183.605 Machine Learning for Visual Computing Assignment 1

Michael Reiter

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1 Task1

1.1 MNIST Data Set and Feature Selection

We applied a perceptron model to classification of handwritten digits. MNIST is a standard data set with 28×28 pixel images of handwritten digits. We selected the digits 0 and 7 and 500 training images for each class for our classification task, as they are reasonably diffenrent. To chose the right feature we plotted all regionprops in a scatter plot matrix,see figure 1 and selected solodity and eccentricity. To compare the batch and online version we used an equivalent number of maximal iterations. This means we took used the size of a batch times the maximum number of iterations in the batch case as the maximum number of iterations for the online algorithm. In our case we took 500000 and 500.

1.1.1 Features without transform

We used once the batch training algorithm and once the online training algorithm to classify the data directly from these to features. The connfusion matrix evaluated on a separate test set gives us following perfomance:

For the online training algorithm evaluated on the test set we got 385 correctly classified results of 400 test images. We got the following confusion matrix:

n=400	classified as 1	classified as 7
Digit 0	185	15
Digit 7	0	200

For the batch training algorithm evaluated on the same test set we got 383 correctly classified results of 400 test images. We got the following confusion matrix:

n=400	classified as 1	classified as 7
Digit 0	187	13
Digit 7	4	200

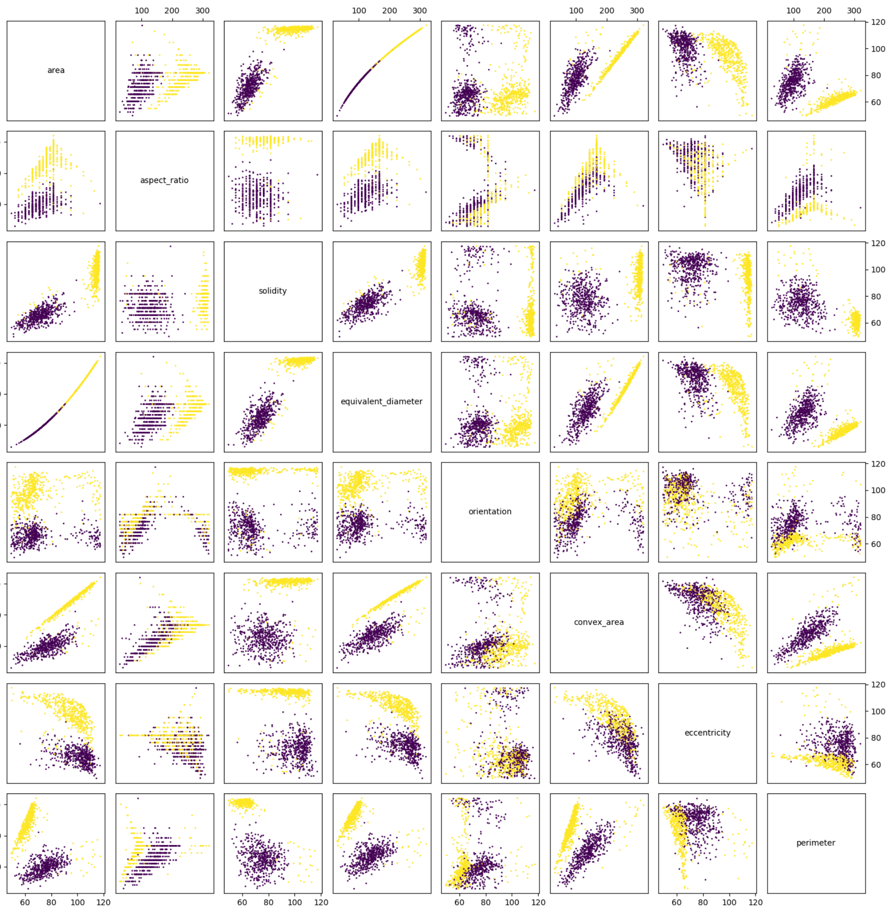


Figure 1: Scatter plot matrix of all region properties

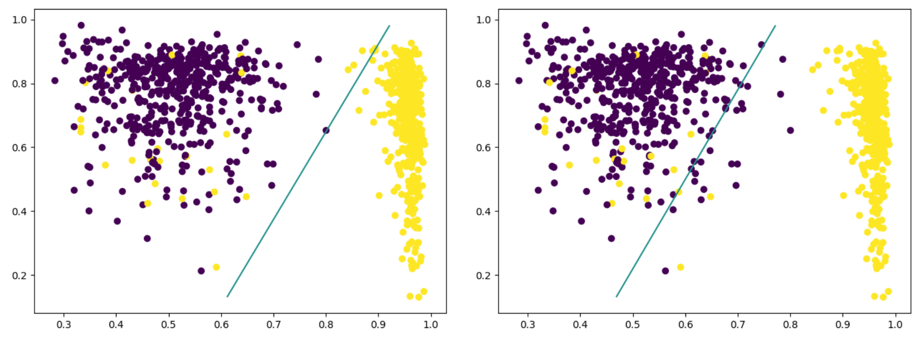


Figure 2: On the decision boundry of the trained perceptron. On the left for the online algorithm on the right for the batch algorithm.

