$$\frac{d}{dx}\left(\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\left(\vec{w}\right)\right)\right)\right)=\vec{f}$$

Funciones de forma

$$\begin{aligned}
N_1 &= (1-\epsilon-\eta-\phi)(2(1-\epsilon-\eta-\phi)-1)\\
N_2 &= \epsilon(2\epsilon-1)\\
N_3 &= \eta(2\eta-1)\\
N_4 &= \phi(2\phi-1)\\
N_5 &= 4\epsilon\eta\\
N_6 &= 4\eta\phi\\
N_7 &= 4\epsilon\phi\\
N_8 &= 4\epsilon(1-\epsilon-\eta-\phi)\\
N_9 &= 4\eta(1-\epsilon-\eta-\phi)\\
N_{10} &= 4\phi(1-\epsilon-\eta-\phi)
\end{aligned} \tag{1}$$

Step 2: Interpolation

$$\alpha \approx N_1\alpha_1+N_2\alpha_2+N_3\alpha_3+N_4\alpha_4+N_5\alpha_5+N_6\alpha_6+N_7\alpha_7+N_8\alpha_8+N_9\alpha_9+N_{10}\alpha_{10} \tag{2}$$

$$\beta \approx N_1\beta_1+N_2\beta_2+N_3\beta_3+N_4\beta_4+N_5\beta_5+N_6\beta_6+N_7\beta_7+N_8\beta_8+N_9\beta_9+N_{10}\beta_{10} \tag{3}$$

$$\gamma \approx N_1\gamma_1+N_2\gamma_2+N_3\gamma_3+N_4\gamma_4+N_5\gamma_5+N_6\gamma_6+N_7\gamma_7+N_8\gamma_8+N_9\gamma_9+N_{10}\gamma_{10} \tag{4}$$

$$\alpha \approx \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 & N_9 & N_{10} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \\ \alpha_9 \\ \alpha_{10} \end{bmatrix} \tag{5}$$

$$\beta \approx \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 & N_9 & N_{10} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \end{bmatrix} \tag{6}$$

$$\gamma \approx \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 & N_9 & N_{10} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \\ \gamma_7 \\ \gamma_8 \\ \gamma_9 \\ \gamma_{10} \end{bmatrix} \tag{7}$$

$$\begin{aligned} \alpha &\approx \mathbf{N}\boldsymbol{\alpha} \\ \beta &\approx \mathbf{N}\boldsymbol{\beta} \\ \gamma &\approx \mathbf{N}\boldsymbol{\gamma} \end{aligned} \tag{8}$$

$$
\begin{aligned}
\vec{w} &= \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \\
&\approx \begin{bmatrix} \mathbf{N}\boldsymbol{\alpha} \\ \mathbf{N}\boldsymbol{\beta} \\ \mathbf{N}\boldsymbol{\gamma} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{N} & 0 & 0 \\ 0 & \mathbf{N} & 0 \\ 0 & 0 & \mathbf{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma} \end{bmatrix} \\
\vec{w} &\approx \mathbf{N}^* \, \mathbf{W}
\end{aligned} \tag{9}
$$

Step 3: Model approximation

$$\frac{d}{dx}\left(\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\left(\vec{w}\right)\right)\right)\right) = \vec{f} \tag{10}$$

$$\frac{d}{dx}\left(\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\left(\mathbf{N}^* \, \mathbf{W}\right)\right)\right)\right) = \vec{f} \tag{11}$$

$$R = -\frac{d}{dx}\left(\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\left(\mathbf{N}^* \, \mathbf{W}\right)\right)\right)\right) + \vec{f} \tag{12}$$

Step 4: W.R.M.

$$\int\int\int wR \, dxdydz = 0 \tag{13}$$

$$\int_V wR \, dV = 0 \tag{14}$$

$$\mathbf{W}^* := \begin{bmatrix} \mathbf{W} & 0 & 0 \\ 0 & \mathbf{W} & 0 \\ 0 & 0 & \mathbf{W} \end{bmatrix} \tag{15}$$

$$\int_V \mathbf{W}^* \left[-\frac{d}{dx}\left(\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\left(\mathbf{N}^* \, \mathbf{W}\right)\right)\right)\right) + \vec{f}\right] dV = 0 \tag{16}$$

Step 5: Galerkin

$$\mathbf{W} = \mathbf{N}^T$$

$$\mathbf{W}^* = \begin{bmatrix} \mathbf{N}^T & 0 & 0 \\ 0 & \mathbf{N}^T & 0 \\ 0 & 0 & \mathbf{N}^T \end{bmatrix} = \mathbf{N}^{*T} \tag{17}$$

$$\int_V \mathbf{N}^{*T} \left[-\frac{d}{dx} \left(\frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \left(\mathbf{N}^* \, \mathbf{W} \right) \right) \right) \right) + \vec{f} \right] dV = 0 \tag{18}$$

$$\int_V \left[-\mathbf{N}^{*T} \frac{d}{dx} \left(\frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \left(\mathbf{N}^* \, \mathbf{W} \right) \right) \right) \right) + \mathbf{N}^{*T} \vec{f} \right] dV = 0 \tag{19}$$

$$-\int_V \mathbf{N}^{*T} \frac{d}{dx} \left(\frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \left(\mathbf{N}^* \, \mathbf{W} \right) \right) \right) \right) dV + \int_V \mathbf{N}^{*T} \vec{f} \, dV = 0 \tag{20}$$

Strong Form:

$$\left(\int_V \mathbf{N}^{*T} \frac{d}{dx} \left(\frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \left(\mathbf{N}^* \right) \right) \right) \right) dV \right) \mathbf{W} = \int_V \mathbf{N}^{*T} \vec{f} \, dV \tag{21}$$

