This question paper consists of 9 printed pages, each of which is identified by the Code Number COMP5450M01

******* VERSION WITH ANSWERS INCLUDED *******

School of Computing

January 2009

KRR: KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

Answer THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentences using *Propositional Logic*:

i) Susan cannot take both Physics and Latin. [1 mark]

Answer:
$\neg(SP \wedge SL)$

ii) On clear nights when the moon is full I eat cheese but never leave my bedroom. [2 marks]

Answer:
$(CN \wedge MF) \rightarrow (EC \wedge \neg LB)$

b) Translate the following sentences into *Propositional Tense Logic*:

i) If you have not bought a ticket you will not be able to board the train. [2 marks]

Answer:
$\neg \mathbf{P}\, \textit{buy\_ticket} \rightarrow \neg \mathbf{F}\, \textit{board\_train}$

ii) I have never visited Russia and if I do it will be after graduating. [2 marks]

Answer:
$\neg \mathbf{P}\, \textit{Visit\_Russia} \wedge \mathbf{G}(\textit{Visit\_Russia} \rightarrow \mathbf{P}\, \textit{Graduate})$

c) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

i) Humans are the only featherless bipeds. [2 marks]

Answer:
$\forall x[(\neg \mathsf{Feathered}(x) \wedge \mathsf{Biped}(x)) \rightarrow \mathsf{Human}(x)]$

ii) A committee must have at least two members. [2 marks]

Answer:
$\forall x[\mathsf{Committee}(x) \rightarrow \exists y \exists z[\mathsf{Member}(y, x) \wedge \mathsf{Member}(z, x) \wedge \neg(y = z)]]$

d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with a predicate P and a binary relation R. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$; and the denotations of P and R are is as follows:

- $\delta(P) = \{a, b, c\}$
- $\delta(R) = \{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\}$

Which of the following formulae are satisfied by this model? [2 marks]

F1. $\exists x[R(x, x)]$
F2. $\forall x \exists y[R(x, y)]$
F3. $\forall x[P(x) \rightarrow \exists y[R(x, y)]]$
F4. $\forall xyz[(R(x, y) \wedge R(y, z)) \rightarrow R(x, z)]$

Answer:
Only F4 is satisfied. (2 marks if exactly this given. 1 mark if one formula given wrongly. 0, if two or more wrong, since this could be guesswork.)

e) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid:

$$\forall x[\neg P(x)],\ \ \forall x[P(x) \vee Q(x)] \ \vdash \ \forall z[Q(z)]$$ [7 marks]

Answer:
The sequent is valid as shown by the following proof:

```
              AXIOM                                         AXIOM
----------------------------------             ---------------------------------
Pk, Ax[-Px], Ax[Px v Qx] |- Pk, Qk     and     Qk Ax[-Px], Ax[Px v Qx] |- Pk, Qk
---------------------------------------------------------------------------------[v |-]
         Pk v Qk, Ax[-Px], Ax[Px v Qx]  |-  Pk, Qk
         -------------------------------------------[A |-]
                  Ax[Px v Qx],  Ax[-Px] |- Pk, Qk
              -------------------------------------[- |-]
              -Pk, Ax[Px v Qx], Ax[-Px] |- Qk
              ----------------------------------[A |-]
                   Ax[-Px], Ax[Px v Qx] |- Qk
                   --------------------------------[|- A]
                   Ax[-Px], Ax[Px v Qx] |- Az[Qz]
```

There is basically mark for each correct rule application. Some credit may be given for almost correct rule applications.

[20 marks total]

Question 2

a) Consider the following scenario:

Alice finds herself in strange hallway. On a glass table is a key and a bottle marked "DRINK ME". Behind a curtain is a tiny locked door. She drinks the contents of the bottle and shrinks, so that she is small enough to go through the door. But she has left the key on the table and cannot reach it. On the verge of tears, she then finds a small cake labelled "EAT ME" and also a white fan.

