This question paper consists
of 7 printed pages, each of
which is identified by the
Code Number COMP5450M01

******* VERSION WITH ANSWERS INCLUDED *******

School of Computing

January 2013

KRR: KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

Answer ALL THREE questions.

This is an open book examination. Candidates may take with them into the examination room any written or printed material. Reproduction or simple rephrasing of course notes is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentence using *Propositional Logic*:

I go shopping on Mondays, Wednesdays and Fridays. [2 marks]

Answer:
$(M \vee W \vee F) \rightarrow S$. 1 mark if almost correct.

b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

i) Every sock in the drawer is green or blue. [2 marks]

Answer:
$\forall x[(\mathsf{Sock}(x) \wedge \mathsf{InDrawer}(x)) \rightarrow (\mathsf{Green}(x) \vee \mathsf{Blue}(x))]$

ii) John ate two pies. [2 marks]

Answer:
$\exists x \exists y[\mathsf{Pie}(x) \wedge \mathsf{Pie}(y) \wedge \neg(x = y) \wedge \mathsf{Ate}(\mathsf{john}, x) \wedge \mathsf{Ate}(\mathsf{john}, y)$

iii) Hugo admires everyone who is richer than himself. [2 marks]

Answer:
$\forall x[\mathsf{Richer}(x, \mathsf{hugo}) \rightarrow \mathsf{Admires}(\mathsf{hugo}, x)]$.

iv) No sensible cat chases every mouse it sees. [2 marks]

Answer:
$\neg \exists x[Cat(x) \wedge Sensible(x) \wedge \forall y[Sees(x, y) \rightarrow Chases(x, y)]]$

c) Give the result of applying the 1st-order *binary resolution* inference rule to the following pairs of clauses:

i) $\{\ P(X),\ Q(X)\ \}, \qquad \{\ \neg Q(a),\ S(Y)\ \}$ [1 mark]

Answer:
$\{P(a), S(Y)\}$

ii) $\{\ G(a, X),\ \neg F(X, j(X))\ \}, \qquad \{\ F(g(a), Z),\ K(Z)\ \}$ [2 marks]

Answer:
$\{G(a, g(a)),\ K(j(g(a))\}$

d) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid:

$$A \wedge B, \quad A \rightarrow \forall x[C(x)] \quad \vdash \quad \forall x[C(x) \vee D(x)]$$

[7 marks]

Answer:
The sequent is valid, as shown by the following proof:

```
      Axiom                                             Axiom
----------------------                  ------------------------------------
A, B |-  A, C(k), D(k)                  A, B, Ax[C(x)], C(k)  |-  C(k), D(k)
------------------------ [- |-]         ------------------------------------ [A |-]
A, B, -A  |-  C(k), D(k)          and   A, B, Ax[C(x)]  |-  C(k), D(k)
------------------------------------------------------------------------ [v |-]
           A, B, -A v Ax[C(x)]  |-  C(k), D(k)
           ------------------------------------ [-> rewrite]
           A, B, A -> Ax[C(x)]  |-  C(k), D(k)]
           ------------------------------------- [|- v]
           A, B, A -> Ax[C(x)]  |-  C(k) v D(k)]
           ---------------------------------------- [|- A]
           A, B, A -> Ax[C(x)]  |-  Ax[C(x) v D(x)]
           ----------------------------------------- [& |-]
           A & B, A -> Ax[C(x)]  |-  Ax[C(x) v D(x)]
```

1 mark for each correct rule application. For full marks complete proof is required with Axioms at the top. Some credit may be given for almost correct rule applications.

[20 marks total]

Question 2

a) Give a representation of the following statements in *Propositional Tense Logic*:

- If you drank the potion you will die. [2 marks]
  Answer:
  $\mathbf{P}\phi \rightarrow \mathbf{F}\psi$
- If you drink the potion you will later die. [2 marks]
  Answer:
  $\mathbf{G}(\phi \rightarrow \mathbf{F}\psi)$

b) A robot is used to carry items in a warehouse. All items in the experiment are either large or small (so any item that is not large is small). The ability of the robot to pick up and carry items is restricted by the following conditions:

- The space of the warehouse is divided into various locations, which the robot can move between. The robot can only pick up an item if the item is at the same location as the robot.
- The robot can carry either one large item or up to two small items — *not* one large and one small item. (Remember that any non-large item is small.)

