（一） $\frac{\partial x^T b}{\partial \mathrm{x}}$

$$x^T b = x_1 b_1 + x_2 b_2 + \cdots + x_n b_n$$

$$\frac{\partial x^T b}{\partial \mathrm{x}} = \frac{\partial (x_1 b_1 + x_2 b_2 + \cdots + x_n b_n)}{\partial (x_1, x_2, \cdots, x_n)^T} = (b_1, b_2, \cdots, b_n)^T = b$$

(二) $\frac{\partial a^T x b}{\partial \mathrm{x}}$

$$a^T x = \begin{pmatrix} a_1 x_{11} + a_2 x_{21} + a_n x_{n1} & a_1 x_{12} + a_2 x_{22} + a_n x_{n2} & \cdots & a_1 x_{1n} + a_2 x_{2n} + a_n x_{nn} \end{pmatrix}$$

$$a^T x b = (a_1 x_{11} + a_2 x_{21} + a_n x_{n1}) b_1 + (a_1 x_{12} + a_2 x_{22} + a_n x_{n2}) b_2 + \cdots + (a_1 x_{1n} + a_2 x_{2n} + a_n x_{nn}) b_n$$

$$\frac{\partial a^T x b}{\partial \mathrm{x}} = \begin{matrix} a_1 b_1 & \cdots & a_1 b_n \\ \vdots & \ddots & \vdots \\ a_n b_1 & \cdots & a_n b_n \end{matrix} = a b^T$$

(三)

(四)迹的导数

1. $\frac{\partial \mathrm{tr}(\mathrm{x})}{\partial \mathrm{x}}$

$$\mathrm{tr}(\mathrm{x}) = x_{11} + x_{22} + \cdots + x_{nn}$$

$$\frac{\partial \mathrm{tr}(\mathrm{x})}{\partial \mathrm{x}} = \frac{\partial (x_{11} + x_{22} + \cdots + x_{nn})}{\partial \begin{bmatrix} x_{11} & \cdots & x_{22} \\ \vdots & \ddots & \vdots \\ x_{1n} & \cdots & x_{nn} \end{bmatrix}} = I$$

2. $\frac{\partial \mathrm{tr}(\mathrm{XA})}{\partial \mathrm{X}}$

$$\mathrm{XA} = \begin{bmatrix} \sum_j^n x_{1j} a_{j1} & \cdots & \sum_j^n x_{1j} a_{jn} \\ \vdots & \ddots & \vdots \\ \sum_j^n x_{nj} a_{j1} & \cdots & \sum_j^n x_{nj} a_{jn} \end{bmatrix}$$

$$\mathrm{tr}(\mathrm{XA}) = \sum_j^n x_{1j} a_{j1} + \sum_j^n x_{2j} a_{j2} + \cdots + \sum_j^n x_{nj} a_{jn}$$

$$\frac{\partial \mathrm{tr}(\mathrm{XA})}{\partial \mathrm{X}} = A^T$$