This question paper consists of 8 printed pages, each of which is identified by the Code Number COMP5450M.

A non-programmable calculator may be used.
Answer All Questions.
This is an open book examination.
Any written or printed material is permitted.

School of Computing

January 2017

COMP5450M

KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

PLEASE DO NOT REMOVE THIS PAPER FROM THE EXAM ROOM

Answer ALL THREE questions

The marks available for each part of each question are clearly indicated.

 TURN OVER FOR QUESTIONS

Question 1

(a) Translate the following sentence into *Propositional Logic*:

- I play tennis on Mondays and Wednesdays. **[2 marks]**
  Answer: $(M \vee W) \rightarrow T$ or $(M \rightarrow T) \wedge (W \rightarrow T)$

(b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

(i) No country is more beautiful than New Zealand. **[2 marks]**
Answer: $\neg\exists x[\mathsf{Country}(x) \wedge \mathsf{MoreBeatiful}(x, \mathsf{nz})]$

(ii) Every sweet in the bag is red or purple. **[2 marks]**
Answer: $\forall x[(\mathsf{Sweet}(x) \wedge \mathsf{InBag}(x)) \rightarrow (\mathsf{Red}(x) \vee \mathsf{Purple}(x))]$

(iii) No student owns more than one car. **[2 marks]**
Answer: $\neg\exists x\exists y\exists z[\mathsf{Student}(x) \wedge \mathsf{Car}(y) \wedge \mathsf{Car}(z) \wedge \neg(y = z) \wedge \mathsf{Owns}(x, y) \wedge \mathsf{Owns}(x, z)$

(iv) Tom hates everyone except himself. **[2 marks]**
Answer: $\forall x[(\mathsf{Person}(x) \wedge \neg(x = \mathsf{tom})) \leftrightarrow \mathsf{Hates}(\mathsf{tom}(x))]$

(c) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid: **[6 marks]**

$$\forall x[R(a, x) \rightarrow S(x)],\ \forall x[R(a, x)]\ \vdash\ \forall x[S(x)]$$

Answer: The sequent is valid, as shown by the following proof:

```
       Axiom
------------------------------------
Ax[Rax->Sx], Rab, Ax[Rax] |- Rab, Sb                      Axiom
-------------------------------------            -------------------------------
-Rab, Ax[Rax->Sx], Rab, Ax[Rax] |- Sb   and  Sb, Ax[Rax->Sx], Rab, Ax[Rax] |- Sb
--------------------------------------------------------------------------------
             -Rab v Sb,  Ax[Rax -> Sx],  Rab, Ax[Rax]  |-  Sb
             ------------------------------------------------
             Rab -> Sb,  Ax[Rax -> Sx],  Rab, Ax[Rax]  |-  Sb
             ------------------------------------------------
                   Ax[Rax -> Sx],  Rab, Ax[Rax]  |-  Sb
                   -------------------------------------
                        Ax[Rax -> Sx],  Ax[Rax]  |-  Sb
                    ------------------------------------
                    Ax[Rax -> Sx],  Ax[Rax]  |-  A x[Sx]
```

1 mark for each correct rule application. For full marks complete proof is required with Axioms at the top. Some credit may be given for almost correct rule applications.

(d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with an unary predicate P and a binary relation T. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$, and the denotations of P and T are as follows:

- $\delta(P) = \{a, b\}$
- $\delta(T) = \{\langle a, c\rangle, \langle b, c\rangle, \langle b, d\rangle, \langle d, c\rangle, \langle e, e\rangle\}$

Which of the following formulae are satisfied by this model? **[4 marks]**

F1. $\forall x \exists y[T(x, y)]$

F2. $\exists x[P(x) \wedge T(x, x)]$

F3. $\forall x[P(x) \rightarrow \exists y[T(x, y)]]$

F4. $\forall x \forall y \forall z[(T(x, y) \wedge T(y, z)) \rightarrow T(x, z)]$

Answer: F3, and F4 are satisfied. 1 mark for each correct.

