \oplus 1$

**$p \neq top$**

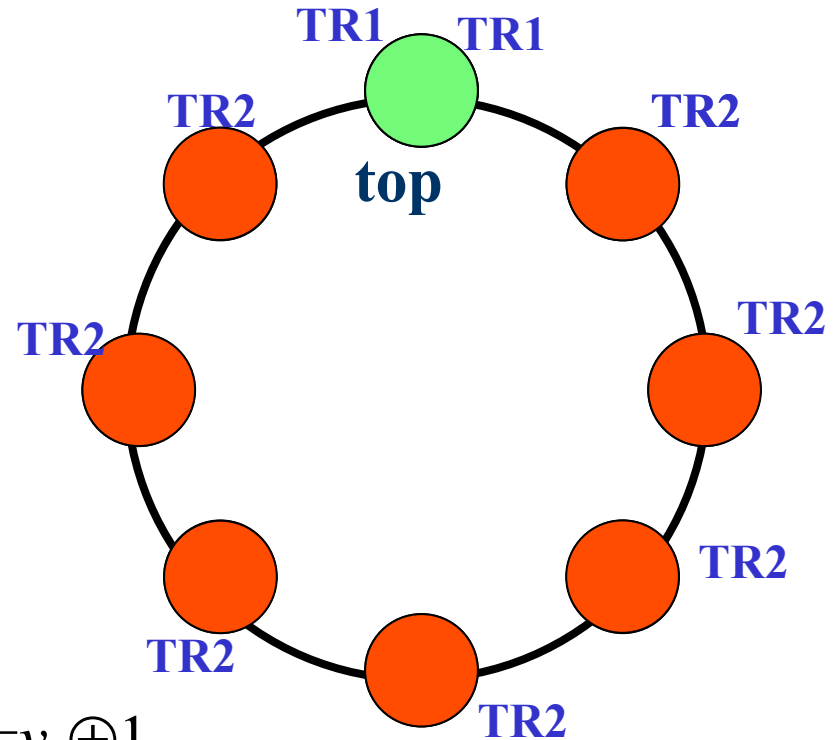
TR2 :  $(v_p \neq v_g) \rightarrow v_p := v_g$

# Anneau à jeton

Auto-stabilisant ?

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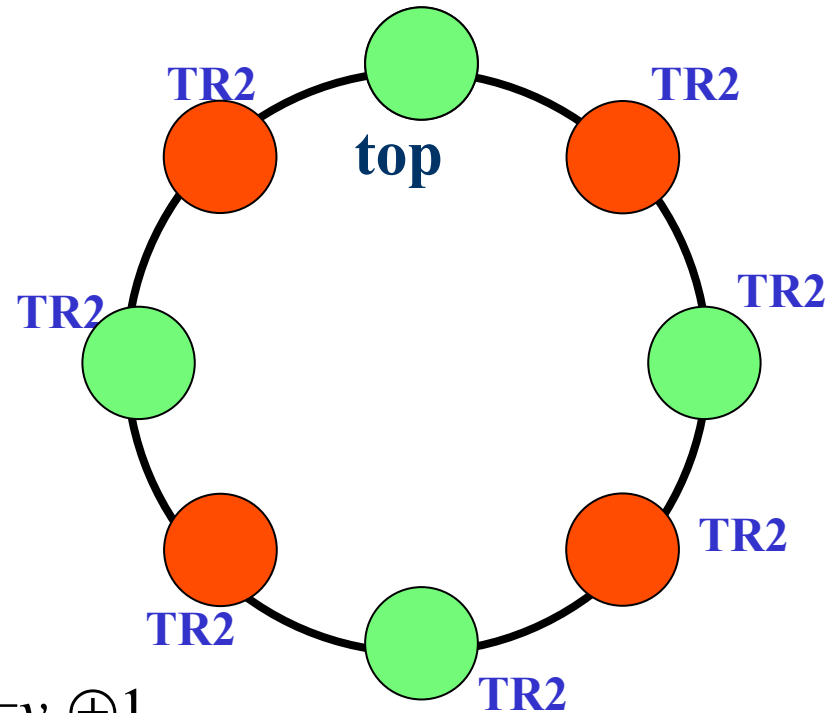
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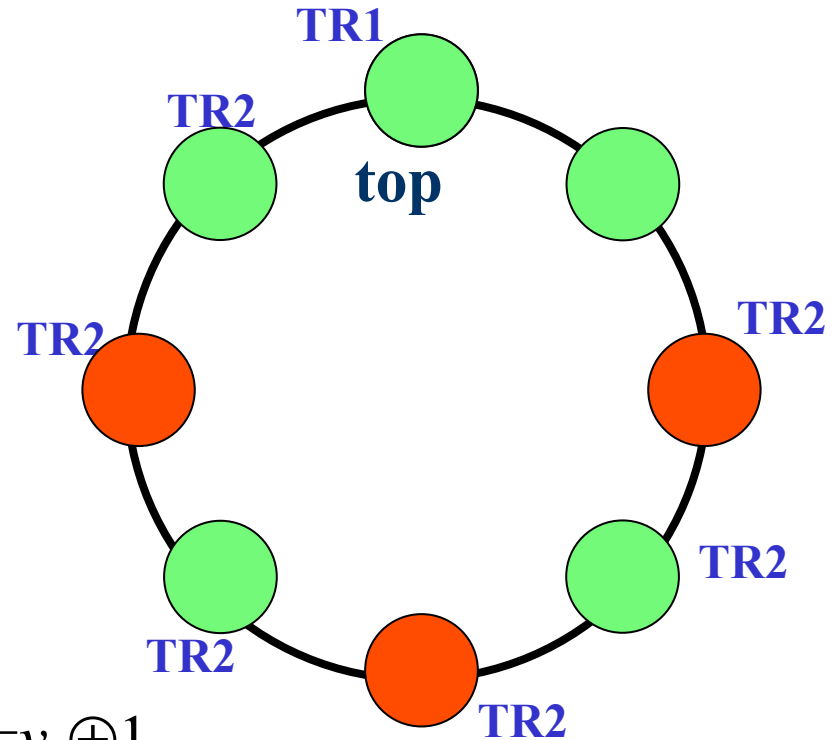
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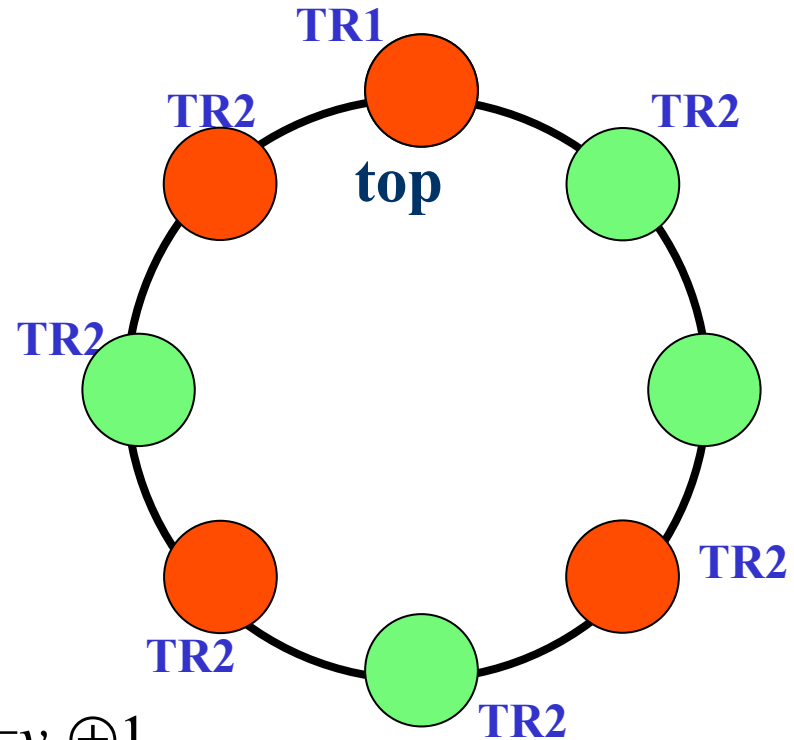
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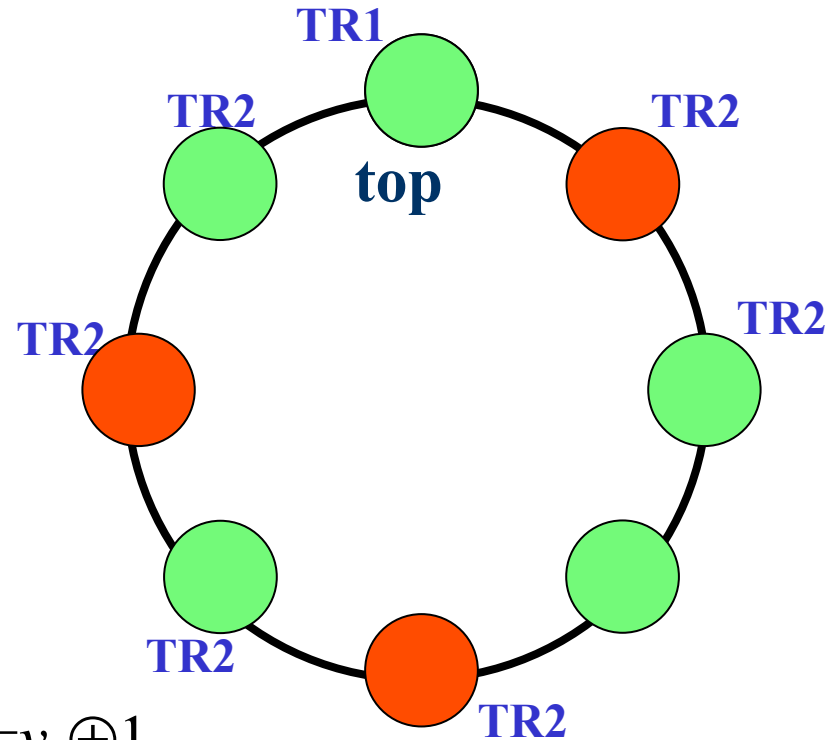
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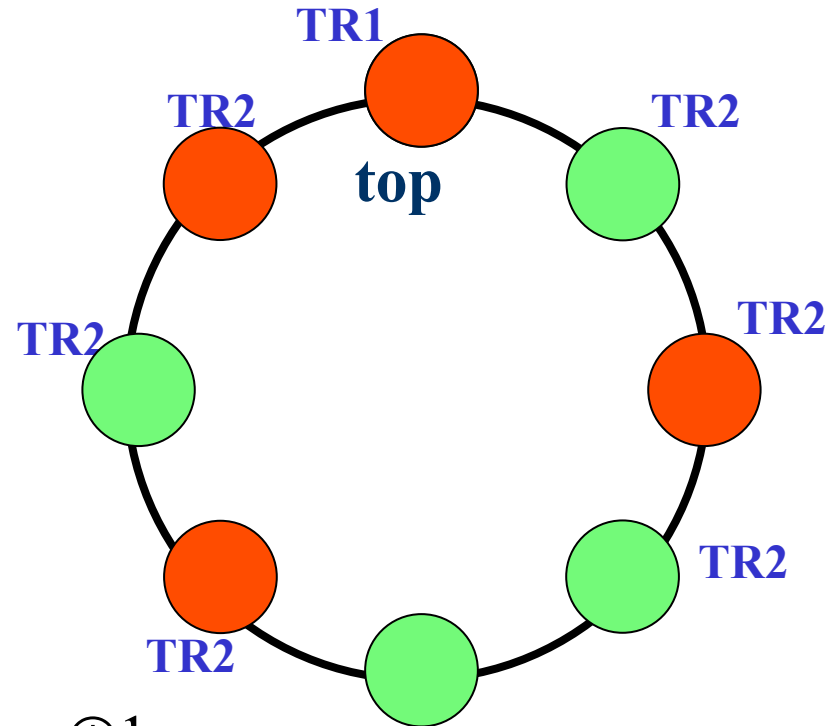
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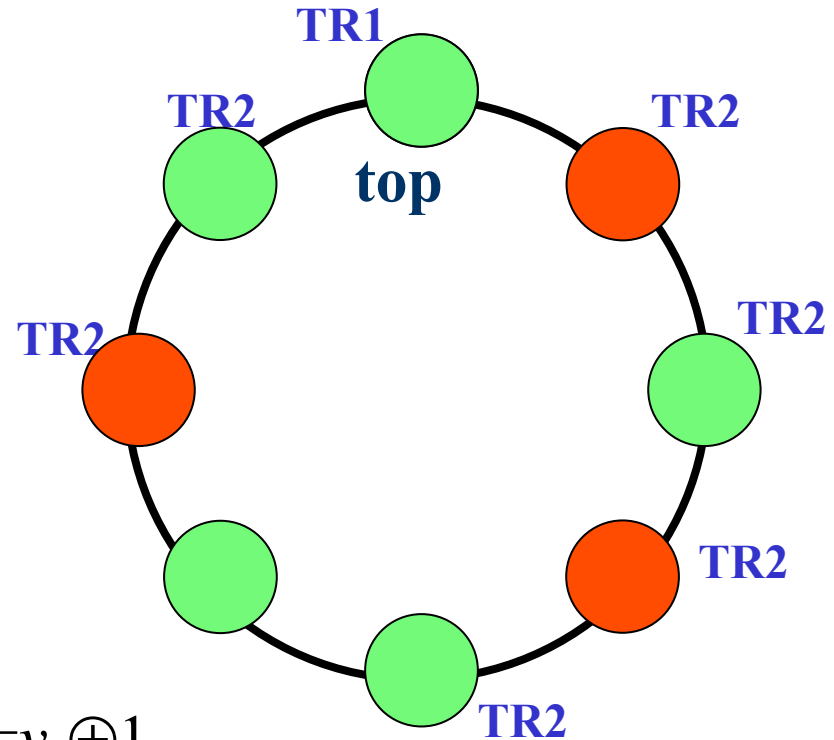
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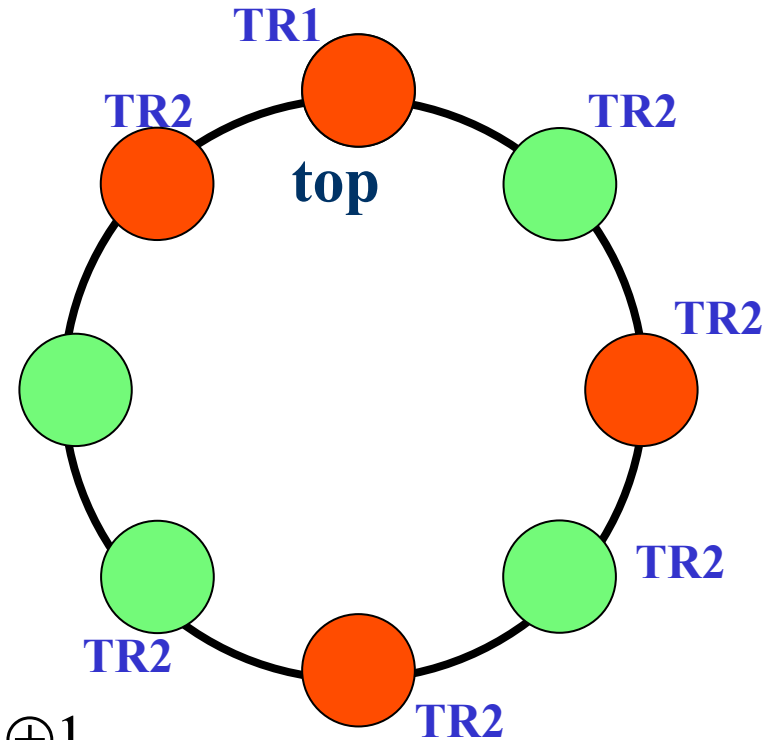
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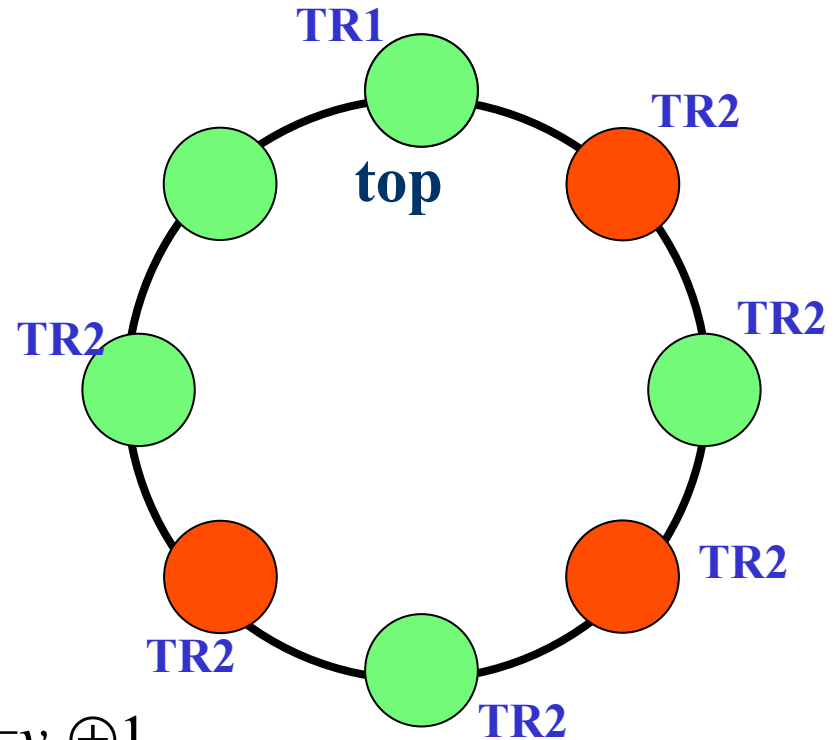
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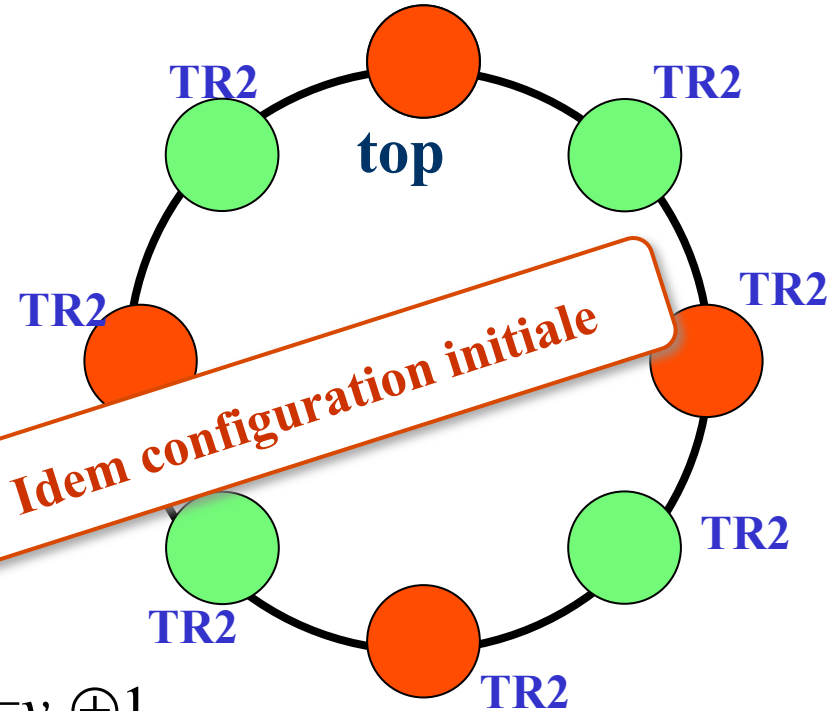
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Exercice : Montrer qu'il se produit la même chose avec un nombre impair de processeurs.

