

**Module Title:** Knowledge Representation  
and Reasoning

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**School of Computing**

**Semester 1 2019/20**

**Calculator instructions:**

- You **are** allowed to use a non-programmable calculator only from the following list of approved models in this examination: **Casio FX-82, Casio FX-83, Casio FX-85.**

**Dictionary instructions:**

- A basic English dictionary is available to use: raise your hand and ask an invigilator, if you need it.

**Examination Information**

- There are **8** pages to this examination.
- There are **2 hours** to complete the examination.
- Answer **all 3** questions.
- The number in brackets [ ] indicates the marks available for each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this examination paper is **60**.
- You are allowed to use annotated materials.

**Please do not remove this paper from the exam venue.**

**\*\* With Solutions \*\***

## Question 1

- (a) Translate the following sentence into *Propositional Logic*: [2 marks]

I go to the park on Sundays unless it is raining.

**Answer:**  $Sunday \rightarrow (Park \leftrightarrow \neg Raining)$ .

Also allow:  $(Sunday \wedge \neg Raining) \rightarrow Park$  or something equivalent to that.

- (b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

- (i) John found a green beetle. [2 marks]

**Answer:**  $\exists x[Found(john, x) \wedge Beetle(x) \wedge Green(x)]$

- (ii) Rabbits don't like chocolate, except for some old rabbits. [2 marks]

**Answer:**  $\forall x[(Rabbit(x) \wedge Likes(x, choc) \rightarrow Old(x)]$

- (iii) All friends of Mary know each other. [2 marks]

**Answer:**  $\forall x \forall y[(Friend(x, mary) \wedge Friend(y, mary)) \rightarrow Knows(x, y)]$

- (iv) One of my uncles does not like any of his own children. [2 marks]

**Answer:**  $\exists x[UncleOf(x, me) \wedge \forall y[ChildOf(y, x) \rightarrow \neg Likes(x, y)]]$

- (c) Give an English sentence that captures the meaning of the following formula of first-order logic in a concise and natural way: [2 marks]

$$\begin{aligned} \forall x[ & Bicycle(x) \rightarrow \\ & \exists y \exists z[ \neg(y = z) \wedge Wheel(y) \wedge Wheel(z) \wedge \\ & HasPart(x, y) \wedge HasPart(x, z) \wedge \\ & \forall w[(HasPart(x, w) \wedge Wheel(w)) \rightarrow (w = y \vee w = z)] ] ] \end{aligned}$$

**Answer:** Bicycles have two wheels. (1 mark for a sentence that is correct but long-winded and/or un-natural.)

- (d)  $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$  is a model for a first-order language with two unary predicates  $P$  and  $Q$  and a binary relation predicate  $R$ . The domain of  $\mathcal{M}$  is the set  $\{a, b, c, d, e\}$ , and the denotation of the predicates is:

- $\delta(P) = \{a, b, c\}$
- $\delta(Q) = \{c, d, e\}$
- $\delta(R) = \{\langle a, d \rangle, \langle b, e \rangle, \langle c, c \rangle\}$

Which of the following formulae are satisfied by this model? [2 marks]

F1.  $\exists x[P(x) \wedge Q(x)]$

F2.  $\exists x \exists y [\neg(x = y) \wedge \forall z [\neg R(z, x) \wedge \neg R(z, y)]]$

F3.  $\forall x [P(x) \rightarrow \exists y [R(x, y) \wedge Q(y)]]$

F4.  $\neg \exists x \exists y [Q(x) \wedge Q(y) \wedge R(x, y)]$

**Answer:** F1 yes, F2 yes, F3 yes, F4 no. (2 marks if exactly these given. 1 mark if one missed out or one extra given. 0, if two or more wrong, since this could be guesswork.)

