Knowledge Representation: Probabilistic Graphs

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Knowledge Representation and Reasoning Module

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Slides based on material from Daphne Koller, Adrian Weller, Zoubin Ghahramani, Henrik I. Christensen, and Karteek Alahari.

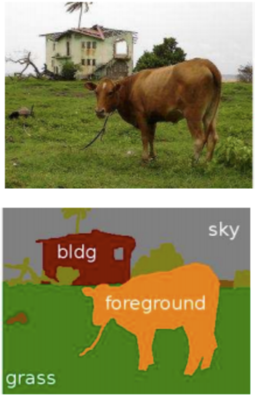
What is this class about ?

Introduction to **probabilistic graphical models** and modeling the uncertainty.

Introduction to ***ProbLog* Framework** .

Motivation

Image Segmentation



e.g., [He et al., 2004; Shotton et al., 2006; Gould et al., 2009]

Task : pixels $\rightarrow$ pixel labels {building, grass, cow, sky}

- many variables
- uncertainty of the correct answer, *i.e.* label

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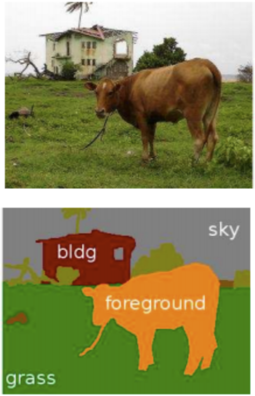

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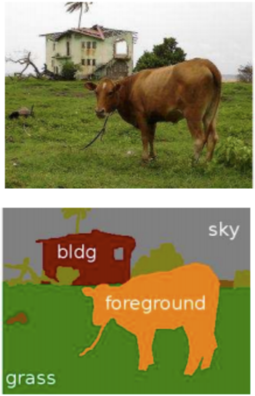

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Task : pixels $\rightarrow$ pixel labels {building, grass, cow, sky}

- many variables
- uncertainty of the correct answer, *i.e.* label

Probabilistic Graphical Models provide a framework to address these problems.

Why probabilistic ?

To model **uncertainty**, due to :

- partial knowledge of state of the world
- noisy observations
- cases/phenomena not observed by the model

Probabilistic theory provides :

- standalone representation with clear semantics
- reasoning patterns
- learning methods

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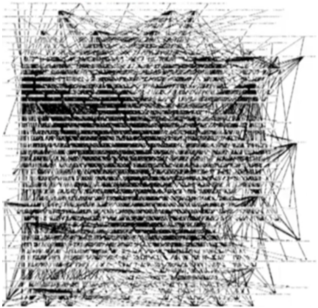
Why graphical ?

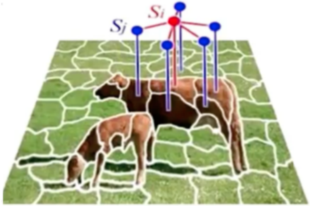
Intersection of ideas from probability theory and computer science.

For representing large number of variables

Random variables $Y_1, Y_2, \ldots Y_n$

Graphical models capture uncertainty through joint distribution $P(Y_1, Y_2, \ldots, Y_n)$.





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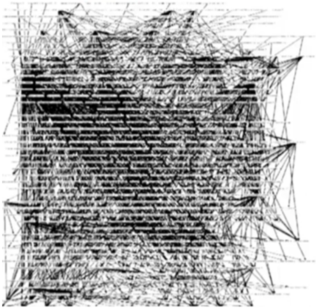
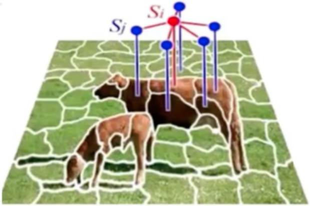