*$p = top$*

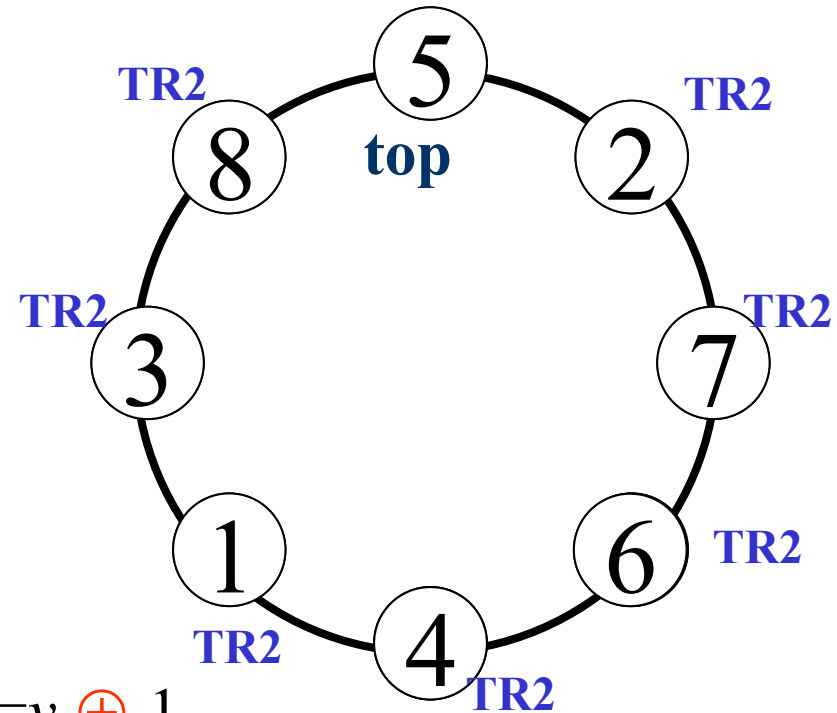
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# Anneau à jeton

*Algorithme Auto-stabilisant de circulation de jeton de Dijkstra*



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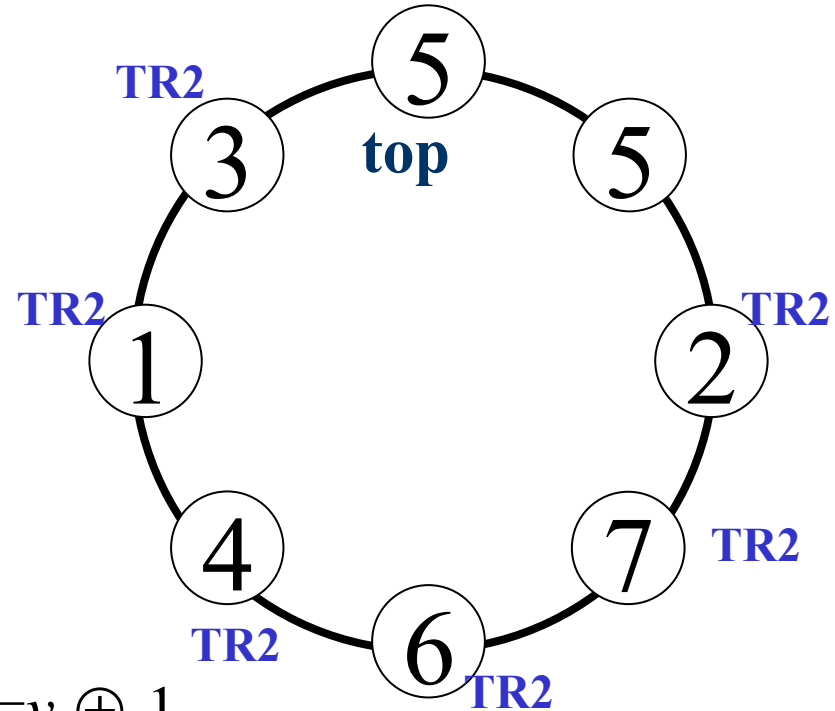
TR1 :  $(v_p = v_g)$   $\rightarrow v_p := v_g \oplus_k 1$

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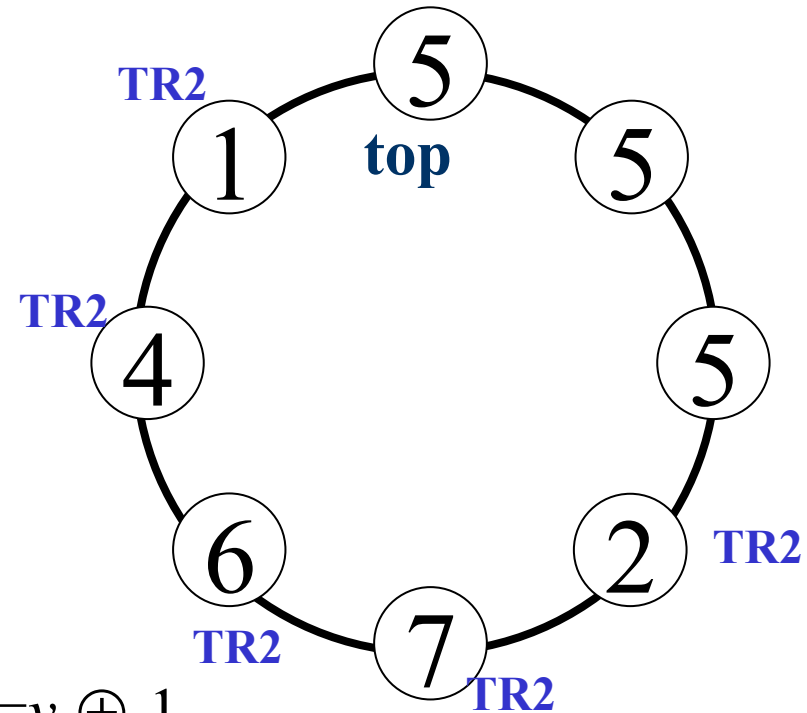
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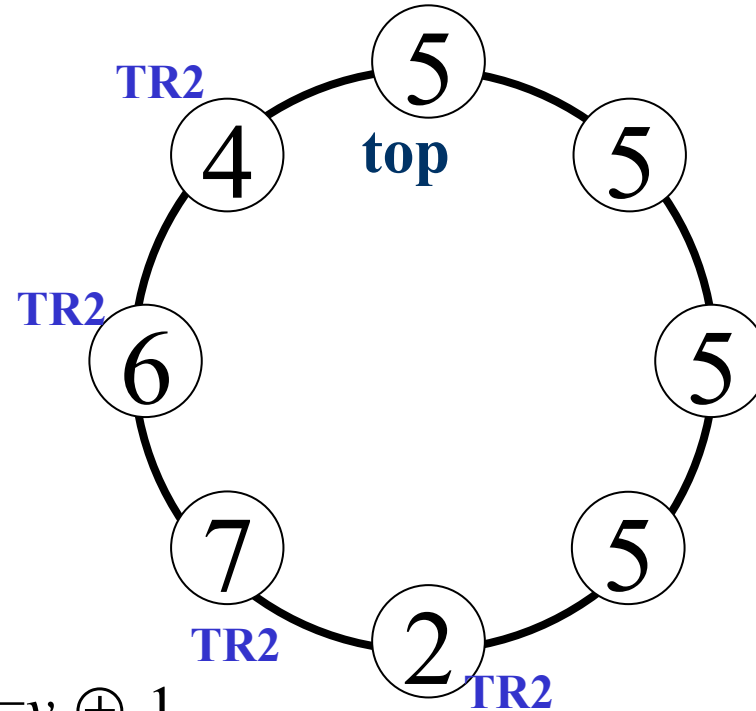
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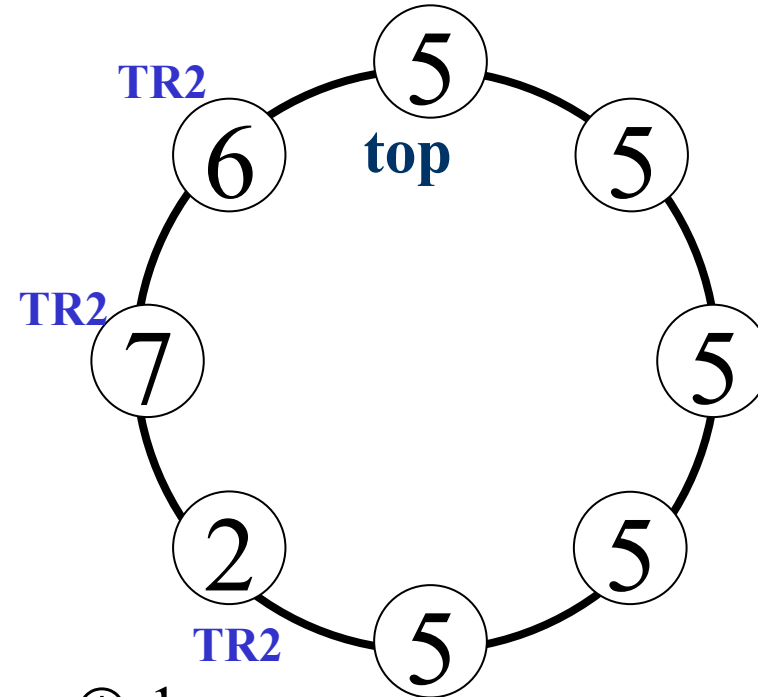
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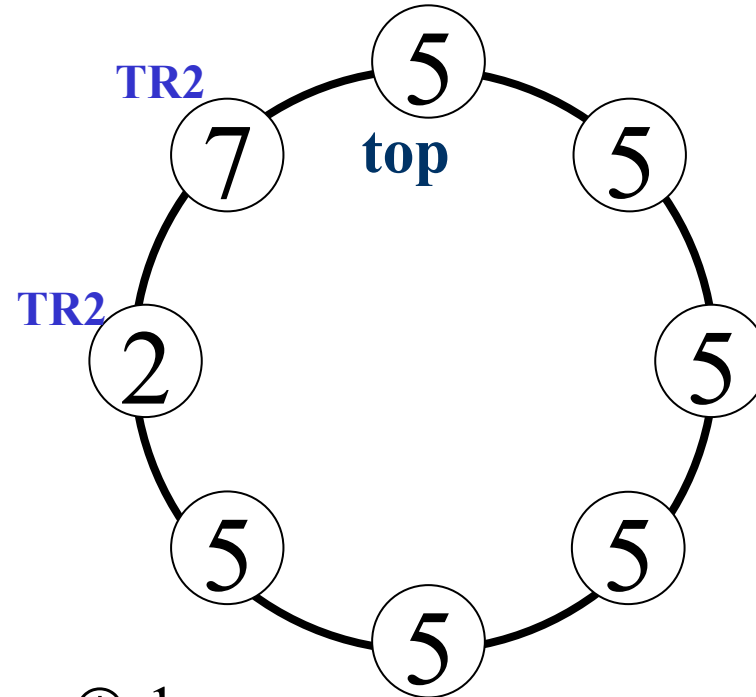
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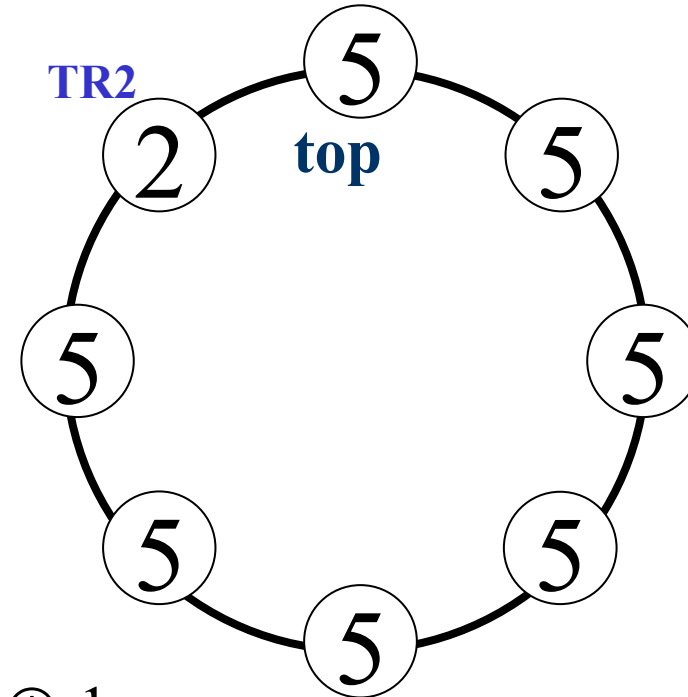
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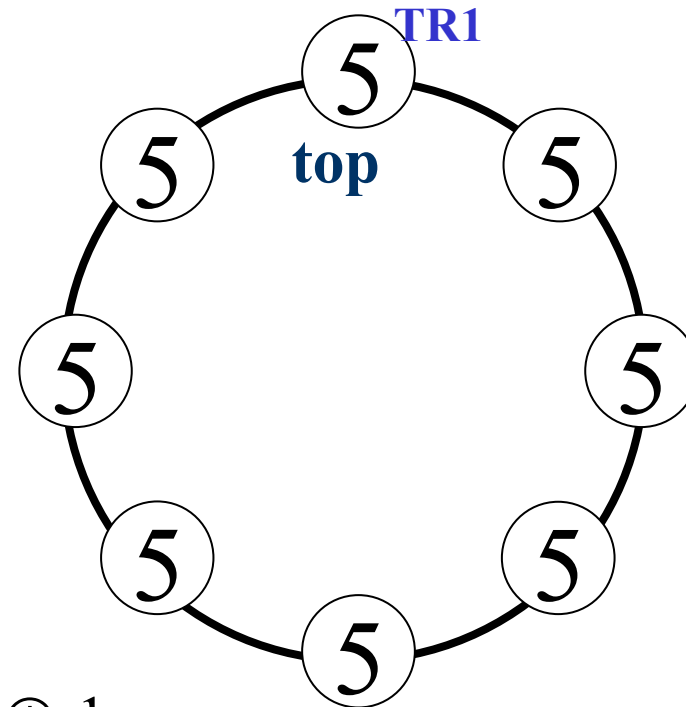
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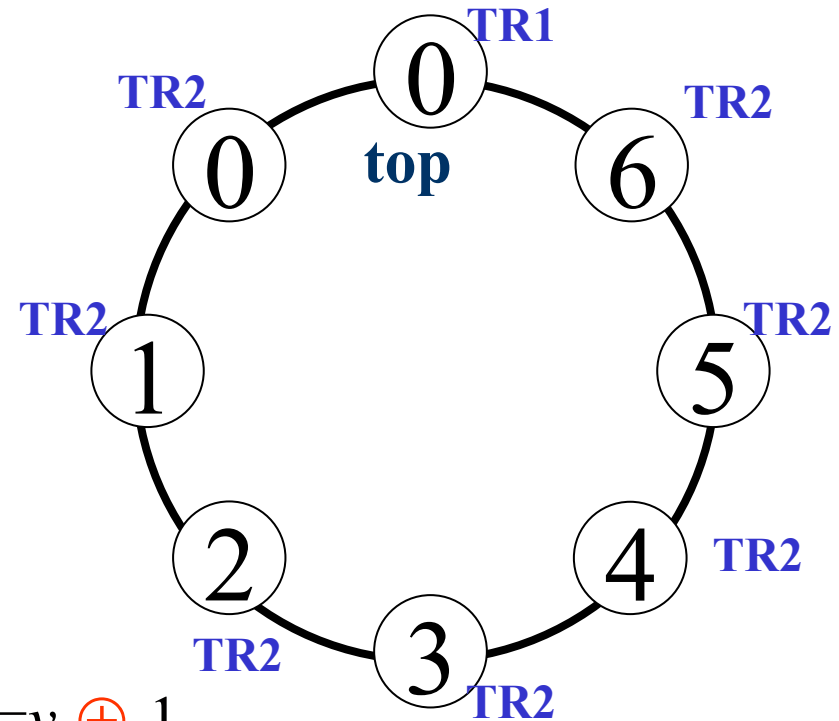
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Valeur de  $k$  ?

