Final Exam of Error-Correcting Codes

Jan. 6, 2022

1. (8%) Describe the conditions of checking whether a convolutional code is catastrophic or not by
   i) transfer function matrix
   and
   ii) state diagram.

2. Consider a rate 1/2 convolutional code C with generator sequences $\overline{g_1} = (111), \overline{g_2} = (101)$.
   (a) (5%) Please draw the diagram of a nonsystematic encoder (NSC) for C.
   (b) (5%) Please draw the diagram of a recursive systematic encoder (RSC) for C.
   (c) (5%) Please draw the state diagram of the code in 2(b).

3. (a) (5%) Let $L_U(u) = \log \frac{P_U(u=+1)}{P_U(u=-1)}$. Please express $P_U(u=+1)$ and $P_U(u=-1)$ in terms of $L_U(u)$.
   (b) (5%) Please express $L(u_1 \oplus u_2)$ in terms of $L(u_1)$ and $L(u_2)$.
   (c) (5%) Approximate $L(u_1 \oplus u_2)$ by the min sum algorithm.
   (d) (5%) Let $y = x + w$, where $x \in \{+1, -1\}$ and w is Gaussian distributed with zero mean and variance σ^2. Find the LLR of a received value y obtained from the channel.

4. (7%) Prove that the final survivor in the Viterbi algorithm is the maximum likelihood path.

5. (a) (5%) Provide the reasons of using RSC instead of NSC in the parallel turbo code.
   (b) (5%) Provide the reasons of using a large random interleaver in the parallel turbo code.

6. Consider a parallel turbo code C with component RSC codes C_1 and C_2. Suppose that both C_1 and C_2 have code rates of 1/2.
   (a) (5%) What is the code rate R of C?
   (b) (5%) What shall we do about C_1 and C_2 if we want to increase the rate R?
   (c) (5%) What assumption about the LLRs is used in the EXIT chart analysis?
   (d) (5%) How can we explain the turbo cliff region in the BER performance of the turbo code using EXIT chart analysis?

7. Consider a (t, s)-regular LDPC (low-density parity-check) code C.
   (a) (5%) What is the code rate of C?
   (b) (5%) Describe the message-passing decoding of C over the AWGN channel.

8. Consider a irregular LDPC code. The degree-distribution polynomial for the variable node is $\lambda(X) = \sum_{i=1}^{d_v} \lambda_i X^{i-1}$ where λ_i denotes the fraction of all edges connected to degree-i variable nodes. The degree-distribution polynomial for the check node is $\rho(X) = \sum_{i=1}^{d_c} \rho_i X^{i-1}$ where ρ_i denotes the fraction of all edges connected to degree-i check nodes. Let m, n, E and R be the number of check nodes, the number of variable nodes, the total number of edges in the graph and code rate respectively.
   (a) (5%) Express E as a function of n and $\lambda(X)$.
   (b) (5%) Express R as a function of $\rho(X)$ and $\lambda(X)$.

m
n H