This question paper consists
of 7 printed pages, each of
which is identified by the
Code Number COMP5450M01

******* VERSION WITH ANSWERS INCLUDED *******

School of Computing

May/June 2011

KRR: KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

Answer ALL THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentence using *Propositional Logic*:

i) Unless the tap is on and the valve is open, the water will not flow. [2 marks]
Answer:
$(TO \wedge VO) \vee \neg WF$. 1 mark if almost correct.

b) Translate the following sentences into *Propositional Tense Logic*:

i) If you have lost your ticket, you will either be given a replacement or a refund. [2 marks]
Answer:
$\mathbf{P}LT \rightarrow (\mathbf{F}\textit{Replacement} \vee \mathbf{F}\textit{Refund})$

ii) When I reached the station, I had already lost my bag. [2 marks]
Answer:
$\mathbf{P}(RS \wedge \mathbf{P}LB)$

c) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

i) John and Mary both live in the same town. [2 marks]
Answer:
$\exists x[\mathsf{Town}(x) \wedge \mathsf{LivesIn}(\mathsf{john}, x) \wedge \mathsf{LivesIn}(\mathsf{mary}, x)]$

ii) Any person who trusts no one deceives themself. [2 marks]
Answer:
$\forall x[\neg\exists y[\mathsf{Trusts}(x, y)] \rightarrow \mathsf{Deceives}(x, x)]$

iii) Tom has a friend all of whose sisters are rich. [2 marks]
Answer:
$\exists x[\mathsf{Friend}(\mathsf{tom}, x) \wedge \forall y[\mathsf{IsSisterOf}(y, x) \rightarrow \mathsf{Rich}(y)]$

d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with two binary relations S and T. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$, and the denotations of S and T are as follows:

- $\delta(S) = \{\langle a, a \rangle\}$
- $\delta(T) = \{\langle a, b \rangle, \langle a, c \rangle, \langle a, d \rangle, \langle a, e \rangle\}$

For each of the following formulae, state whether it is satisfied by the model $\mathcal{M}$: [2 marks]

F1. $\exists x \exists y[S(x, y)]$

F2. $\exists x \forall y[S(x, y) \vee T(x, y)]$

F3. $\exists x \forall y[T(x, y)]$

F4. $\neg\forall x[\neg S(x, x)]$

Answer:
F1, F2 and F4 are satisfied. (2 marks if exactly these given. 1 mark if one missed out or one extra given. 0, if two or more wrong, since this could be guesswork.)

e) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid:

$$\forall x[P(x) \wedge G(x)],\ H(a) \ \vdash\ \forall x[F(x) \vee G(x)] \wedge H(a)$$ [6 marks]

Answer:
The sequent is valid, as shown by the following proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{Axiom}}{\forall x[P(x) \wedge G(x)],\ P(b),\ G(b),\ H(a) \ \vdash\ F(b),\ G(b)}}{\forall x[P(x) \wedge G(x)],\ P(b) \wedge G(b),\ H(a) \ \vdash\ F(b),\ G(b)}\wedge\vdash}{\forall x[P(x) \wedge G(x)],\ H(a) \ \vdash\ F(b),\ G(b)}\forall\vdash}{\forall x[P(x) \wedge G(x)],\ H(a) \ \vdash\ F(b) \vee G(b)}\vdash\vee}{\forall x[P(x) \wedge G(x)],\ H(a) \ \vdash\ \forall x[F(x) \vee G(x)]}\vdash\forall \qquad \dfrac{\textit{Axiom}}{\forall x[P(x) \wedge G(x)],\ H(a) \ \vdash\ H(a)}}{\forall x[P(x) \wedge G(x)],\ H(a) \ \vdash\ \forall x[F(x) \vee G(x)] \wedge H(a)}\vdash\wedge$$

There is basically 1 mark for each correct rule application and 1 mark for reducing all branches to axioms. Some credit may be given for almost correct rule applications.

[20 marks total]

Question 2

a) A Situation Calculus theory describes the possible actions of a cleaning robot. The vocabulary of the theory includes fluents of the forms:

$has(agent, item)$, $located(\textit{item-or-agent,room})$, $connected(room1, room2)$, $\textit{clean-floor}(room)$

and constants denoting various items such as broom, $\mathsf{litter\text{-}bin}$, etc..

