This question paper consists of 9 printed pages, each of which is identified by the Code Number COMP5450M.

A non-programmable calculator may be used.
Answer All Questions.
This is an open book examination.
Any written or printed material is permitted.

School of Computing

January 2018

COMP5450M

KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

PLEASE DO NOT REMOVE THIS PAPER FROM THE EXAM ROOM

Answer ALL THREE questions

The marks available for each part of each question are clearly indicated.

Question 1

(a) Translate the following sentence into *Propositional Logic*:

- My hat is in the hall or the kitchen, unless I left it in the car. **[2 marks]**
  Answer: $\mathit{HatHall} \vee \mathit{HatKitchen} \vee \mathit{LeftHatCar}$.

(b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

(i) No dog is pink. **[2 marks]**
Answer: $\neg\exists x[\mathsf{Dog}(x) \wedge \mathsf{Pink}(x)]$

(ii) A greedy child ate two cakes. **[2 marks]**
Answer:
$\exists xyz[\mathsf{Child}(x) \wedge \mathsf{Greedy}(x) \wedge \mathsf{Ate}(x, y) \wedge \mathsf{Ate}(x, z) \wedge \mathsf{Cake}(y) \wedge \mathsf{Cake}(z) \wedge \neg(y = z)]$

(iii) The same shrine was visited by every blind pilgrim **[2 marks]**
Answer: $\exists x[\mathsf{Shrine}(x) \wedge \forall y[(\mathsf{Pilgrim}(y) \wedge \mathsf{Blind}(y)) \rightarrow \mathsf{Visited}(y, x)]\ ]$

(iv) Every poor man has a beard or shaves himself. **[2 marks]**
Answer: $\forall x[(\mathsf{Man}(x) \wedge \mathsf{Poor}(x)) \rightarrow (\exists y[\mathsf{Beard}(y) \wedge \mathsf{Has}(x, y)] \vee \mathsf{Shaves}(x, x))]$

(c) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid: **[6 marks]**

$$\vdash\ \neg\forall x[R(a, x) \wedge \neg R(x, b)]$$

Answer: The sequent is valid as shown by the following proof:

```
                 AXIOM
 -----------------------------------------
  Ax[Rax & -Rxb], Rab, -Rbb,  Raa |-  Rab
 -------------------------------------------[& |-]
  Ax[Rax & -Rxb], Rab & -Rbb, Raa |-  Rab
 ------------------------------------------[A |-]
    Ax[Rax & -Rxb],  Raa |-  Rab
   -------------------------------------[- |-]
    Ax[Rax & -Rxb],  Raa, -Rab |-
 ------------------------------------[& |-]
 Ax[Rax & -Rxb],  Raa & -Rab |-
 ------------------------------------[A |-]
          Ax[Rax & -Rxb] |-
       ---------------------------[|- -]
         |-  -Ax[Rax & -Rxb]
```

There is basically a mark for each correct rule application. Some credit may be given for almost correct rule applications.

(d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with a binary relation symbol 'T'. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$, and the denotation of T is:

- $\delta(T) = \{\langle a, b\rangle, \langle b, c\rangle, \langle c, d\rangle, \langle d, e\rangle, \langle e, a\rangle\}$

Which of the following formulae are satisfied by this model? **[4 marks]**

F1. $\forall x \exists y[T(x, y)]$

F2. $\exists y[T(x, y) \wedge T(y, x)]$

F3. $\forall x \exists y[T(y, x)]$

F4. $\forall x \forall y \forall z[(T(x, y) \wedge T(y, z)) \rightarrow T(x, z)]$

Answer: F1, and F3 are satisfied. 1 mark for each correct.

[Question 1 total: 20 marks]

Question 2

(a) The *RoboFix* robot repair service acquires non-functional robots, repairs them and sells them to the general pubic. *RoboFix* uses *Situation Calculus* (as specified in the COMP5450M slides) in an AI reasoning system to plan its repairing operations.