Observing this predicament, a logician sets out to help Alice by representing the perplexing situation in *Situation Calculus*. The logician produces the following partial specification:

Effect axioms:

$\forall s[\mathsf{Poss}(\mathsf{eat\_cake}, s) \rightarrow \mathsf{Holds}(\mathsf{alice\_tall}, \mathsf{result}(\mathsf{eat\_cake}, s))]$

$\forall s[\mathsf{Poss}(\mathsf{use\_fan}, s) \rightarrow \mathsf{Holds}(\mathsf{alice\_small}, \mathsf{result}(\mathsf{use\_fan}, s))]$

$\forall s[\mathsf{Poss}(\mathsf{unlock}, s) \rightarrow \neg\mathsf{Holds}(\mathsf{door\_locked}, \mathsf{result}(\mathsf{unlock}, s))]$

$\forall s[\mathsf{Poss}(\mathsf{through\_small\_door}, s) \rightarrow \mathsf{Holds}(\mathsf{in\_Wonderland}, \mathsf{result}(\mathsf{through\_small\_door}, s))]$

$\forall s \forall x[\mathsf{Poss}(\mathsf{pick\_up}(x), s) \rightarrow \mathsf{Holds}(\mathsf{have}(x), \mathsf{result}(\mathsf{pick\_up}(x), s))]$

Initial state: $\mathsf{Holds}(\mathsf{alice\_small}, \mathsf{s0}) \wedge \mathsf{Holds}(\mathsf{door\_locked}, \mathsf{s0})$

To complete this representation, you must supply the following further information:

i) Give a *domain* axiom stating that there can be no situation in which Alice is both tall and small. [1 mark]

Answer:

```
all s ( -(holds( tall,s ) & holds( small, s) ) ).
```

ii) Give suitable *precondition* axioms specifying conditions under which each of the five actions can be performed. [5 marks]

Answer:

```
all s (holds( have(cake), s) -> poss(eat_cake,s)).
all s (holds( have(fan), s) -> poss(use_fan,s)).
all s (holds( have(key), s) -> poss(unlock,s)).
all s ((holds( alice_small_, s) & -holds(door_locked, s))-> poss(through_small_door,s)).
all s ( holds(alice_tall,s) -> poss(pickup(key),s))
all s ( holds(alice_small,s) -> poss(pickup(cake),s))
all s ( holds(alice_small,s) -> poss(pickup(fan),s))
```

iii) Give appropriate *frame* axioms for the $\mathsf{have}(x)$ fluent. [4 marks]

Answer:

```
all s all x ( holds(have(x),s) <-> holds( have(x), result(eat_cake,s)) ).
all s all x ( holds(have(x),s) <-> holds( have(x), result(use_fan,s))).
all s all x ( holds(have(x),s) <-> holds( have(x), result(unlock,s))).
all s all x ( holds(have(x),s) <-> holds( have(x), result(through_small_door,s))).
all s all x all y ( holds(have(x),s) -> holds( have(x), result(take(y),s)) ).
```

iv) Give a sequence of actions that will result in the fluent in_Wonderland holding in the final state. [2 marks]

Answer:

`pick_up(fan)`, `eat_cake`, `pick_up(key)`, `use(fan)`, `unlock`, `through_small_door`

b) A *Default Logic* theory contains the classical facts:

C1. Owns_computer(Bill)

C2. Computing_Student(Linus)

C3. $\neg\mathsf{Likes}(\mathsf{Linus}, \mathsf{Windows})$
C4. $\neg\exists x[(\mathsf{Runs}(x, \mathsf{Windows}) \vee \mathsf{Runs}(x, \mathsf{Linux})) \wedge \mathsf{Runs}(x, \mathsf{MacOS})]$

and the following default rules:

D1. $\mathsf{Computing\_student}(x)\ :\ \mathsf{Owns\_computer}(x)\ /\ \mathsf{Owns\_computer}(x)$
D2. $\mathsf{Computing\_student}(x)\ :\ \mathsf{Runs}(x, \mathsf{Linux})\ /\ \mathsf{Runs}(x, \mathsf{Linux})$
D3. $\mathsf{Owns\_computer}(x)\ :\ \neg\mathsf{Computing\_student}(x),\ \mathsf{Runs}(x, \mathsf{Windows})\ /\ \mathsf{Runs}(x, \mathsf{Windows})$
D4. $\mathsf{Owns\_computer}(x) \wedge \neg\mathsf{Likes}(x, \mathsf{Windows}) : \mathsf{Runs}(x, \mathsf{MacOS})\ /\ \mathsf{Runs}(x, \mathsf{MacOS})$

i) Explain the meaning of the default rule **D3**. [2 marks]

Answer:
If someone owns a computer then, unless they are a computing student, one can assume they run windows as long as it is consistent to do so.

ii) Determine how many extensions this theory has. For each extension state which instances of the $\mathsf{Runs}(x, y)$ relation are true in that extension and explain how each of these instances can be derived from the axioms and default rules. [6 marks]

Answer:
The theory has two extensions.