(e) Use the *Sequent Calculus* to show that the following sequent is valid: [6 marks]

$$\forall x[P(x) \wedge Q(x)], (P(a) \wedge Q(b)) \rightarrow R \vdash R$$

You should only use rules from the following rule set, which was presented in the lecture slides, to construct your proof:

$$\frac{\text{Axiom}}{\alpha, \Gamma \vdash \alpha, \Delta}$$

$$\frac{\alpha, \beta, \Gamma \vdash \Delta}{(\alpha \wedge \beta), \Gamma \vdash \Delta} [\wedge \vdash]$$

$$\frac{\Gamma \vdash \alpha, \Delta \quad \text{and} \quad \Gamma \vdash \beta, \Delta}{\Gamma \vdash (\alpha \wedge \beta), \Delta} [\vdash \wedge]$$

$$\frac{\alpha, \Gamma \vdash \Delta \quad \text{and} \quad \beta, \Gamma \vdash \Delta}{(\alpha \vee \beta), \Gamma \vdash \Delta} [\vee \vdash]$$

$$\frac{\Gamma \vdash \alpha, \beta, \Delta}{\Gamma \vdash (\alpha \vee \beta), \Delta} [\vdash \vee]$$

$$\frac{\Gamma \vdash \alpha, \Delta}{\neg \alpha, \Gamma \vdash \Delta} [\neg \vdash]$$

$$\frac{\Gamma, \alpha \vdash \Delta}{\Gamma \vdash \neg \alpha, \Delta} [\vdash \neg]$$

$$\frac{\Gamma, \neg \alpha \vee \beta \vdash \Delta}{\Gamma, \alpha \rightarrow \beta \vdash \Delta} [\rightarrow \vdash r.w.]$$

$$\frac{\Gamma \vdash \neg \alpha \vee \beta, \Delta}{\Gamma \vdash \alpha \rightarrow \beta, \Delta} [\vdash \rightarrow r.w.]$$

$$\frac{\forall x[\Phi(x)], \Phi(k), \Gamma \vdash \Delta}{\forall x[\Phi(x)], \Gamma \vdash \Delta} [\forall \vdash]$$

$$\frac{\Gamma \vdash \Phi(k), \Delta}{\Gamma \vdash \forall x[\Phi(x)], \Delta} [\vdash \forall]^\dagger$$

$^\dagger$  where  $\kappa$  cannot occur anywhere in the lower sequent.

**Answer:** The sequent is valid as shown by the following proof:

$$\frac{\frac{\frac{\text{Axiom}}{\forall x[P(x) \wedge Q(x)], P(a), Q(a) \vdash P(a), R} [\wedge \vdash] \quad \frac{\forall x[P(x) \wedge Q(x)], P(a) \wedge Q(a) \vdash P(a), R}{\forall x[P(x) \wedge Q(x)] \vdash P(a), R} [\vee \vdash]}{\forall x[P(x) \wedge Q(x)] \vdash (P(a) \wedge Q(b)), R} [\neg \vdash] \quad \frac{\frac{\frac{\text{Axiom}}{\forall x[P(x) \wedge Q(x)], P(a), Q(a) \vdash P(b), R} [\wedge \vdash] \quad \frac{\forall x[P(x) \wedge Q(x)], P(b) \wedge Q(b) \vdash Q(b), R}{\forall x[P(x) \wedge Q(x)] \vdash Q(b), R} [\vee \vdash]}{\forall x[P(x) \wedge Q(x)], \neg(P(a) \wedge Q(b)) \vdash R} [\neg \vdash]}{\frac{\forall x[P(x) \wedge Q(x)], \neg(P(a) \wedge Q(b)) \vee R \vdash R}{\forall x[P(x) \wedge Q(x)], (P(a) \wedge Q(b)) \rightarrow R \vdash R} [\rightarrow \vdash r.w.]} \quad \frac{\text{Axiom}}{\forall x[P(x) \wedge Q(x)], R \vdash R} [\vee \vdash]$$

There is basically a mark for each correct rule application. Some credit may be given for almost correct rule applications.

[Question 1 total: 20 marks]

## Question 2

- (a) (i) Give the set of *clausal* formulae (i.e. formulae in *conjunctive normal form*) corresponding to the following propositional formulae: [4 marks]

$$\neg\neg P, (P \rightarrow Q), ((Q \wedge R) \rightarrow S), \neg(R \rightarrow S)$$

**Answer:**

$$\{\{P\}, \{\neg P, Q\}, \{\neg Q, \neg R, S\}, \{R\}, \{\neg S\}\}$$

- (ii) Give a proof that these formulae are inconsistent using *binary propositional resolution*. [4 marks]

**Answer:**

1. $\{P\}$	6. $\{Q\}$	1+2
2. $\{\neg P, Q\}$	7. $\{\neg R, S\}$	3+6
3. $\{\neg Q, \neg R, S\}$	8. $\{S\}$	4+7
4. $\{R\}$	9. $\emptyset$	5+8
5. $\{\neg S\}$		

- (b) Translate the following sentence into *Propositional Tense Logic*: [2 marks]

I found your wallet after you had left.