The domain of the calculus includes the following kinds of entity: robots, robot parts, monetary values in *credits*, robot model types (e.g. R2-Droid, Nexus-6, Terminator-II) and part types (e.g. CPU, Vis-System, Morality-Chip, Oil-Tank, ...). Each robot is of a particular model and each part is of a particular type for a particular model. A fully functional robot has a fixed value depending on its model. Replacement parts can be bought from the manufacturer at a cost depending on the part type and robot model.

All robots of a particular model require the same part types, and should have exactly one part of each of these types. Since robots have huge numbers of parts, the *RoboFix* database only keeps track of working parts that are in stock but are not fitted to a robot. These robots have very reliable self-testing mechanisms; so, when a robot is not-functional, accurate information is stored about all types of part that are either faulty or missing for that robot; and it may be assumed that all other parts that the model normally has are present and functional. Thus, if one robot of model m has a faulty/missing part of type t, and there is another model m robot in stock which does not have part type t recorded as faulty/missing, then it will be possible to reclaim the required part from the second robot and subsequently fit it to the first.

The *fluents* used in the *RoboFix Situation Calculus* representation are:

$\mathsf{Credits}(n)$	The *RoboFix* account has n credits (cannot be negative).
$\mathsf{InStock}(x)$	A particular robot or part x is currently held in stock.
$\mathsf{RobotModel}(r, m)$	r is a robot of model m.
$\mathsf{FaultyOrMissing}(r, t)$	Robot r, has its part of type t faulty or missing.
$\mathsf{PartType}(p, t, m)$	p is a spare part of type t for robot model m.
$\mathsf{PartCost}(t, m, v)$	Part type t for robot model m costs v credits.
$\mathsf{RobotModel\_Value}(m, v)$	A functional robot of model m is worth v credits.

And the action terms are:

$\mathbf{Buy}(p)$	Buy part p for its standard cost (must have sufficient credits).
$\mathbf{Sell}(r)$	Sell robot r, which must be fully functional, for its standard value.
$\mathbf{Reclaim}(p, r)$	Reclaim part p from robot r.
$\mathbf{Fit}(p, r)$	Fit part p to robot r.

The *RoboFix* representation also allows arithmetic operations and comparisons to be applied to credit values (e.g. $v = v_1 + v_2$, $v_1 \geq v_2$).

(i) Give *precondition* axioms for the **Buy** and **Sell** actions. **[4 marks]**

(ii) Give *effect* axioms for the **Reclaim** and **Fit** actions. **[4 marks]**

(iii) Give a *frame* axiom for the $\mathsf{FaultyOrMissing}$ fluent that captures the condition that a faulty or missing part will remain faulty or missing when any action is carried out, except for the action that fits a part of the required type. **[2 marks]**

Hint: Rather than giving axioms for every action, I suggest that you use a variable to refer to an arbitrary action, but exclude the case where the fluent will change.

Answer:

(i) Precondition axiom for **Buy**:
$Poss(\mathbf{Buy}(p), s) \leftarrow$
$\quad \exists t m v_1 v_2[\mathsf{PartType}(p,t,m) \wedge \mathsf{PartCost}(t,m,v_1) \wedge \mathsf{Credits}(v_2) \wedge v_2 \geq v_1]$

Precondition axiom for **Sell**:
$Poss(\mathbf{Sell}(r), s) \leftarrow Holds(\mathsf{InStock}(r), s) \wedge \neg\exists t[Holds(\mathsf{FaultyOrMissing})(r,t), s)]$

(ii) Effect axioms for **Reclaim**:
$Holds(\mathsf{InStock}(p), result(\mathbf{Reclaim}(p,r), s) \leftarrow Poss(\mathbf{Reclaim}(p,r), s)$

$Holds(\mathsf{FaultyOrMissing}(r,t), result(reclaim(p,r), s) \leftarrow$
$\quad Poss(\mathbf{reclaim}(p,r), s) \wedge \exists m[\mathsf{PartType}(p,t,m)]$

Effect axioms for **Fit**:
$\neg Holds(\mathsf{InStock}(p), result(\mathbf{Fit}(p,r), s) \leftarrow Poss(\mathbf{Fit}(p,r), s)$

