

This question paper consists of 8 printed pages, each of which is identified by the Code Number COMP5450M.

A non-programmable calculator may be used.
Answer All Questions.
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School of Computing

January 2017

COMP5450M

KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

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EXAM ROOM**

Answer **ALL THREE** questions

The marks available for each part of each question are clearly indicated.

Question 1

(a) Translate the following sentence into *Propositional Logic*:

- I play tennis on Mondays and Wednesdays.

[2 marks]

Answer: $(M \vee W) \rightarrow T$ or $(M \rightarrow T) \wedge (W \rightarrow T)$

(b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

- (i) No country is more beautiful than New Zealand.

[2 marks]

Answer: $\neg \exists x [\text{Country}(x) \wedge \text{MoreBeautiful}(x, \text{nz})]$

- (ii) Every sweet in the bag is red or purple.

[2 marks]

Answer: $\forall x [(\text{Sweet}(x) \wedge \text{InBag}(x)) \rightarrow (\text{Red}(x) \vee \text{Purple}(x))]$

- (iii) No student owns more than one car.

[2 marks]

Answer: $\neg \exists x \exists y \exists z [\text{Student}(x) \wedge \text{Car}(y) \wedge \text{Car}(z) \wedge \neg(y = z) \wedge \text{Owns}(x, y) \wedge \text{Owns}(x, z)]$

- (iv) Tom hates everyone except himself.

[2 marks]

Answer: $\forall x [(\text{Person}(x) \wedge \neg(x = \text{tom})) \leftrightarrow \text{Hates}(\text{tom}(x))]$

(c) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid: [6 marks]

$$\forall x [R(a, x) \rightarrow S(x)], \forall x [R(a, x)] \vdash \forall x [S(x)]$$

Answer: The sequent is valid, as shown by the following proof:

Axiom		Axiom
-----		-----
Ax [Rax \rightarrow Sx], Rab, Ax [Rax] - Rab, Sb		Ax [Rax \rightarrow Sx], Rab, Ax [Rax] - Sb
-----	and	-----
-Rab, Ax [Rax \rightarrow Sx], Rab, Ax [Rax] - Sb		-Rab \vee Sb, Ax [Rax \rightarrow Sx], Rab, Ax [Rax] - Sb
-----		-----
		Rab \rightarrow Sb, Ax [Rax \rightarrow Sx], Rab, Ax [Rax] - Sb

		Ax [Rax \rightarrow Sx], Rab, Ax [Rax] - Sb

		Ax [Rax \rightarrow Sx], Ax [Rax] - Sb

		Ax [Rax \rightarrow Sx], Ax [Rax] - Ax [Sx]

1 mark for each correct rule application. For full marks complete proof is required with Axioms at the top. Some credit may be given for almost correct rule applications.

(d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with an unary predicate P and a binary relation T . The domain of \mathcal{M} is the set $\{a, b, c, d, e\}$, and the denotations of P and T are as follows:

- $\delta(P) = \{a, b\}$
- $\delta(T) = \{\langle a, c \rangle, \langle b, c \rangle, \langle b, d \rangle, \langle d, c \rangle, \langle e, e \rangle\}$

Which of the following formulae are satisfied by this model?

[4 marks]

F1. $\forall x \exists y [T(x, y)]$

F2. $\exists x [P(x) \wedge T(x, x)]$

F3. $\forall x [P(x) \rightarrow \exists y [T(x, y)]]$

F4. $\forall x \forall y \forall z [(T(x, y) \wedge T(y, z)) \rightarrow T(x, z)]$

Answer: F3, and F4 are satisfied. 1 mark for each correct.

[Question 1 total: 20 marks]

Question 2

(a) Give a representation of the following statements in *Propositional Tense Logic*:

- Mary has bought a ticket and will go to the festival. [2 marks]

Answer: $\mathbf{P}(\text{MaryBuyTicket}) \wedge \mathbf{F}(\text{MaryGoFestival})$

- I shall visit Manchester and after that I shall visit Liverpool. [2 marks]

Answer: $\mathbf{F}(\text{IVM} \wedge \mathbf{FIVL})$

(b) The *Situation Calculus* is to be used to describe the actions of a robot vacuum cleaner and the changes that result from these actions. As well as the special predicate *holds* and the *result* function, the vocabulary used includes fluents *clean*(*r*), and *dirty*(*r*) that specify whether room *r* is clean or dirty and an action **move**(*r*₁, *r*₂), which takes place when the vacuum cleaner moves from room *r*₁ to room *r*₂.