[Question 1 total: 20 marks]

Question 2

(a) Give a representation of the following statements in *Propositional Tense Logic*:

- Mary has bought a ticket and will go to the festival. **[2 marks]**
  Answer: $\mathbf{P}(\mathsf{MaryBuyTicket}) \wedge \mathbf{F}(\mathsf{MaryGoFestival})$
- I shall visit Manchester and after that I shall visit Liverpool. **[2 marks]**
  Answer: $\mathbf{F}(\mathsf{IVM} \wedge \mathbf{F}\mathsf{IVL})$

(b) The *Situation Calculus* is to be used to describe the actions of a robot vacuum cleaner and the changes that result from these actions. As well as the special predicate *holds* and the *result* function, the vocabulary used includes fluents *clean*(r), and *dirty*(r) that specify whether room r is clean or dirty and an action $\mathbf{move}(r_1, r_2)$, which takes place when the vacuum cleaner moves from room r_1 to room r_2.

Using this vocabulary, specify *frame axioms* that ensure that the $\mathbf{move}(r_1, r_2)$ action does not alter the state of the rooms r_1 and r_2, with respect to them being clean or dirty: both rooms will remain in the state they were in before the **move** action. **[4 marks]**

Answer: $holds(clean(r_1), s) \rightarrow holds(clean(r_1),\ result(\mathbf{move}(r_1, r_2), s))$

$holds(clean(r_2), s) \rightarrow holds(clean(r_1),\ result(\mathbf{move}(r_1, r_2), s))$

$holds(dirty(r_1), s) \rightarrow holds(dirty(r_1),\ result(\mathbf{move}(r_1, r_2), s))$

$holds(dirty(r_2), s) \rightarrow holds(dirty(r_1),\ result(\mathbf{move}(r_1, r_2), s))$

1 mark for formulae that are not correctly formed but are along the right lines. 2 marks for one or more correctly formed frame axioms but not capturing requirements properly. 3 marks if most requirements captured. 4 marks for capturing all the specified requirements.

(c) For each of the following *Prolog* queries, give the value of the variable `X` after the query has been executed:

(i) `?- X = 2+3.` **[1 mark]**

(ii) `?- A + B = 2 + 3, X is A * B.` **[1 mark]**

(iii) `?- L = [[a,b],[c,d], L = [L1,L2], L2 = [X,_].` **[1 mark]**

(iv) `?- L = [up,down], setof([A,B], (member(A,L), member(B,L)), X).` **[2 marks]**

Answer:

```
i) 2+3
ii) 6
iii) c
iV) [[up,up],[up,down],[down,up],[down,down]]
```

(d) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function, conv. The constants (a, b and c) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

(i) $\neg\mathsf{EQ}(\mathsf{a}, \mathsf{conv}(\mathsf{a})))$ **[1 mark]**

(ii) $\forall x[\mathsf{P}(x, \mathsf{a}) \rightarrow \mathsf{NTPP}(x, \mathsf{b})]$ **[1 mark]**

(iii) $\mathsf{DC}(\mathsf{a}, \mathsf{b}) \wedge \mathsf{TPP}(\mathsf{a}, \mathsf{conv}(\mathsf{b}))$ **[2 marks]**

(iv) $\mathsf{DC}(\mathsf{a}, \mathsf{b}) \wedge \mathsf{DC}(\mathsf{b}, \mathsf{c}) \wedge \mathsf{DC}(\mathsf{a}, \mathsf{b}) \wedge \mathsf{NTPP}(\mathsf{a}, \mathsf{conv}(\mathsf{sum}(\mathsf{b}, \mathsf{c})))$ **[3 marks]**

Answer: Possible diagrams are as follows:

```
(i)     -----------
        |  a      |
        |     -----
        |     |
        |     -----
        |         |
        -----------


(ii)    ----------------
        |              |
        |   -------    |
        |   | a  |     |
        |   -------    |
        |         b    |
        ----------------


        -------------------
(iii)   |                 |
        |     -------------
        |     |
        | b   |       -----
        |     |       | a |
        |     |       -----
        |     |
        |     -------------
        |                 |
        -------------------

(iV)  ----------              ---------
       |        |    ------    |       |
       |   b    |    | a  |    |   c   |
       |        |    ------    |       |
       ----------               ---------
```


[Question 2 total: 20 marks]

Question 3

(a) The RCC-8 relations, DC, EC, PO, TPP, NTPP, TPPi, NTPPi and EQ form a mutually exhaustive and pairwise disjoint set of relations that can hold between spatial regions. Suppose that we have a situation involving regions a, b and c, such that the relations $\mathsf{PO}(\mathsf{a},\mathsf{b})$ and $\mathsf{NTPP}(\mathsf{b},\mathsf{c})$ hold.