The robot is programmed to plan possible behaviours by automated reasoning using an encoding into *Situation Calculus* of the pre-conditions and effects of possible actions that can be performed. As well as the standard *Holds* and *Poss* predicates and the *result* function of Situation Calculus, the encoding includes the following domain-specific vocabulary:

- Action: $\mathbf{pickup}(x)$ — the robot picks up item x.
- Fluent: $\mathsf{Location}(l)$ — the robot is at location l.
- Fluent: $\mathsf{Located}(x, l)$ — item x is at location l.
- Fluent: $\mathsf{Carrying}(x)$ — the robot is carrying item x.
- Fixed property: $\mathsf{Large}(x)$ — item x is large.

Using the Situation Calculus representation, specify the following axioms:

i) An effect axiom for the **pickup** action. [2 marks]

ii) A frame action for the Location fluent in relation to the **pickup** action. [2 marks]

iii) A precondition axiom for the **pickup** action. [4 marks]

Answer:

- Effect axiom:

$$\mathit{Holds}(\mathsf{Carrying}(x), \mathit{result}(\mathbf{pickup}(x), s)) \leftarrow \mathit{Poss}(\mathbf{pickup}(x), s)$$

- Frame axiom:

$$\mathit{Holds}(\mathsf{Location}(l), \mathit{result}(\mathbf{pickup}(x), s)) \leftarrow \mathit{Holds}(\mathsf{Location}(l), s)$$

- Precondition axiom:

$$\begin{aligned} poss(\mathbf{pickup}(r,x),s) \leftarrow \ (\ & \exists l[\mathit{Holds}(\mathsf{Located}(r,l),s) \wedge \mathit{Holds}(\mathsf{Located}(x,l),s)] \wedge \\ & \neg\exists y[\mathsf{Large}(y) \wedge \mathit{Holds}(\mathsf{Carrying}(r,y),s)] \wedge \\ & \neg(\mathsf{Large}(x) \wedge \exists y[\mathit{Holds}(\mathsf{Carrying}(r,y),s)]) \wedge \\ & \neg\exists z_1 \exists z_2[\neg(x=y) \wedge \mathit{Holds}(\mathsf{Carrying}(r,z_1),s) \wedge \mathit{Holds}(\mathsf{Carrying}(r,z_1),s)] \ ) \end{aligned}$$

c) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the convex hull function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, draw a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\mathsf{DC}(a,b) \wedge \mathsf{NTPP}(a, \mathsf{conv}(b))$ [2 marks]

ii) $\neg\mathsf{P}(a,b) \wedge \neg\mathsf{P}(b,a) \wedge \exists x[\mathsf{P}(x,a) \wedge \mathsf{P}(x,b)]$ [2 marks]

iii) $\neg\mathsf{EQ}(a, \mathsf{conv}(a)) \wedge \mathsf{EQ}(b, \mathsf{conv}(b)) \wedge \mathsf{EQ}(c, \mathsf{conv}(c)) \wedge$
$\mathsf{DC}(a,b) \wedge \mathsf{DC}(a,c) \wedge \mathsf{EC}(b,c) \wedge \mathsf{PO}(a, \mathsf{conv}(\mathsf{sum}(b,c)))$ [4 marks]

Answer:
Some possibilities are shown below; these are not necessarily unique; others accepted as appropriate.

```
 i)   -------------------         lose a mark for each condition
      |                 |         not satisfied
      |     -------------
      |     |
      | b   |   -----
      |     |   | a |
      |     |   -----
      |     |
      |     ---------------
      |                   |
      ---------------------


ii)

      ---------\  /--------  Just need to show a and b overlapping
      |         \/        |
      |         /\        |
      |   a    /  \   b   |
      |        \  /       |
      |         \/        |
      |         /\        |
      ---------/  \--------

iii)
            |--\    /--|
      |\    |   \--/   |
      | \   | a       /  /--------
      |  \   \-------/  /        |
      |   \            /         |    lose a mark for each condition
      |    \          /          |    not satisfied
      |     \        /           |
      |  b   \      /            |
      |       \    /      c      |
      |        \  /              |
      |         \/               |
      |          \               |
      |-----------\---------------
```

[20 marks total]

Question 3

a) Suppose that we incorporate *Default Logic* rules into the *Situation Calculus* formalism. Write down a default rule stating that: unless we have reason to believe an agent has an item we can assume he/she does not have it. [2 marks]

Answer:

$: \neg Holds(\mathsf{has}(a, x), s) \ / \ \neg Holds(\mathsf{has}(a, x), s)$

b) A *Description Logic* theory of family relationships includes the primitive concepts Male and Female, and the relations $\mathsf{hasSibling}$ and $\mathsf{hasChild}$.