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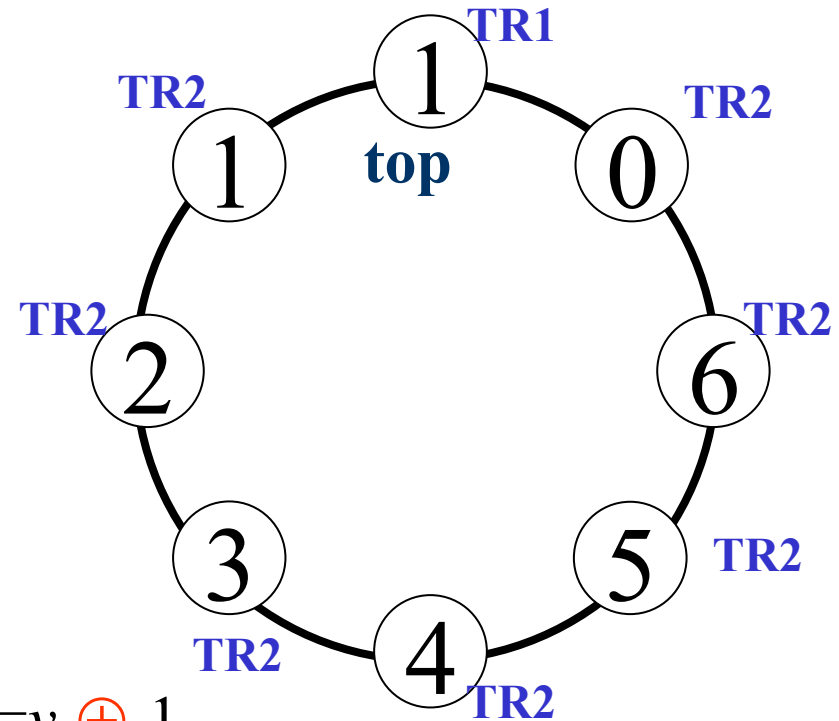
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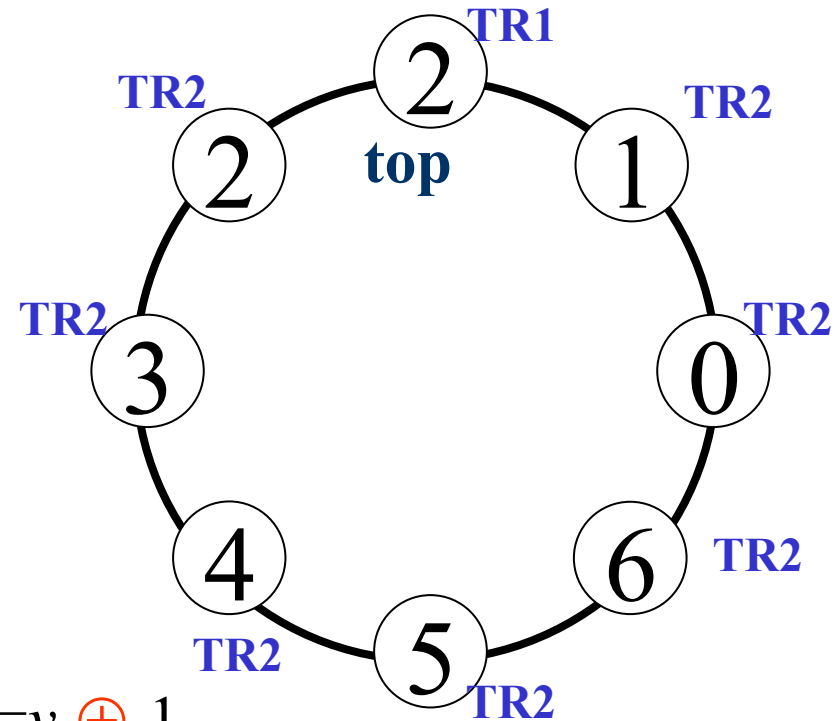
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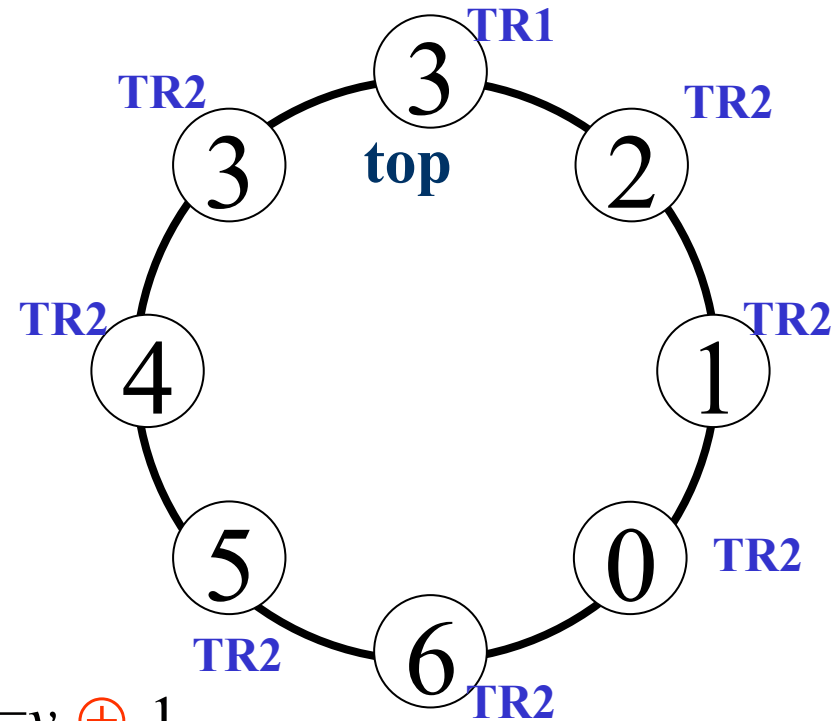
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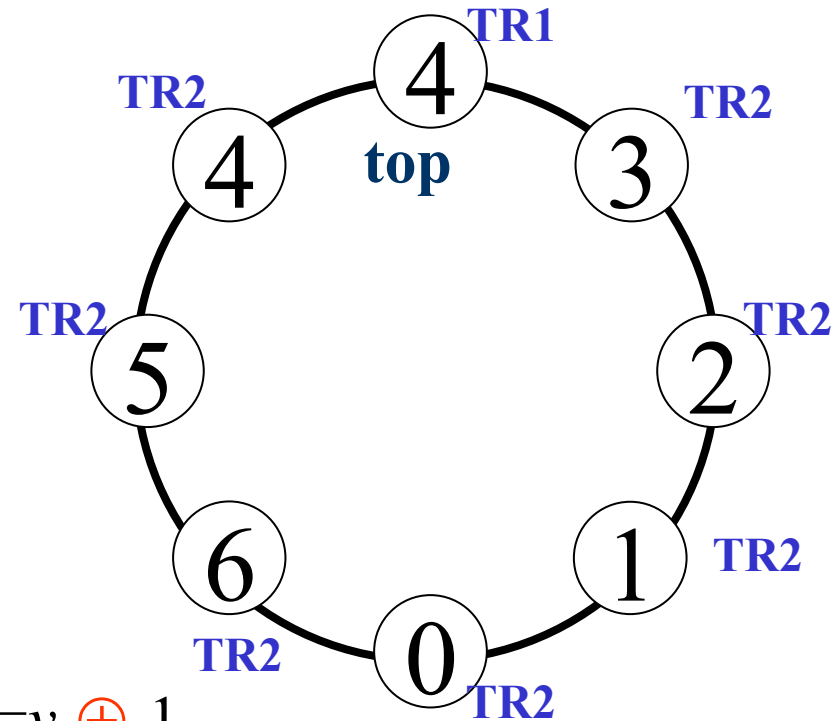
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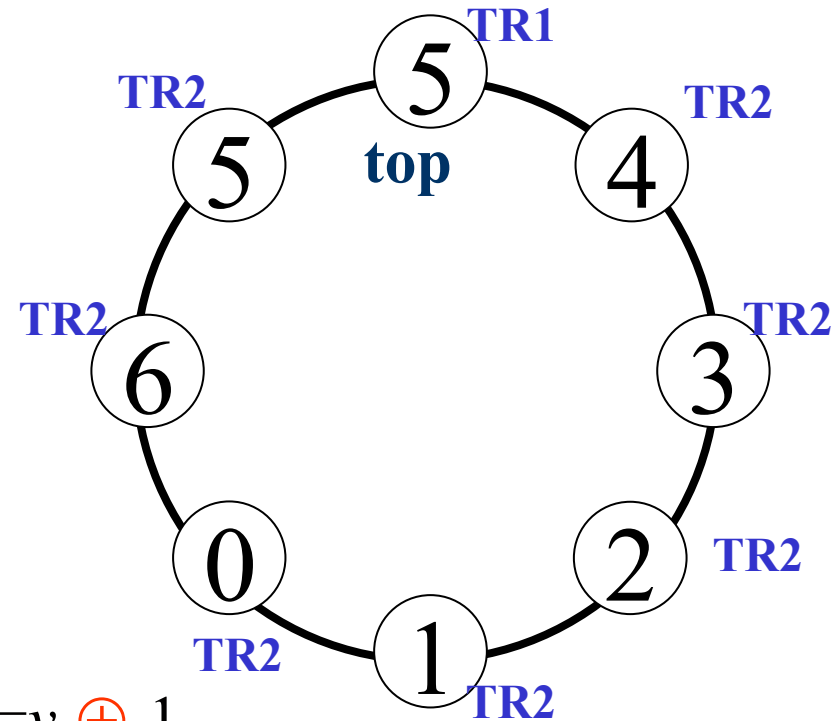
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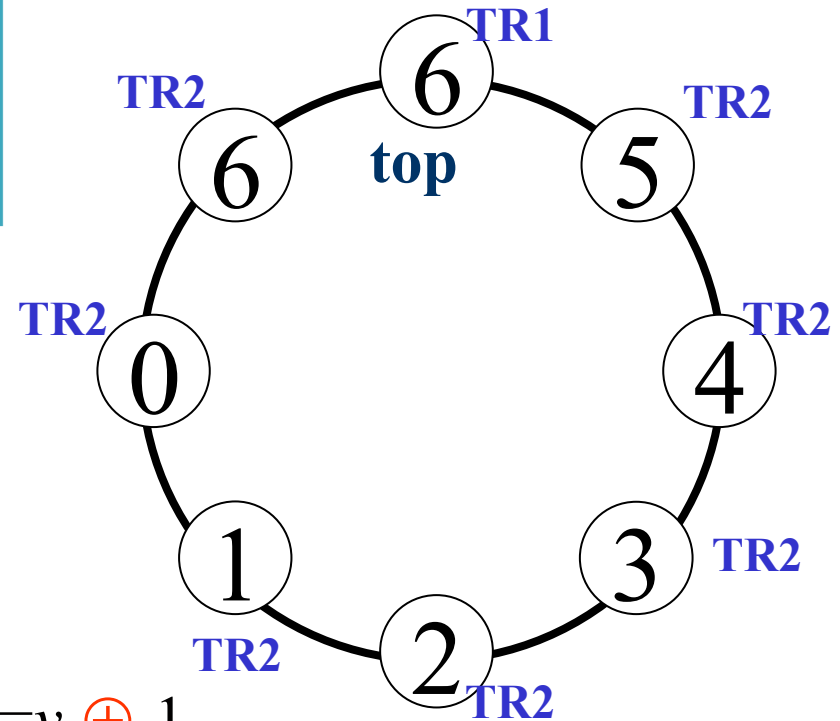
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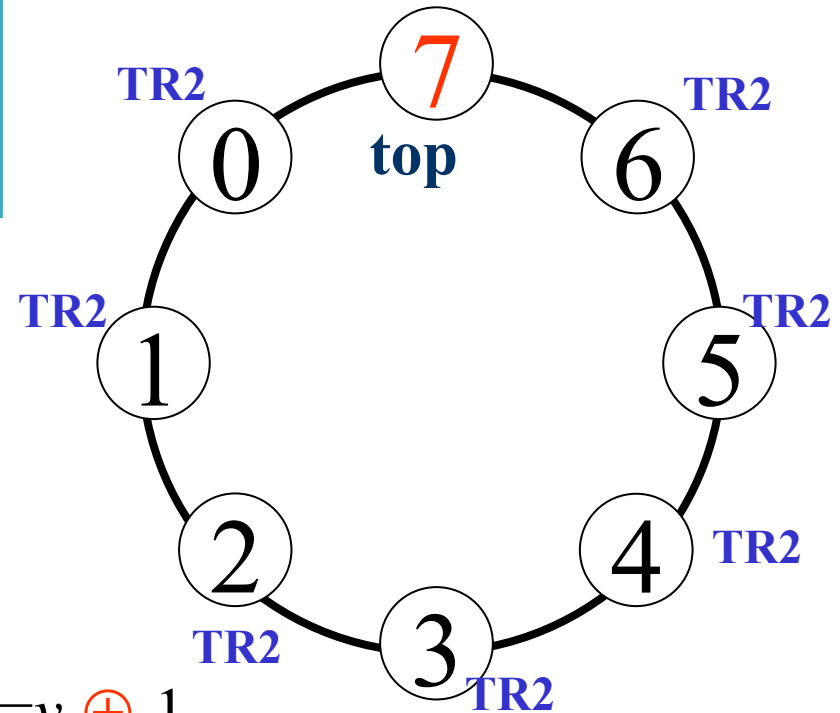
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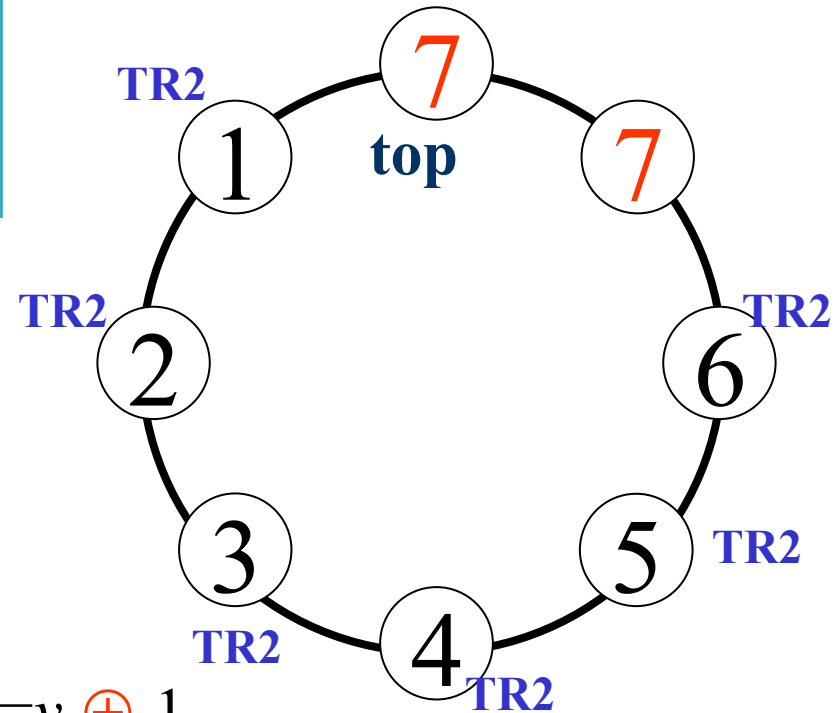
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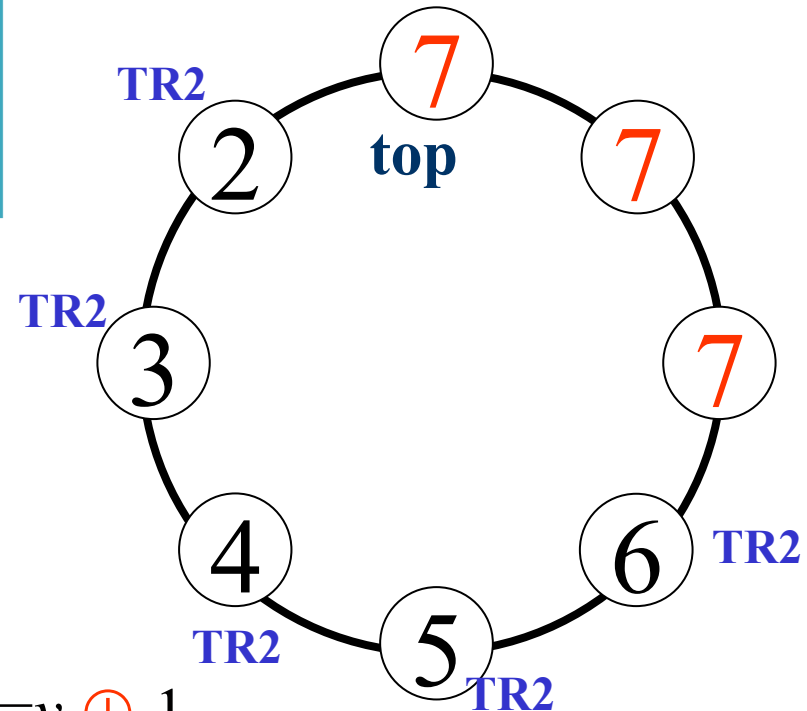
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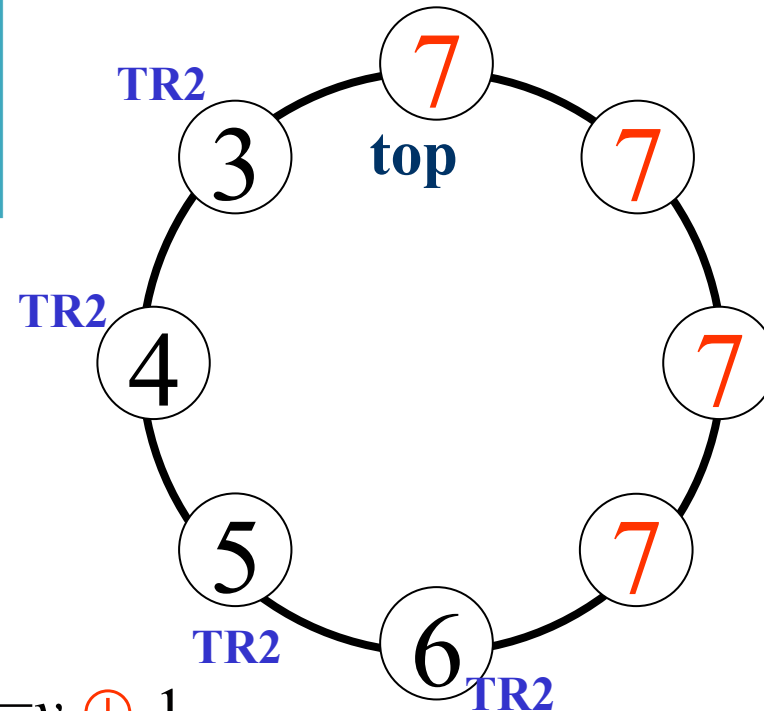
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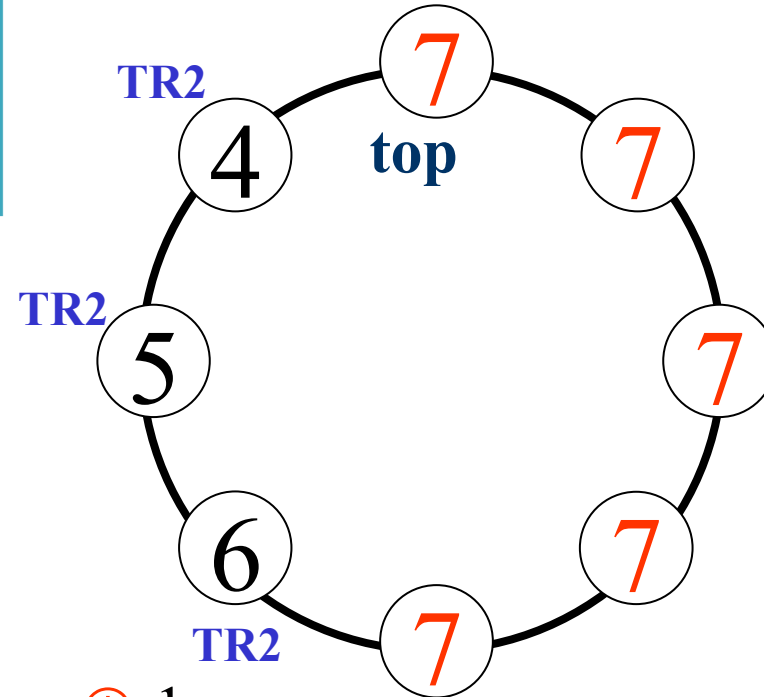
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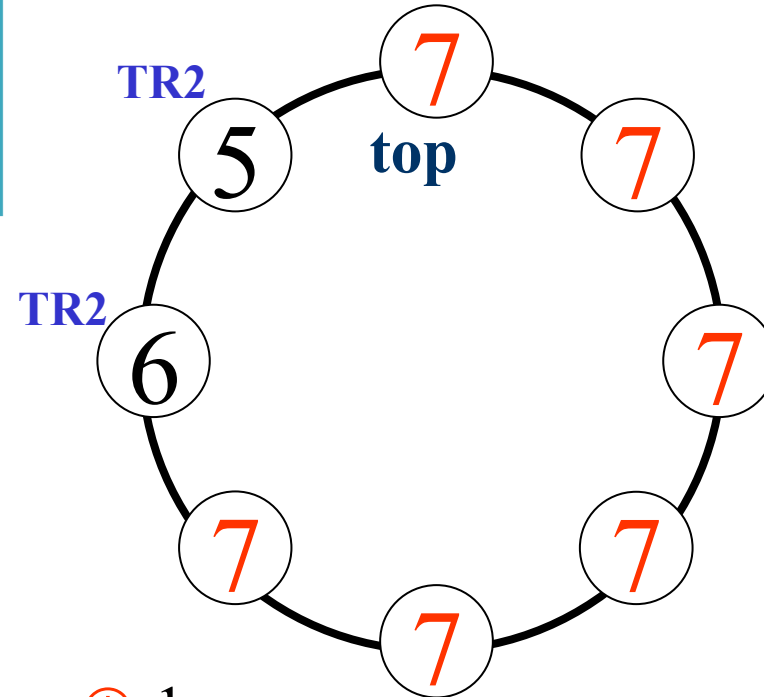
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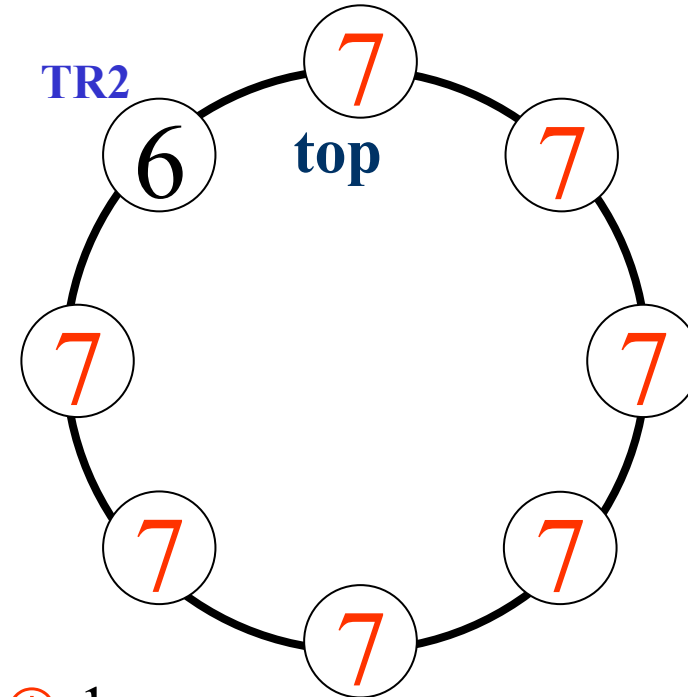
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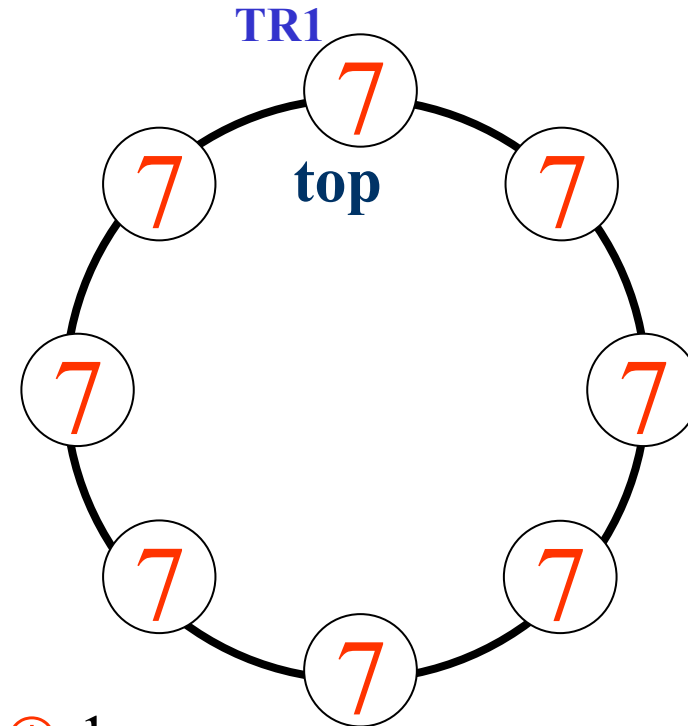
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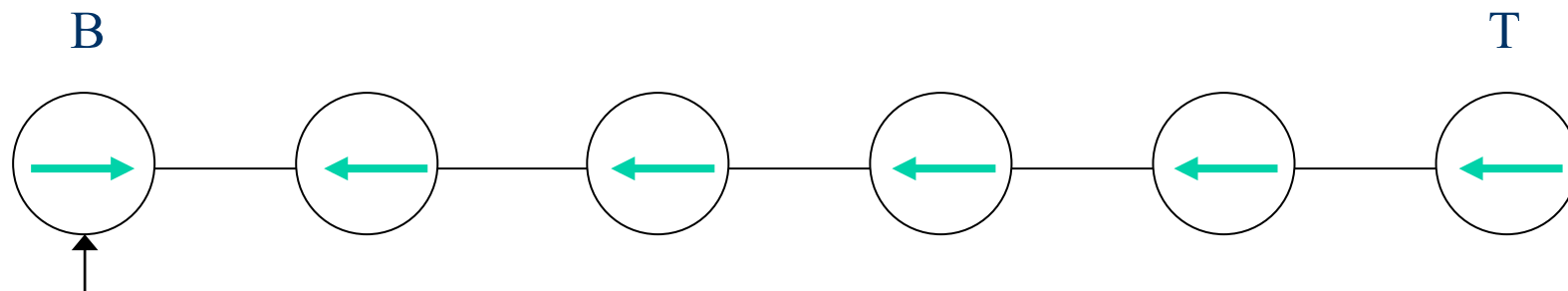
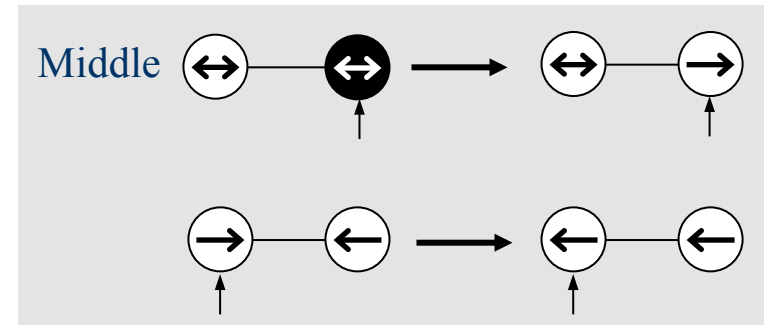
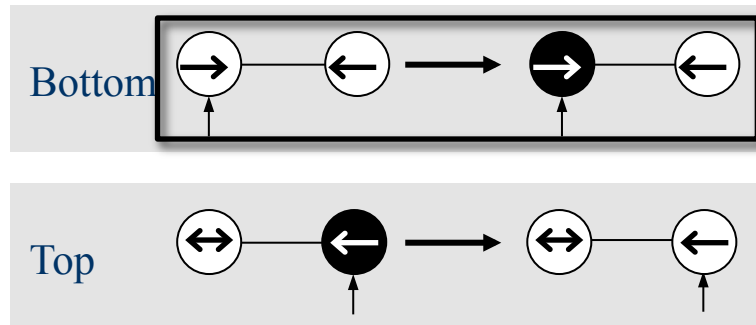
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**Temps de Stabilisation :  $O(n)$**