The vocabulary also includes the following actions:

$\mathbf{pickup}(agent, item)$, $\mathbf{drop}(agent, item)$, $\mathbf{move}(agent, room1, room2)$, $\mathbf{sweep}(agent, room)$.

Among others, the theory contains the axiom:

A) $\forall\ a\ i\ [\mathsf{Holds}(has(a, i), \mathsf{result}(\mathbf{pickup}(a, i))) \ \leftarrow\ \mathsf{Poss}(\mathbf{pickup}(a, i), s)]$

i) What kind of axiom is **A**? [1 mark]

Answer:
It is an *effect* axiom.

ii) Using the Poss predicate, give a suitable *precondition axiom* specifying the conditions under which an action $\mathbf{sweep}(a, r)$ is possible. [2 marks]

Answer:
$Poss(\mathbf{sweep}(a, r), s) \leftarrow \mathsf{Holds}(located(a, r), s) \wedge \mathsf{Holds}(has(a, \mathsf{broom}), s)$

iii) Give a suitable *effect axiom* specifying the effect of carrying out the $\mathbf{sweep}(a, r)$ action. [2 marks]

Answer:
$\forall\ a\ r\ [\mathsf{Holds}(\textit{clean-floor}(r), \mathsf{result}(\mathbf{sweep}(a, r), s)) \ \leftarrow\ \mathsf{Poss}(\mathbf{sweep}(a, r), s)]$

iv) Give a *frame* axiom which specifies that: when an agent moves from one room to another, this does not affect the state of the floor of any room. [2 marks]

Answer:
$\forall\ a\ r\ r_1\ r_2\ [\mathsf{Holds}(\textit{clean-floor}(r), \mathsf{result}(\mathbf{move}(a, r_1, r_2))) \ \leftrightarrow \mathsf{Holds}(\textit{clean-floor}(r), s)]$

v) How might the $\textit{clean-floor}(r)$ fluent be affected by a *ramification* of one of the actions other than $\mathbf{sweep}(a, r)$? [2 marks]

Answer:
A drop action might cause a mess, causing a *clean-floor* fluent to no longer hold.

b) Translate the following sentences into *Description Logic*. You should use concept and relation names that make clear your intended interpretation of these symbols:

i) Mushrooms that are purple or green are poisonous. [2 marks]

Answer:
$(\mathsf{Mushroom} \sqcap (\mathsf{Purple} \sqcup \mathsf{Green})) \sqsubseteq \mathsf{Poisonous}$

ii) Every boy who owns a bicycle is happy. [2 marks]

Answer:
$(\mathsf{Boy} \sqcap \exists \mathsf{owns.Bicycle}) \sqsubseteq \mathsf{Happy}$

c) Jane's holiday began with a stay in Paris and ended with a stay in Amsterdam. Jane's bicycle tour began immediately after she left Paris and ended when she reached Amsterdam.

i) Encode the given information in terms of the relations of *Allen's Interval Calculus* (you should give 4 interval relation facts). [2 marks]

Answer:
Begins(paris, holiday), Ends(amsterdam, holiday),
MetBy(bicycle-tour, paris), Meets(bicycle-tour, amsterdam).

ii) Show how *compositional reasoning* can be used to calculate that Jane's bicycle tour took place During her holiday. Give details of each composition rule that you apply and how the results of these rules are combined. Depending on your representation of the given facts you may need to convert some of your relations to their converse before applying compositional inference. If you wish, you may illustrate your calculations diagrammatically.

[5 marks]

Answer:
Essentially: (met-by ; starts) = {overlapped-by, ends, during}
(meets ; ends) = {overlaps, starts, during}
Intersecting these two gives {during}. Diagrams can be used to illustrate the structure of the network relating the four intervals and also to show how the required compositions are determined.