**Answer:**  $P(IFoundWallet \wedge P YouLeave)$

- (c) An AI specialist wants to model using *Situation Calculus* a situation in which a group of agents each possess various precious jewels of different colours, which they exchange with each other. Each agent wants to acquire any jewel that is of a colour that they do not possess, and, if they only have one jewel of a particular colour, they want to keep that jewel. Two agents will exchange jewels with each other only if: one of the two receives a jewel that they want to acquire; and neither of them gives a jewel that they want to keep.

The only *fluent* used in the representation will be:

- $has(a, j)$  — agent  $a$  has jewel  $j$ .

The following predicate will also be used:

- $Colour(j, c)$  — jewel  $j$  is of colour  $c$ .

(This is a normal predicate, not a fluent, since the colour of a jewel cannot change.)

The crucial part of the Situation Calculus representation will be axiomatisation of the action of two agents exchanging jewels. This action will be represented as follows:

- $Exchange(a_1, j_1, a_2, j_2)$  — agent  $a_1$  gives jewel  $j_1$  to agent  $a_2$ ,  
and agent  $a_2$  gives jewel  $j_2$  to agent  $a_1$ .

- (i) Give an *effect* axiom specifying all changes of fluents caused by the **Exchange** action. [3 marks]

**Answer:**

$$\begin{aligned} & ( \text{holds}(\text{has}(a_1, j_2), \text{result}(\text{Exchange}(a_1, j_1, a_2, j_2), s)) \wedge \\ & \quad \text{holds}(\text{has}(a_2, j_1), \text{result}(\text{Exchange}(a_1, j_1, a_2, j_2), s)) \wedge \\ & \quad \neg \text{holds}(\text{has}(a_1, j_1), \text{result}(\text{Exchange}(a_1, j_1, a_2, j_2), s)) \wedge \\ & \quad \neg \text{holds}(\text{has}(a_2, j_2), \text{result}(\text{Exchange}(a_1, j_1, a_2, j_2), s)) \\ & ) \leftarrow \text{poss}(\text{Exchange}(a_1, i_1, a_2, i_2), s) \end{aligned}$$

Note: this describes the changes in the only fluent, has, after an **Exchange** action (on condition that the action is possible).

- (ii) Give an appropriate *pre-condition* axiom for the **Exchange** action, which captures the conditions required for an exchange to take place. [4 marks]

**Answer:**  $\text{poss}(\text{Exchange}(a_1, i_1, a_2, i_2), s) \leftarrow$

$$\begin{aligned} & \quad \# \text{ agents must have the jewel they are giving} \\ & \quad \text{holds}(\text{has}(a_1, i_1), s) \wedge \text{holds}(\text{has}(a_2, i_2), s) \wedge \\ & \quad \# \text{ the jewels have certain colours} \\ & \quad \text{Colour}(j_1, c_1) \wedge \text{Colour}(j_2, c_2) \wedge \\ & \quad \# \text{ each agent has another jewel of the colour they are giving} \\ & \quad \exists j_3 [\neg(j_3 = j_1) \wedge \text{Colour}(j_3, c_1) \wedge \text{holds}(\text{has}(a_1, j_3), s)] \wedge \\ & \quad \exists j_3 [\neg(j_3 = j_2) \wedge \text{Colour}(j_3, c_2) \wedge \text{holds}(\text{has}(a_2, j_3), s)] \wedge \\ & \quad \# \text{ one of the agents does not have a jewel of the colour they are receiving} \\ & \quad (\neg \exists j_3 [\text{Colour}(j_3, c_1) \wedge \text{holds}(\text{has}(a_2, j_3), s)] \\ & \quad \vee \\ & \quad \neg \exists j_3 [\text{Colour}(j_3, c_2) \wedge \text{holds}(\text{has}(a_1, j_3), s)]) \end{aligned}$$

- (iii) Give a suitable *frame axiom* which specifies what will stay the same after the **Exchange** action. [3 marks]

**Answer:**  $\text{holds}(\text{has}(a, j), \text{result}(\text{Exchange}(a_1, j_1, a_2, j_2), s))$

$$\begin{aligned} & \quad \leftrightarrow \\ & \quad (\text{holds}(\text{has}(a, j), s) \wedge (\neg((a = a_1) \vee (a = a_2)) \vee \neg((j = j_1) \vee (j = j_2)))) \end{aligned}$$

Note: this says that has relations where either the agent or the jewel is not involved in the **Exchange** stay the same, before and after the **Exchange**. It is OK to have  $\leftarrow$  instead of  $\leftrightarrow$ . The formula with  $\leftarrow$  is the limited (but commonly used) form of the Frame axiom which applies when reasoning about what stays the same after an action takes place.