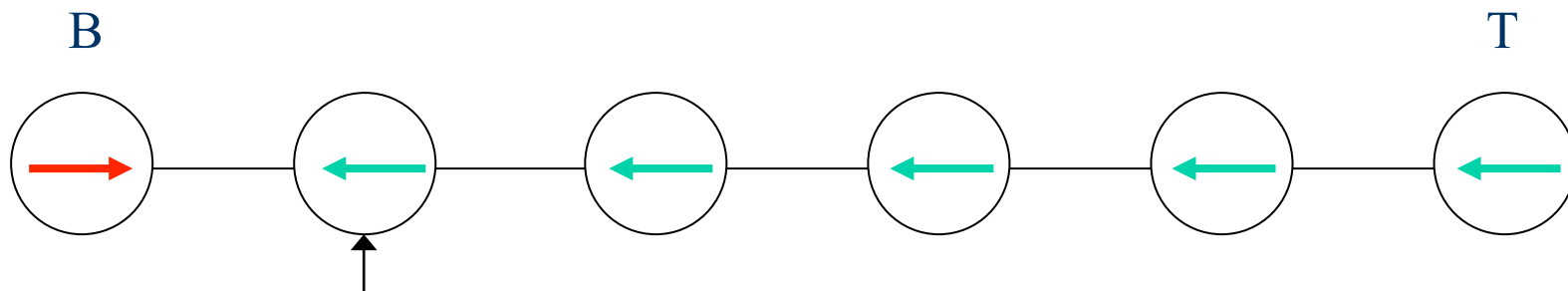
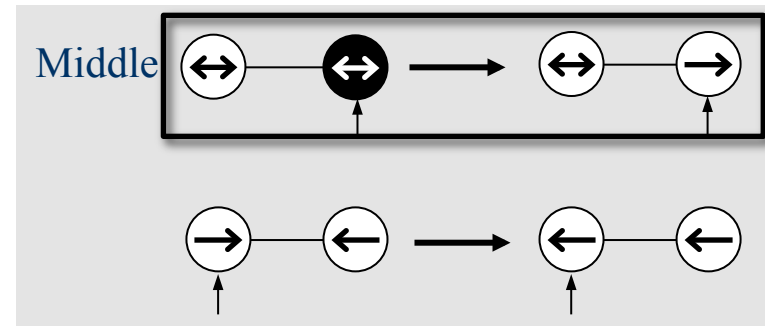
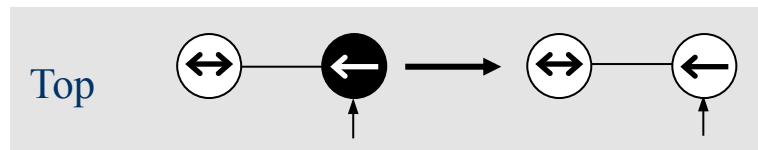
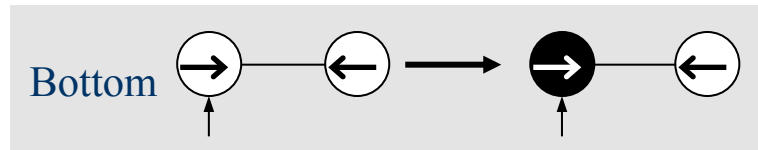


 [Dijkstra 74]



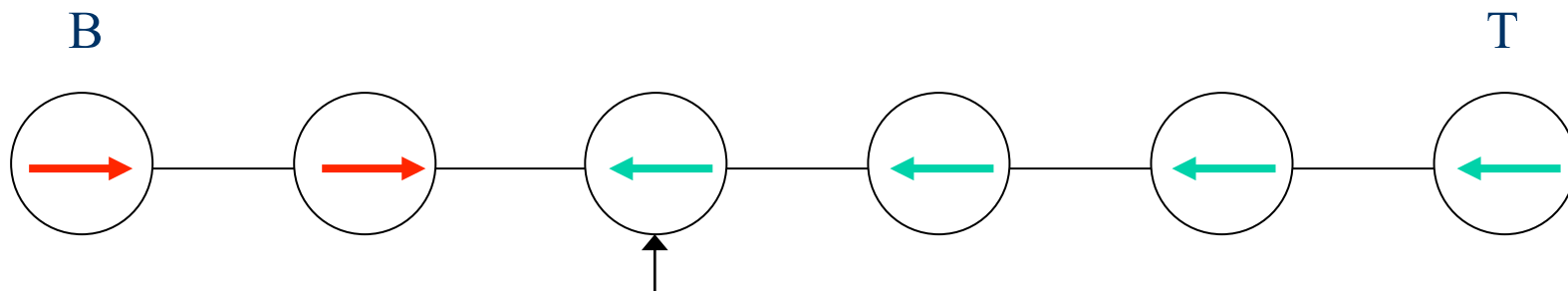
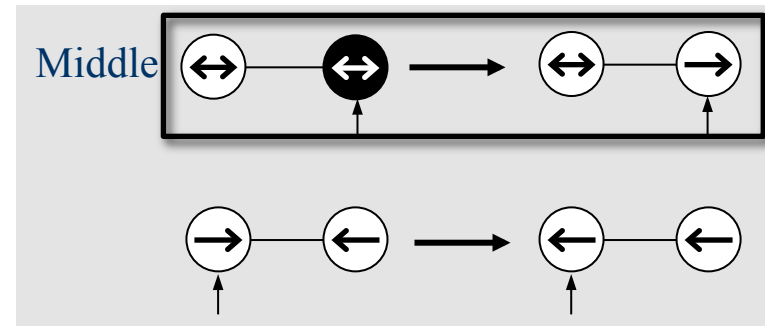
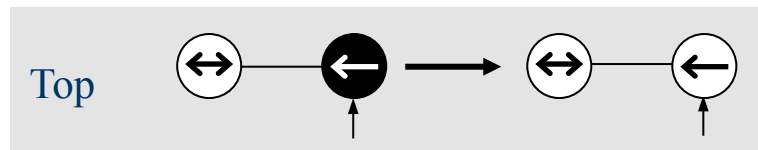
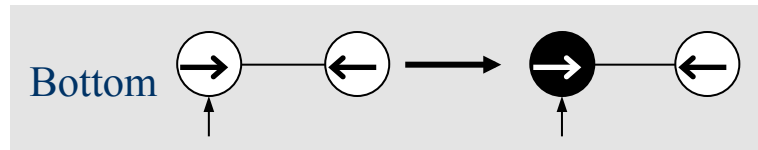


 [Dijkstra 74]





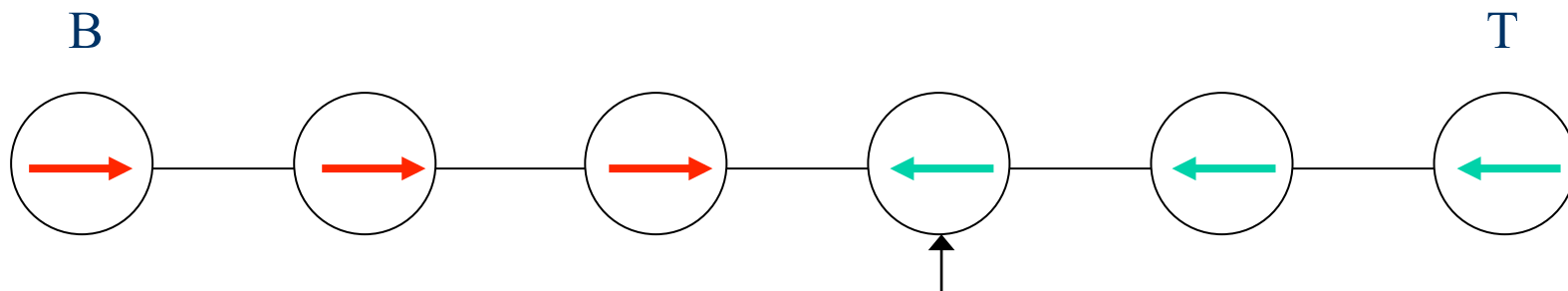
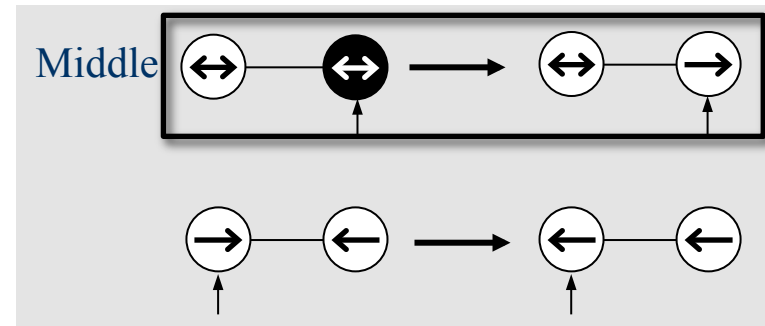
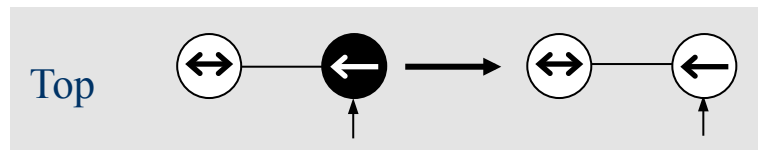
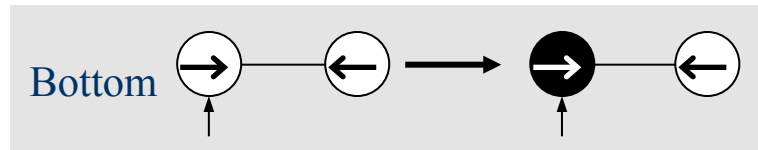
 [Dijkstra 74]