$\neg Holds(\mathsf{FaultyOrMissing}(r,t), result(\mathbf{Fit}(p,r), s) \leftarrow Poss(\mathbf{fit}(p,r), s) \wedge \exists m[\mathsf{PartType}(p,t,m)]$

(iii) $Holds(\mathsf{FaultyOrMissing}(r,t), result(\alpha, s)) \leftarrow$
$\quad Holds(\mathsf{FaultyOrMissing}(r,t), s) \wedge \neg\exists p \exists m[\mathsf{PartType}(p,t,m) \wedge \alpha = \mathbf{Fit}(p,r)]$

(b) The *RoboFix* robot repair service (as described in part (a) of this question) found that it was not earning any credits because its AI system was neither executing any actions to reclaim parts nor selling any robots. Eventually, the problem was identified as due to the way that only facts about faulty or missing parts of robots were being stored, not facts about working parts. It was decided that two *Default Logic* rules needed to be added to supplement the normal classical rules used with *Situation Calculus*.

(i) Briefly explain, in 2 or 3 sentences, why some *Default Reasoning* is required in order for the robot repair system described in part (a) to work correctly. **[2 marks]**

Answer: In classical logic, one cannot infer anything from the absence of information. In this case we cannot infer that a part is in working order, just because we do not have information that it is faulty.

(ii) Give a default rule which will be required in order to infer that working parts may be reclaimed from a robot that is not fully functional. Give a brief explanation (1 or 2 sentences) of what the rule says. **[2 marks]**

Answer: $: \neg Holds(\mathsf{FaultyOrMissing}(r,t),s))/\neg Holds(\mathsf{FaultyOrMissing}(r,t),s)$

or simply $: \neg\mathsf{FaultyOrMissing}(r,t)\ /\ \mathsf{FaultyOrMissing}(r,t)$

(Variables that are not explicitly quantified are implicitly universal.)

Unless it is inconsistent that (in a situation s) a robot does have a faulty/missing part of a particular type, it can be inferred that it does not have a faulty/missing part of that type.

(We also need to infer that if a robot has a faulty part of a given type, then any other robot of the same model must have a part of that type. But this can be done via a classical axiom because all the faulty parts are recorded.)

(iii) Give a default rule which will be required in order to infer that a robot may be sold. Give a brief explanation (1 or 2 sentences) of what the rule says. **[2 marks]**

Answer: $: \neg\exists t[Holds(\mathsf{FaultyOrMissing}(r,t),s)]/\neg\exists t[Holds(\mathsf{FaultyOrMissing}(r,t),s)]$

(or simply $: \neg\exists t[\mathsf{FaultyOrMissing}(r,t)]\ /\ \neg\exists t[\mathsf{FaultyOrMissing}(r,t)]$)

Unless it is inconsistent that (in a situation s) a robot has no faulty/missing parts, it can be inferred that it has no faulty/missing parts. (The fact that *all* parts are functional if no fault is known cannot be derived from the previous rule which concerns individual parts.)

Note: Though it is based on the same scenario as Q2(a) on the previous page, correct answers to Q2(a) are not necessary in order to answer this part (Q2(b)). The answers for Q2(b)(i) and Q2(b)(ii), may be given either in terms of default rules involving *Situation Calculus* formulae of the form $Holds(f,s)$ or, more simply as default rules in which one or more of the *fluents* of the *RoboFix* representation are taken as ordinary 1st-order logic predicates.

(c) Translate the following sentences into *propositional tense logic*:

(i) I have been to Disneyland but will never go again. **[2 marks]**

Answer: $\mathbf{P}GD \wedge \neg\mathbf{F}GD$

(ii) I shall visit London and after that Paris. **[2 marks]**
Answer: $\mathbf{F}(VL \wedge \mathbf{F}VP)$

[Question 2 total: 20 marks]

Question 3

(a) For each of the following *Prolog* queries, give the value of the variable `X` after the query has been executed:

(i) `?- X = 2*3.` **[1 mark]**

(ii) `?- A = [a,b,c,d,e], [_|B] = A, [X|_] = B.` **[1 mark]**

(iii)
```
?- A = [1,2,3,4,5], B = [2,4,6,8],
   setof( I, (member(I,A), \+(member(I,B))), X ).
```
[2 marks]