Using this vocabulary, specify *frame axioms* that ensure that the **move**(*r*₁, *r*₂) action does not alter the state of the rooms *r*₁ and *r*₂, with respect to them being clean or dirty: both rooms will remain in the state they were in before the **move** action.

[4 marks]

Answer: $\text{holds}(\text{clean}(r_1), s) \rightarrow \text{holds}(\text{clean}(r_1), \text{result}(\text{move}(r_1, r_2), s))$

$\text{holds}(\text{clean}(r_2), s) \rightarrow \text{holds}(\text{clean}(r_1), \text{result}(\text{move}(r_1, r_2), s))$

$\text{holds}(\text{dirty}(r_1), s) \rightarrow \text{holds}(\text{dirty}(r_1), \text{result}(\text{move}(r_1, r_2), s))$

$\text{holds}(\text{dirty}(r_2), s) \rightarrow \text{holds}(\text{dirty}(r_1), \text{result}(\text{move}(r_1, r_2), s))$

1 mark for formulae that are not correctly formed but are along the right lines. 2 marks for one or more correctly formed frame axioms but not capturing requirements properly. 3 marks if most requirements captured. 4 marks for capturing all the specified requirements.

(c) For each of the following *Prolog* queries, give the value of the variable X after the query has been executed:

- (i) `?- X = 2+3.` [1 mark]

- (ii) `?- A + B = 2 + 3, X is A * B.` [1 mark]

- (iii) `?- L = [[a,b],[c,d]], L = [L1,L2], L2 = [X,_].` [1 mark]

- (iv) `?- L = [up,down], setof([A,B], (member(A,L), member(B,L)), X).` [2 marks]

Answer:

i) 2+3

ii) 6

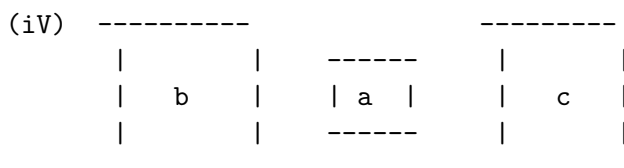
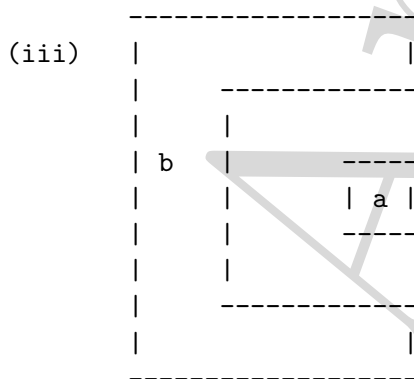
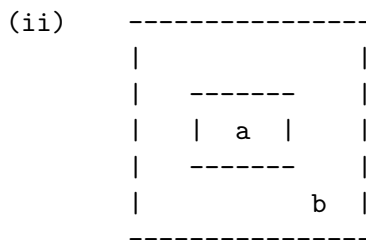
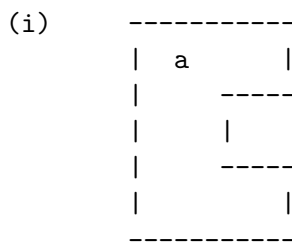
iii) c

iv) `[[up,up],[up,down],[down,up],[down,down]]`

- (d) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function, *conv*. The constants (a, b and c) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

- (i) $\neg \text{EQ}(\text{a}, \text{conv}(\text{a}))$ [1 mark]
 (ii) $\forall x [\text{P}(x, \text{a}) \rightarrow \text{NTPP}(x, \text{b})]$ [1 mark]
 (iii) $\text{DC}(\text{a}, \text{b}) \wedge \text{TPP}(\text{a}, \text{conv}(\text{b}))$ [2 marks]
 (iv) $\text{DC}(\text{a}, \text{b}) \wedge \text{DC}(\text{b}, \text{c}) \wedge \text{DC}(\text{a}, \text{b}) \wedge \text{NTPP}(\text{a}, \text{conv}(\text{sum}(\text{b}, \text{c})))$ [3 marks]

Answer: Possible diagrams are as follows:



[Question 2 total: 20 marks]

Answers

Question 3

- (a) The RCC-8 relations, DC, EC, PO, TPP, NTPP, TPPi, NTPPi and EQ form a mutually exhaustive and pairwise disjoint set of relations that can hold between spatial regions. Suppose that we have a situation involving regions a , b and c , such that the relations $PO(a, b)$ and $NTPP(b, c)$ hold.

