This question paper consists
of 7 printed pages, each of
which is identified by the
Code Number COMP5450M01

******* VERSION WITH ANSWERS INCLUDED *******

School of Computing

January 2012

KRR: KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

Answer ALL THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentence using *Propositional Logic*:

i) If you lose or damage this equipment you must pay for it, unless you have insurance. [2 marks]
Answer:
$((L \vee D) \wedge \neg I) \rightarrow P$. 1 mark if almost correct.

b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

i) John's sister kissed Mary's brother. [2 marks]
Answer:
$\exists x \exists y[\mathsf{Sister}(john, x) \wedge \mathsf{Brother}(mary, y) \wedge \mathsf{Kissed}(x, y)]$

ii) Only one apple was eaten. [2 marks]
Answer:
$\exists x[(\mathsf{Apple}(x) \wedge \mathsf{Eaten}(x)) \wedge \forall y[(\mathsf{Apple}(y) \wedge \mathsf{Eaten}(y)) \rightarrow y = x]]$

c) Which of the following formula can be satisfied in a model in which there is only one element?

i) $\forall x[R(x, x)]$
ii) $\forall x \forall y[R(x, y)]$
iii) $\exists x \exists y[\neg(x = y)]$ [1 mark]

Answer:
(i) and (ii) can be satisfied but not (iii).

d) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid:

$$\forall x[\neg R(a, x)] \ \vdash \ \forall x[\neg \forall y[R(y, x)]]$$ [6 marks]

Answer:
The sequent is valid, as shown by the following proof:

```
                   AXIOM
          ------------------------
 Ax[-R(a,x)], Ay[R(y,k)], R(a,k)  |-  R(a,k)
------------------------------------------------- [- |-]
 Ax[-R(a,x)], -R(a,k), Ay[R(y,k)], R(a,k) |-
------------------------------------------------- [A |-]
 Ax[-R(a,x)], Ay[R(y,k)], R(a,k) |-
------------------------------------------------- [A |-]
 Ay[R(y,k)), Ax[-R(a,x)] |-
------------------------------------------------- [|- -]
 Ax[-(R(a,x)))] |-  -Ay[R(y,k)]
------------------------------------------------- [|- A]
 Ax[-R(a,x)] |- Ax[-(all(y,R(y,x))))]
```

There is basically 1 mark for each correct rule application (including AXIOM) Some credit may be given for almost correct rule applications.

e) Demonstrate that the following entailment is valid using *First-Order Binary Resolution*.

$$P(a),\ Q(b),\ \forall x \forall y[(P(x) \wedge Q(y)) \rightarrow R(x, y)] \ \vdash \ \exists y[R(a, y)]$$

Your demonstration should take the following form:

i) State the entailment problem as an equivalent consistency checking problem. Give the set S of formulae whose consistency should be checked. [1 mark]

ii) Transform S into a set of formulae in *clausal form*, with variables *standardised apart* [3 marks]

iii) Give a sequence of *binary resolution* inferences that proves the set inconsistent. [3 marks]

Answer:
i) The problem is equivalent to checking consistency of

$$\{P(a),\ Q(b),\ \forall x \forall y[(P(x) \wedge Q(y)) \rightarrow R(x,y)],\ \neg\exists y[R(a,y)]\}$$

ii) The clausal form (standardised apart) is:

$$\{\ \underset{1}{\{P(a)\}},\ \underset{2}{\{Q(b)\}},\ \underset{3}{\{\neg P(X), \neg Q(Y), R(X,Y)\}},\ \underset{4}{\{\neg R(a,Z)\}}\ \}$$

iii) The binary resolution proof is:

- Resolve 1 and 3 gives 5: $\{\neg Q(Y), R(a,Y)\}$
- Resolve 2 and 5 gives 6: $\{R(a,b)\}$
- Resolve 6 and 4 gives: $\{\}$. Q.E.D.

