This question paper consists
of 9 printed pages, each of
which is identified by the
Code Number COMP5450M

****** VERSION WITH MARK SCHEME INCLUDED ******

School of Computing

January 2008

KRR: KNOWLEDGE REPRESENTATION AND REASONING

Time allowed: 2 hours

Answer THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentences using *propositional* logic:

i) On Saturdays and Sundays I go to the park if it is fine. [1 mark]

Answer:
$((Sat \vee Sun) \wedge Fine) \rightarrow Park$

ii) Janet cannot take both logic and astrophysics unless she does not take maths. [1 mark]

Answer:
$(JL \wedge JP) \rightarrow \neg JM$

b) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with two binary relations R and S. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$; and the denotations of the relation R is as follows:

$$\delta(R) \ = \ \{\langle a, b\rangle, \langle b, c\rangle, \langle c, d\rangle, \langle d, a\rangle\}$$

i) Which of the following formulae are satisfied by this model: [2 marks]

F1. $\exists x \forall y[R(x, y)]$
F2. $\forall x \exists y[R(x, y)]$
F3. $\forall xyz[(R(x, y) \wedge R(y, z)) \rightarrow R(x, z)]$
F4. $\forall xyz[(R(x, y) \wedge R(y, z)) \rightarrow \exists w[R(z, w) \wedge R(w, x)]]$

Answer:
F2 and F4 are satisfied. (2 marks if exactly these given. 1 mark if one missed out or one extra given. 0, if two or more wrong, since this could be guesswork.)

ii) Given that the model satisfies the formula

$$S(x, y) \leftrightarrow (R(x, y) \vee R(y, x) \vee (x = y)) \ ,$$

Specify the denotation of the S relation, $\delta(S)$. [2 marks]

Answer:
This is just the reflexive and symmetric closure of R — i.e.
$\delta(S) = \ \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle, \langle d, d\rangle, \langle a, b\rangle, \langle b, c\rangle, \langle c, d\rangle, \langle d, a\rangle, \langle b, a\rangle, \langle c, b\rangle, \langle d, c\rangle, \langle a, d\rangle\}$

c) Translate the following sentences into first-order predicate logic (using equality where necessary):

i) No sensible cat chases every mouse it sees. [2 marks]

Answer:
$\neg\exists x[Cat(x) \wedge Sensible(x) \wedge \forall y[Sees(x, y) \rightarrow Chases(x, y)]]$

ii) Every beardless man shaves himself unless he is rich. [3 marks]

Answer:
$\forall x[(\mathsf{Man}(x) \wedge \neg\mathsf{Bearded}(x)) \rightarrow (\mathsf{Shaves}(x, x) \vee \mathsf{Rich}(x))]$

iii) No student owns more than one car. [3 marks]

Answer:
$\forall xyz[(\mathsf{Student}(x) \wedge \mathsf{Car}(y) \wedge \mathsf{Car}(z) \wedge \mathsf{Owns}(x, y) \wedge \mathsf{Owns}(x, z)) \rightarrow (y = z)]$

d) Using the *Sequent Calculus* (as specified in the KRR course notes), determine whether the following sequent is valid:

$$\forall x[P(x) \rightarrow \forall y[R(x, y)]], \quad P(a) \ \vdash \ \forall x[R(a, x)]$$ [6 marks]

Answer:
The sequent is valid as shown by the following proof:

```
       AXIOM                                                   AXIOM
 -----------------------------------                 ------------------------------------------
 A x [Px -> Ay[Rxy], Pa  |-  Pa, Rab                 A x [Px -> Ay[Rxy], Ay[Ray], Rab, Pa |- Rab
-------------------------------------[- |-]          ------------------------------------------[A |-]
 A x [Px -> Ay[Rxy], -Pa, Pa  |-  Rab                A x [Px -> Ay[Rxy], Ay[Ray], Pa |- Rab
---------------------------------------------------------------------------------------[v |-]
                   A x [Px -> Ay[Rxy], (-Pa v Ay[Ray]), Pa  |-   Rab
                  --------------------------------------------------[-> rw]
                  A x [Px -> Ay[Rxy], (Pa -> Ay[Ray]), Pa  |-   Rab
                  -------------------------------------------------[A |-]
                       A x [Px -> A y[Rxy]],  Pa |-   Rab
                      --------------------------------------[|- A]
                       A x [Px -> A y[Rxy]],  Pa |-  Ax[Rax]
```

There is basically mark for each correct rule application. Some credit may be given for almost correct rule applications.

