Question 1

a) i) $\textsf{See-you-Tuesday} \leftrightarrow \neg\textsf{Go-cycling}$ (1 mark)
$\neg\textsf{Go-cycling} \rightarrow \textsf{See-you-Tuesday}$ also OK.

ii) $(S \wedge \neg H) \rightarrow (P \vee B)$ (2 marks)

b) i) $\mathcal{M}_1$ only. (1 mark)

ii) $\mathcal{M}_1$ and $\mathcal{M}_3$. (1 mark)

iii) $\mathcal{M}_1$ and $\mathcal{M}_2$. (1 mark)

iv) All four models. (1 mark)

c) i) $\forall x[\neg\textsf{Like}(x, \textsf{me}) \rightarrow \neg\textsf{Like}(\textsf{me}, x)]$ (2 marks)

ii) $\forall x[\textsf{Number}(x) \rightarrow \exists y[\textsf{Number}(y) \wedge (y > x)]]$ (2 marks)

iii) $\exists x[\textsf{Clown}(x) \wedge \textsf{Juggle}(x) \wedge \textsf{Unicycle}(x)]$
$\wedge \forall xy[(\textsf{Clown}(x) \wedge \textsf{Juggle}(x) \wedge \textsf{Unicycle}(x) \wedge \textsf{Clown}(y) \wedge \textsf{Juggle}(y) \wedge \textsf{Unicycle}(y))$
$\rightarrow (x = y)]$ (3 marks)

d)

```
                                 P, Qb, P |- Qa
                                 -------------------- (|- -)
                                 P, Qb |- Qa, -P
                                ---------------------- (|- -)
                                 P |-  -Qb, Qa, -P
                                ---------------------- (|- A)
                                P |- Ax[-Qx], Qa, -P
                                ---------------------- (- |-)
     Axiom                      -Ax[-Qx], P |- Qa, -P
---------------                ----------------------- (Erw)
-P, P |- Qa, -P    and          Ex[Qx], P |- Qa, -P
------------------------------------------------------ (v |-)
-P v Ex[Qx], P  |- Qa, -P
------------------------------ (|- v)
-P v Ex[Qx], P  |- Qa v -P
------------------------------ (->rw)
P -> Ex[Qx], P  |-  Qa  v -P
```

(5 marks — 1 for each correct rule application up to 5, -1 for each mistake)

The sequent is invalid since one branch is left open after all possible rules have been applied. (1 mark)

Question 2

a) i) $\neg\exists t[t < \mathsf{now} \wedge \mathsf{Meet}(\mathsf{susan}, \mathsf{tom}, t)]$ (1 mark)

ii) $\neg\exists t[t < \mathsf{now} \wedge \mathsf{Arrive}(\mathsf{charles}, t)] \rightarrow \neg\exists t[\mathsf{now} < t \wedge \mathsf{Arrive}(\mathsf{charles}, t)]$ (2 marks)

b) i) $\mathbf{P}\mathit{Drank} \rightarrow \mathbf{G}\mathit{sleep}$ (1 mark).

ii) $\mathbf{G}(\mathit{Work\_Bookshop} \vee \mathbf{P}\mathit{Graduate})$ (2 marks).

c) A complete specification is given below. Students will be given 1 or 2 marks for each correct formula, depending on its complexity, up to a maximum of 14 available marks.

```
Domain axioms:
food( pie )
adjacent( lounge, kitchen)

Initial Situation
Holds( hungry(tom), s0))
Holds( location(tom, lounge), s0 )
Holds( location(pie, kitchen), s0 )

Precondition axioms:
Possible( eat(a,i), s ) <- Holds( hungry(a,s) ), & Holds( located(a,l), s) &
                           Holds( Holds( located(i,l), s) & food( i ).

Possible( move(a,l1,l2) s ) <- Holds( location(a, l1, s) ) & adjacent( l1, l2)

Effect axioms:
Holds( -hungry(a), result(eat(a,i),s) ) <- possible( eat(a,i), s)
Holds( gone(a), result(eat(a,i),s) )    <- possible( eat(a,i), s)

Holds( located(a,l2), result(move(a,l1,l2),s) )  <- possible( move(a,l1,l2), s)

Frame Axioms:
Holds( located(a,l), result(eat(a,x),s) )   <-> Holds( located(a,l), s)
Holds( hungry(a), result(move(a,l1,l2),s) ) <-> Holds( hungry(a), s)
Holds( eaten(i), result(act,s) ) <- Holds( eaten(i), s)
```