Step 6: Integration by Parts

$$\int_V \mathbf{N}^{*T} \frac{d}{dx} \left(\frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \mathbf{N}^* \right) \right) \right) dV$$

$$\int U \, dV = UV - \int V \, dU \tag{22}$$

$$U = \mathbf{N}^{*T} \qquad dV = \frac{d}{dx} \left(\frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \mathbf{N}^* \right) \right) \right) \tag{23}$$

$$dU = \frac{d}{dx} \mathbf{N}^{*T} \qquad V = \frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \mathbf{N}^* \right) \right) \tag{24}$$

$$\left[\mathbf{N}^{*T} \cdot \frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \mathbf{N}^* \right) \right) \right] \Bigg|_V - \int_V \frac{d}{dx} \left(EI \frac{d}{dx} \left(\frac{d}{dx} \mathbf{N}^* \right) \right) \cdot \frac{d}{dx} \mathbf{N}^{*T} \, dV \tag{25}$$

Going back:

$$\left(\left[\mathbf{N}^{*T}\cdot\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\mathbf{N}^*\right)\right)\right]\bigg|_V-\int_V\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\mathbf{N}^*\right)\right)\cdot\frac{d}{dx}\mathbf{N}^{*T}\,dV\right)\mathbf{W}=\int_V\mathbf{N}^{*T}\vec{f}\,dV \tag{26}$$

Weak form:

$$\left(-\int_V\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\mathbf{N}^*\right)\right)\cdot\frac{d}{dx}\mathbf{N}^{*T}\,dV\right)\mathbf{W}=\int_V\mathbf{N}^{*T}\vec{f}\,dV-\left[\mathbf{N}^{*T}\cdot\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\mathbf{N}^*\mathbf{W}\right)\right)\right]\bigg|_V \tag{27}$$

$$\mathbf{K}=-\int_V\frac{d}{dx}\left(EI\frac{d}{dx}\left(\frac{d}{dx}\mathbf{N}^*\right)\right)\cdot\frac{d}{dx}\mathbf{N}^{*T}\,dV \tag{28}$$

$$\mathbf{b}=\int_V\mathbf{N}^{*T}\vec{f}\,dV \tag{29}$$

Step 6.5: Defining Matrix Components

Sistema Local Final:

$$\mathbf{KX} = \mathbf{b}$$

$$\mathbf{K}_{30x30} = EIJ \begin{bmatrix} \boldsymbol{\mu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\mu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\mu} \end{bmatrix}$$

$$J = \begin{vmatrix} x_2 - x_1 & x_3 - x_1 & x_4 - x_1 \\ y_2 - y_1 & y_3 - y_1 & y_4 - y_1 \\ z_2 - z_1 & z_3 - z_1 & z_4 - z_1 \end{vmatrix}$$

$$\boldsymbol{\mu}_{10x10} = \begin{bmatrix} A & E & 0 & 0 & -F & 0 & -F & G & F & F \\ E & B & 0 & 0 & -H & 0 & -H & I & H & H \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -F & -H & 0 & 0 & C & 0 & J & -K & -C & -J \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -F & -H & 0 & 0 & J & 0 & C & -K & -J & -C \\ G & I & 0 & 0 & -K & 0 & -K & D & K & K \\ F & H & 0 & 0 & -C & 0 & -J & K & C & J \\ F & H & 0 & 0 & -J & 0 & -C & K & J & C \end{bmatrix}$$

$$A = -\frac{1}{192c_2^2}(4c_1 - c_2)^4 - \frac{1}{24c_2}(4c_1 - c_2)^3 - \frac{1}{3840c_2^3}(4c_1 - c_2)^5 + \frac{1}{3840c_2^3}(4c_1 + 3c_2)^5$$

$$B = -\frac{1}{192c_2^2}(4c_1+c_2)^4 + \frac{1}{24c_2}(4c_1+c_2)^3 + \frac{1}{3840c_2^3}(4c_1+c_2)^5 - \frac{1}{3840c_2^3}(4c_1-3c_2)^5$$

$$C = \frac{4}{15}c_2^2$$

$$D = \frac{1}{192c_2^2}(4c_2-c_1)^4 - \frac{1}{3840c_2^3}(4c_2-c_1)^5 + \frac{1}{7680c_2^3}(4c_2+8c_1)^5 - \frac{7}{7680c_2^3}(4c_2-8c_1)^5 + \frac{1}{768c_2^3}(-8c_1)^5 - \frac{c_1}{96c_2^3}(4c_2-8c_1)^4 + \frac{2c_1-1}{192c_2^3}(-8c_1)^4$$

$$E = \frac{8}{3}c_1^2 + \frac{1}{30}c_2^2$$

$$F = \frac{2}{3}c_1c_2 - \frac{1}{30}c_2^2$$

$$G = -\frac{16}{3}c_1^2 - \frac{4}{3}c_1c_2 - \frac{2}{15}c_2^2$$

$$H = \frac{2}{3}c_1c_2 + \frac{1}{30}c_2^2$$

$$I = -\frac{16}{3}c_1^2 - \frac{2}{3}c_2^2$$
$$J = \frac{2}{15}c_2^2$$
$$K = -\frac{4}{3}c_1 c_2$$
$$c_1 = \frac{1}{(x_2 - x_1)^2}$$
$$c_2 = \frac{1}{x_2 - x_1}(4x_1 + 4x_2 - 8x_8)$$

==============

$$\mathbf{b}_{30x1} = \frac{J}{120}\begin{bmatrix} \boldsymbol{\tau} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\tau} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\tau} \end{bmatrix} \vec{f}$$
$$\boldsymbol{\tau}_{10x1} = \begin{bmatrix} 59 \\ -1 \\ -1 \\ -1 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$