[20 marks total]

Question 2

a) Translate the following sentence into *Description Logic*. You should use concept and relation names that make clear your intended interpretation of these symbols:

Animals that are either huge, venomous or have a nasty bite are dangerous. [3 marks]

Answer:
$(\mathsf{Animal} \sqcap (\mathsf{Huge} \sqcup \mathsf{Venomous} \sqcup \exists \mathsf{has}.(\mathsf{Bite} \sqcap \mathsf{Nasty}))) \sqsubseteq \mathsf{Dangerous}$

b) Consider the following formula of *Propositional Tense Logic*:

$$\mathbf{PF}\phi \rightarrow (\mathbf{P}\phi \vee \mathbf{F}\phi)$$

State whether this formula is necessarily true according to the usual semantics for tense logic, and briefly explain your answer. [2 marks]

Answer:
No it is not necessarily true. If ϕ were true at the present time but at no other time, then $\mathbf{PF}\phi$ would be true but neither $\mathbf{P}\phi$ nor $\mathbf{F}\phi$, would be true, so the formula as a whole would be false.

c) Production of a certain device involves a number of operations, which must be carried out according to a schedule that satisfies a number of temporal constraints.

More specifically the *production period* (P) starts with *assembly* (A) and ends with *calibration* (C). The assembly period (A) starts with component fitting (F). *Calibration* (C) begins immediately after *component fitting* (F) is completed.

i) Encode the given information in terms of the relations of *Allen's Interval Calculus* (you should give 4 interval relation facts). [2 marks]

Answer:
Starts(A,P), Ends(C, P),
Starts(F,A), Meets(F, C).

ii) Show how *compositional reasoning* can be used to calculate the temporal relation that holds between the intervals of *assembly* (A) and *calibration* (C). Give details of each composition rule that you apply and how the results of these rules are combined. Depending on your representation of the given facts you may need to convert some of your relations to their converse before applying compositional inference. If you wish, you may illustrate your calculations diagrammatically. [5 marks]

Answer:
Essentially: (starts;ended-by) = {overlaps, meets, before}
(started-by;meets) = {includes,ended-by,overlaps}
Intersecting these two gives {overlaps}.
Diagrams may be used to illustrate the structure of the network relating the four intervals and also to show how the required compositions are determined.

d) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the convex hull function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, draw a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\exists r[\mathsf{P}(r, a) \wedge \mathsf{P}(r, b) \wedge \mathsf{P}(r, c)]$ [2 marks]

ii) $\mathsf{TPP}(a, b) \wedge \mathsf{DC}(c, a) \wedge \mathsf{EC}(c, b)$ [2 marks]

iii) $\mathsf{P}(a, b) \wedge \mathsf{P}(b, c) \wedge \mathsf{P}(c, a)$ [2 marks]

iv) $\mathsf{NTPP}(a, b) \wedge \exists x[\mathsf{DC}(x, b) \wedge \mathsf{P}(x, \mathsf{conv}(a)) \wedge \mathsf{P}(x, \mathsf{conv}(b))]$ [2 marks]

Answer:
Some possibilities are shown below; these are not necessarily unique; others accepted as appropriate.

```
 i)      ----------
        |  a      |
        |      ----------------
        |      |  |       b   |
        |      |  |           |
        |   |-----------|     |
        |   |  |  |     |     |
        |   |  ---------------
        |   |     |     |
       -----------      |
            |        c  |
            -------------
```

```
          -------------
ii)       |         b  |
         |------|     |------
         | a    |     |  c  |     This one is pretty easy.
         |      |     |     |
         |------|     |------
         |            |
         --------------
```

```
          ------------
iii)       | a,b,c     |
          |           |  a, b and c are all the same region
          |           |
          |           |
          |           |
          -------------
```

```
          -------------------------------
iv)      | b                            |
          |   ----------------------    |  a is ntpp of b
          |   |         a          |    |  and they have overlapping
          |   |   |------------|   |    |  concavities
          |   |   |            |   |    |
          |   |   |  |-----|   |   |    |
          |   |   |  |     |   |   |    |
          |   ----   |     |   -----    |
          ------------     --------------
```

[20 marks total]

Question 3

a) The rules of the Tiddlywinks World Championship (TWC) state that a person is eligible to represent a given country if *at least one* of the following conditions are satisfied:

i) the person was born in that country;
ii) at least one of the person's parents was born in that country;
iii) at least two of the person's grandparents were born in that country;
iv) nobody interested in tiddlywinks was born in that country.