[20 marks total]

Question 3

a) Convert the following formulae into *clausal form*:

i) $(P \vee Q) \rightarrow \neg(A \wedge B)$ [2 marks]

Answer:
$\{\ \{\neg P, \neg A, \neg B\},\ \ \{\neg Q, \neg A, \neg B\}\}$

ii) $\forall x[\exists y \exists z[R(y, z) \wedge (S(x, y) \vee S(x, z))]]$ [2 marks]

Answer:
$\{\{R(f(x), g(x))\},\ \{S(x, f(x)), S(x, g(x))\}\}$

b) Give a *most general unifier* of the following pair of terms (where X, Y and Z are variables):

$$q(r(X), X) \qquad q(Y, s(Z))$$ [2 marks]

Answer:
$X \Rightarrow s(Z),\ Y \Rightarrow r(s(Z))$ [1 for each correct]

c) A *Prolog* database contains facts of the forms: `lives(X, Y)`, meaning that `X` is a person who lives in town or city `Y`; and `in_country(X, Y)`, meaning that the town or city `X` is located in the country `Y`.

Briefly explain the meaning of the predicate `xxxxxxx( X, S )` as defined below and suggest a suitable meaningful name for it. [2 marks]

```
xxxxxxx( X, S ) :-
              setof( Y, (lives( X, T1 ),
                         lives( Y, T2 ),
                         in_country( T1, C ),
                         in_country( T2, C )
                        ),
                     S ).
```

Answer:
The predicate will be true when S is a list giving the set of people who live in the same country as X. The predicate could be called `compatriots` (or `people_in_same_country`).

d) A *Default Logic* theory Θ contains the classical facts

C1 $\mathsf{Bird}(\mathsf{bob})$
C2 $\mathsf{Bird}(\mathsf{martin})$
C3 $\mathsf{Lives}(\mathsf{bob}, \mathsf{antarctic})$
C4 $\forall x[\mathsf{Bird}(x) \rightarrow \mathsf{HasWings}(x)]$
C5 $\forall x[\mathsf{Penguin}(x) \rightarrow \neg\mathsf{Flies}(x)]$

and the default rules

D1 $\mathsf{HasWings}(x) : \mathsf{Flies}(x)\ /\ \mathsf{Flies}(x)$
D2 $\mathsf{Bird}(x) \wedge \mathsf{Lives}(x, \mathsf{antarctic}) : \mathsf{Penguin}(x)\ /\ \mathsf{Penguin}(x)$

i) Is $\mathsf{Flies}(\mathsf{bob})$ true in some, all or no, *extensions* of Θ? Briefly explain your answer, mentioning which default rule or rules are applied to generate each extension. [3 marks]

Answer:
This is true in one extension and not true in another. We can infer classically from C4 that Bob has wings. If we then apply D1 we infer that Bob flies. However, if we apply D2 before D1 we would infer that Bob is a Penguin, so by C5 Bob does not fly, which blocks subsequent application of D1.

ii) Which atomic facts about martin are true in all *extensions* of Θ? Briefly explain your answer. [3 marks]

Answer:
Bird(martin) is given. HasWings(martin) follows classically from C2 and C4. Flies(martin) then follows from D1. D2 cannot be applied, since we cannot infer that martin lives in the antarctic.

e) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the convex hull function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, draw a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\exists x \exists y[\mathsf{DC}(x, y) \wedge \mathsf{EC}(x, \mathsf{a}) \wedge \mathsf{EC}(y, \mathsf{a}) \wedge \mathsf{P}(\mathsf{sum}(x, y), \mathsf{conv}(\mathsf{a}))]$ [2 marks]

ii) $\mathsf{EQ}(\mathsf{a}, \mathsf{conv}(\mathsf{a}))) \wedge \neg\mathsf{EQ}(\mathsf{b}, \mathsf{conv}(\mathsf{b})) \wedge \mathsf{TPP}(\mathsf{a}, \mathsf{b})$ [2 marks]

iii) $\forall x[\mathsf{C}(x, \mathsf{a}) \rightarrow \mathsf{O}(x, \mathsf{b})]$ [2 marks]

Answer:
Some possibilities are shown below; these are not necessarily unique; others accepted as appropriate.

```
              -----   -----
i)          | a   |  |    |
            |     ----    |   a has two separate concavities.
            |             |
            |             |
            |    |---|    |
            |    |   |    |
            ------   ----


             -------------
             |  b        |
             |        ----
             |        |
ii)          |-----   |      a convex, b concave,  TPP(a,b)
             | a  |   |
             |    |   |
             |-----   |
             |        ----
             |           |
             -------------


             ---------------
iii)         |             |
             |   ----      |   a is a non tangential proper part of b
             |   |a |    b |
             |   ----      |
             ---------------
```

[20 marks total]