CS 430/585
Computer Graphics I

Polygon Clipping and Filling

Week 3, Lecture 5

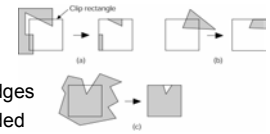
David Breen, William Regli and Maxim Peysakhov
Geometric and Intelligent Computing Laboratory
Department of Computer Science
Drexel University
http://gicl.cs.drexel.edu

Outline

- Polygon clipping
  - Sutherland-Hodgman,
  - Weiler-Atherton
- Polygon filling
  - Scan filling polygons
  - Flood filling polygons
  - Pattern filling polygons
- Introduction and discussion of homework #2

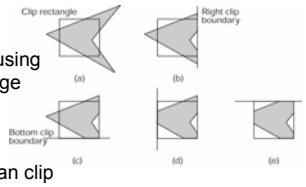
Polygon Clipping

- Lots of different cases
- Issues
  - Edges of polygon need to be tested against clipping rectangle
  - May need to add new edges
  - Edges discarded or divided
  - Multiple polygons can result from a single polygon



The Sutherland-Hodgman Polygon-Clipping Algorithm

- Divide and Conquer
- Idea:
  - Clip single polygon using single infinite clip edge
  - Repeat 4 times
- Note the generality:
  - 2D convex n-gons can clip arbitrary n-gons
  - 3D convex polyhedra can clip arbitrary polyhedra

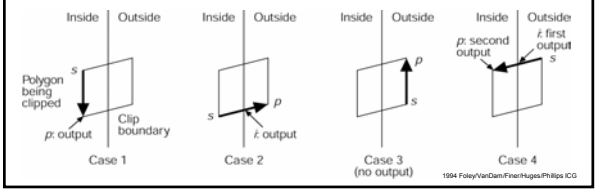


Sutherland-Hodgman Algorithm

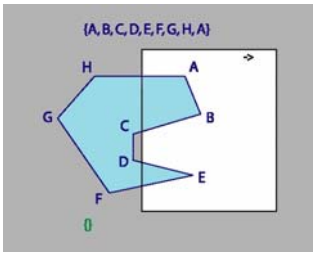
- Input:
  - $v_1, v_2, \ldots v_n$ the vertices bounding the polygon
  - A single infinite clip edge
- Output:
  - $v'_1, v'_2, \ldots v'_n$, vertices of the clipped polygon
- Do this 4 (or *n*) times

Sutherland-Hodgman Algorithm

- Move around polygon from v_n to v_1 and back to v_n
- Check v_i, v_{i-1} wrt the clip edge
- There are 4 cases:

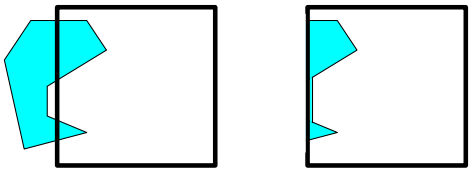


Sutherland-Hodgman Algorithm



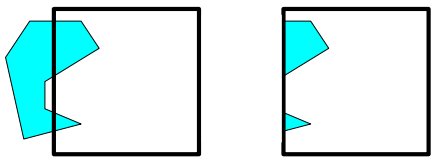
Issues with Sutherland-Hodgman Algorithm

- Clipping of the concave polygon
- Can produce two CONNECTED areas



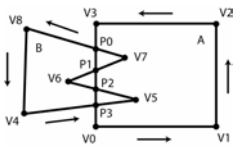
Weiler-Atherton Algorithm

- General clipping algorithm for concave polygons with holes
- Produces multiple polygons (with holes)



Weiler-Atherton Algorithm

- Given polygons A and B as linked list of vertices (counter-clockwise order)
- Find all edge intersections & place in list
- Insert intersection nodes
- Nodes point to A & B
- Determine in/out status of vertices



Weiler-Atherton Algorithm: Union

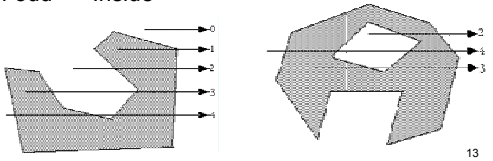
- Find a vertex of A outside of B
- Traverse linked list
- At each intersection point switch to other polygon
- Do until return to starting vertex
- All visited vertices and nodes define union'ed polygon

Weiler-Atherton Algorithm: Intersection

- Start at intersection point
  - If connected to an "inside" vertex, go there
  - Else step to an intersection point
- Traverse linked list
- At each intersection point switch to other polygon and remove intersection point from list
- Do until return to starting intersection point
- If intersection list not empty, pick another one
- All visited vertices and nodes define and'ed polygon

Point P Inside a Polygon?