Question 3

a) i) Equivalent to checking consistency of:

$$\{\neg Q,\ (Q \vee R \vee T),\ \neg(R \wedge \neg Q),\ \neg(Q \vee T)\}\ \text{(1 mark)}$$

ii) Clausal form: $\{\{Q, R, T\},\ \{\neg R, Q\},\ \{\neg Q\},\ \{\neg T\}\}$ (2 Marks)

iii) Resolution Proof:

```
1) Q, R, T
2) -R, Q
3) -Q
4) -T
5) R, T    (from 1 & 3)
6) R       (from 4 & 5)
7) Q       (from 2 & 6)
8) {}      (from 3 & 7)
```

Variant proofs are also possible. (4 Marks — ∼1 each rule)

b) $\{P(k)\}$, $\{\neg H(a, f(a))\}$, $\{G(X, b), P(a)\}$
(3 marks — one for each)

c) The system cannot be both sound and complete. (2 marks)

d) The rule states that in the absence of information to the contrary, one can infer that a friend of a friend of any person is also a friend of that person. (2 marks)

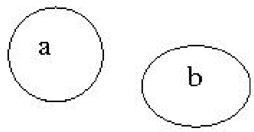
e) i) There are 2 extensions. (1 mark)

ii) A holds in all extensions. B and C hold in some (but not all) extensions, and D holds in no extensions. (4 marks)

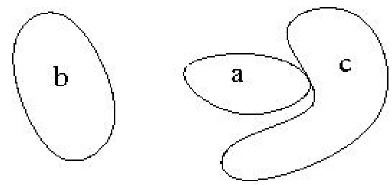
iii) There are no extensions. An extension must be closed under applicable default rules and contain only formulae derivable by classical deduction and well-founded default inference. **D4** is applicable to any extension derived from the original theory Θ (since A is true in all extensions and $\neg D$ is clearly not derivable). Thus we must infer E to ensure default closure. But then we can derive ¬D from **C3** and this undercuts **D4**, so the application of **D4** is not well-founded. (2 marks)

Question 4

a) Many variants are possible as long as the formulae are satisfied. Typical diagrams would be:



i)



ii)

(1 mark for i) and 3 marks for ii).)

b) i) **C1** $(\mathsf{Begins}; \mathsf{Ended\text{-}by}) = \{\mathsf{Overlaps}, \mathsf{Meets}, \mathsf{Before}\}$ (2 marks)

C2 $(\mathsf{Contains}; \mathsf{Meets}) = \{\mathsf{Contains}, \mathsf{Ended\text{-}by}, \mathsf{Overlaps}\}$ (2 marks)

R1) $\mathsf{Begins}(sunny, morning)$ (1 mark)

R2) $\mathsf{Ended\text{-}by}(morning, read)$ (1 mark)

R3) $\mathsf{Contains}(sunny, coffee)$ (1 mark)

R4) $\mathsf{Meets}(coffee, newspaper)$ (1 mark)

ii) $\mathsf{Begins}(sunny, morning); \mathsf{Ended\text{-}by}(morning, read)$
$$\Longrightarrow \{\mathsf{Overlaps}, \mathsf{Meets}, \mathsf{Before}\}(sunny, read)$$
$\mathsf{Contains}(sunny, coffee); \mathsf{Meets}(coffee, read)$
$$\Longrightarrow \{\mathsf{Contains}, \mathsf{Ended\text{-}by}, \mathsf{Overlaps}\}(sunny, read)$$
$\{\mathsf{Overlaps}, \mathsf{Meets}, \mathsf{Before}\}(sunny, read) \wedge \{\mathsf{Contains}, \mathsf{Ended\text{-}by}, \mathsf{Overlaps}\}(sunny, read)$
$$\Longrightarrow \mathsf{Overlaps}(sunny, read)$$

Thus we derive: $\mathsf{Overlaps}(sunny, read)$
(4 marks)

c) i) $\mathsf{Human} \sqsubseteq (\neg\mathsf{Aquatic} \sqcap \mathsf{Mammal})$ (2 marks)

ii) $\exists\mathsf{hasChild.Human} \sqsubseteq \mathsf{Human}$ (2 marks)