This question paper consists
of 5 printed pages, each of
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School of Computing

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AI34: KNOWLEDGE REPRESENTATION AND REASONING

Time allowed: 2 hours

Answer THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentences using *propositional* logic:

i) I will see you on Tuesday, unless I go cycling. [1 mark]

ii) If it is sunny but not too hot I shall go to the park or the beach. [2 marks]

b) Consider a fist order language whose vocabulary consists of a single binary relation, R, and four models: $\mathcal{M}_1 = \langle \mathcal{D}, \delta_1 \rangle$, $\mathcal{M}_2 = \langle \mathcal{D}, \delta_2 \rangle$, $\mathcal{M}_3 = \langle \mathcal{D}, \delta_3 \rangle$ and $\mathcal{M}_4 = \langle \mathcal{D}, \delta_4 \rangle$, which all have the same domain of individuals, $\mathcal{D} = \{a, b, c\}$). The denotation functions for each of the models are as follows:

$\mathcal{M}_1$: $\delta_1(R) = \{\langle a,a\rangle,\ \langle a,b\rangle,\ \langle a,c\rangle,\ \langle b,a\rangle,\ \langle b,b\rangle,\ \langle b,c\rangle,\ \langle c,a\rangle,\ \langle c,b\rangle,\ \langle c,c\rangle\}$
$\mathcal{M}_2$: $\delta_2(R) = \{\langle b,a\rangle,\ \langle b,b\rangle,\ \langle b,c\rangle\}$
$\mathcal{M}_3$: $\delta_2(R) = \{\langle a,b\rangle,\ \langle b,a\rangle,\ \langle c,a\rangle\}$
$\mathcal{M}_4$: $\delta_3(R) = \{\langle a,a\rangle\}$

For each of the following formulae, state which of the models $\mathcal{M}_1 - \mathcal{M}_4$ satisfy it, if any:

i) $\forall x \forall y [R(x,y)]$ [1 mark]

ii) $\forall x \exists y [R(x,y)]$ [1 mark]

iii) $\exists x \forall y [R(x,y)]$ [1 mark]

iv) $\exists x \exists y [R(x,y)]$ [1 mark]

c) Translate the following sentences into first-order predicate logic (using equality where necessary):

i) I don't like anyone who doesn't like me. [2 marks]

ii) There is no highest prime number. [2 marks]

iii) Only one clown can both juggle and ride a unicycle. [3 marks]

d) Using the *Sequent Calculus* (as specified in the AI34 course notes), determine whether the following sequent is valid:

$$P \rightarrow \exists x[Q(x)],\quad P \quad \vdash \quad (Q(a) \vee \neg P)$$ [6 marks]

[20 marks total]

Question 2

a) A first-order temporal language includes predicates of the form $P(x, t)$ and $R(x, y, t)$, where the variable t denotes the time at which the property or relation holds. It also includes an ordering relation '$<$' over time points and a constant now denoting the present time. Use this language to represent each of the following statements:

   i) Susan has never met Tom. [1 mark]

   ii) If Charles has not arrived yet, he will never arrive. [2 marks]

b) Translate the following sentences into *Propositional Tense Logic*:

   i) If Alethea drank the potion she will sleep forever. [1 mark]

   ii) I will continue work at the bookshop at least until after I graduate. [2 marks]

c) Consider the following scenario:

> Tom is in the lounge, which is adjacent to the kitchen. In the kitchen is a pie. Tom is hungry.

Give a representation of this scenario in *Situation Calculus* including axioms governing the actions that are likely to occur. Formulate your representation using the following vocabulary:

- Fixed predicates: $\mathit{food}(x)$, $\mathit{adjacent}(l1, l2)$
- Fluents: $\mathit{hungry}(x)$, $\mathit{located}(x, l)$, $\mathit{eaten}(x)$
- Actions: $\mathbf{eat}(x, i)$, $\mathbf{move}(x, l1, l2)$.