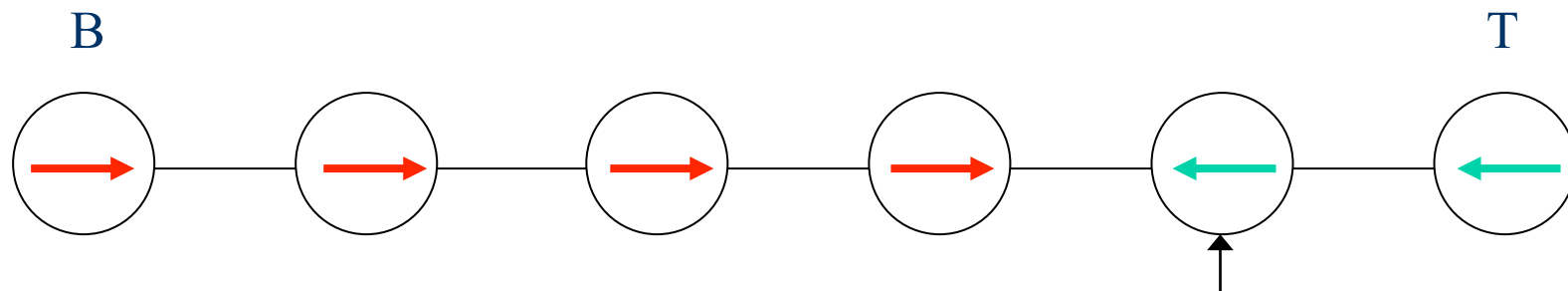
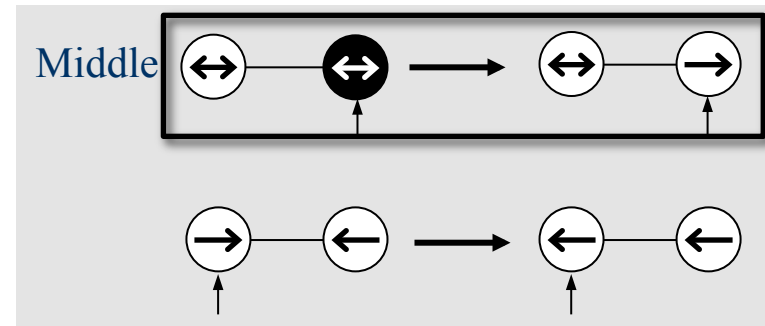
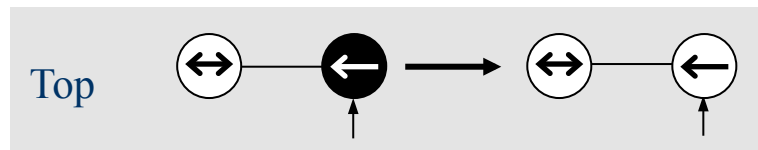
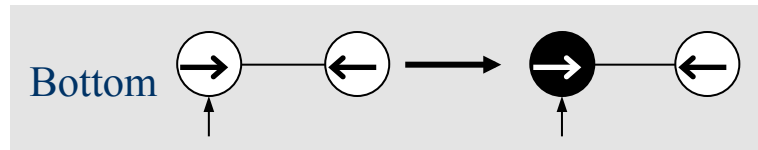


 [Dijkstra 74]



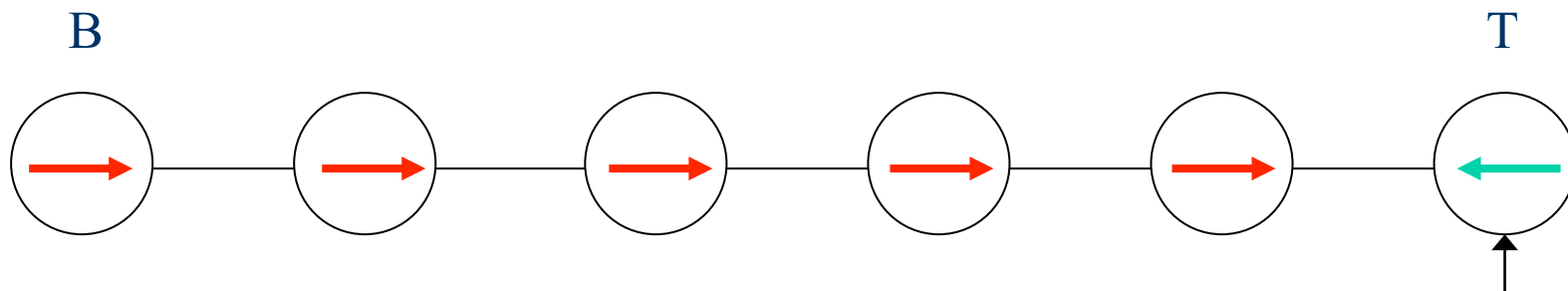
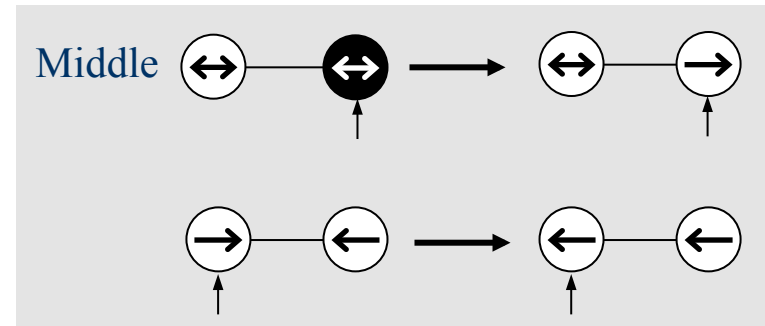
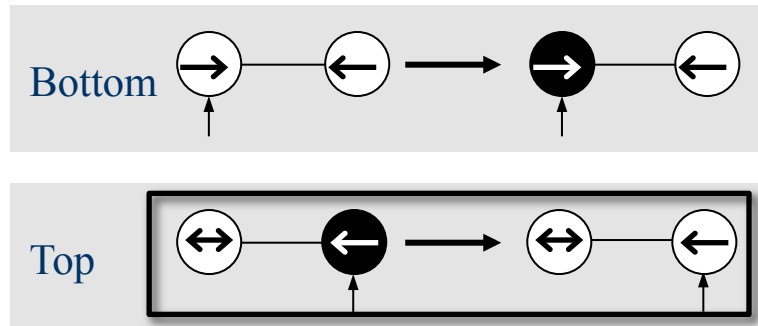


 [Dijkstra 74]



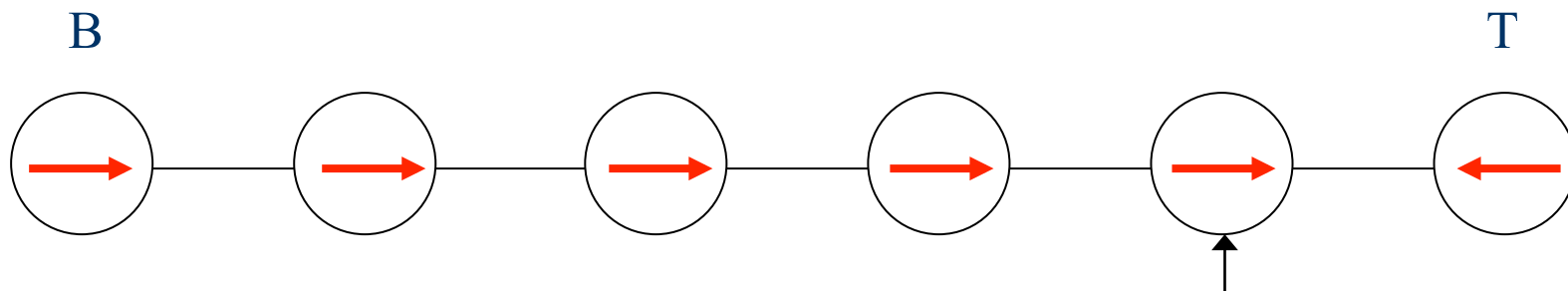
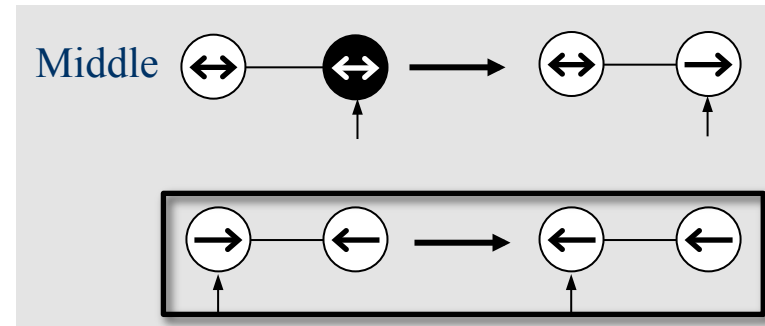
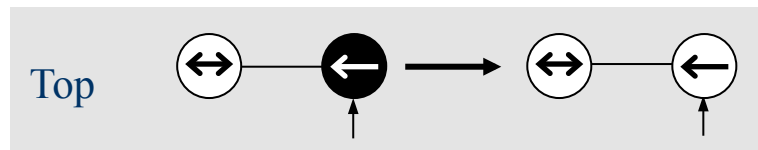
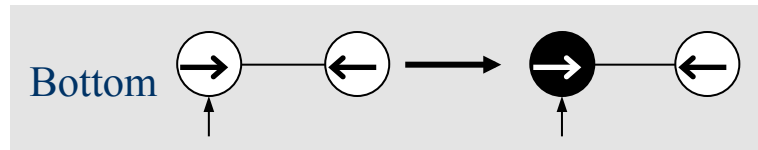


 [Dijkstra 74]



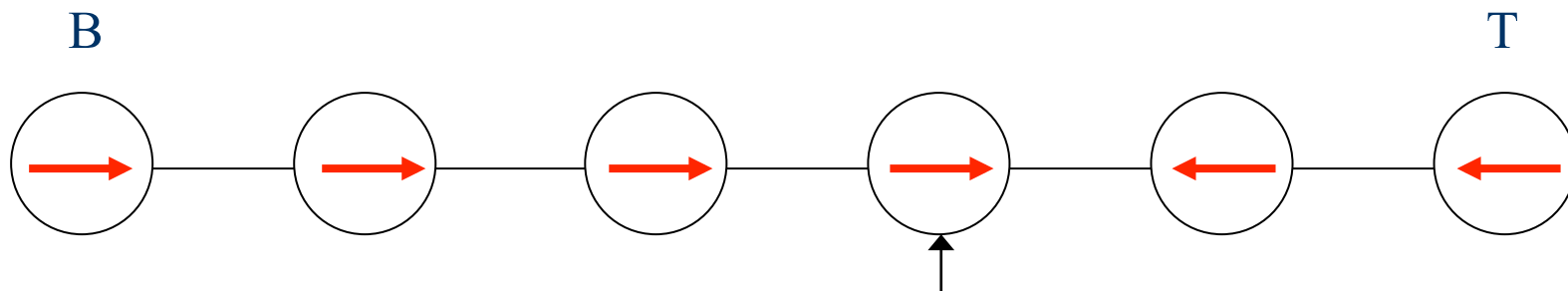
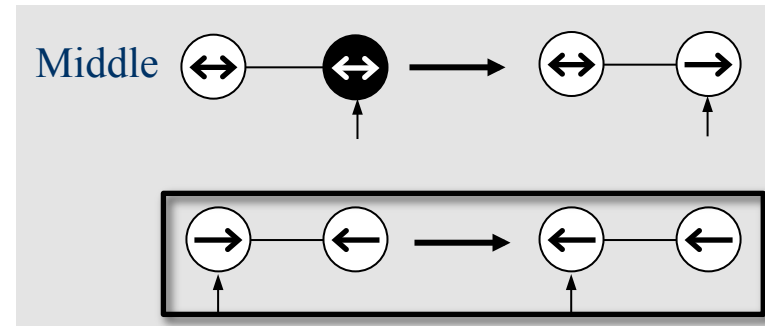
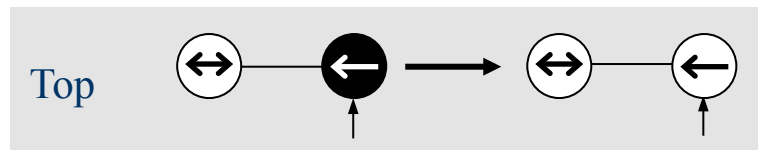
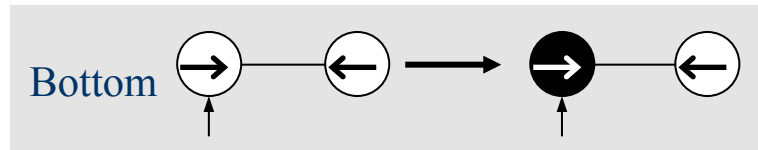


 [Dijkstra 74]



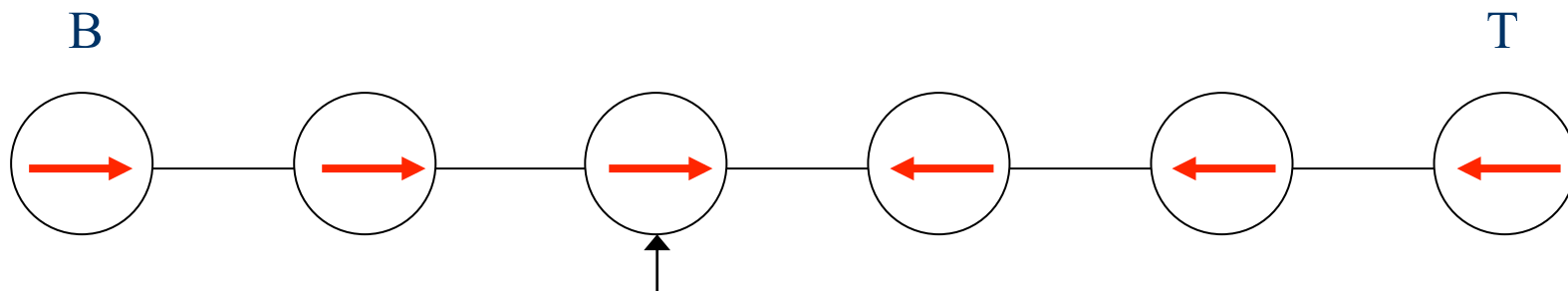
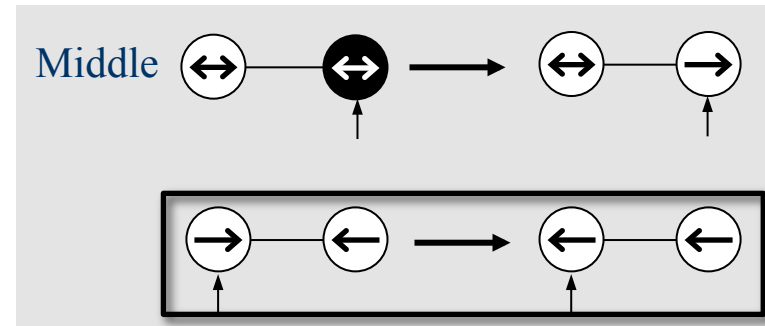
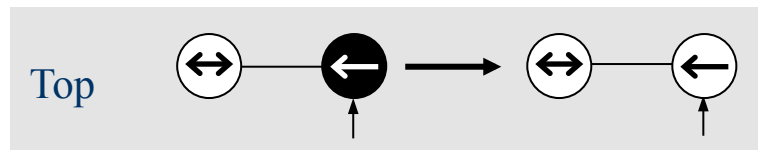
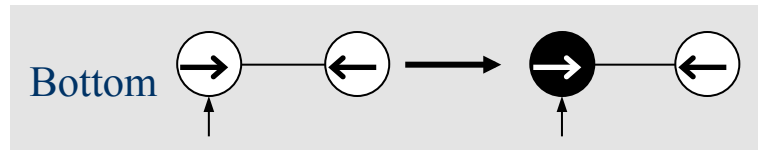


 [Dijkstra 74]



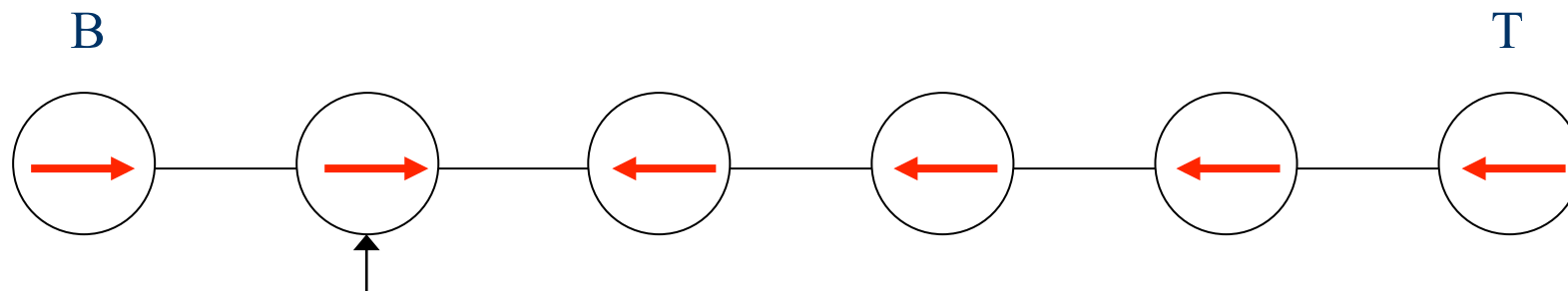
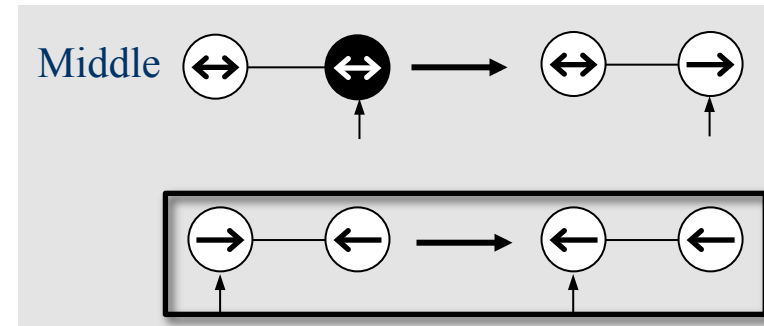
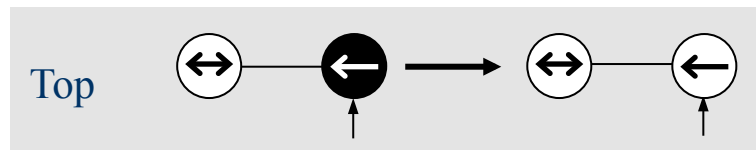
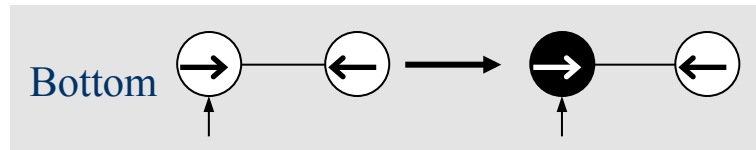