- Connect P with another point P` that you know is outside polygon
- Intersect segment PP` with polygon edges
- Watch out for vertices!
- If # intersections is even (0 is even) => Outside
- If odd => Inside



Do two edges intersect?

Cyrus-Beck Algorithm

Line $P(t) = P_0 + t(P_1-P_0)$

Point on the edge P_{Ei}

$$N_i \bullet [P(t)-P_{Ei}] = 0$$

$$N_i \bullet [P_0 + t(P_1-P_0)-P_{Ei}] = 0$$

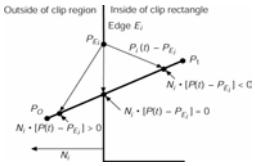
$$N_i \bullet [P_0-P_{Ei}] + N_i \bullet t[P_1-P_0] = 0$$

$$Let\ D = (P_1-P_0)$$

$$t = \frac{N_i \bullet [P_0 - P_{Ei}]}{-N_i \bullet D}$$

Make sure

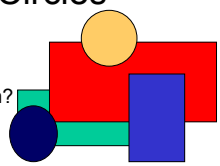
1. $D \neq 0$, or $P_1 \neq P_0$
2. $N_i \bullet D \neq 0$ *lines are not parallel*



Is $0 \leq t \leq 1$ for both edges?

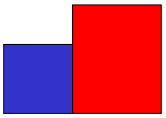
Filling Primitives: Rectangles, Polygons & Circles

- Two part process
  - Which pixels to fill?
  - What values to fill them with?
- Idea: ***Coherence***
  - *Spatial*: pixels are the same from pixel-to-pixel and scan-line to scan line;
  - *Span*: all pixels on a span get the same value
  - *Scan-line*: consecutive scan lines are the same
  - *Edge*: pixels are the same along edges



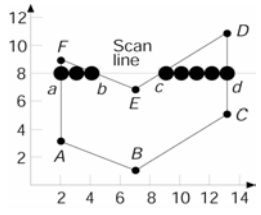
Scan Filling Primitives: Rectangles

- Easy algorithm
  - Fill from x_{min} to x_{max}
    Fill from y_{min} to y_{max}
- Issues
  - What if two adjacent rectangles share an edge?
  - Color the boundary pixels twice?
  - Rules:
    - Color only interior pixels
    - Color left and bottom edges

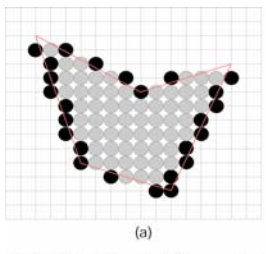


Scan Filling Primitives: Polygons

- Observe:
  - FA, DC intersections are integer
  - FE, ED intersections are not integer
- For each scan line, how to figure out which pixels are inside the polygon?



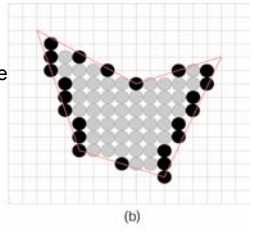
Scan Filling Polygons



- Idea #1: use midpoint algo on each edge, keeping track of extrema points
- Note: many extrema pixels lie outside the polygon
- Why: midpoint algo has no sense of in/out

Scan Filling Polygons

- Idea #2: draw pixels only strictly inside
  - Find intersections of scan line with edges
  - Sort intersections by increasing *x* coordinate
  - Fill pixels on inside based on a parity bit
    - B_p initially even
    - Invert at each intersect
    - Draw with odd, do not draw when even



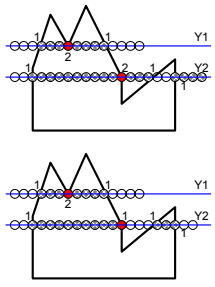
Span extrema Other pixels in the span

Scan Filling Polygons

- Issues with Idea #2:
  - If at a fractional x value, how to pick which pixels are in interior?
  - Intersections at integer pixel coordinates?
  - Shared vertices?
  - Vertices that define a horizontal edge?

