This question paper consists of 4 printed pages, each of which is identified by the Code Number COMP333501

Special Requirements

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School of Computing

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AI34: KNOWLEDGE REPRESENTATION AND INFERENCE

Time allowed: 2 hours

Answer THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Question 1

a) Represent the following sentences using *propositional* logic:

   i) Tom is neither tall nor short. [1 mark]

   ii) I only go shopping on Mondays, Wednesdays or Fridays. [2 marks]

   iii) Every Sunday I either go cycling or climbing unless it rains. [2 marks]

b) A model $\mathcal{M}$ for a propositional language is represented by the following structure:

$$\{A = \mathbf{f},\ B = \mathbf{t},\ C = \mathbf{f},\ D = \mathbf{f}\}$$

   i) What truth-value is assigned to the formula $(B \wedge D) \rightarrow A$ by $\mathcal{M}$? [1 mark]

   ii) Is the formula $\neg(A \vee \neg D)$ satisfied by $\mathcal{M}$? [1 mark]

c) Give a simpler formula that is logically equivalent to $(\neg P \rightarrow \neg Q)$. [1 mark]

d) Translate the following sentences into 1st-order predicate logic (using equality where necessary):

   i) Someone ate every pie on the table. [2 marks]

   ii) At least two students wrote to each other. [2 marks]

e) A formal theory of family relationships includes the properties Male and Female and a relation $\mathsf{MarriedTo}$. Give a first order formula asserting a significant semantic connection between these concepts. [2 marks]

f) Using the *Sequent Calculus* (as specified in the AI34 course notes), prove that the following sequent is valid:

$$S \rightarrow \forall x[K(x)],\quad S \quad \vdash \quad \forall x[K(x) \vee R(x)]$$ [6 marks]

[20 marks total]

Question 2

a) A 1st-order temporal language contains predicates $\mathit{Activate}(a,t)$ and $\mathit{Access}(a,t)$. These predicates mean that bank account a is respectively activated or accessed at time point t. The language also includes a strict temporal ordering relation $<$ and allows quantification to be applied to both account number and time variables.

Explain in English the meaning of the following formula of this language:

$$\forall a \forall t[\mathit{Access}(a,t) \rightarrow \exists t'[(t' < t) \wedge \mathit{Activate}(a,t')]]$$ [2 marks]

b) Translate the following sentences into *Propositional Tense Logic*:

i) I have only eaten haggis when in Scotland. [2 marks]

ii) I shall visit Paris and after that I shall visit Rome. [2 marks]

c) Let ϕ stand for the proposition 'The photocopier is broken'. Explain in English the meaning of the tense logic formulae:

i) $\mathbf{GF}\phi$ [2 marks]

ii) $\mathbf{P}\phi \wedge \mathbf{H}(\mathbf{P}\phi \rightarrow \phi)$ [2 marks]

d) A Situation Calculus theory makes use of fluents of the forms:

$\mathit{robot\_has}(\mathit{item})$ $\quad$ $\mathit{on\_floor}(\mathit{item}, \mathit{room})$ $\quad$ $\mathit{locked}(\mathit{door})$
$\mathit{robot\_location}(\mathit{room})$ $\quad$ $\mathit{connects}(\mathit{door}, \mathit{room}_1, \mathit{room}_2)$

The theory includes constants referring to items, one of which is key.

The theory also describes the behaviour of a robot in terms of the following actions:

$\mathbf{pick\_up}(\mathit{object})$ $\quad$ $\mathbf{unlock}(\mathit{door})$ $\quad$ $\mathbf{move\_to}(\mathit{room})$

i) Suppose an initial situation, s_0, is described as follows:

$\mathsf{Holds}(\mathit{connects}(\mathsf{door1}, \mathsf{hall}, \mathsf{lounge}), s_0)$ $\quad$ $\mathsf{Holds}(\mathit{connects}(\mathsf{door2}, \mathsf{hall}, \mathsf{study}), s_0)$
$\mathsf{Holds}(\neg \mathit{locked}(\mathsf{door1}), s_0)$ $\quad$ $\mathsf{Holds}(\mathit{locked}(\mathsf{door2}), s_0)$
$\mathsf{Holds}(\mathit{on\_floor}(\mathsf{key}, \mathsf{lounge}), s_0)$ $\quad$ $\mathsf{Holds}(\mathit{robot\_location}(\mathsf{hall}), s_0)$

Give a sequence of actions that will result in the goal $\mathit{robot\_location}(\mathsf{study})$ being satisfied. [2 marks]

ii) Give an *effect* axiom specifying the result of carrying out the **unlock** action. [2 marks]

iii) For each of the actions **pick_up** and **move_to** specify a *precondition* axiom stating the conditions under which the action is possible. [4 marks]

iv) Write down a *frame* axiom stating that the **move_to** action does not affect the *locked* fluent. [2 marks]

[20 marks total]