[20 marks total]

Question 2

a) Translate the following sentences into *Propositional Tense Logic*:

- I shall not go out before the baby-sitter arrives. [2 marks]
  Answer:
  $\mathbf{G}(\neg\mathsf{Go\text{-}Out} \vee \mathbf{P}(\mathsf{Bsit\text{-}Arr}))$
- Until the earthquake, Leeds was always beautiful, except when it was raining. [2 marks]
  Answer:
  $\mathbf{H}(\neg(\mathbf{P}\mathsf{Earthquake} \vee \mathsf{Earthquake}) \rightarrow (\mathsf{Leeds\_beautiful} \vee \mathsf{Raining}))$. (Some variants will be allowed.)

b) Consider the following scenario:

A robotic delivery vehicle operates in a town in which a number of plazas (i.e. public open spaces) are connected by straight roads. Each of the plazas has from one to four roads leading from it to one of the other squares. Every road runs either North-South or West-East.

The vehicle starts its operation in a plaza known as 'base', carrying a number of packages, each of which is addressed with the name of one of the other plazas.

The vehicle's goal is to deliver all its packages to the correct plaza (according to its address) and return to base.

In order to achieve this vehicle can perform only three types of action:

- turn 90° clockwise,
- travel forward (in the direction it is facing) along a road to a connected plaza,
- drop a package (if addressed to the plaza where the vehicle is located).

After carrying out any of these actions, the vehicle will always be located at one of the plazas and facing one of the four compass directions (North, South East or West).

This scenario is to be represented in *Situation Calculus*. A background theory of fixed facts describing a very simple town layout has been specified as follows:

$$\mathsf{Connected}(base, plaza1, north) \ \wedge \ \mathsf{Connected}(plaza1, base, south)$$
$$\mathsf{Connected}(base, plaza2, south) \ \wedge \ \mathsf{Connected}(plaza2, base, north)$$
$$\mathsf{Address}(package1, plaza1) \ \wedge \ \mathsf{Address}(package2, plaza2)$$
$$\mathsf{Package}(x) \leftrightarrow ((x = package1) \vee (x = package2))$$

It has also been decided that the theory should be formulated in terms of three *fluents*: $\mathsf{Loc}(l)$, giving the location of the vehicle; $\mathsf{Facing}(d)$, giving the direction the vehicle is facing; and $\mathsf{Delivered}(p)$, which holds when package p has been delivered.

Your task is to give the rest of the *Situation Calculus* theory. Your axioms should be correct for any town fitting the general scenario description, not just the given set of Connected and Address, relations. Your representation should include the following:

i) Specification of what *fluents* hold in the initial situation (s_0), assuming that the vehicle initially faces north. [2 marks]

ii) Specification of suitable *precondition* axioms for each of the actions. [4 marks]

iii) Specification of suitable *effect* axioms for each of the actions. [4 marks]

iv) Specification of *frame* axioms for each of the fluents. [4 marks]

v) Definition of a predicate $\mathsf{Goal}(s)$ which is true just in case situation s satisfies the goal conditions. The definition should apply to any town layout fitting the general scenario description. [2 marks]

Answer:
A full answer is given below. Partial marks can of course be obtained for partially correct or incomplete answers.

```
% i) Initial Conditions
Holds( Located(base), s0)
Holds( Facing(north), s0)
Holds( -Delivered(package1), s0)
Holds( -Delivered(package2), s0)

% ii) Preconditions
Poss(drop(p), s)   <- Holds( (Address(p,loc) & located(loc) & -Delivered(p)), s )
Poss( forward, s ) <- Holds( (Located(loc) & facing(d)), s) & Ex[Connected(loc,x,d)]
% (turn90 is always possible)

% iii) Effects
Holds( Delivered(p), result(drop(p),s))
Holds( (Located(newloc) & Facing(d)), result( forward, s) )
           <- (Holds( (Located(oldloc) & Facing(d)), s) & Connected(oldloc,newloc,d))

Holds( Facing(east), result(turn90,s) ) <- Holds( Facing(north), s)
Holds( Facing(south), result(turn90,s) ) <- Holds( Facing(east), s)
Holds( Facing(west), result(turn90,s) ) <- Holds( Facing(south), s)
Holds( Facing(north), result(turn90,s) ) <- Holds( Facing(west), s)

% iv) Frame axioms
Holds( Delivered( p ), result( forward,s ) <-> Holds( Delivered(p), s)
Holds( Delivered( p ), result( turn90,s ) <-> Holds( Delivered(p), s)

Holds( Located( loc ), result( turn90,s ) <-> Holds( Located(loc), s)
Holds( Located( loc ), result( drop(x),s ) <-> Holds( Located(loc), s)

Holds( Facing( d ), result( forward,s ) <-> Holds( Facing(d), s)
Holds( Facing( d ), result( drop(x),s ) <-> Holds( Facing(d), s)

% v) GOAL axiom
Goal( s ) <-> ( Ax[Package(x) -> Holds(  Delivered(x), s)] )
```