Answer:
```
i) 2*3
ii) b
iii) [1,3,5]
```

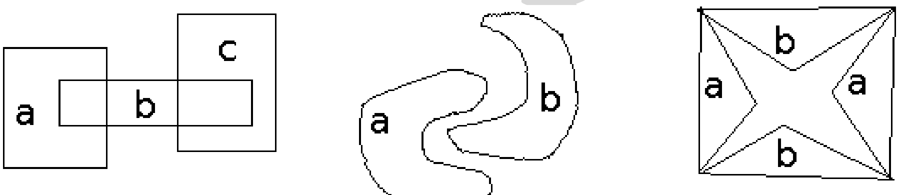
(b) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function, conv. The constants (a, b and c) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

(i) $\mathsf{PO}(\mathsf{a},\mathsf{b}) \wedge \mathsf{PO}(\mathsf{b},\mathsf{c}) \wedge \mathsf{DC}(\mathsf{a},\mathsf{c})$ **[2 marks]**

(ii) $\mathsf{PO}(\mathsf{a},\mathsf{conv}(\mathsf{b})) \wedge \mathsf{PO}(\mathsf{conv}(\mathsf{a}),\mathsf{b}) \wedge \mathsf{DC}(\mathsf{a},\mathsf{b})$ **[2 marks]**

(iii) $\mathsf{EC}(\mathsf{a},\mathsf{b}) \wedge (\mathsf{conv}(\mathsf{a}) = \mathsf{conv}(\mathsf{b}))$ **[2 marks]**

Answer: Possible diagrams are as follows:



(i) (ii) (iii)

(c) A *Description Logic* theory of family relationships includes the primitive concepts Male and Female, and the relations $\mathsf{hasSibling}$ and $\mathsf{hasChild}$.

Give a description logic formula which defines each of the following concepts in terms of the primitives:

(i) Sister **[2 marks]**

(ii) $\mathsf{Only\_Child}$ **[2 marks]**

(iii) Uncle **[2 marks]**

Answer:

(i) $\mathsf{Sister} \equiv \mathsf{Female} \sqcap \exists \mathsf{hasSibling}.\top$

(ii) $\mathsf{Only\_Child} \equiv \neg\exists\mathsf{hasSibling}.\top$

(iii) $\mathsf{Uncle} \equiv \mathsf{Male} \sqcap \exists\mathsf{hasSibling}.\exists\mathsf{hasChild}.\top$

(d) This question concerns a *Fuzzy Logic* in which the following definitions of *linguistic modifiers* are specified:

$$\mathsf{somewhat}(\phi) = \phi^{1/2} \qquad \mathsf{somewhat}(\phi) = \phi^{1/4} \qquad \mathsf{very}(\phi) = \phi^2$$

The logic is used to describe Leo the lion, who possesses certain characteristics to the following degrees:

$$\mathsf{Large}(\mathsf{leo}) = 0.9 \qquad \mathsf{Fierce}(\mathsf{leo}) = 0.7 \qquad \mathsf{Clever}(\mathsf{leo}) = 0.25$$

Translate the following sentences into fuzzy logic and also give the fuzzy truth value of each proposition (under the standard fuzzy interpretation of the Boolean connectives):

(i) Leo is very large and somewhat fierce but not clever. **[2 marks]**

(ii) Leo is very very clever. **[2 marks]**

Answer:

A) $\mathsf{very}(\mathsf{Large}(\mathsf{leo})) \wedge \mathsf{somewhat}(\mathsf{Fierce}(\mathsf{leo})) \wedge \neg\mathsf{Clever}(\mathsf{leo})$ (1 mark)

Truth value $= Min\{0.9^2, 0.7^{1/2}, (1-0.25)\} = Min\{0.81, 0.837, 0.75\} = 0.75$ (1 mark)

B) $\mathsf{very}(\mathsf{very}(\mathsf{Clever}(\mathsf{leo})))$ (1 mark)

Truth value $= ((0.25)^2)^2 = 0.00391$ (1 mark)

[Question 3 total: 20 marks]

[Grand total: 60 marks]