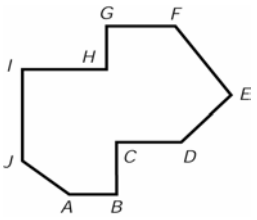
How to handle vertices?

- Problem:
  - vertices are counted twice
- Solution:
  - If both endpoints at a vertex are on the same side of the scan line, count it twice
  - If both endpoints are on different sides of a scan line, count it once
  - Compare current y value with y value of other endpoint



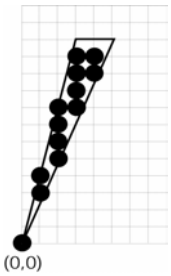
How to handle horizontal edges?

- Idea: don't count vertices
  - On AB, A is at y_{min} for JA; AB does not contribute, B_p is odd and draw AB
  - Edge BC has y_{min} at B, but AB does not contribute, B_p becomes even and drawing stops
  - At J, IJ has y_{min} but JA does not, so B_p becomes odd and span drawn to BC
  - The span that goes from IJ to C sees no change at C,
  - etc.



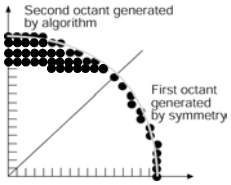
How to handle slivers?

- When the scan area does not have an "interior"
- Solution: use anti-aliasing
- But, to do so will require softening the rules about drawing only interior pixels



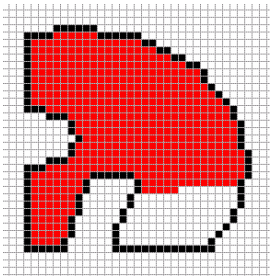
Scan-Filling Polygon

Scan Filling Curved Objects



- Hard in general case
- Easier for circles and ellipses.
- Use midpoint Alg to generate boundary points.
- Fill in horizontal pixel spans
- Use symmetry

Boundary-Fill Algorithm



- Start with some internal point (x,y)
- Check neighbors for filled or border color
- Color neighbors.
- Continue recursively

4 Connected Boundary-Fill Alg

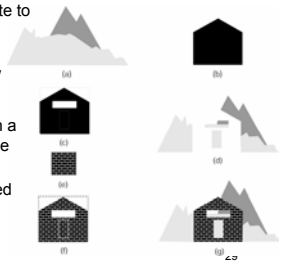
```
Void BoundaryFill4( int x, int y, int fill,
  int bnd)
{
   If Color(x,y) != fill and Color(x,y) != bnd
   {
     SetColor(x,y) = fill;
     BoundaryFill4(x+1, y, fill, bnd);
     BoundaryFill4(x, y +1, fill, bnd);
     BoundaryFill4(x-1, y, fill, bnd);
     BoundaryFill4(x, y -1, fill, bnd);
   }
}
```


Boundary-Fill Algorithm

- Issues with recursive boundary-fill algorithm:
  - May make mistakes if parts of the space already filled with the Fill color
  - Requires very big stack size
- More efficient algorithms
  - First color contiguous span along one scan line
  - Only stack beginning positions of neighboring scan lines

Pattern Filling

- Via Scan Conversion
  - Copy pixel in pattern template to location in polygon
  - The entire screen is 'tiled' and the polygon is a window
- Without Scan Conversion
  - Create patterned template in a rectangular work area (off the screen)
  - Copy this template as needed
- Handle overwrites with masking



Homework #2

- Modify homework #1
- Add "moveto" and "lineto" commands
- They will define closed polygons
- Perform union and intersection operations on polygons
- Display edges with Bresenham code

Course Status

So far everything is a straight line!

- How to model 2D curved objects?
  - Representation
    - Circles
    - Types of 2D Curves
    - Parametric Cubic Curves
    - Bézier Curves, (non)uniform, (non)rational
    - NURBS
  - Drawing of 2D Curves
    - Line drawing algorithms for complex curves
    - DeCasteljeau, Subdivision, De Boor