This question paper consists
of 4 printed pages, each of
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Examination for the degree of BSc or BA

AI34: KNOWLEDGE REPRESENTATION AND INFERENCE

Time allowed: 2 hours

Answer *three* questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Question 1

a) Represent the following sentences using *propositional* logic:

i) On Saturdays, if it is sunny I go into town or out to the countryside. [2 marks]

ii) Janet cannot take both logic and astrophysics unless she also takes maths. [2 marks]

b) A model $\mathcal{M}$ for a propositional language is represented by the following structure:

$$\{P = \mathbf{f},\ Q = \mathbf{t},\ R = \mathbf{f}\}$$

i) What truth-value is assigned by $\mathcal{M}$ to the formula $\neg(P \vee Q) \rightarrow R$? [1 mark]

ii) Does $\mathcal{M}$ satisfy the formula $(Q \wedge \neg R) \wedge (P \vee R)$? [1 mark]

c) Translate the following sentences into 1st-order predicate logic (using equality where necessary):

i) Every boy who owns a yo-yo is happy. [2 marks]

ii) If any two regions overlap they must share a common part. [3 marks]

d) i) Using the predicates $\mathbf{InSameClass}(x, y)$, $\mathbf{Taller}(x, y)$ and equality, represent in 1st-order predicate logic the sentence: "John is the tallest boy in his class." [2 marks]

ii) Write down a 1st-order formula representing a fundamental property of the relation $\mathbf{Taller}(x, y)$. [2 marks]

e) Using the *Sequent Calculus* (as specified in the AI34 course notes), prove that the following sequent is valid:

$$\forall x[P(x) \vee Q(x)],\ \forall x[\neg P(x)]\ \vdash\ \forall x[Q(x)]$$ [5 marks]

[Total 20 marks]

Question 2

a) Use *propositional tense logic* to represent the sentence:

I have not watched the film nor read the book and I shall not watch the film unless I have read the book. [2 marks]

b) Let ϕ stand for the condition 'the alarm is sounding', ψ stand for 'the security system has been reset' and θ stand for 'the security system is in an alert state'. A security system is designed to satisfy the following formula specified in tense logic:

$$\mathbf{G}(\mathbf{P}\phi \rightarrow (\theta \vee \mathbf{P}\psi))$$

i) Give an interpretation of the formula in English. [2 marks]

ii) Assuming the formula is satisfied, is it possible for there to be some time in the future when the alarm is sounding but the system is not in an alert state? Explain your answer briefly. [2 marks]

c) A Situation Calculus theory contains fluents of the forms

$has(agent, item)$, $located(item\text{-}or\text{-}agent, room)$, $adjacent(room1, room2)$, $clean\text{-}floor(room)$

and constants denoting various items such as broom, $\mathsf{litter\text{-}bin}$, etc..

It also has the following actions

$\mathbf{pickup}(agent, item)$, $\mathbf{drop}(agent, item)$, $\mathbf{move}(agent, room1, room2)$, $\mathbf{sweep}(agent, room)$.

Among others, the theory contains the axiom **A**:

$$\forall\ a\ r\ [\mathsf{Holds}(clean\text{-}floor(r), \mathsf{result}(\mathbf{sweep}(a,r)))\ \leftarrow\ \mathsf{Poss}(\mathbf{sweep}(a,r), s)]$$

i) What kind of axiom is **A**? [1 mark]

ii) Using the Poss predicate, give a suitable *precondition axiom* specifying the conditions under which an action $\mathbf{sweep}(a, r)$ is possible. [2 marks]

iii) Give (again using Poss) a suitable *precondition axiom* specifying the conditions under which $\mathbf{move}(a, r1, r2)$ is possible. [2 marks]

iv) Give a *frame* axiom which specifies that: when an agent moves into a room, this does not affect the state of the floor of that room. [2 marks]

v) How might the $clean\text{-}floor(r)$ fluent be affected by a *ramification* of one of the actions other than $\mathbf{sweep}(a, r)$? [2 marks]

vi) Suppose that we incorporate *Default Logic* rules into the situation calculus formalism. Write down a default rule stating that: unless we have reason to believe an agent has an item we can assume he/she doesn't. [2 marks]

d) Let $\mathcal{TL}$ be a 1st-order temporal language containing propositional constants (e.g. *Error_Sig*, *Explode*, *Shut_Down* etc.), time variables, $t_1, \ldots, t_n$, which may be quantified over, a strict temporal ordering relation $<$, a constant 'now' denoting the current time, and a construct $\mathsf{HoldsAt}(\phi, t)$ meaning that proposition ϕ is true at time t.