(Probabilistic) Graphical Models

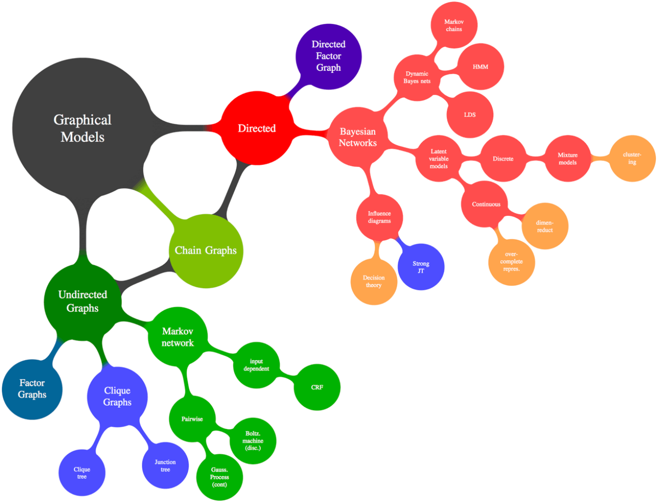


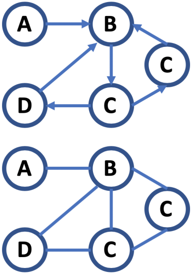
Figure – Image by David Barber

Graphs

The Basics

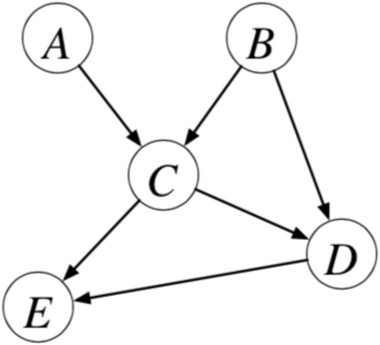
Basics concepts :

- Vertices/Nodes
- Edges
- Directed & Undirected
- Degree & In/Out Degree
- Neighbours & Parent/Child
- Walk, Path, and Cycle



Probabilistic Graphs

Knowledge Representation



Nodes represent **random variables**.

Edges represent **dependencies** between variables.

Directed Acyclic Graphical Model

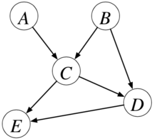
Bayesian Networks

A **Bayesian network** is specified by a DAG $= (V, \overrightarrow{E})$ with :

- one node $i \in V$ for each random variable X_i
- one conditional probability distribution (CPD) per node, $p(x_i | x_{Pa(i)})$, specifying the variable's probability conditioned on its parents' values

So the probability distribution which describes the DAG is :

$$p(x_i, ..., x_n) = \prod_{i \in V} p(x_i | x_{Pa(i)})$$



A DAG Model, as the one shown above, corresponds to the joint probability distribution :

$$p(A, B, C, D, E) = p(A)p(B)p(C|A, B)p(D|B, C)p(E|C, D)$$

Inference in a Graphical Model

Basic concepts

- Consider inference of $p(x, y)$ we can formulate this as :

$$p(x, y) = p(x|y)p(y) = p(y|x)p(x)$$

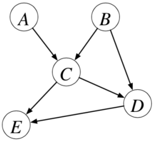
- We can further marginalize

$$p(y) = \sum_{x'} p(y|x')p(x')$$

- Using *Bayes Rule* we can reverse the inference

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

Inference in a Graphical Model



Consider the graph shown above, which represents :

$$p(A,B,C,D,E) = p(A)p(B)p(C|A,B)p(D|B,C)p(E|C,D)$$

Inference : Evaluates the probability distribution over some set of variables, given the values of another set of variables.

For example, how can we compute $p(A|C=c)$? Assume each variable is binary.

Inference in a Graphical Model

$$p(A|C = c) = \frac{p(A,C=c)}{p(C=c)}$$

$p(C = c) = \sum_A p(A, C = c)$

Naive method :

$p(A, C = c) = \sum_{B,D,E} p(A, B, C = c, D, E)$

More efficient method :

$p(A, C = c)$

$= \sum_{B,D,E} p(A)p(B)p(C = c|A, B)p(D|B, C = c)p(E|C = c, D)$

$= \sum_B p(A)p(B)p(C = c|A, B) \sum_D p(D|B, C = c) \sum_E p(E|C = c, D)$

$= \sum_B p(A)p(B)p(C = c|A, B)$

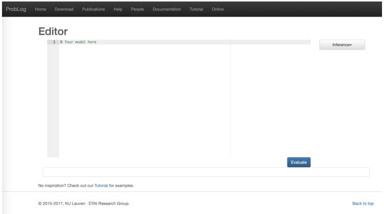
Introduction to *ProbLog*

Used for :