In one extension we have: Runs(Bill,Windows), which is derived directly from **C1** and **D3**; and Runs(Linus,Linux), which is derived from **C1** and **D2**.

In the second extension we have : Runs(Bill,Windows), which is derived (as in the other extension) from **C1** and **D3**; and Runs(Linus,MacOS). The latter derived as follows: first use **C2** and **D1** to infer Owns_computer(Linux), and then use this and **C3** with **D4** to derive Runs(Linus,MacOS).

[20 marks total]

Question 3

a) Convert the following formulae into *clausal form*:

i) $\neg((\neg P \wedge (Q \vee R)) \rightarrow \neg(\neg S \rightarrow P))$ [2 marks]

Answer:

$\{\{-P\},\ \{Q, R\},\ \{S, P\}\}$

ii) $\forall x \exists y [P(x) \rightarrow (Q(x, y) \wedge R(y, x))]$ [2 marks]

Answer:

$\{\{-P(x),\ Q(x, f(x))\},\ \{\{-P(x),\ R(f(x), x)\}\}$

b) Compute the *most general unifier* (if any) of the following pairs of expressions, showing your working. Also state the unified expression or explain why there is no unifier. You should *not* rename variables before unifying. Variables are shown in italics. [4 marks]

i) f(v, g(u, h(w,s)), s, w, 42))
f(w, g(42,z), 45, h(u, 24),u))

ii) h(f(k(w,w), w), w)
h(f(y,y), v)

Answer:

i) {w⇒ h(33,24), u⇒ 42, z⇒ h(h(33,24),45), s⇒ 45, v⇒ h(42,24), s⇒ 24}
unified expression is
f(h(42,33),g(42,h(h(42,33),45),45,h(42,33),42)
(3 marks for this)

ii) The expressions do not unify since w occurs in k(w,w) – 1 mark)

c) Compute all the factors of the following clause:

$$\{\ \mathsf{T}(44, y),\ \mathsf{T}(33, x),\ \mathsf{T}(x, y),\ \neg\mathsf{T}(y, x),\ \neg\mathsf{T}(a, x)\ \}$$ [3 marks]

Answer:

One mark for each correct factor, one mark off (min zero) for each incorrect one.

[T(33,44),T(44,y), ¬ T(y,44) ¬ T(a,44)]

[T(44,33), T(33,33),¬ T(33,33) ¬ T(a,33)]

[T(44,a), T(33,x),T(x,a), ¬ T(a,x)]

d) Describe the relative strengths and weaknesses of following inference systems: (a) the Gentzen Sequent Calculus for First Order Predicate Calculus (FOPC), (b) Resolution for FOPC, (c) Composition in the relational calculus. You should consider the expressiveness of the representation language, the naturalness of the proofs, and the efficiency of inference (giving reasons why inference is more or less efficient). [4 marks]

Answer:

A mark will be given for each sensible point made. Some examples are:

Gentzen system allows full FOPC, more natural; better for doing hand proofs.

Resolution requires special normal form, but this reduces search space and increases efficiency. Unification in resolution delays choice of universally quantified variable instantiation, compared to Gentzen.

Composition on relation calculi is a more restricted language than FOPC or clausal form. More efficient (decidable) but less expressive, but adequate for certain tasks (eg some spatial/temporal calculi).

e) A *Prolog* database contains facts of the forms: `lives(X, Y)`, meaning that `X` is a person who lives in town or city `Y`; and `in_country(X, Y)`, meaning that the town or city `X` is located in the country `Y`.

