This question paper consists of 4 printed pages, each of which is identified by the Code Number COMP333501

May/June 2005

Examination for the degree of BSc or BA

AI34: KNOWLEDGE REPRESENTATION AND INFERENCE

Time allowed: 2 hours

Answer *three* questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Question 1

a) Represent the following sentences using *propositional* logic:

  i) If the forecast is correct it will not rain today or tomorrow. [2 marks]

  ii) On Mondays and Tuesdays I go cycling unless it is raining. [2 marks]

b) Translate the following sentences into 1st-order predicate logic (using equality where necessary):

  i) Every human has a heart. [2 marks]

  ii) Barber Jones shaves every man who does not shave himself. [2 marks]

  iii) Jane has only two children. [2 marks]

c) Give a *binary resolution* proof of the inconsistency of the following set of propositional clauses:

$$\{\{A, \neg B, E\},\ \{\neg A, \neg D\},\ \{B, \neg D\},\ \{D, \neg C\},\ \{C, D\},\ \{\neg E\}\}$$ [5 marks]

d) Using the *Sequent Calculus* (as specified in the AI34 course notes), prove that the following sequent is valid:

$$\forall x[G(x) \rightarrow H],\ \ \neg H \ \ \vdash \ \ \forall x[\neg G(x)]$$ [5 marks]

[Total 20 marks]

Question 2

a) For each of the following tense logic formulae state whether it is necessarily true according to the usual semantics for tense logic, and briefly explain your answer.

i) $\mathbf{F}\phi \rightarrow \mathbf{PF}\phi$ [2 marks]

ii) $\mathbf{PF}\phi \rightarrow (\mathbf{P}\phi \vee \mathbf{F}\phi)$ [2 marks]

iii) $(\mathbf{PG}(\phi \rightarrow \psi) \wedge \phi) \rightarrow \psi$ [2 marks]

b) The Situation Calculus is used to define a theory describing the actions of a robot that is used to move objects around a factory.

Some of the objects that must be moved are 'heavy'. The robot can pick up any item. However, if it is carrying a heavy item it cannot move to an adjacent room. To move a heavy item, the robot must pick it up and load it onto a trolley. It can then hook itself up to the trolley. While the robot is hooked up, whenever it moves from one location to another, the trolley and any object on the trolley will also move with the robot. The robot can also move while carrying any non-heavy item even if it is also hooked up to the trolley.

Together with the usual Holds and Poss predicates, the theory is stated in terms of the following fluents:

located(*item/robot/trolley, room*)	*carrying*(*item*)	*heavy*(*item*)	*stored-in*(*item, room*)
adjacent(*room1*, *room2*)	*on-trolley*(*item*)	*hooked-up*	

The constants include robot, trolley as well as further constants denoting room and various items such as hammer, anvil, etc.. Items that are carried or on the trolley are also considered as located in the room occupied by the robot or the trolley.

The theory incorporates the following actions:

pickup(*item*)	**load**(*item*)	**hook-up**	**move**($room1, room2$)
drop(*item*)	**un-load**(*item*)	**un-hook**	

i) Using the specified vocabulary, give a suitable *precondition axiom* specifying the conditions under which an action **move**($room1, room2$) is possible. [2 marks]

ii) Give (using the specified vocabulary) one or more *effect* axioms, for the **move**($room1, room2$) action. Your axiom(s) should take into account the different effects which occur depending on whether or not the robot is hooked-up to the trolley. [4 marks]

iii) Give a *frame* axiom that specifies which instances of the *located* fluent will remain constant when a **move**($room1, room2$) action takes place. [3 marks]

iv) Suppose that we incorporate *Default Logic* rules into the situation calculus formalism. Write down a default rule stating that: unless we have reason to believe otherwise, an item will be located at the place where it is stored. [2 marks]

c) A 1st-order temporal language contains predicates $Activate(a, t)$, $Approve(a, t)$, These predicates mean that account a is activated or (respectively) approved at time point t. The language also includes a strict temporal ordering relation $<$ and a constant 'now' denoting the current time. Quantification may be applied to both account number and time variables.