- Draw diagrams illustrating each of the RCC-8 relations that could hold between regions a, and c. **[3 marks]**

  Answer: There are three possibilities. The student should draw three simple diagrams corresponding to each of the cases where: $\mathsf{PO}(a,c)$, $\mathsf{TPP}(a,c)$ and $\mathsf{NTPP}(a,c)$.

- State which subset of the RCC-8 is the composition $\mathsf{PO;NTPP}$. **[1 mark]**

  Answer: $\{\mathsf{PO,TPP,NTPP}\}$

(b) Represent the following sentences in *Description Logic*:

(i) Every human is either male or female. **[1 mark]**

Answer: $\mathsf{Human} \sqsubseteq (\mathsf{Male} \sqcup \mathsf{Female})$

(ii) Nothing is both male and female. **[1 mark]**

Answer: $(\mathsf{Male} \sqcap \mathsf{Female}) \equiv \bot$

(iii) A curry is a stew with a spicy ingredient. **[2 marks]**

Answer: $\mathsf{Curry} \sqsubseteq$ (or $\equiv$) $\mathsf{Stew} \sqcap \exists \mathsf{hasIngredient.Spicy}$

(iv) Only humans can have human children. **[2 marks]**

Answer: $\exists \mathsf{hasChild.Human} \sqsubseteq \mathsf{Human}$

(c) A *default theory* Θ contains the following formulae:

	Classical Facts		Default Rules
C1	$\mathsf{Person(Lilly} \wedge \mathsf{Drinks\_Milk(Lilly)}$	**D1**	$\mathsf{Person}(x)\ :\ \mathsf{Adult}(x)\ /\ \mathsf{Adult}(x)$
C2	$\neg\exists x[\mathsf{Adult}(x) \wedge \mathsf{Child}(x)]$	**D2**	$\mathsf{Drinks\_Milk}(x)\ :\ \mathsf{Child}(x)\ /\ \mathsf{Child}(x)$

(i) Interpret **D1** in English. **[2 marks]**

Answer: In the absence of any contradictory information, any person can be assumed to be an adult.

(ii) For each of the following formulae, state whether it is true in *none*, *some but not all*, or *all* extensions of Θ? **[4 marks]**

A. $\mathsf{Adult(Lilly)}$ C. $\mathsf{Adult(Lilly)} \vee \mathsf{Child(Lilly)}$

B. $\mathsf{Child(Lilly)}$ D. $\mathsf{Adult(Lilly)} \wedge \mathsf{Child(Lilly)}$

Answer: A. Some; B. Some; C. All; D. None

(d) This question concerns a *Fuzzy Logic* based on the usual first-order predicate logic syntax, but with additional *linguistic modifiers* used as propositional operators with the following semantics: $\mathsf{quite}(\phi) = \phi^{\frac{1}{2}}$ $\mathsf{very}(\phi) = \phi^2$.

The logic is used to describe Sam, who possesses certain characteristics to the following degrees: $\mathsf{Sick}(\mathsf{sam}) = 0.3$, $\mathsf{Sad}(\mathsf{sam}) = 0.8$.

Translate each of the following sentences into the fuzzy logic representation and also give the fuzzy truth value of each proposition (using the standard fuzzy interpretation of the Boolean connectives):

(A) Sam is neither sick nor sad. **[2 marks]**

(B) Sam is very sick and quite sad. **[2 marks]**

Answer:

A) $\neg\mathsf{Sick}(\mathsf{sam})) \wedge \neg\mathsf{Sad}(\mathsf{sam})$ (1 mark)
Truth value $= Min(1 - 0.7, 1 - 0.8) = 0.2$ (1 mark)

B) $\mathsf{very}(\mathsf{Sick}(\mathsf{sam})) \wedge \mathsf{quite}(\mathsf{Sad}(\mathsf{sam}))$ (1 mark)
Truth value $= Min\{0.3^2, (0.8)^{\frac{1}{2}}\} = Min\{0.09, 0.894\} = 0.09$ (1 mark)

[Question 3 total: 20 marks]

[grand total: 60 marks]