Question 3

a) Give a *binary resolution* proof of the inconsistency of the following set of propositional clauses:

$$\{\ \{P, Q, R\},\ \{\neg P, Q\},\ \{\neg Q\},\ \{\neg R, S, T\},\ \{Q, \neg T\},\ \{T, \neg S\}\ \}$$ [5 marks]

b) Give a *most general unifier* of the following pair of terms (where X, Y and Z are variables):

$$f(X, g(Y)) \qquad\qquad f(h(Z), Z)$$ [2 marks]

c) Give the result of applying the 1st-order *binary resolution* inference rule to the following pairs of clauses:

i) $\{\ P(X),\ Q(X)\ \}, \quad \{\ \neg Q(a)\ \}$ [1 mark]

ii) $\{\ G(a, X),\ \neg F(X, j(X))\ \}, \quad \{\ F(g(a), Z),\ K(Z)\ \}$ [2 marks]

d) A *default theory* Θ contains the classical facts

C1 $\mathsf{Drinks\_Milk(Daisy)}$
C2 $\neg\exists x[\mathsf{Adult}(x) \wedge \mathsf{Child}(x)]$

and the default rules:

D1 $:\ \mathsf{Adult}(x)\ /\ \mathsf{Adult}(x)$
D2 $\mathsf{Drinks\_Milk}(x)\ :\ \mathsf{Child}(x)\ /\ \mathsf{Child}(x)$

i) Interpret **D2** in English. [2 marks]

ii) Which of the following facts are true in *some* extension of Θ?

1. $\mathsf{Adult(Daisy)}$
2. $\mathsf{Child(Daisy)}$
3. $\mathsf{Adult(Daisy)} \vee \mathsf{Child(Daisy)}$
4. $\mathsf{Adult(Daisy)} \wedge \mathsf{Child(Daisy)}$ [2 marks]

iii) Which of the facts 1–4, given in the last part, are true in *every* extension of Θ? [2 marks]

e) A *default theory* contains the classical formulae:

C1 $\mathsf{A}(i)$
C2 $\forall x[\mathsf{D}(x) \rightarrow \neg\mathsf{B}(x)]$

and the following default rules:

D1 $\mathsf{A}(x) : \mathsf{B}(x)\ /\ \mathsf{C}(x)$
D2 $\mathsf{C}(x) : \mathsf{D}(x)\ /\ \mathsf{D}(x)$

A default reasoning algorithm applies the **D1** rule to the fact $\mathsf{A}(i)$ to derive $\mathsf{C}(i)$. It then applies **D2** to $\mathsf{C}(i)$ to derive $\mathsf{D}(i)$.

Explain briefly whether the conclusion $\mathsf{D}(i)$ is warranted. [2 marks]

f) Write down a *non-normal* default rule corresponding to the commonsense assumption that a tree will not normally be found inside a building, unless you have reason to believe it is a bonsai tree. [2 marks]

[20 marks total]

Question 4

a) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the *convex hull* function, conv. The quantifiers range over non-empty spatial regions. For each formula *draw* a configuration of the regions a, b and (where relevant) c, which satisfies the formula:

i) $\mathsf{EC}(a,b) \wedge \mathsf{TPP}(a, \mathsf{conv}(b))$ [2 marks]

ii) $\neg\mathsf{P}(a,b) \wedge \neg\mathsf{P}(a,c) \wedge \mathsf{NTPP}(a, \mathsf{sum}(b,c))$ [2 marks]

iii) $\forall x[\mathsf{C}(x,a) \rightarrow \mathsf{O}(x,b)]$ [2 marks]

b) The following questions concern compositional reasoning in terms of the relational partition given by the RCC-8 set of topological relations, $\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{NTPP}, \mathsf{TPPi}, \mathsf{NTPPi}, \mathsf{EQ}\}$:

i) Draw simple diagrams illustrating all possible RCC-8 relations that can hold between regions a and c, if we know that $\mathsf{EC}(a,b)$ and $\mathsf{NTPP}(b,c)$. [2 marks]

ii) Give the relation, expressed as a subset of the RCC-8 relations, that is equivalent to each of the following compositions:

1) $(\mathsf{EC};\mathsf{NTPP})$ [2 marks]

2) $(\mathsf{EC};\mathsf{TPPi})$ [2 marks]

iii) Suppose a spatial configuration among regions a, b, c and d is described by the following facts:

$$\mathsf{EC}(a,b), \quad \mathsf{NTPP}(b,c) \quad \mathsf{EC}(a,d), \quad \mathsf{DC}(c,d)\ .$$

Given that $(\mathsf{EC};\mathsf{DC}) = \{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPPi}, \mathsf{NTPPi}\}$, use compositional reasoning to derive the RCC-8 relation that must hold between a and c. [2 marks]

c) A *Description Logic* theory of family relationships includes the primitive concepts Male and Female, and the relations hasSibling and hasChild.

Give a description logic formula which defines each of the following concepts in terms of the primitives:

i) Sister [2 marks]

ii) Only_Child [2 marks]

iii) Uncle [2 marks]

[20 marks total]