[20 marks total]

Question 3

a) Use *binary resolution* to determine whether the following propositional entailment is valid:

$$(P \rightarrow \neg R),\ ((P \vee Q) \rightarrow (R \vee S)),\ (P \vee \neg T)\ \vdash\ (S \vee \neg T)\ .$$

In order to do this you must:

i) re-formulate the entailment problem as a consistency checking problem; [1 mark]
Answer:
The sequent is valid iff
$\{\ (P \rightarrow \neg R),\ ((P \vee Q) \rightarrow (R \vee S)),\ (P \vee \neg T),\ \neg(S \vee \neg T)\ \}$ is inconsistent.

ii) transform the formulae into clausal (i.e. conjunctive normal) form; [3 marks]
Answer:
The CNF form is $\{\{\neg P, \neg R\}, \{\neg P, R, S\}, \{\neg Q, R, S\}, \{P, \neg T\}, \{\neg S\}, \{T\}\ \}$

iii) carry out a binary resolution proof to show inconsistency. [4 marks]
Answer:
4 if all correct -1 for each incorrect rule application. Proof should look like:

```
 1.  {-p, -r}
 2.  {-p,r,s}
 3.  {-q,r,s}
 4.  {p, -t}
 5.  {-s}
 6.  {t}
 7.  {p}   (4+7)
 8.  {-r}  (1+8)
 9.  {r,s} (2+7)
10.  {s}   (8+9)
11   {}    (5+10)
```

b) Consider the following formulae, which assert that the binary relation R is symmetric, transitive and total:

F1. $\forall xy[R(x, y) \rightarrow R(y, x)]$
F2. $\forall xy[(R(x, y) \wedge R(y, z)) \rightarrow R(x, z)]$
F3. $\forall x\exists y[P(x, y)]$

Prove by binary resolution that these formulae entail that R is also reflexive — i.e. prove F4:

F4 $\forall x[R(x, x)]$.

To solve this you will need to:

- put each of the formulae F1–3, together with the negation of F4, into *clausal normal form.* [4 marks] **Answer:**

  Put into CNF:

  - $\{\neg Rxy, Ryx\}$
  - $\{\neg Ruv, \neg Rvw, Ruw\}$
  - $R(z, f(z)\}$
  - $\neg R(c1, c2)$

- Give a binary resolution proof. (This should only require a sequence of four applications of the binary resolution rule; but a slightly longer proof is also acceptable.) [4 marks]

Answer:

```
1 [] -R(x,y)|R(y,x).
2 [] -R(x,y)| -R(y,z)|R(x,z).
3 [] R(x,$f1(x)).
4 [] -R($c1,$c1).
5 [binary,1.1,3.1] R($f1(x),x).
13 [binary,2.3,4.1] -R($c1,x)| -R(x,$c1).
17 [binary,13.1,3.1] -R($f1($c1),$c1).
18 [binary,17.1,5.1] $F.
```

c) Use *Description Logic* to represent the following axioms:

i) The only frogs that are poisonous are blue or yellow. [2 marks]

Answer:
$(\mathsf{Frog} \sqcap \mathsf{Poisonous}) \sqsubseteq (\mathsf{Blue} \sqcap \mathsf{Yellow})$

ii) Every dog with a kind and reliable owner is happy. [2 marks]

Answer:
$(\mathsf{Dog} \sqcap \exists\,\mathsf{owned\text{-}by}.(\mathsf{Kind} \sqcap \mathsf{Reliable})) \sqsubseteq \mathsf{Happy}$

[20 marks total]