- computing marginal probabilities given evidences
- learn form partial interpretations
- solve inference tasks
- solve decision theoretic problems
- model uncertainty
- express complex probabilistic models

Installation

- Online Editor
  https://dtai.cs.kuleuven.be/problog/editor.html



- Import as a python package

```
pip3 install problog --user
```

- Standalone tool

ProbLog Program

A *ProbLog* program consists of two parts :

1. a set of ground probabilistic facts
   i.e. ***p : :f***
2. a logic program, a set of logic rules and non-probabilistic facts

More examples : https://dtai.cs.kuleuven.be/problog/tutorial/basic/08_rule_probs.html

Coding in *ProbLog*

Example *Heads*

Calculate the probability of both coins being head.

$$P(\text{allHeads}) = P(\text{heads1}) * P(\text{heads2})$$

```
% Probabilistic facts:
0.5::heads1.
0.6::heads2.

% Rules:
allHeads :- heads1, heads2.

% Queries:
query(allHeads).
```

Coding in *ProbLog*

Example *Heads*

Calculate the probability of neither of the coins being head.

$$P(\text{noHeads}) = (1 - P(\text{heads1})) * (1 - P(\text{heads2}))$$

```
% Probabilistic facts:
0.5::heads1.
0.6::heads2.

% Rules:
noHeads :- \+heads1, \+heads2.

% Queries:
query(noHeads).
```

Coding in *ProbLog*

Example *Heads*

Calculate the probability of at least one of the two coins is head; one of the two is bend(biased).

$$P(\text{someHeads}) = (1 - P(\text{noHeads}))$$
$$P(\text{noHeads}) = (1 - P(\text{heads1})) * (1 - P(\text{heads2}))$$

```
% Probabilistic facts:
0.5::heads1.
0.6::heads2.

% Rules:
someHeads :- heads1.
someHeads :- heads2.

% Queries:
query(someHeads).
```

Coding in *ProbLog*

Example *Smokers*

```
0.3::stress(X) :- person(X).
0.2::influences(X,Y) :- person(X), person(Y).
smokes(X) :- stress(X).
smokes(X) :- friend(X,Y), influences(Y,X), smokes(Y).

0.4::asthma(X) :- smokes(X).

person(angelika).
person(joris).
person(jonas).
person(dimitar).

friend(joris,jonas).
friend(joris,angelika).
friend(joris,dimitar).
friend(angelika,jonas).
```

Coding in *ProbLog*

Example *Smokers*

What is the probability of Dimitar smoking ?

```
0.3::stress(X) :- person(X).
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query(smokes(dimitar)).
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query(smokes(dimitar)).
```

$$P(\text{smokes}(\text{Dimitar})) = P(\text{stress}(\text{Dimitar}))$$

Coding in *ProbLog*

Example *Smokers*

What is the probability of Angelika smoking ?

```
0.3::stress(X) :- person(X).
0.2::influences(X,Y) :- person(X), person(Y).
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query(smokes(angelika)).
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Coding in *ProbLog*

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What is the probability of Angelika smoking ?

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friend(joris,angelika).
friend(joris,dimitar).
friend(angelika,jonas).

query(smokes(angelika)).
```

$$P(\text{smokes}(\text{Angelika})) =$$
$$1 - \left[\left(1 - P(\text{stress}(\text{Angelika}))\right) * \left(1 - \left(P(\text{influences}(\text{Jonas}, \text{Angelika})) * P(\text{smokes}(\text{Jonas}))\right)\right)\right]$$

Useful Resources

- *ProbLog* Editor : https://dtai.cs.kuleuven.be/problog/editor.html
- *ProbLog* python package installation : https://problog.readthedocs.io/en/latest/install.html#installing-python
- Python installation : https://docs.python.org/3.5/using/windows.html
- Use *ProbLog* as a python library : https://dtai.cs.kuleuven.be/problog/tutorial/advanced/01_python_interface.html
- *ProbLog* Documentation : https://problog.readthedocs.io/en/latest/
- *ProbLog* examples : https://dtai.cs.kuleuven.be/problog/tutorial/basic/01_coins.html