This question paper consists
of 5 printed pages, each of
which is identified by the
Code Number COMP5450M01

School of Computing

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KRR: KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

Answer THREE questions.

This is an open notes examination. Candidates may take with them into the examination room their lecture notes, photocopies and handouts, but no text books. Reproduction or simple rephrasing is unlikely to win credit in any question.

Turn over for question 1

Question 1

a) Represent the following sentences using *Propositional Logic*:

i) Susan cannot take both Physics and Latin. [1 mark]

ii) On clear nights when the moon is full I eat cheese but never leave my bedroom. [2 marks]

b) Translate the following sentences into *Propositional Tense Logic*:

i) If you have not bought a ticket you will not be able to board the train. [2 marks]

ii) I have never visited Russia and if I do it will be after graduating. [2 marks]

c) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

i) Humans are the only featherless bipeds. [2 marks]

ii) A committee must have at least two members. [2 marks]

d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with a predicate P and a binary relation R. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$; and the denotations of P and R are is as follows:

- $\delta(P) = \{a, b, c\}$
- $\delta(R) = \{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\}$

Which of the following formulae are satisfied by this model? [2 marks]

F1. $\exists x[R(x, x)]$
F2. $\forall x \exists y[R(x, y)]$
F3. $\forall x[P(x) \rightarrow \exists y[R(x, y)]]$
F4. $\forall xyz[(R(x, y) \wedge R(y, z)) \rightarrow R(x, z)]$

e) Using the *Sequent Calculus* (as specified in the module notes), determine whether the following sequent is valid:

$$\forall x[\neg P(x)], \quad \forall x[P(x) \vee Q(x)] \ \vdash \ \forall z[Q(z)]$$ [7 marks]

[20 marks total]

Question 2

a) Consider the following scenario:

Alice finds herself in strange hallway. On a glass table is a key and a bottle marked "DRINK ME". Behind a curtain is a tiny locked door. She drinks the contents of the bottle and shrinks, so that she is small enough to go through the door. But she has left the key on the table and cannot reach it. On the verge of tears, she then finds a small cake labelled "EAT ME" and also a white fan.

Observing this predicament, a logician sets out to help Alice by representing the perplexing situation in *Situation Calculus*. The logician produces the following partial specification:

Effect axioms: $\forall s[\mathsf{Poss}(\mathsf{eat\_cake}, s) \rightarrow \mathsf{Holds}(\mathsf{alice\_tall}, \mathsf{result}(\mathsf{eat\_cake}, s))]$

$\forall s[\mathsf{Poss}(\mathsf{use\_fan}, s) \rightarrow \mathsf{Holds}(\mathsf{alice\_small}, \mathsf{result}(\mathsf{use\_fan}, s))]$

$\forall s[\mathsf{Poss}(\mathsf{unlock}, s) \rightarrow \neg\mathsf{Holds}(\mathsf{door\_locked}, \mathsf{result}(\mathsf{unlock}, s))]$

$\forall s[\mathsf{Poss}(\mathsf{through\_small\_door}, s) \rightarrow \mathsf{Holds}(\mathsf{in\_Wonderland}, \mathsf{result}(\mathsf{through\_small\_door}, s))]$

$\forall s \forall x[\mathsf{Poss}(\mathsf{pick\_up}(x), s) \rightarrow \mathsf{Holds}(\mathsf{have}(x), \mathsf{result}(\mathsf{pick\_up}(x), s))]$

Initial state: $\mathsf{Holds}(\mathsf{alice\_small}, \mathsf{s0}) \wedge \mathsf{Holds}(\mathsf{door\_locked}, \mathsf{s0})$

To complete this representation, you must supply the following further information:

i) Give a *domain* axiom stating that there can be no situation in which Alice is both tall and small. [1 mark]

ii) Give suitable *precondition* axioms specifying conditions under which each of the five actions can be performed. [5 marks]

iii) Give appropriate *frame* axioms for the $\mathsf{have}(x)$ fluent. [4 marks]

iv) Give a sequence of actions that will result in the fluent $\mathsf{in\_Wonderland}$ holding in the final state. [2 marks]

b) A *Default Logic* theory contains the classical facts:

C1. $\mathsf{Owns\_computer}(\mathsf{Bill})$
C2. $\mathsf{Computing\_Student}(\mathsf{Linus})$
C3. $\neg\mathsf{Likes}(\mathsf{Linus}, \mathsf{Windows})$
C4. $\neg\exists x[(\mathsf{Runs}(x, \mathsf{Windows}) \vee \mathsf{Runs}(x, \mathsf{Linux})) \wedge \mathsf{Runs}(x, \mathsf{MacOS})]$

and the following default rules:

D1. $\mathsf{Computing\_student}(x) \;:\; \mathsf{Owns\_computer}(x) \;/\; \mathsf{Owns\_computer}(x)$
D2. $\mathsf{Computing\_student}(x) \;:\; \mathsf{Runs}(x, \mathsf{Linux}) \;/\; \mathsf{Runs}(x, \mathsf{Linux})$
D3. $\mathsf{Owns\_computer}(x) \;:\; \neg\mathsf{Computing\_student}(x),\; \mathsf{Runs}(x, \mathsf{Windows}) \;/\; \mathsf{Runs}(x, \mathsf{Windows})$
D4. $\mathsf{Owns\_computer}(x) \wedge \neg\mathsf{Likes}(x, \mathsf{Windows}) : \mathsf{Runs}(x, \mathsf{MacOS}) \;/\; \mathsf{Runs}(x, \mathsf{MacOS})$

i) Explain the meaning of the default rule **D3**. [2 marks]

ii) Determine how many extensions this theory has. For each extension state which instances of the $\mathsf{Runs}(x, y)$ relation are true in that extension and explain how each of these instances can be derived from the axioms and default rules. [6 marks]

[20 marks total]

Question 3

a) Convert the following formulae into *clausal form*:

   i) $\neg((\neg P \wedge (Q \vee R)) \rightarrow \neg(\neg S \rightarrow P))$ [2 marks]

   ii) $\forall x \exists y [P(x) \rightarrow (Q(x,y) \wedge R(y,x))]$ [2 marks]

b) Compute the *most general unifier* (if any) of the following pairs of expressions, showing your working. Also state the unified expression or explain why there is no unifier. You should *not* rename variables before unifying. Variables are shown in italics. [4 marks]

   i) f(v, g(u, h(w,s)), s, w, 42))
   f(w, g(42,z), 45, h(u, 24),u))

   ii) h(f(k(w,w), w), w)
   h(f(y,y), v)

c) Compute all the factors of the following clause:

$$\{\ \mathsf{T}(44, y),\ \mathsf{T}(33, x),\ \mathsf{T}(x, y),\ \neg\mathsf{T}(y, x),\ \neg\mathsf{T}(a, x)\ \}$$ [3 marks]

d) Describe the relative strengths and weaknesses of following inference systems: (a) the Gentzen Sequent Calculus for First Order Predicate Calculus (FOPC), (b) Resolution for FOPC, (c) Composition in the relational calculus. You should consider the expressiveness of the representation language, the naturalness of the proofs, and the efficiency of inference (giving reasons why inference is more or less efficient). [4 marks]

e) A *Prolog* database contains facts of the forms: `lives(X, Y)`, meaning that `X` is a person who lives in town or city `Y`; and `in_country(X, Y)`, meaning that the town or city `X` is located in the country `Y`.

   Define the following Prolog predicates:

   i) a predicate `compatriot(X, Y)`, which is true if persons `X` and `Y` live in the same country. [2 marks]

   ii) a predicate `city_inhabitants(C,L)`, such that if `C` is the name of a town or city, then `L` is the set of all people living in that city. [2 marks]

   iii) a predicate `uninhabiited(C)`, which is true if `C` is the name of a town or city in which no one lives (according to the facts in the Prolog database). [1 marks]

[20 marks total]

Question 4

a) Give a possible informal rendering in English of the following description logic expression: [1 mark]

$$\mathsf{Male} \sqcap \exists \mathsf{hasChild.Person} \sqcap \forall \mathsf{hasChild.}(\neg \mathsf{Male})$$

b) Translate the following sentences into *Description Logic*. You should use concept and relation names that make clear your intended interpretation these symbols:

i) Students are either undergraduate-students or postgraduate-students. [1 mark]

ii) Postgraduates have a bachelors degree. [1 mark]

iii) Brilliant students have all grades of A. [2 marks]

c) Consider the following project plan for a student,which is formalised in terms of Allen's temporal interval relations and the $\mathsf{Occurs}(\mathit{event}, \mathit{interval})$ relation:

- $\mathsf{Occurs}(\mathit{related\text{-}work\text{-}reading},\ r)$
- $\mathsf{Occurs}(\mathit{dissertation\text{-}writing},\ w)$
- $\mathsf{Occurs}(\mathit{design\text{-}implementation},\ i)$
- $\mathsf{Occurs}(\mathit{evaluation},\ e)$
- $\mathsf{Overlaps}(r, w)$
- $\mathsf{Contains}(w, i)$
- $\mathsf{Overlapped\text{-}by}(e, i)$
- $\mathsf{Before}(r, e)$

i) Draw a directed graph showing the temporal relationships holding between the four intervals r, w, i, e. If no temporal relationship has been specified, then label the arc with $\top$, denoting the disjunction of all the Allen relationships. [2 marks]

ii) For any arcs labelled with $\top$, use compositional reasoning to determine a more restricted set of relations which may hold on that arc. [5 marks]

d) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC), together with the spatial sum function and the convex hull function, conv. The quantifiers range over non-empty spatial regions. For each of the following formulae, draw a configuration of the regions (labelled a, b and c as appropriate) which satisfies the formula:

i) $\mathsf{PO(a,b)} \wedge \mathsf{EC(b,c)} \wedge \mathsf{NTPP(a,c)}$ [2 marks]

ii) $\forall x[\mathsf{PO}(x, \mathsf{a}) \rightarrow \mathsf{PO}(x, \mathsf{b})]$ [2 marks]

iii) $\mathsf{P(a, conv(b))} \wedge \neg \mathsf{P(a,b)}$ [2 marks]

iv) $\mathsf{EQ(c, sum(a,b))} \wedge \neg \mathsf{SCON(c)}$ [2 marks]

[20 marks total]