Define the following Prolog predicates:

i) a predicate `compatriot(X, Y)`, which is true if persons `X` and `Y` live in the same country. [2 marks]

ii) a predicate `city_inhabitants(C,L)`, such that if `C` is the name of a town or city, then `L` is the set of all people living in that city. [2 marks]

iii) a predicate `uninhabiited(C)`, which is true if `C` is the name of a town or city in which no one lives (according to the facts in the Prolog database). [1 marks]

Answer:

```
compatriot( X, Y ) :-
           lives( X, T1 ),
           lives( Y, T2 ),
           in_country( T1, C ),
           in_country( T2, C ).

 city_inhabitants( C, L ) :-
          setof( X, lives(X,C), L ).

  uninhabited( C ) :- city_inhabitants( C, [] ).
```

[20 marks total]

Question 4

a) Give a possible informal rendering in English of the following description logic expression: [1 mark]

$$\mathsf{Male} \sqcap \exists\mathsf{hasChild.Person} \sqcap \forall\mathsf{hasChild.}(\neg\mathsf{Male})$$

Answer:
The concept of a father without sons (or who only has daughers).

b) Translate the following sentences into *Description Logic*. You should use concept and relation names that make clear your intended interpretation these symbols:

i) Students are either undergraduate-students or postgraduate-students. [1 mark]

ii) Postgraduates have a bachelors degree. [1 mark]

iii) Brilliant students have all grades of A. [2 marks]

Answer:
Student $\equiv$ UGStudent $\sqcup$ PGStudent (perhaps also UGStudent $\sqcap$ PGStudent $= \bot$)

PGStudent $\sqsubseteq \exists$ hasdegree.BachelorDegree

BrilliantStudent $\sqsubseteq \forall$ hasgrade.A-grade

c) Consider the following project plan for a student,which is formalised in terms of Allen's temporal interval relations and the $\mathsf{Occurs}(\mathit{event}, \mathit{interval})$ relation:

- $\mathsf{Occurs}(\mathit{related\text{-}work\text{-}reading},\ r)$
- $\mathsf{Occurs}(\mathit{dissertation\text{-}writing},\ w)$
- $\mathsf{Occurs}(\mathit{design\text{-}implementation},\ i)$
- $\mathsf{Occurs}(\mathit{evaluation},\ e)$
- $\mathsf{Overlaps}(r, w)$
- $\mathsf{Contains}(w, i)$
- $\mathsf{Overlapped\text{-}by}(e, i)$
- $\mathsf{Before}(r, e)$

i) Draw a directed graph showing the temporal relationships holding between the four intervals r, w, i, e. If no temporal relationship has been specified, then label the arc with $\top$, denoting the disjunction of all the Allen relationships. [2 marks]

Answer:

```
    /--> w
   /    ^ \
  |   o/   \contains
  |   /  T  v
T |  r<-----i
  |  \      ^
  |   \<    /
  |    \   /oi
  |     v /
   \     e
    \----
```

ii) For any arcs labelled with $\top$, use compositional reasoning to determine a more restricted set of relations which may hold on that arc. [5 marks]

Answer:
e→w arc: overlapped-by, during, ends

r → i arc: before, overlaps, meets

d) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the convex hull function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, draw a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\mathsf{PO}(\mathsf{a},\mathsf{b}) \wedge \mathsf{EC}(\mathsf{b},\mathsf{c}) \wedge \mathsf{NTPP}(\mathsf{a},\mathsf{c})$ [2 marks]

ii) $\forall x[\mathsf{PO}(x,\mathsf{a}) \rightarrow \mathsf{PO}(x,\mathsf{b})]$ [2 marks]

iii) $\mathsf{P}(\mathsf{a},\mathsf{conv}(\mathsf{b})) \wedge \neg\mathsf{P}(\mathsf{a},\mathsf{b})$ [2 marks]

iv) $\mathsf{EQ}(\mathsf{c},\mathsf{sum}(\mathsf{a},\mathsf{b})) \wedge \neg\mathsf{SCON}(\mathsf{c})$ [2 marks]

Answer:
Some possibilities are shown below; these are not necessarily unique; others accepted as appropriate.

```
i)    No configuration is possible.


          ---     |----      ---
ii)       |a|     | b  |     |c |
          ---     |----      ----


          ------------
          |  b        |
          |        ----
          |        |
iii)      |        | |----
          |        | | a  |
          |        | |----
          |        |
          |        ----
          |           |
          ------------


           ---     |----
iv)        |a|     | b  |
           ---     |----

           where c=sum(a,b)
```

[20 marks total]