System:

$$x_{k+1} = A x_k + B u_k + \varepsilon_k$$

$$y_k = C x_k + D u_k + v_k$$

$$\varepsilon \equiv N(0, R)$$
$$v \equiv N(0, Q)$$
$$x_0 \equiv N(\_,\_)$$
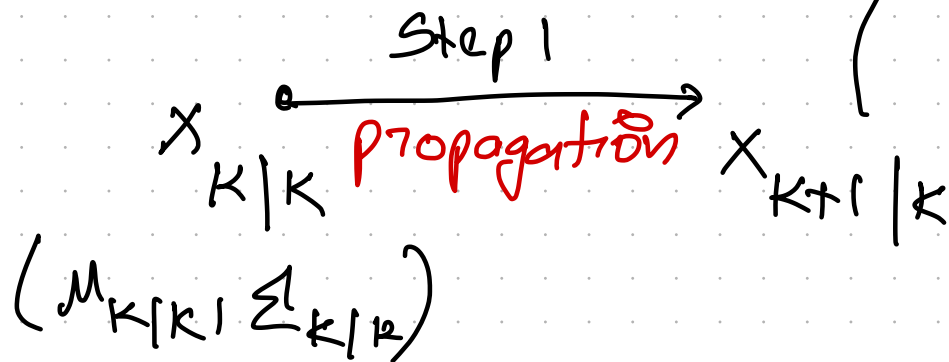
$$X_{k|k} = P(x_k \mid y_1 \cdots y_k)$$

$$\equiv N(\mu_{k|k}, \Sigma_{k|k})$$

$X_{k|k}$ — Step 1 — Propagation → $X_{k+1|k}$ — Step 2 / Update → $X_{k+1|k+1}$

$(\mu_{k|k}, \Sigma_{k|k})$

$$\hookrightarrow P(x_{k+1} \mid y_1 \cdots y_{k+1})$$
$$=$$

Step 1 $\left(\mu_{k+1|k} , \Sigma_{k+1|k} \right) \equiv X_{k+1|k}$

$$X_{k+1|k} = A X_{k|k} + B U_k + \varepsilon_k$$

$$E\left(X_{k+1|k} \right) = E\left(A X_{k|k} + B U_k + \varepsilon_k \right)$$

$$\mu_{k+1|k} = A \mu_{k|k} + B U_k$$

$$\Sigma_{k+1|k} = A \Sigma_{k|k} A^T + R$$

Step 2 : $\hat{X} + Y$

$$\underline{\text{Step 2}} \quad \text{Given} \quad x_{k+1|k} \equiv N\left(\mu_{k+1|k}, \Sigma_{k+1|k}\right)$$

$$\mu_{k+1|k+1}, \Sigma_{k+1|k+1} \equiv x_{k+1|k+1}$$

$$y_{k+1} = C x_{k+1} + v_{k+1} \qquad \left[D = 0\right]$$

$$\underline{\mu_{k+1|k+1}} = \mu_{k+1|k} + K\left(y_{k+1} - C\mu_{k+1|k}\right)$$

$$K = \Sigma_{k+1|k} C^T \left(C \Sigma_{k+1|k} C^T + Q\right)^{-1}$$

$$p\left(x_{k+1} \mid y_1 \cdots y_{k+1}\right) \equiv N\left(\mu_{k+1|k+1}, \Sigma_{k+1|k+1}\right)$$

$$\underline{\Sigma_{k+1|k+1}} = \left(\Sigma_{k+1|k}^{-1} + C^T Q^{-1} C \right)^{-1}$$

"inverse covariances add together"

$$\Sigma_{new}^{-1} = \Sigma_{old}^{-1} + noise^T$$

Camara

LIDAR

$\dfrac{U(k)}{100 \text{ Hz}}$

10 Hz

50 Hz

$X^o_{k|k} \longrightarrow X_{k+1|k}$

$X_{k+c|k+c}$

K_c

K_L