- Draw diagrams illustrating each of the RCC-8 relations that could hold between regions a , and c . [3 marks]

Answer: There are three possibilities. The student should draw three simple diagrams corresponding to each of the cases where: $PO(a, c)$, $TPP(a, c)$ and $NTPP(a, c)$.

- State which subset of the RCC-8 is the composition $PO;NTPP$. [1 mark]

Answer: $\{PO, TPP, NTPP\}$

- (b) Represent the following sentences in *Description Logic*:

- (i) Every human is either male or female. [1 mark]

Answer: $\text{Human} \sqsubseteq (\text{Male} \sqcup \text{Female})$

- (ii) Nothing is both male and female. [1 mark]

Answer: $(\text{Male} \sqcap \text{Female}) \equiv \perp$

- (iii) A curry is a stew with a spicy ingredient. [2 marks]

Answer: $\text{Curry} \sqsubseteq (\text{or } \equiv) \text{Stew} \sqcap \exists \text{hasIngredient.Spicy}$

- (iv) Only humans can have human children. [2 marks]

Answer: $\exists \text{hasChild.Human} \sqsubseteq \text{Human}$

- (c) A *default theory* Θ contains the following formulae:

Classical Facts	Default Rules
C1 $\text{Person}(\text{Lilly}) \wedge \text{Drinks_Milk}(\text{Lilly})$	D1 $\text{Person}(x) : \text{Adult}(x) / \text{Adult}(x)$
C2 $\neg \exists x [\text{Adult}(x) \wedge \text{Child}(x)]$	D2 $\text{Drinks_Milk}(x) : \text{Child}(x) / \text{Child}(x)$

- (i) Interpret **D1** in English. [2 marks]

Answer: In the absence of any contradictory information, any person can be assumed to be an adult.

- (ii) For each of the following formulae, state whether it is true in *none*, *some but not all*, or *all* extensions of Θ ? [4 marks]

- | | |
|---------------------------------|---|
| A. $\text{Adult}(\text{Lilly})$ | C. $\text{Adult}(\text{Lilly}) \vee \text{Child}(\text{Lilly})$ |
| B. $\text{Child}(\text{Lilly})$ | D. $\text{Adult}(\text{Lilly}) \wedge \text{Child}(\text{Lilly})$ |

Answer: A. Some; B. Some; C. All; D. None

- (d) This question concerns a *Fuzzy Logic* based on the usual first-order predicate logic syntax, but with additional *linguistic modifiers* used as propositional operators with the following semantics: $\text{quite}(\phi) = \phi^{\frac{1}{2}}$ $\text{very}(\phi) = \phi^2$.

The logic is used to describe Sam, who possesses certain characteristics to the following degrees: $\text{Sick}(\text{sam}) = 0.3$, $\text{Sad}(\text{sam}) = 0.8$.

Translate each of the following sentences into the fuzzy logic representation and also give the fuzzy truth value of each proposition (using the standard fuzzy interpretation of the Boolean connectives):

(A) Sam is neither sick nor sad. [2 marks]

(B) Sam is very sick and quite sad. [2 marks]

Answer:

A) $\neg\text{Sick}(\text{sam}) \wedge \neg\text{Sad}(\text{sam})$ (1 mark)

Truth value = $\text{Min}(1 - 0.3, 1 - 0.8) = 0.2$ (1 mark)

B) $\text{very}(\text{Sick}(\text{sam})) \wedge \text{quite}(\text{Sad}(\text{sam}))$ (1 mark)

Truth value = $\text{Min}\{0.3^2, (0.8)^{\frac{1}{2}}\} = \text{Min}\{0.09, 0.894\} = 0.09$ (1 mark)

[Question 3 total: 20 marks]

[grand total: 60 marks]