 [Dijkstra 74]



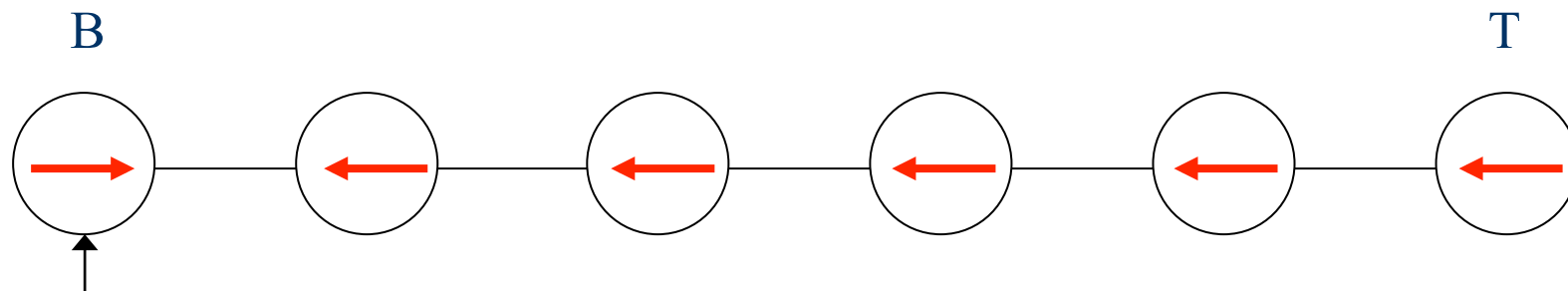
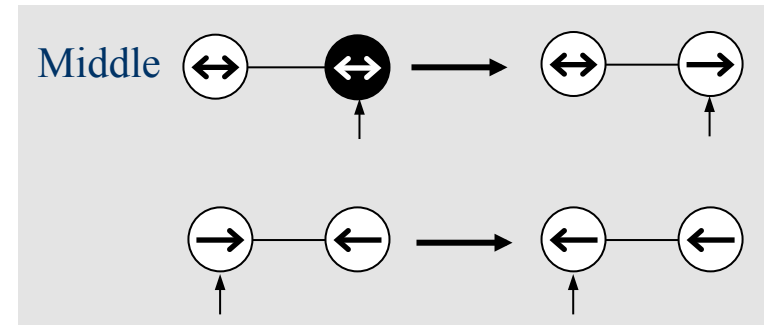
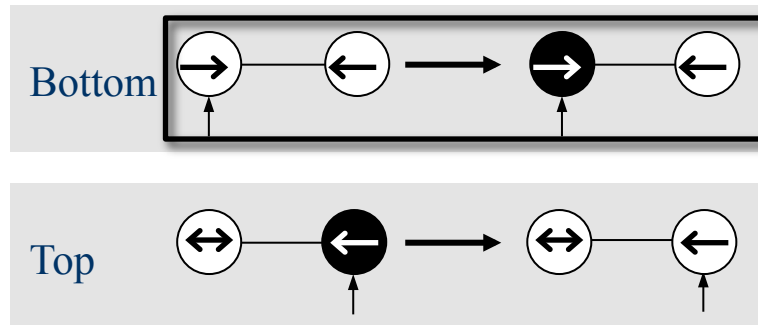


 [Dijkstra 74]





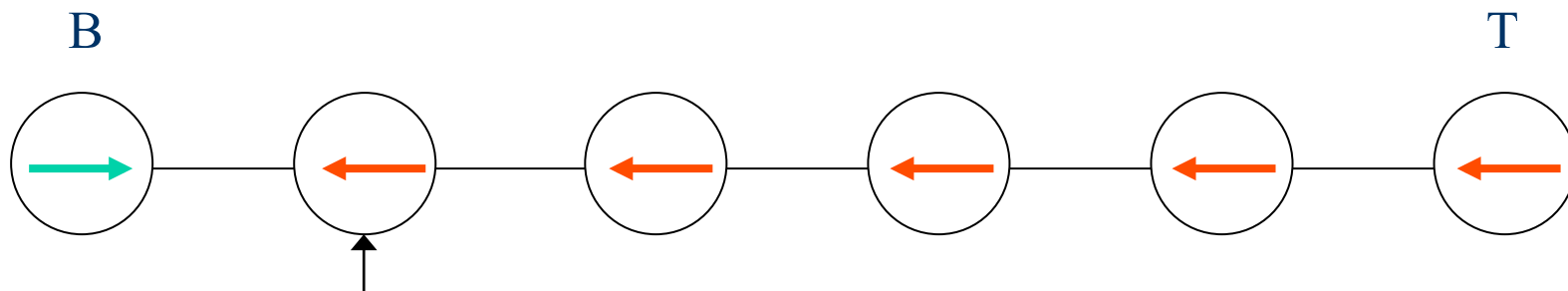
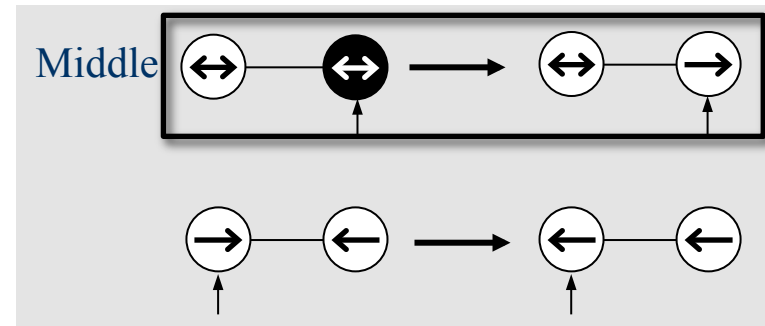
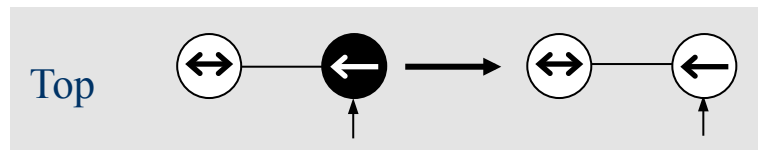
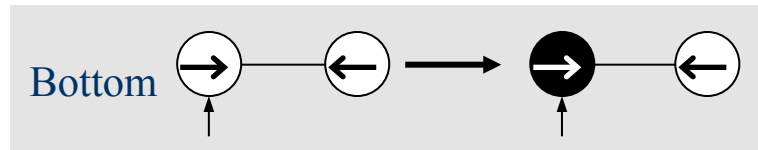
 [Dijkstra 74]





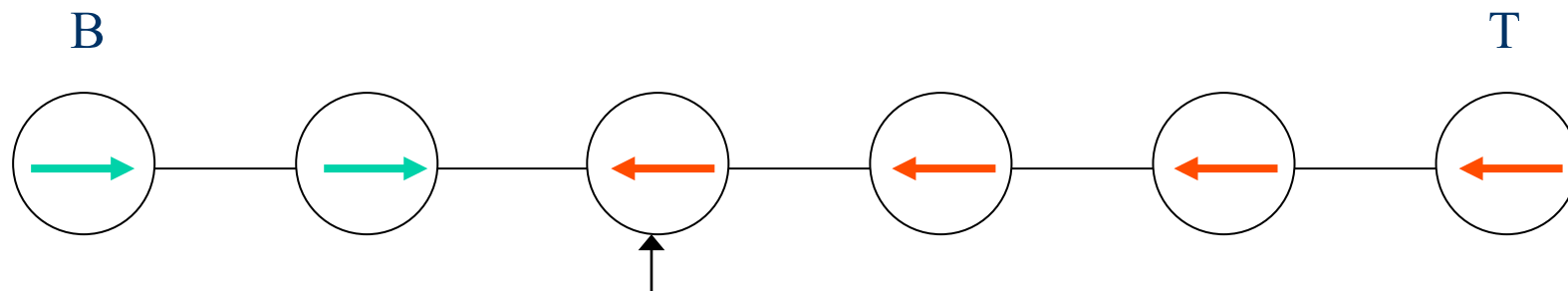
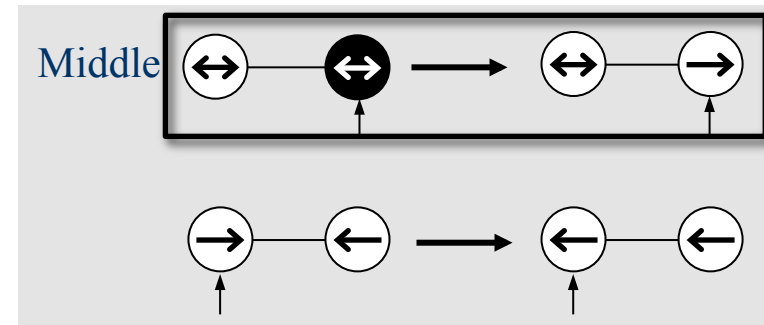
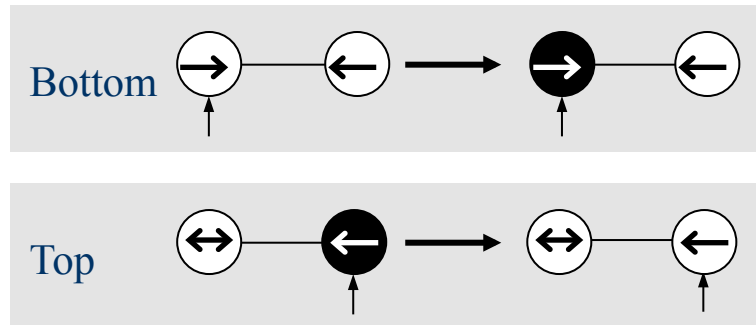


 [Dijkstra 74]



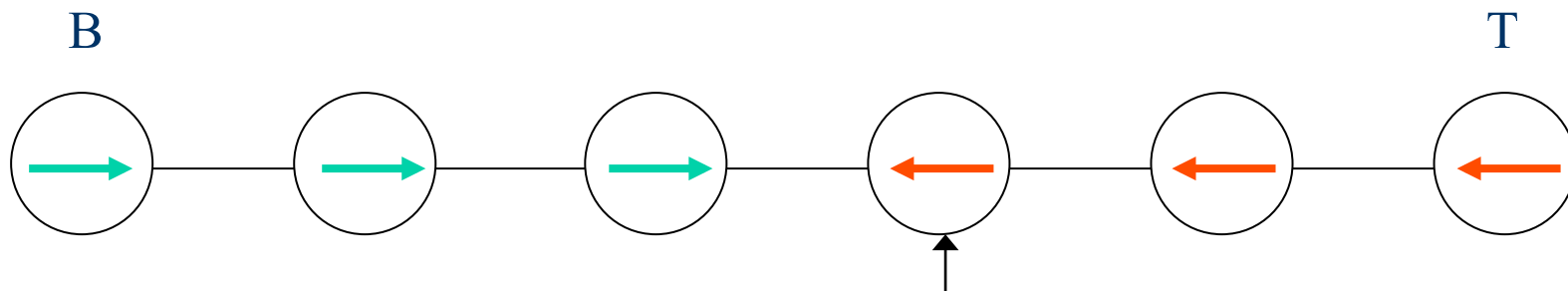
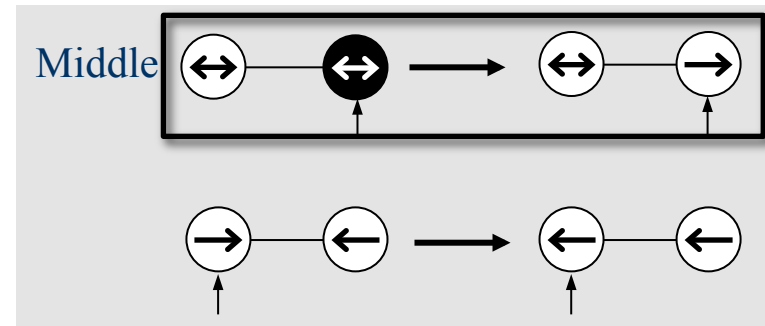
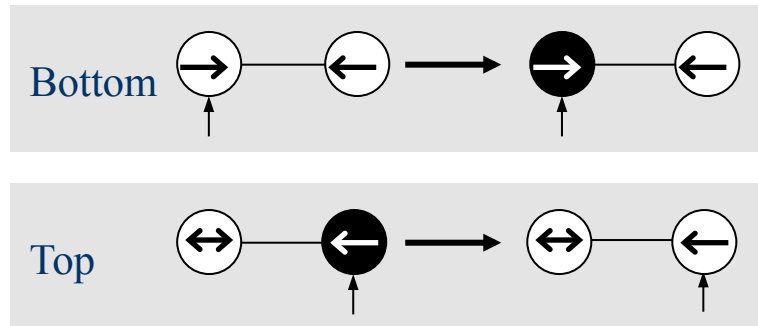


 [Dijkstra 74]



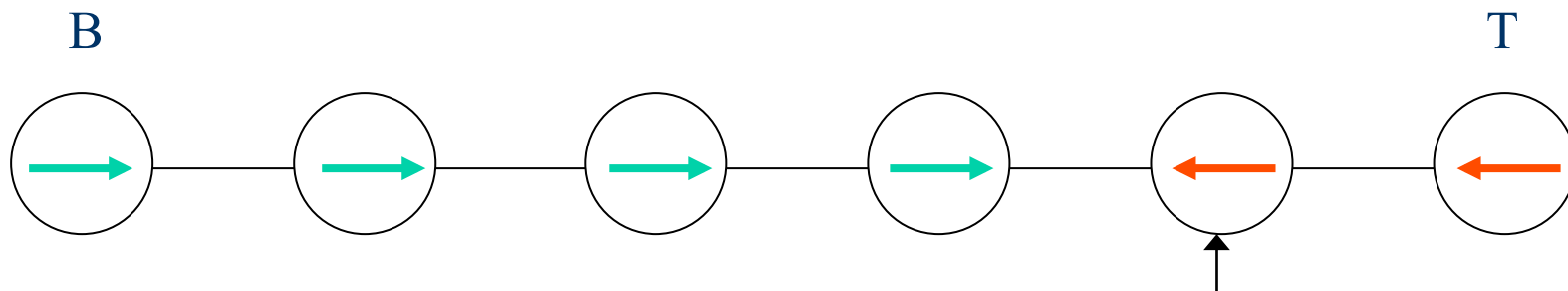
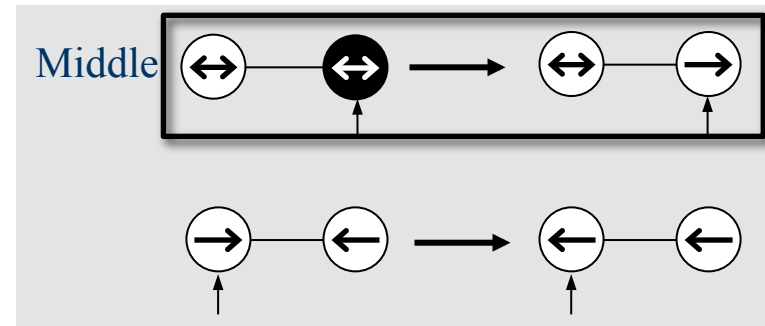
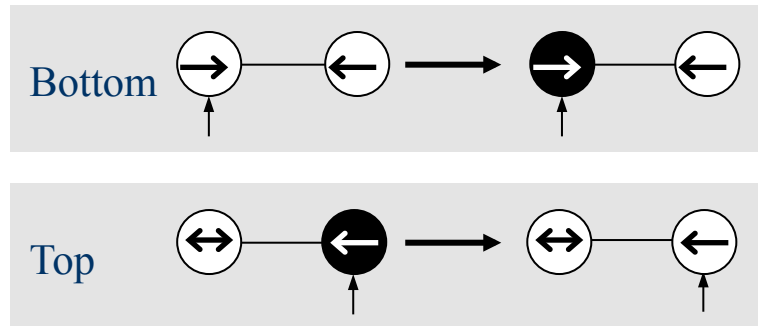


 [Dijkstra 74]



