3D - Rotations

Euler's rotation theorem

$$[\quad] \qquad \underset{3\times3}{[\quad]} \rightarrow (\theta, \omega)$$

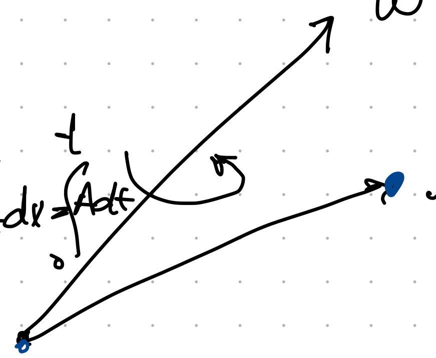
q

R Roll

Axis - Angle rep.

$$\omega = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

1 (deg/s)



$$\dot{x}(t) = A\,x(t)$$

$$\frac{dx}{dt} = A\,x(t) = \int_0^t \frac{1}{x}\,dx = \int_0^t A\,dt$$

$$\boxed{x(t) = e^{At} \cdot x(0)}$$

$$r \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

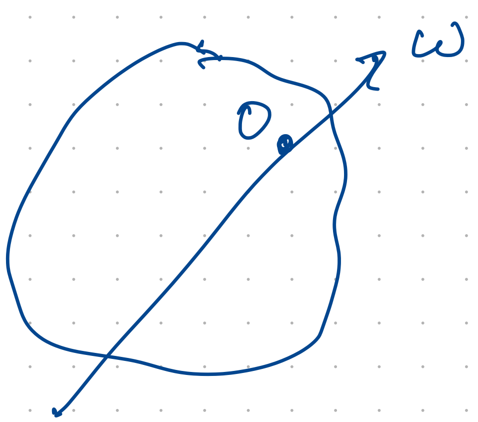
θ

$$\dot{r}(t) = \omega \times r$$

$$\boxed{\dot{r}(t) \equiv \hat{\omega}\, r(t)}$$

$$\rightarrow r(\theta) = \underbrace{\exp(\hat{\omega}\,\theta)}\, r(0)$$

$$\rightarrow r(\theta) = \underbrace{R(\hat{\omega}, \theta)}\, r(0) \leftarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



$$R \cdot p = (1)\, p$$

$$\pi^* = T \pi^*$$

Quaternions (Encoding a rotation)

Point $x \xrightarrow{\text{Transform}} (0, x) = p$

$$q = (v_0, v) \equiv \left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\, \omega\right)$$

$$p' = q \cdot p \cdot q^{-1}$$

$^{(4)}p' \leftarrow p^{(4)}$

$\downarrow$

(ω, θ)

(4)

$$q'(p')q'^{-1}$$

ω', θ'

$$\underbrace{q' q}\, p\, \underbrace{q^{-1} q'^{-1}}$$

$$s\, p\, s^{-1}$$

$q \quad q'$

s