The corresponding decision boundries are shown in figure 2. We see that the data is not linearly seperable, because there are yellow points inside the purple "cloud" In theory a perceptron is able to detect a linearly seperable set, because if the set is linearly seperable, then the algorithm is guaranteed to converge. Moreover we can bound the number of training steps from above. But as this bound depends on a separating hyperplane and the training set, we can not know beforehand how big this upper bound is.

1.1.2 Classification with feature transform

We performed the same experiments as above, but first applied following feature transform to our data. $(x, y) \mapsto (1, x, y, x^2, y^2, xy)$

For the online training algorithm evaluated on the test set we got 385 correctly classified results of 400 test images. We got the following confusion matrix:

n=400	classified as 1	classified as 7
Digit 0	186	14
Digit 7	0	200

For the batch training algorithm evaluated on the same test set we got 383 correctly classified results of 400 test images. We got the following confusion matrix:

n=400	classified as 1	classified as 7
Digit 0	187	13
Digit 7	4	200

The corresponding decision boundries are shown in figure 3. It's not very surprising that the feature transform doesn't help the classification. as there are yellow points, correspondig to the digit 0, inbetween purple points. Such a simple feature transform cannot separete this points, as the new decision boundry is just a quadratic.

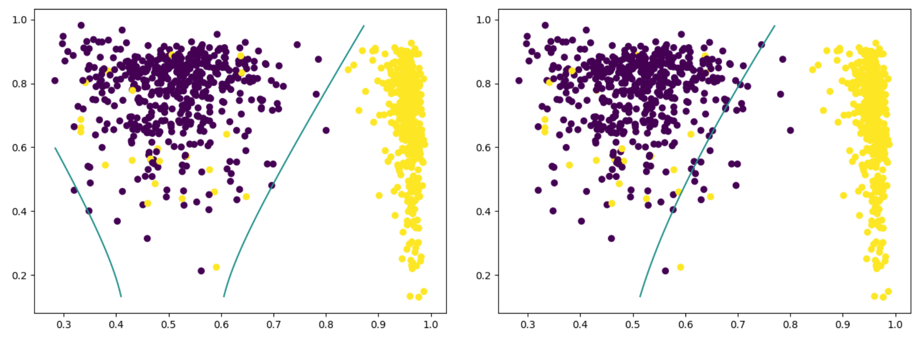


Figure 3: On the decision boundry of the trained perceptron. On the left for the online algorithm on the right for the batch algorithm.

1.1.3 Classification directly with image data

2 Task2

2.1 LMS vs. closed Form

We followed the instructions and used the function below with $G = 6$ to determine our y values for the experimental setup.

$$f(x) = 2x^2 - Gx + 1 \tag{1}$$

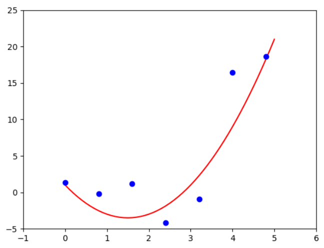
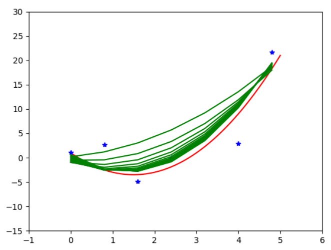


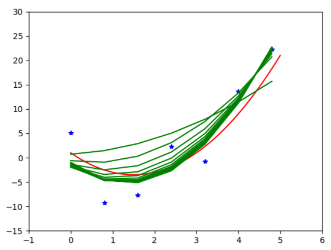
Figure 4: Setup and an example of t_i values with additional noise