In order to maintain their rules, the International Tiddlywinks Association (ITA) maintains a *Prolog* database of the place of birth, and parentage of the entire population of the world. They also have complete records of all people who have ever shown any interest in tiddlywinks.

More specifically the database contains a complete set of facts of the forms:

- `born(` *person* `,` *country* `)`
- `has_parent(` *person1* `,` *person2* `)`
- `interested(` *person* `)`

Write a *Prolog* definition of a predicate `eligible/2` that takes two arguments and can be used to determine who is eligible to represent which country, according to the ITA rules. In other words, the query `?- eligible(N,C).` should successively return every person's name (`N`) and country (`C`), such that person `N` is eligible to represent country `C` at the TWC.
(It is suggested that you formulate your predicate definition by means of four clauses, corresponding to each of the conditions (i–iv) given above.) [10 marks]

Answer:

```
eligible(P,C) :- born(P,C).                                        2 marks

eligible(P,C) :- has_parent( P, PP ), born( PP, C ).               2 marks

eligible(P,C) :- has_parent( P, PP1 ), has_parent( P, PP2 ),       4 marks
                 has_parent( PP1, GP1 ), has_parent( PP2, GP2 ),
                 \+( GP1 = GP2 ),
                 born( GP1, C ), born( GP2, C ).

eligible(P,C) :- \+( ( interested(X), born(X, C) ) ).              2 marks
```

b) This question concerns a *Fuzzy Logic* in which the following definitions of *linguistic modifiers* are specified:

$$\mathsf{somewhat}(\phi) = \phi^{1/2} \qquad \mathsf{slightly}(a) = \phi^{1/4} \qquad \mathsf{very}(\phi) = \phi^2$$

The logic is used to describe a dragon by the name of Attor, who possesses certain characteristics to the following degrees:

$$\mathsf{Large}(\mathsf{attor}) = 0.9 \qquad \mathsf{Fierce}(\mathsf{attor}) = 0.7 \qquad \mathsf{Clever}(\mathsf{attor}) = 0.25$$

i) Translate each of the following sentences into fuzzy logic and also give the fuzzy truth value of each proposition (under the standard fuzzy interpretation of the Boolean connectives):

A) Attor is very large and somewhat fierce but not clever. [2 marks]

B) Attor is very very clever. [2 marks]

Answer:

A) $\mathsf{very}(\mathsf{Large}(\mathsf{attor})) \wedge \mathsf{somewhat}(\mathsf{Fierce}(\mathsf{attor})) \wedge \neg\mathsf{Clever}(\mathsf{attor})$ (1 mark)

Truth value $= Min\{0.9^2, 0.7^{1/2}, (1 - 0.25)\} \; = \; Min\{0.81, 0.837, 0.75\} \; = 0.75$ (1 mark)

B) $\mathsf{very}(\mathsf{very}(\mathsf{Clever}(\mathsf{attor})))$ (1 mark)

Truth value $= ((0.25)^2)^2 = 0.00391$ (1 mark)

ii) Suppose that we also assert the following conditionals, and in the fuzzy logic we interpret these '*if ... then*' statements as expressing *fuzzy subset* relations:

A) If something is large and fierce then it is somewhat dangerous.

B) If something is clever and fierce then it is very dangerous.

How dangerous is Attor? More specifically, calculate the range of possible fuzzy truth values for the proposition $\mathsf{Dangerous}(\mathsf{attor})$, meaning that Attor is dangerous. Show how you computed this result. [6 marks]

Answer:

Attor is 'large and fierce' to degree 0.7, so according to (A) he must be 'somewhat dangerous' to at least this degree, so he must be dangerous to at least the degree 0.49. (2 marks)

Attor is 'clever and fierce' to degree 0.25, so according to (B) he must be 'very dangerous' to at least this degree, so he must be dangerous to at least the degree 0.5. (2 marks)

Thus the truth value of $\mathsf{Dangerous}(\mathsf{attor})$ is in the range [0.5..1] (2 marks)

[20 marks total]