[Question 2 total: 20 marks]

## Question 3

- (a) For each of the following *Prolog* queries, give the value of the variable  $X$  after the query has been executed:

(i)  $?- Y = 8, Y/2 = X.$  [1 mark]

(ii)  $?- [\_ | Y] = [\text{which, one, is, it}], [\_ , X | \_] = Y.$  [1 mark]

(iii)  $?- L = [1,2], \text{setof}([A,B], (\text{member}(A,L), \text{member}(B,L)), X).$  [2 marks]

**Answer:**

(i)  $8/2$

(ii)  $\text{is}$

(iii)  $[[1,1], [1,2], [2,1], [2,2]]$  (Allow any ordering of the sublists.)

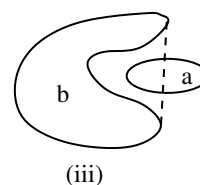
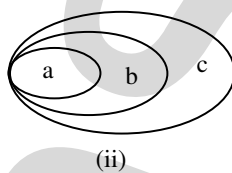
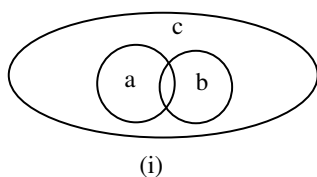
- (b) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function,  $\text{conv}$ . The constants ( $a$ ,  $b$  and  $c$ ) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

(i)  $\text{NTPP}(\text{sum}(a,b), c) \wedge \text{PO}(a,b)$  [2 marks]

(ii)  $P(a,b) \wedge P(b,c) \wedge \text{TPP}(a,c)$  [2 marks]

(iii)  $\text{DC}(a,b) \wedge \text{PO}(a, \text{conv}(b))$  [2 marks]

**Answer:** Possible diagrams are as follows:



- (c) Represent the following statements in *Description Logic*:

(i) Humans are a kind of non-aquatic mammal. [2 marks]

**Answer:**  $\text{Human} \sqsubseteq (\neg \text{Aquatic} \sqcap \text{mammal})$

(ii) Every happy child has a happy friend. [2 marks]

**Answer:**  $(\text{Happy} \sqcap \text{Child}) \sqsubseteq \exists \text{hasFriend.Happy}$

- (d) Give an explanation in English of the following *Default Logic* rule: [2 marks]

$$\text{Friend}(x,y) \wedge \text{Friend}(x,z) : \text{Friend}(y,z) / \text{Friend}(y,z)$$

**Answer:** If someone has two friends then in the absence of information to the contrary one can assume the two are also friends with each other.

- (e) This question concerns a *Fuzzy Logic* in which the following definitions of truth functions for *linguistic modifiers* are specified:

$$\text{quite}(\phi) = \phi^{\frac{1}{2}} \quad \text{very}(\phi) = \phi^2$$

The logic is used to describe Jumbo the elephant, who possesses certain characteristics to the following degrees:

$$\text{Large}(\text{jumbo}) = 0.7 \quad \text{Intelligent}(\text{jumbo}) = 0.36$$

Translate the following sentences into fuzzy logic and also give the fuzzy truth value of each proposition (under the standard fuzzy interpretation of the Boolean connectives):

(i) Jumbo is quite intelligent. [2 marks]

(ii) Jumbo is intelligent and not very large. [2 marks]

**Answer:**

A)  $\text{quite}(\text{Intelligent}(\text{jumbo}))$  (1 mark)

Truth value  $(0.36)^{1/2} = 0.6$  (1 mark)

B)  $\text{Intelligent}(\text{jumbo}) \wedge \neg \text{very}(\text{Large}(\text{jumbo}))$  (1 mark)

Truth value =  $\text{Min}(0.36, 1 - (0.7)^2) = \text{Min}(0.36, 0.51) = 0.36$  (1 mark)

[Question 3 total: 20 marks]

[Grand total: 60 marks]