Question 4

a) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the *convex hull* function, conv. The quantifiers range over non-empty spatial regions. For each formula *draw* a configuration of the regions a, b and (where relevant) c, which satisfies the formula:

i) $\neg\mathsf{EQ}(a, \mathsf{conv}(a)) \wedge \mathsf{DC}(a, b) \wedge \mathsf{TPP}(a, \mathsf{conv}(b))$ [2 marks]

ii) $\forall x[\mathsf{C}(x, a) \rightarrow \mathsf{C}(x, b)] \wedge \mathsf{C}(a, \mathsf{compl}(b)) \wedge \exists z[\mathsf{P}(z, b) \wedge \mathsf{P}(z, d)] \wedge \mathsf{DC}(a, d)$ [3 marks]

Answer:
Various diagrams are possible and straightforward examples are fairly easy to find. Marks will be awarded by checking each conjunct of the given formulae and subtracting 1 for each that is not satisfied by the diagram. (With minimum 0 marks for each part.)

b) This question concerns reasoning with the relational partition given by the RCC-8 topological relations:

$$\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{NTPP}, \mathsf{TPPi}, \mathsf{NTPPi}, \mathsf{EQ}\}\ .$$

Regions a, b, c and d are known to be arranged in the following relations:

$$\mathsf{NTPP}(a, b),\ \mathsf{NTPP}(a, c),\ \mathsf{EC}(b, d),\ \mathsf{EC}(c, d)\ .$$

i) Draw a diagram of the relational network corresponding to this situation. [1 marks]
Answer:
This is trivial, just need to link nodes a, b, c, d according to the given relations.

ii) Use compositional reasoning to find the possible topological relations between a and d. [2 marks]
Answer:
This can be done by computing the composition $\mathsf{NTPP};\mathsf{EC}$, which comes out as DC.

iii) Use compositional reasoning to find the possible topological relations between b and c. Indicate the steps used to obtain this result. [4 marks]
Answer:
This requires two steps.
First compose $(NTPPi; NTPP)$ to get $\{PO, EQ, TPP, TPPi, NTPP, NTPPi\}$.
Then compose $(EC; EC)$ to get $\{DC, EC, EQ, PO, EQ, TPP, TPPi\}$.
The answer is then the intersection of these: $\{PO, EQ, TPP, TPPi\}$
If not completely correct, any diagrams drawn to illustrate compositional possibilities will be taken into account to give partial marks.

iv) Draw a diagram of a possible 2-dimensional situation satisfying the given relations. [2 marks]
Answer:
Any diagram satisfying the relations is fine. 1 mark if three of the four relations is satisfied.

c) An AI system for reasoning about robots implements a default logic inference rule

$$\mathsf{Arm}(x) : \mathsf{Usable}(x), \neg\mathsf{Broken}(x)\ /\ \mathsf{Usable}(x)$$

It also has information that one of the arms of Rob the robot is broken.

To reason about this situation, the system computes default inferences from the following default theory:

$$\begin{aligned}\langle \mathcal{T}, \mathcal{D}\rangle\ =\ \langle\ &\{\mathsf{Arm(rla)},\ \mathsf{Arm(rra)},\ (\mathsf{Broken(rla)}) \vee \mathsf{Broken(rra)}\},\\ &\{\mathsf{Arm}(x) : \mathsf{Usable}(x), \neg\mathsf{Broken}(x)\ /\ \mathsf{Usable}(x)\}\ \rangle,\end{aligned}$$

where rla and rra refer respectively to the robot's left and right arms.

i) What are the extensions this theory? If there are no extensions explain why. If there are one or more, state how many and write down the set of literals (i.e. atomic facts or their negations) that are true in each extension. [3 marks]

Answer:
There is just one extension and the facts true in it are:

$$\{\mathsf{Arm}(\mathsf{rla}),\ \mathsf{Arm}(\mathsf{rra}),\ \mathsf{Usable}(\mathsf{rla}),\ \mathsf{Usuable}(\mathsf{rra})\}$$

ii) Comment briefly on whether the application of default logic in this case produces intuitively reasonable conclusions. [2 marks]

Answer:
Application of default rules gives the counter-intuitive result that both $\mathsf{Usable}(\mathsf{rla})$, $\mathsf{Usuable}(\mathsf{rra})$ are true in the only extension, although we know that one of them must be false.

iii) Give a *classical* axiom involving the $\mathsf{Usable}(x)$ and $\mathsf{Broken}(x)$ predicates that would make the theory more complete. [1 mark]

Answer:
$\forall x[\neg(\mathsf{Usable}(x) \wedge \mathsf{Broken}(x))]$ (Equivalent or alternative answers will be allowed.)

[20 marks total]