Give a description logic formula which defines each of the following concepts in terms of the primitives:

i) Sister [2 marks]

ii) Only_Child [2 marks]

iii) Uncle [2 marks]

Answer:

i) $\mathsf{Sister} \equiv \mathsf{Female} \sqcap \exists \mathsf{hasSibling}.\top$

ii) $\mathsf{Only\_Child} \equiv \neg\exists \mathsf{hasSibling}.\top$

iii) $\mathsf{Uncle} \equiv \mathsf{Male} \sqcap \exists \mathsf{hasSibling}.\exists \mathsf{hasChild}.\top$

c) The following questions concern compositional reasoning in terms of the relational partition given by the RCC-8 set of topological relations:

$$\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{NTPP}, \mathsf{TPPi}, \mathsf{NTPPi}, \mathsf{EQ}\} \ .$$

i) Draw simple diagrams illustrating the possible RCC-8 relations that could hold between a and c if we know that $\mathsf{TPP}(a, b)$ and $\mathsf{TPP}(b, c)$. [3 marks]

ii) Give the disjunctive relation, expressed as a subset of the RCC-8 relations, that is equivalent to the composition $(\mathsf{TPP}; \mathsf{TPP})$. [2 marks]

iii) If, as well as $\mathsf{TPP}(a, b)$ and $\mathsf{TPP}(b, c)$, we also know that $\mathsf{EC}(a, d)$ and $\mathsf{EC}(d, c)$ hold, what can we now infer about the relation between a and c? [1 mark]

iv) What disjunction of RCC-8 relations is equivalent to the composition $(\mathsf{EC}; \mathsf{EC})$? [2 marks]

Answer:

i)

```
  /----------------------\          /----------------------\
 /                        \        /                        \
 |  -------------\         |       |  -------------\         |
 | /              \        |       | /              \        |
 |/                |       |       |/                |       |
 |         /-----\ |       |       | /----\          |       |
 |         |  a  | |       |       |/  a  |    b     |   c   |
 |     b   |     | |       |       |      |          |       |
 |         \-----/ |   c   |       |\-----/          |       |
 |                 |       |       |                 |       |
 |\---------------/       /        |\---------------/       /
 |                       /         |                       /
 \                      /          \                      /
  \--------------------/            \--------------------/
```

ii) $\{\mathsf{TPP}, \mathsf{NTPP}\}$

iii) We then know that TPP must hold.

iv) $\{\ \mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{TPPi}, \mathsf{EQ}\ \}$

d) This question concerns a *Fuzzy Logic* based on the usual first-order predicate logic syntax, but with additional *linguistic modifiers* used as propositional operators with the following semantics:

$$\mathsf{quite}(\phi) = \phi^{\frac{1}{2}} \qquad \mathsf{very}(\phi) = \phi^2$$

The logic is used to describe Tom, who possesses certain characteristics to the following degrees:

$$\mathsf{Tall}(\mathsf{tom}) = 0.6 \qquad \mathsf{Bald}(\mathsf{tom}) = 0.25$$

Translate each of the following sentences into the fuzzy logic representation and also give the fuzzy truth value of each proposition (using the standard fuzzy interpretation of the Boolean connectives):

A) Tom is not very tall. [2 marks]

B) Tom is quite bald and very tall. [2 marks]

Answer:

A) $\neg\mathsf{very}(\mathsf{Tall}(\mathsf{tom}))$ (1 mark)
Truth value $= 1 - (0.6)^2 = 0.64$ (1 mark)

B) $\mathsf{quite}(\mathsf{Bald}(\mathsf{tom})) \wedge \mathsf{very}(\mathsf{Tall}(\mathsf{tom}))$ (1 mark)
Truth value $= Min\{0.25^{\frac{1}{2}}, (0.6)^2\} = Min\{0.5, 0.36\} = 0.36$ (1 mark)

[20 marks total]