2.1.1 What is the resulting weight Vector when using the LMS rule?

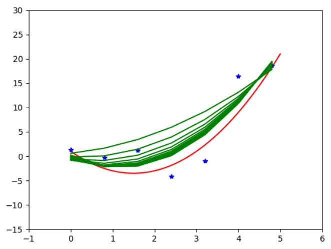
We implemented an online LMS learning rule for regression to determine useful weights to fit the curve. Figure **??** shows three runs with randomised training data. As we can see the fitted curve is moved towards the original curve with each iteration. γ or the learning factor controls how much impact new values have on the resulting weights for the next iteration.



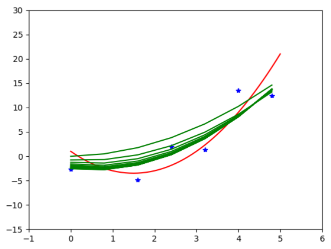
(a)



(b)



(c)



(d)

Figure 5: Visualization of the learning process of **LMS**

2.1.2 How can you determine the optimal $w*$ in closed form? Compare $w*$ with the outcome of the LMS-rule training

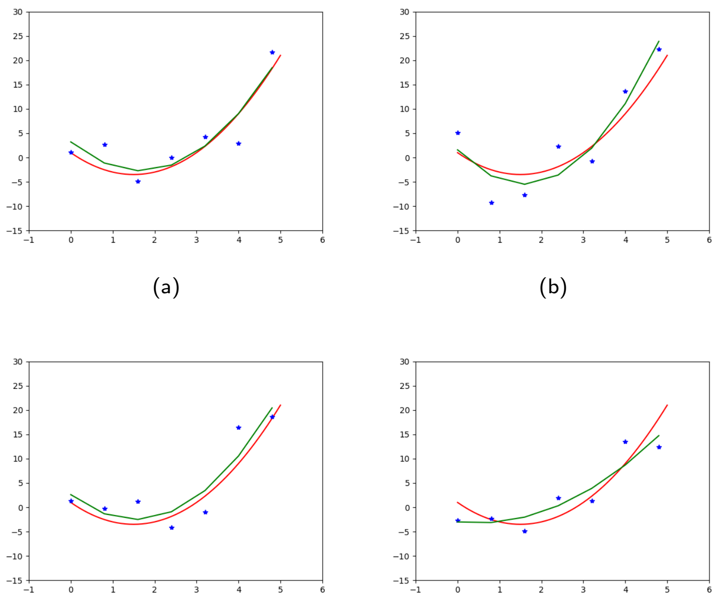
We used the pseudo inverse approach to minify the sum of squares error. As displayed the in the Tables, 1 and 2 we can see the resulting weights and the calculated errors. We can see that the error rate for our four examples with CF are all better without much surprise.

Figure	**LMS**	**CF**
a	97.09004286828198	71.03243449830224
b	112.43543529868002	97.82006516766448
c	109.4839763954856	84.46023189949588
d	61.164485036889054	47.10797373501289

Table 1: error comparisons betweenLMS and CF

2.1.3 Influence of γ

As mentioned earlier γ has to influence if the algorithm can converge. In case $gamma$ is too big, new values get too much weight, thus 'overriding' older ones.



(a) (b) (c) (d)

Figure 6: the same training data as in figure, however this time solved in closed form (**CF**)

See Figure 7

Figure	**LMS**	**CF**
a	2.79641366 -4.06370085 1.6057828	6.10503808 -6.93083294 2.04038233
b	0.90857717 -6.42409915 2.36363687	1.57854501 -8.9662967 2.8358482
c	0.25694124 -4.13859362 1.69469808	2.59983553 -6.64808401 2.1588638
d	-2.58665078 -0.97237072 0.9121616	-3.01226058 -0.92406774 0.96261322

Table 2: resulting weightsLMS and CF

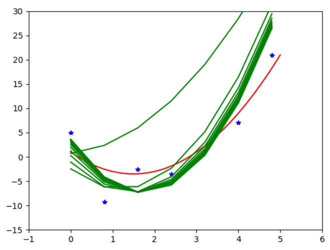


Figure 7: γ has been chosen too high

2.2 Image Data

Unfortunately we did not finish in time, and we will not present this task here.