Use $\mathcal{TL}$ to represent the sentence 'If an error has been signalled the reactor will shut down before it explodes.' [3 marks]

[Total 20 marks]

Question 3

a) Give a *binary resolution* proof of the inconsistency of the following set of propositional clauses:

$$\{\{A, B, C\}, \{\neg A\}, \{\neg B, E, D\}, \{\neg C\}, \{A, \neg E\}, \{C, \neg D\}\}$$ [5 marks]

b) You are evaluating the safety of a robotic surgical instrument. A software verification tool based on *sound* logical reasoning has produced a proof that the software controlling the instrument satisfies a set of operating safety requirements. You then learn that the proof procedure used by the verification tool is *not complete*. Can you be sure that the software really satisfies the safety requirements? Briefly explain your answer. [2 marks]

c) A default theory Θ contains the classical facts

C1 $\mathsf{Adult}(\mathsf{fred})$
C2 $\mathsf{Rich}(\mathsf{fred})$
C3 $\forall x[\mathsf{Works}(x) \rightarrow \mathsf{Stressed}(x)]$

and the default rules:

D1 $\mathsf{Adult}(x) : \mathsf{Works}(x) \ / \ \mathsf{Works}(x)$
D2 $\mathsf{Rich}(x) : \neg\mathsf{Stressed}(x) \ / \ \mathsf{Happy}(x)$

i) Interpret **D1** in English. [2 marks]

ii) Interpret **D2** in English. [2 marks]

iii) Is $\mathsf{Works}(\mathsf{fred})$ true in some, all or no, *extensions* of Θ? Briefly xplain your answer. [2 marks]

iv) Is $\mathsf{Happy}(\mathsf{fred})$ true in some, all or no, *extensions* of Θ? Briefly explain your answer. [2 marks]

Suppose we learn the additional fact $\mathsf{Retired}(\mathsf{fred})$.

v) What simple simple classical axiom should we add to Θ in order to take account of the significance of this fact? [2 marks]

vi) What further facts about fred would now hold in all extensions of the new theory? [2 marks]

d) What kind of inference is illustrated by the following line of reasoning?

No one has ever seen a dragon.
Therefore, dragons do not exist. [1 mark]

[Total 20 marks]

Question 4

a) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the *convex hull* function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, *draw* a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\mathsf{PO}(a, b) \wedge \mathsf{P}(\mathsf{sum}(a, b), c)$ [1 mark]

ii) $\mathsf{DC}(a, b) \wedge \mathsf{TPP}(a, \mathsf{conv}(b))$ [1 mark]

iii) $\neg\exists r[\mathsf{P}(r, a) \wedge \neg\mathsf{P}(r, b)]$ [1 mark]

b) A theory of the geometry of spatial points is formulated using the relation $\mathsf{B}(x, y, z)$, which is true just in case: point y lies on the straight line between points x and z. (Hence, $\mathsf{B}(x, y, z)$ can only hold if all three points are distinct.)

You wish to add to this theory a new relation $\mathsf{Col}(x, y, z)$, which holds just in case x, y and z are *collinear* (which includes the case where two or all points are identical). Give a 1st-order formula which defines $\mathsf{Col}(x, y, z)$ in terms of $\mathsf{B}(x, y, z)$ and '='. [2 marks]

c) The following questions concern compositional reasoning in terms of the relational partition given by the RCC-8 set of topological relations, $\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{TPP}, \mathsf{NTPP}, \mathsf{TPPi}, \mathsf{NTPPi}, \mathsf{EQ}\}$:

i) Draw simple diagrams illustrating the possible RCC-8 relations that could hold between a and c if we know that $\mathsf{EC}(a, b)$ and $\mathsf{TPP}(b, c)$. [2 marks]

ii) Give the disjunctive relation, expressed as a subset of the RCC-8 relations, that is equivalent to the composition $(\mathsf{EC}; \mathsf{TPP})$. [2 marks]

iii) Give the disjunctive relation, expressed as a subset of the RCC-8 relations, that is equivalent to the composition $(\mathsf{TPP}; \mathsf{EC})$. [2 marks]

iv) A spatial configuration of regions satisfies the following relations:
$\mathsf{EC}(a, b)$, $\mathsf{TPP}(b, c)$, $\mathsf{TPP}(a, d)$, $\mathsf{EC}(d, c)$.
Draw a diagram of the network of relations and using compositional reasoning determine the unique RCC-8 relation that must hold between regions a and c. [3 marks]

d) i) Translate the statement 'Whales are aquatic mammals' into *Description Logic*. [2 marks]

ii) Translate the statement 'No edible mushroom is purple' into *Description Logic*. [2 marks]

iii) Use *Description Logic* to define the concept the $\mathsf{Proud\text{-}Father}$ in terms of the concepts Father and $\mathsf{Successful}$ and the role $\mathsf{has\text{-}child}$. [2 marks]

[Total 20 marks]

End