Your representation should include the following:

   i) Specification of any fixed properties and relations that hold. [2 marks]

   ii) Specification of what *fluents* hold in the initial situation (s_0). [2 marks]

   iii) Specification of suitable *precondition* and *effect* axioms for each of the actions. [6 marks]

   iv) Specification of *frame* axioms for each of the fluents. [4 marks]

[20 marks total]

Question 3

a) Use *binary resolution* to show that the following propositional entailment is valid:

$$\neg Q,\ (Q \vee R \vee T),\ \neg(R \wedge \neg Q) \ \vdash\ (Q \vee T)$$

In order to do this you must:

i) re-formulate the entailment problem as a consistency checking problem; [1 mark]

ii) transform the formulae into clausal (i.e. conjunctive normal) form; [2 marks]

iii) carry out a binary resolution proof to show inconsistency. [3 marks]

b) Give all formulae that can be derived by a single application of the first-order *binary resolution* inference rule to the following set of clauses:

$$\{\ \{G(X,Y),\ \neg H(a,Y)\},\quad \{P(X),\ H(X,b)\},\quad \{\neg G(b,f(a))\},\quad \{\neg H(k,Z)\}\ \}$$ [3 marks]

c) The problem of computing entailment in a logical language $\mathcal{L}$ is known to be undecidable. An AI software tool implements an inference system that its developers claim can test the validity of formulae expressed in $\mathcal{L}$. Given any formula of $\mathcal{L}$, the algorithm always returns either '`valid`' or '`invalid`', within a period of 1 minute. What can you say about the soundness and completeness of this inference system? [2 marks]

d) Give an explanation in English of the following *Default Logic* rule:

$$\mathsf{Friend}(x,y) \wedge \mathsf{Friend}(y,z)\ :\ \mathsf{Friend}(x,z)\ /\ \mathsf{Friend}(x,z)$$ [2 marks]

e) A *default theory* Θ contains the classical formulae:

C1. A
C2. $(\mathsf{B} \leftrightarrow \neg\mathsf{C})$
C3. $(\mathsf{E} \rightarrow \neg\mathsf{D})$

and the following default rules:

D1. $(\mathsf{A} \vee \mathsf{D})\ :\ \mathsf{B}\ /\ \mathsf{B}$
D2. $:\mathsf{C}\ /\ \mathsf{C}$
D3. $\mathsf{E}\ :\ \mathsf{D}\ /\ \mathsf{D}$

i) How many *extensions* does Θ have? [1 mark]

ii) For each of the atomic propositions A, B, C and D, state whether it holds in all, some but not all, or no extensions of Θ. [4 marks]

iii) Suppose we form the theory Θ' by adding to Θ an additional default rule:

D4. $\mathsf{A}\ :\ \mathsf{D}\ /\ \mathsf{E}$

State how many extensions Θ' has, and explain why. [2 marks]

[20 marks total]

Question 4

a) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the *convex hull* function, conv. The quantifiers range over non-empty spatial regions. For each formula *draw* a configuration of the regions a, b and (where relevant) c, which satisfies the formula:

i) $\exists x[\mathsf{P}(a, x) \wedge \mathsf{DC}(x, b)]$ [1 mark]

ii) $(\mathsf{DC}(a, \mathsf{conv}(b)) \wedge \mathsf{EC}(a, c) \wedge \mathsf{PO}(a, \mathsf{conv}(c)) \wedge \mathsf{NTPP}(a, \mathsf{conv}(\mathsf{sum}(b, c)))$ [3 marks]

b) This question concerns reasoning with Allen's calculus of temporal interval relations. The basic relations of the calculus (as described in the AI34 notes) are the following:

Before, After, Meets, Met-by, Overlaps, Overlapped-by, Starts, Started-by,
Ends, Ended-by, Contains, During, Equals.

i) Give the disjunction of basic Allen relations that is equivalent to each of the following compositions (it may be helpful to draw diagrams to work these out):

C1. (Begins ; Ended-by) [2 marks]

C2. (Contains ; Meets) [2 marks]

ii) Represent each of the following statements by means of one of Allen's temporal interval relations:

R1. The morning began sunny but didn't end that way. [1 mark]

R2. At some point during in the morning I started reading the newspaper and finished exactly at mid-day. [1 mark]

R3. During the middle of the sunny period I drank some coffee. [1 mark]

R4. It was immediately after finishing the coffee that I started reading the newspaper. [1 mark]

iii) Use *compositional reasoning* to determine the temporal relation between the sunny period and my reading the newspaper. (Depending on how you represented the given facts, you may also have to re-express an interval relation in terms of its converse) [4 marks]

c) Use *Description Logic* to represent the following axioms:

i) Humans are a kind of non-aquatic mammal. [2 marks]

ii) Only humans can have human children. [2 marks]

[20 marks total]