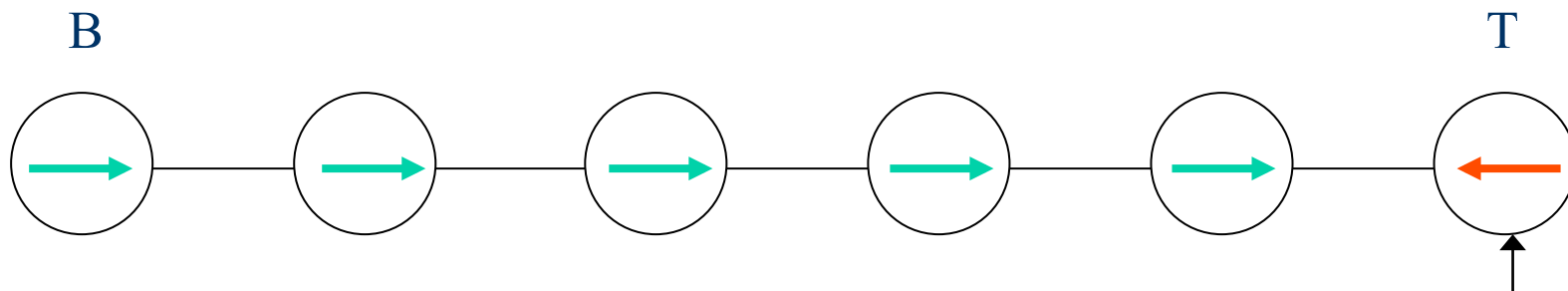
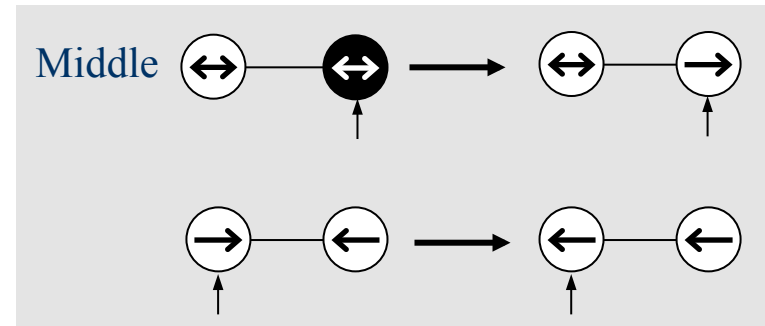
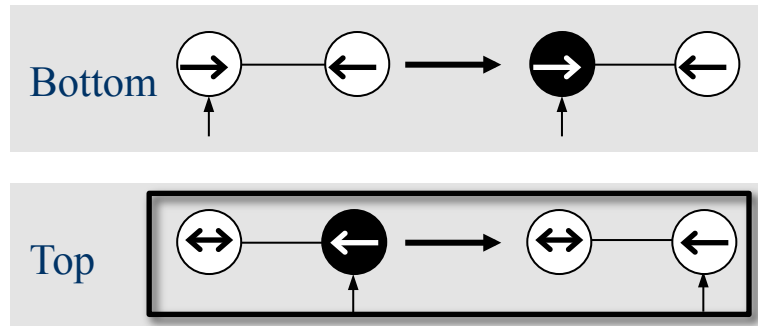


 [Dijkstra 74]



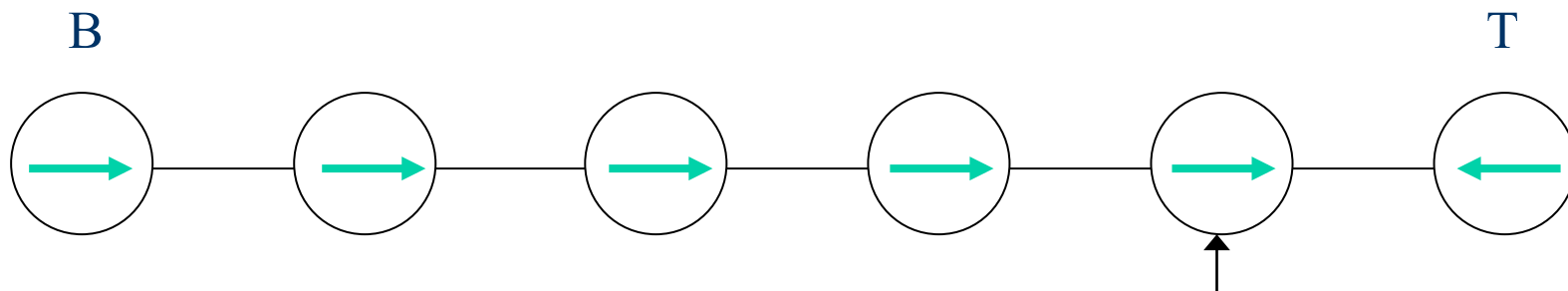
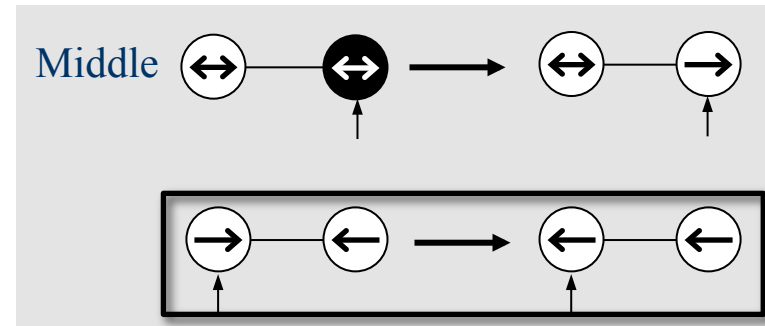
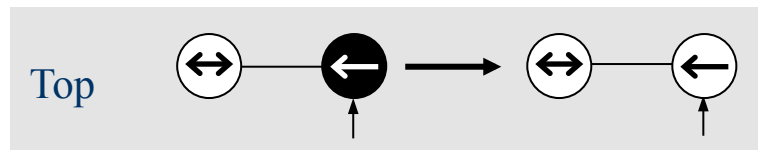
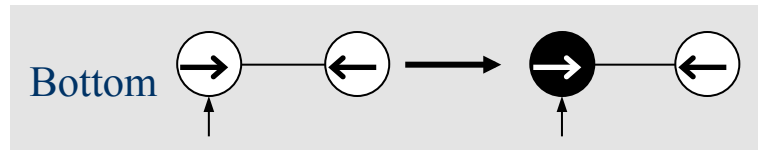


 [Dijkstra 74]



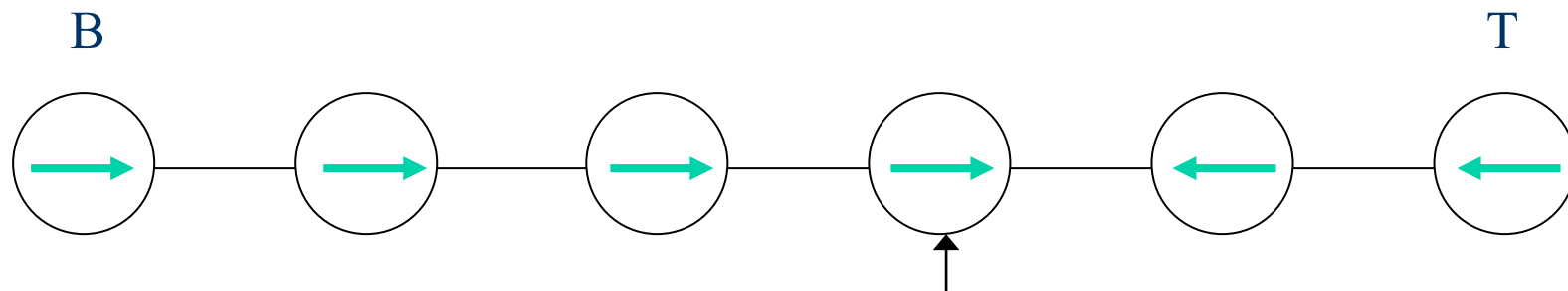
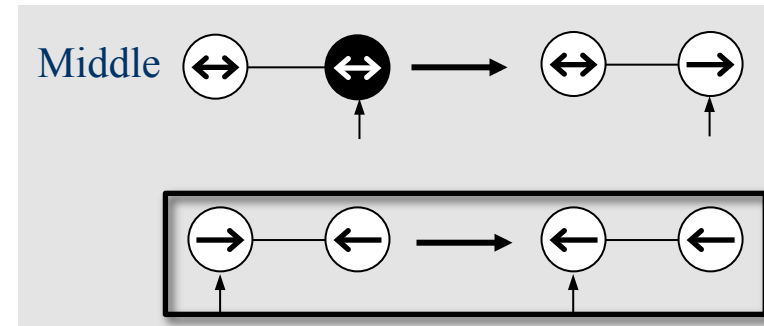
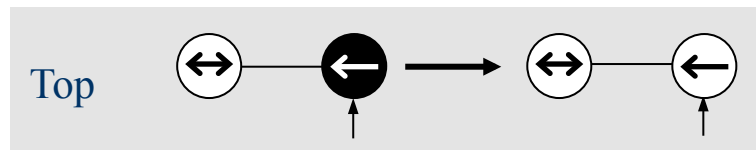
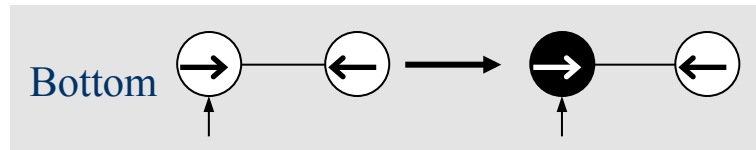


 [Dijkstra 74]



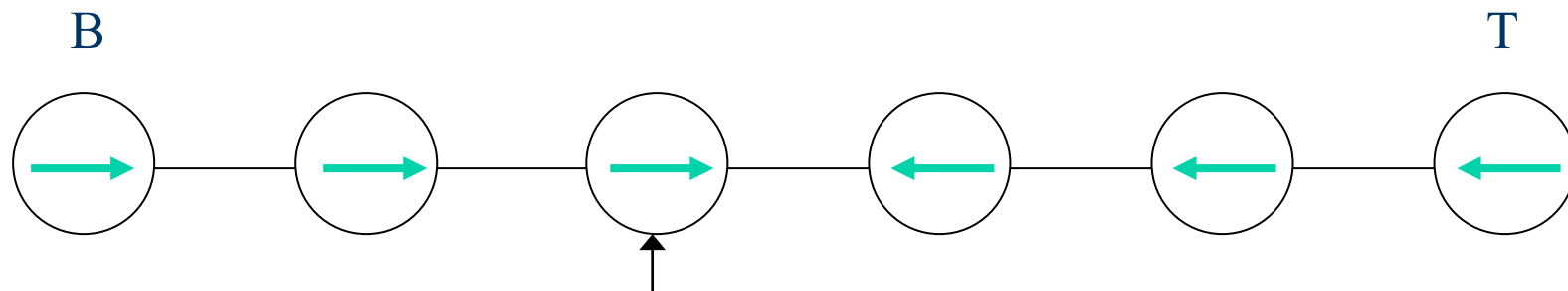
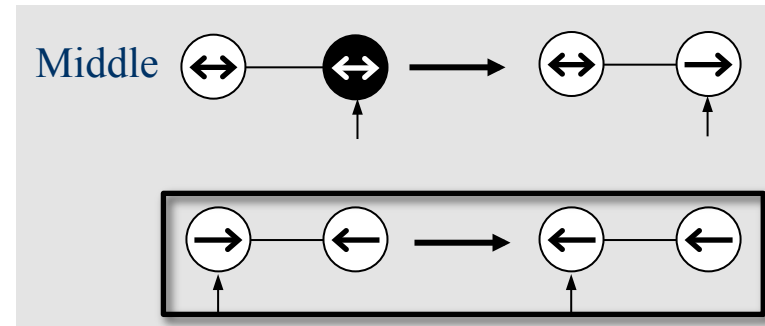
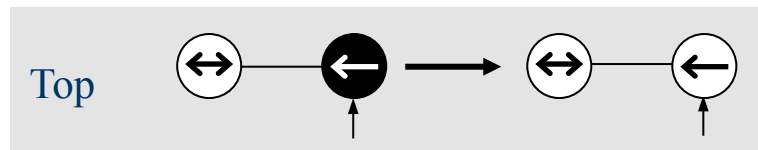
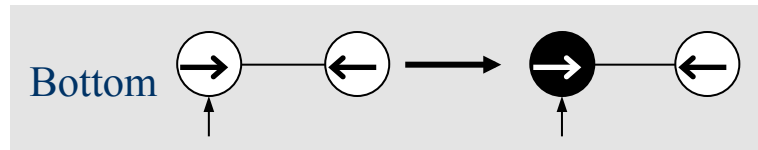


 [Dijkstra 74]





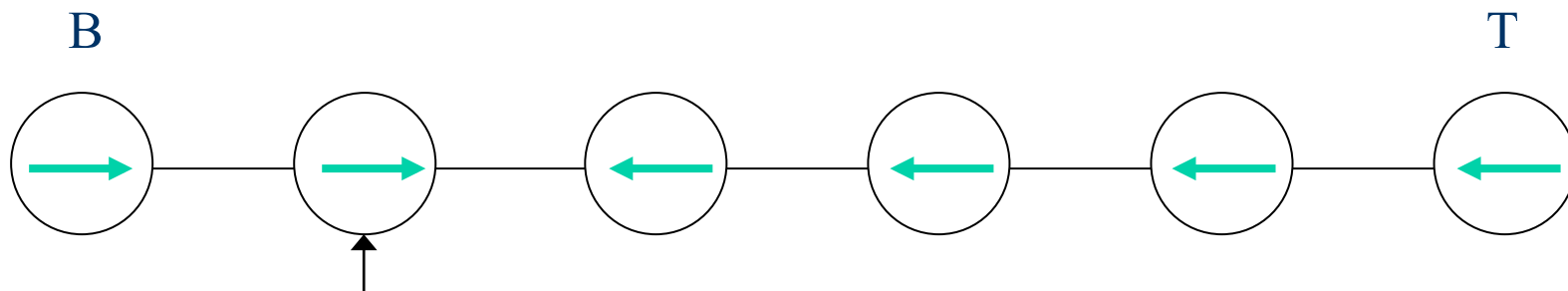
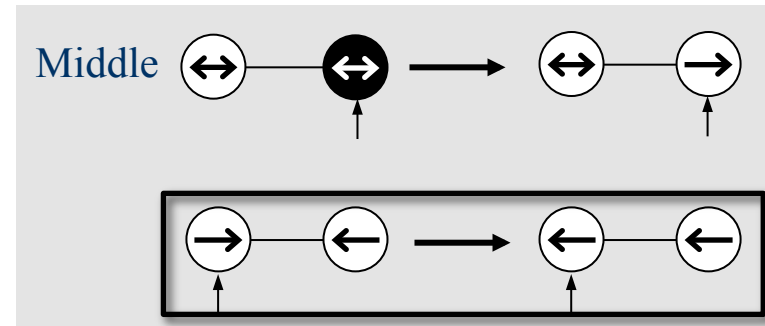
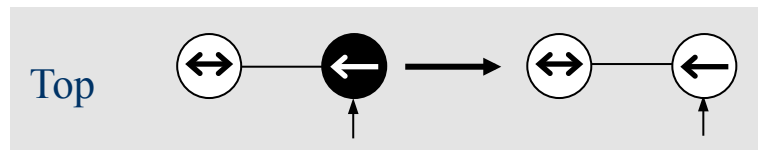
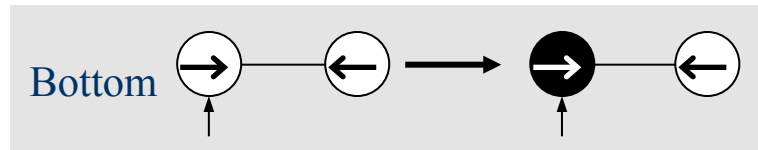
 [Dijkstra 74]





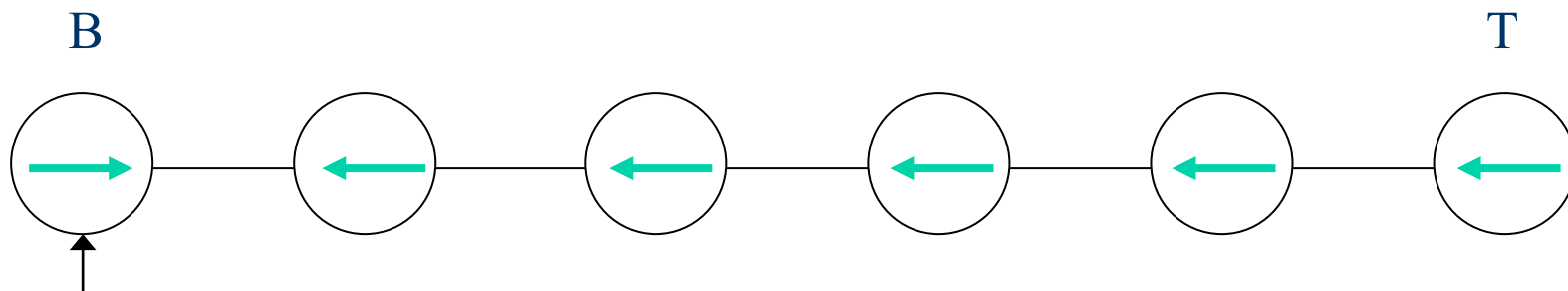
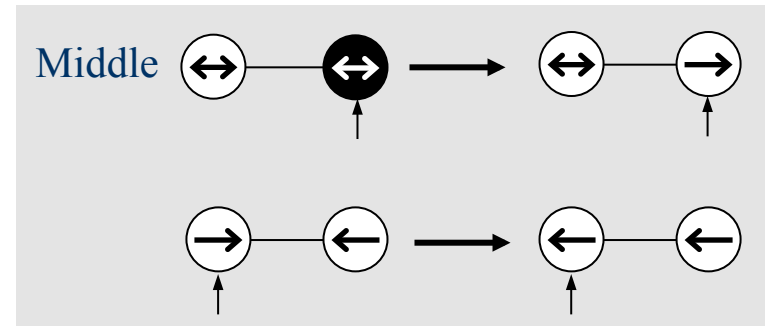
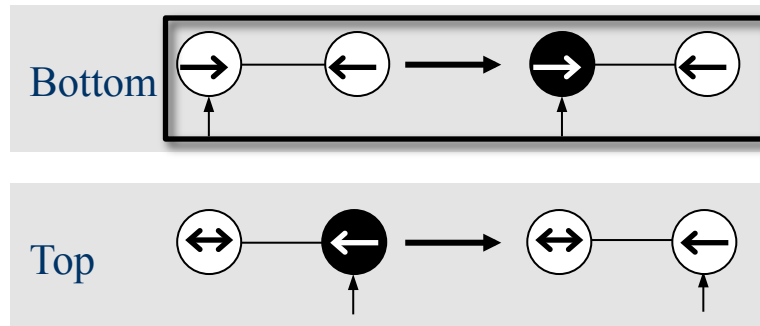


 [Dijkstra 74]





 [Dijkstra 74]

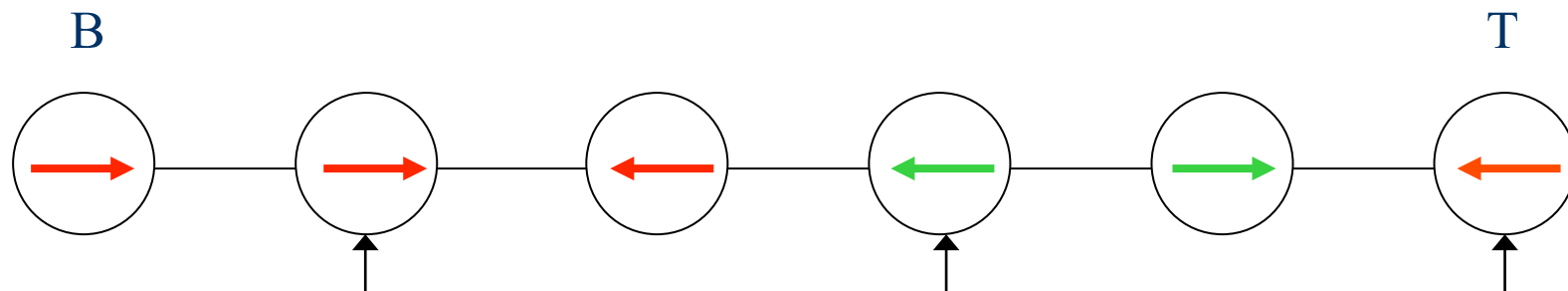
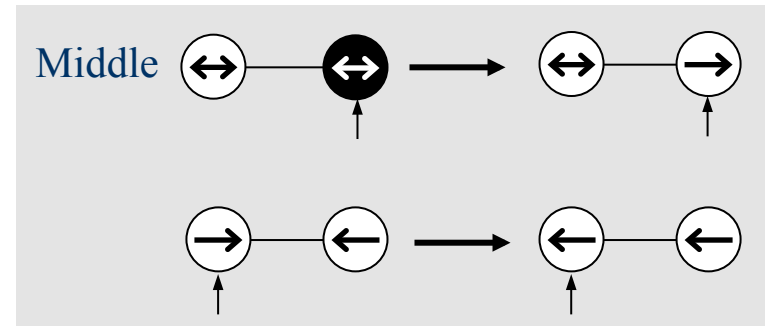
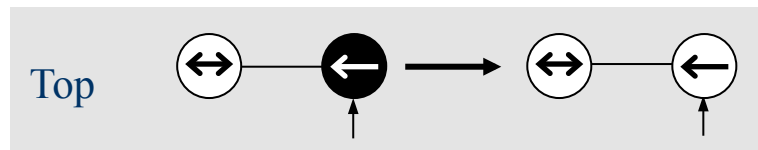
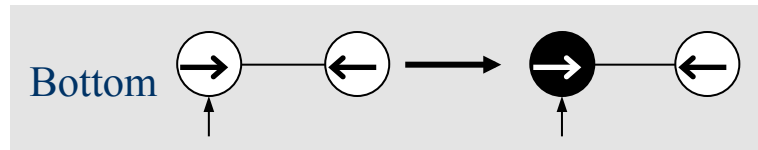




Auto-stabilisation ?