Use this temporal language to represent the sentence:
'Any account that has not already been activated may only be activated once it has been approved.'
[3 marks]

[Total 20 marks]

Question 3

a) A default theory Θ contains the classical facts

C1 $\mathsf{Child}(\mathsf{bart})$
C2 $\mathsf{American}(\mathsf{bart})$
C3 $\mathsf{American}(\mathsf{fred})$
C4 $\neg\exists x[\mathsf{Adult}(x) \wedge \mathsf{Child}(x)]$

and the default rules:

D1 $\mathsf{Child}(x) : \neg\mathsf{OwnsCar}(x) \ / \ \neg\mathsf{OwnsCar}(x)$
D2 $: \mathsf{Adult}(x) \ / \ \mathsf{Adult}(x)$
D3 $\mathsf{American}(x) : \mathsf{OwnsCar}(x) \ / \ \mathsf{OwnsCar}(x)$
D4 $\mathsf{OwnsCar}(x) : \mathsf{PaysTax}(x) \ / \ \mathsf{PaysTax}(x)$

i) Interpret **D1** in English. [2 marks]

ii) Interpret **D2** in English. [1 marks]

iii) Is it possible to derive $\mathsf{PaysTax}(\mathsf{bart})$ by a valid sequence of default inferences? [1 mark]

iv) Is it possible to show that Θ is inconsistent by a sequence of default inferences? [1 mark]

v) Is $\mathsf{OwnsCar}(\mathsf{bart})$ true in some, all or no, *extensions* of Θ? Briefly explain your answer. [2 marks]

vi) Is $\mathsf{Adult}(\mathsf{bart})$ true in some, all or no, *extensions* of Θ? Briefly explain your answer. [2 marks]

vii) What facts about fred are true in all *extensions* of Θ? [3 marks]

viii) Suggest a modification of **D3**, which would prevent it being used to derive certain unwarranted conclusions. [1 mark]

b) Represent the following statements in Description Logic:

i) Every human is either male or female. [1 mark]

ii) A curry is a stew with a spicy ingredient. [2 marks]

iii) Every happy child has a happy friend. [2 marks]

iv) Fanatics never respect each other. [2 marks]

[Total 20 marks]

Question 4

a) Consider the following holiday plan, which is formalised in terms of Allen's temporal interval relations and the $\mathsf{Occurs}(\mathit{event}, \mathit{interval})$ relation:

$\mathsf{Occurs}(\textbf{holiday}, h)$	$\mathsf{Starts}(v, h)$
$\mathsf{Occurs}(\textbf{stay-in-Venice}, v)$	$\mathsf{Ends}(f, h)$
$\mathsf{Occurs}(\textbf{stay-in-Florence}, f)$	$\mathsf{Contains}(h, r)$
$\mathsf{Occurs}(\textbf{read-novel}, r)$	$\mathsf{Meets}(v, f)$

i) What disjunctive Allen relation is the composition ($\mathsf{Starts}; \mathsf{Contains}$) ? [2 marks]

ii) Draw a diagram showing the temporal relationships holding between the period of the whole holiday and the stays in Venice and Florence. [2 marks]

iii) By adding to your diagram, clearly indicate all possible temporal relations that can hold between the **read-novel** event and the three other events in the scenario. [2 marks]

iv) Suppose it turns out that the traveller finishes the novel just before leaving Venice. Write down the Allen relation that must then hold between r and v and the relation that must hold between r and f. [2 marks]

b) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the *convex hull* function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, *draw* a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\mathsf{EC}(a, b) \wedge \mathsf{EC}(b, c) \wedge \mathsf{EC}(a, c)$ [2 marks]

ii) $\forall x[\mathsf{P}(x, a) \rightarrow \mathsf{P}(x, b)]$ [2 marks]

iii) $\neg\mathsf{EQ}(a, \mathsf{conv}(a)))$ [2 marks]

iv) $\exists x \exists y[a = \mathsf{sum}(x, y) \wedge \neg\mathsf{C}(x, y)]$ [2 marks]

c) A theory of the geometry of spatial points in the plane is formulated using the relation $xy = zw$, which is true just in case the distance between points x and y is the same as the distance between points z and w. Use 1st-order logic to define each of the following concepts, in terms of the $xy = zw$ relation:

i) $\mathsf{Equilateral}(x, y, z)$, which is true just in case points x, y and z form an equilateral triangle. [2 marks]

ii) $\mathsf{Square}(x, y, z, w)$, which is true just in case points x, y and z form a square, whose edges are the lines xy, yz, zw and wx. [2 marks]

[Total 20 marks]

End