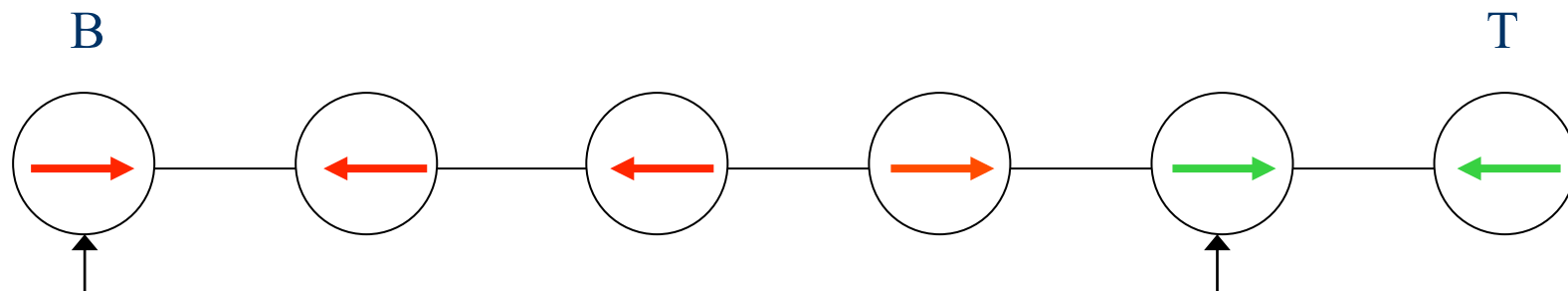
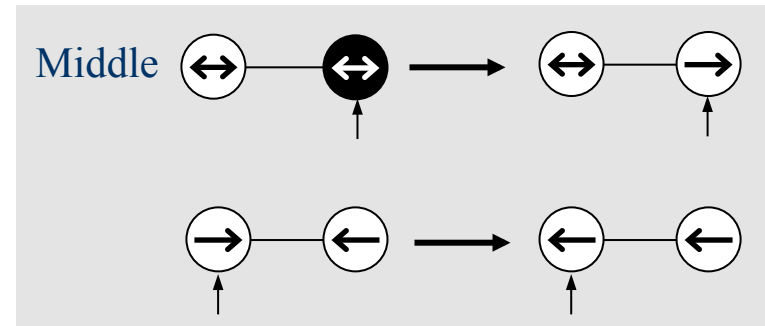
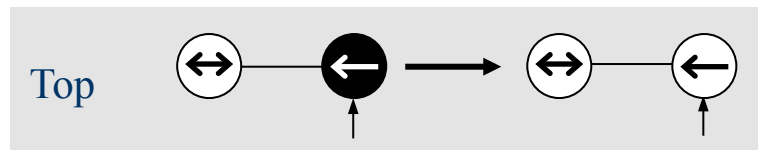
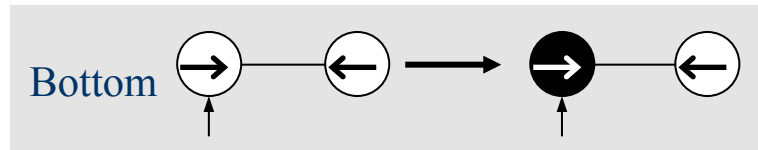


 [Dijkstra 74]



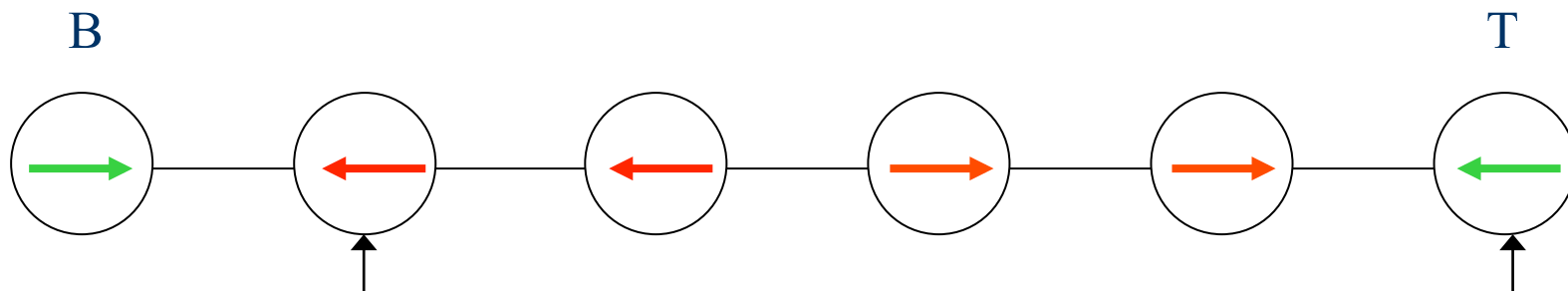
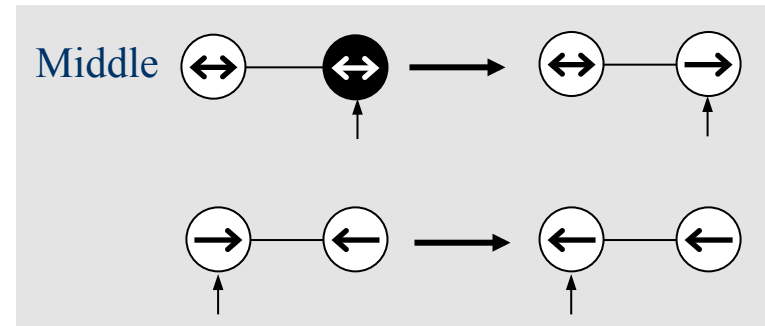
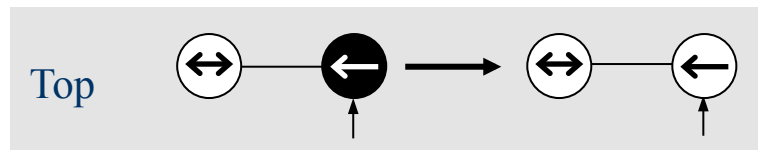
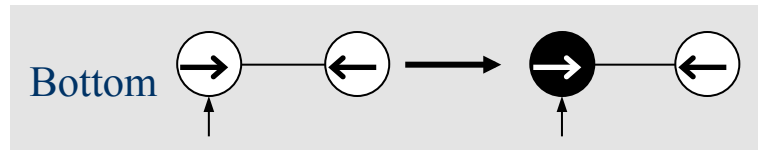


 [Dijkstra 74]



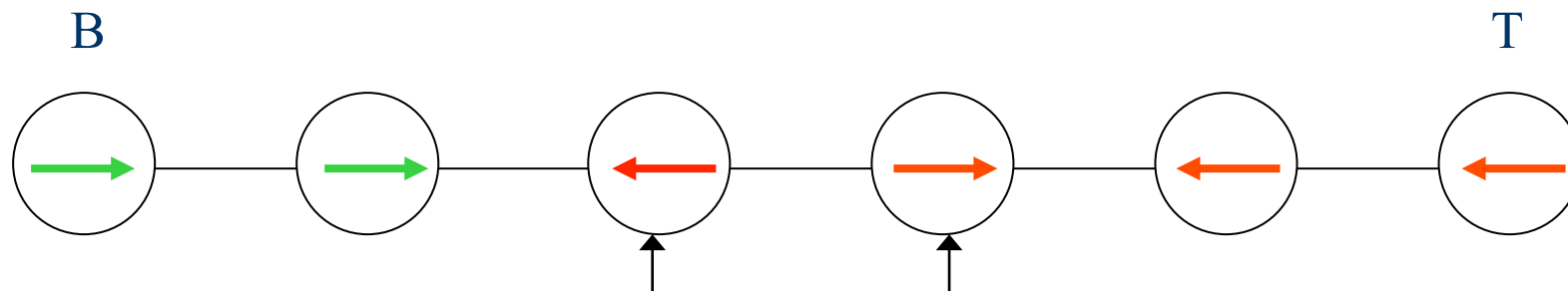
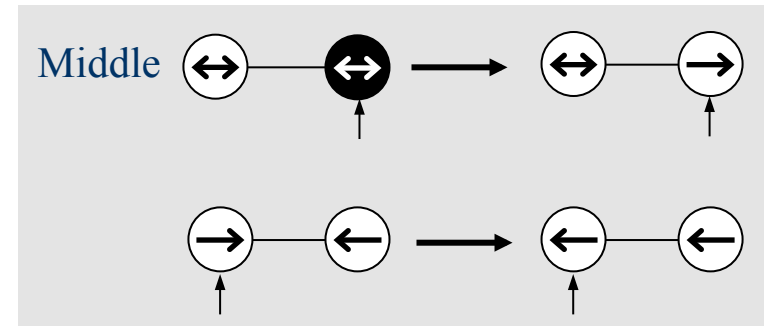
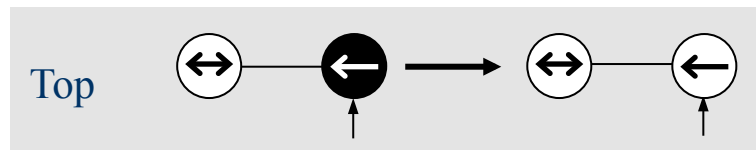
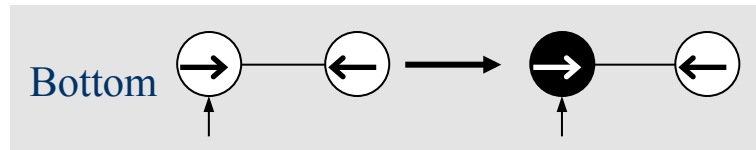


 [Dijkstra 74]



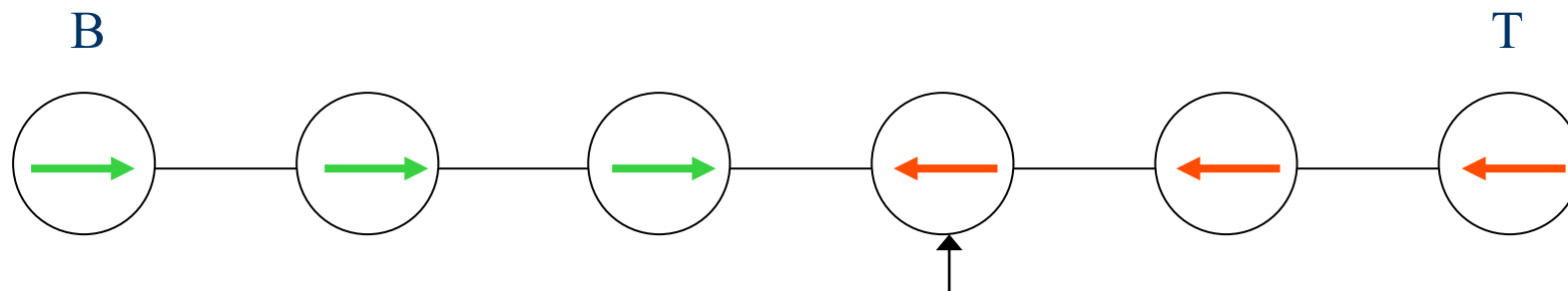
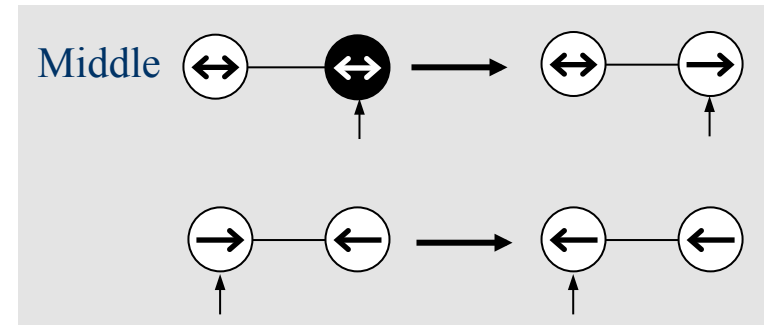
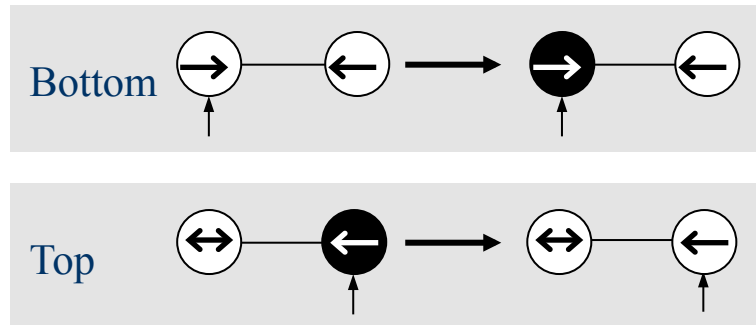


 [Dijkstra 74]





 [Dijkstra 74]

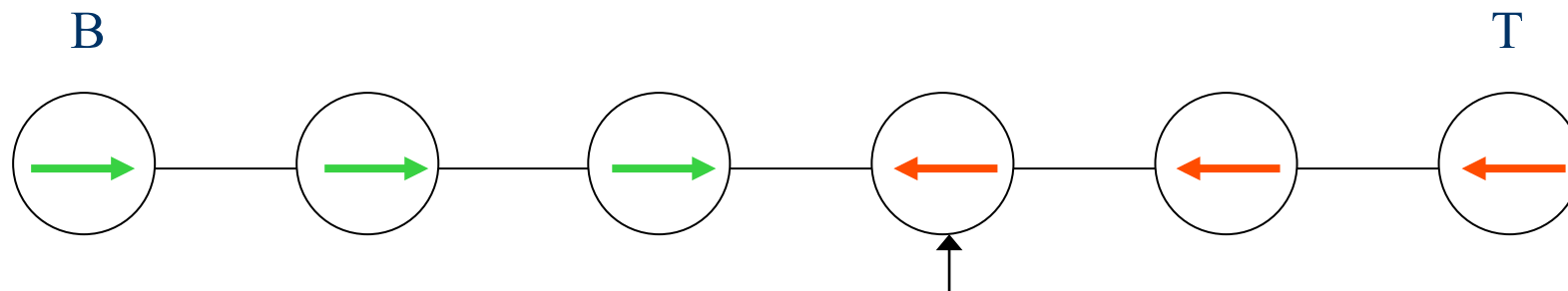
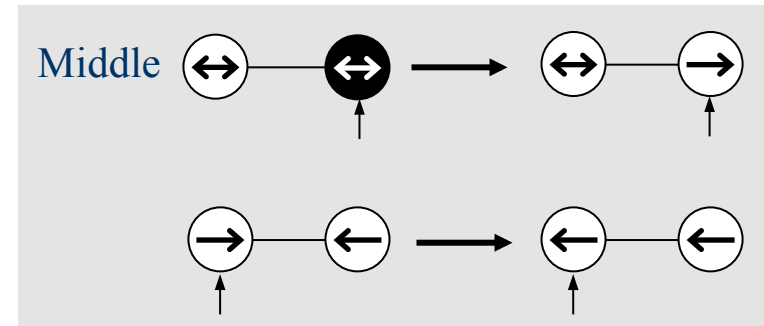
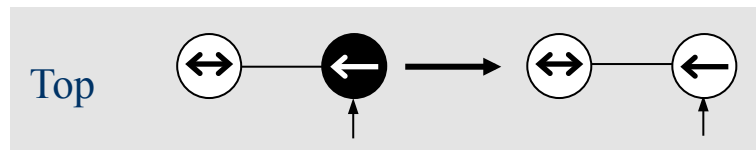
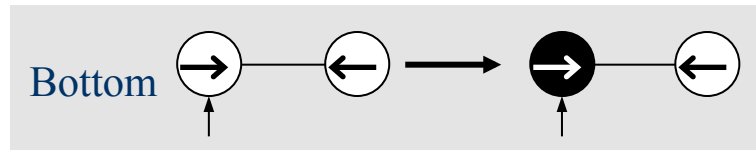


**Stabilisé !**





 [Dijkstra 74]



**Stabilisé !**

**Temps de stabilisation =  $O(n)$**