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Class: Machine Learning

Policy Iteration

Instructor: Matteo Leonetti

Learning outcomes

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- Define and compute the value and action value function for a given policy
- Perform a step of generalized policy iteration

Example: policy

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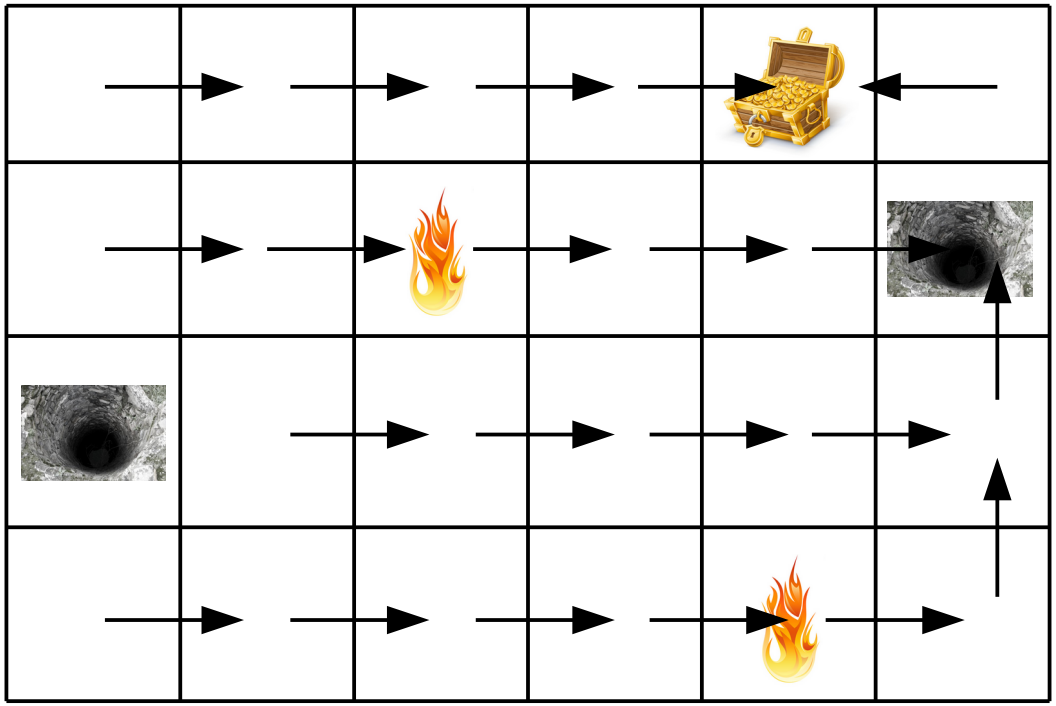

Is this a good policy? How do we establish the value of a policy?

Value

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We define the *value* of a state under a given policy, as the expected return from that state while following the policy:

$$G_t \equiv \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} = R_{t+1} + \gamma G_{t+1}$$

$$v_\pi(s) \equiv E_\pi[G_t | S_t = s] = E_\pi[R_{t+1}] + \gamma E_\pi[G_{t+1}]$$

$$= E_\pi[R_t] + \gamma v(s_{t+1})$$

This is called the *Bellman* equation

Understanding the value

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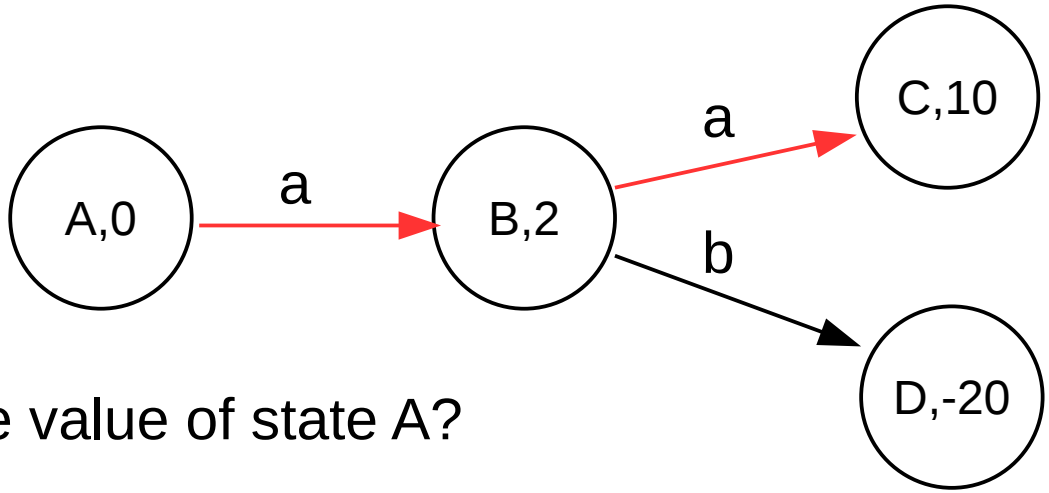

What is the value of state A?

It depends on what the agent is going to do in B (the policy)

Assuming:

$\gamma = 0.5$

$v_\pi(B) = 10$

$\pi(B) = a$

$v_\pi(A) = 2 + \gamma v_\pi(B) = 7$

Understanding the value

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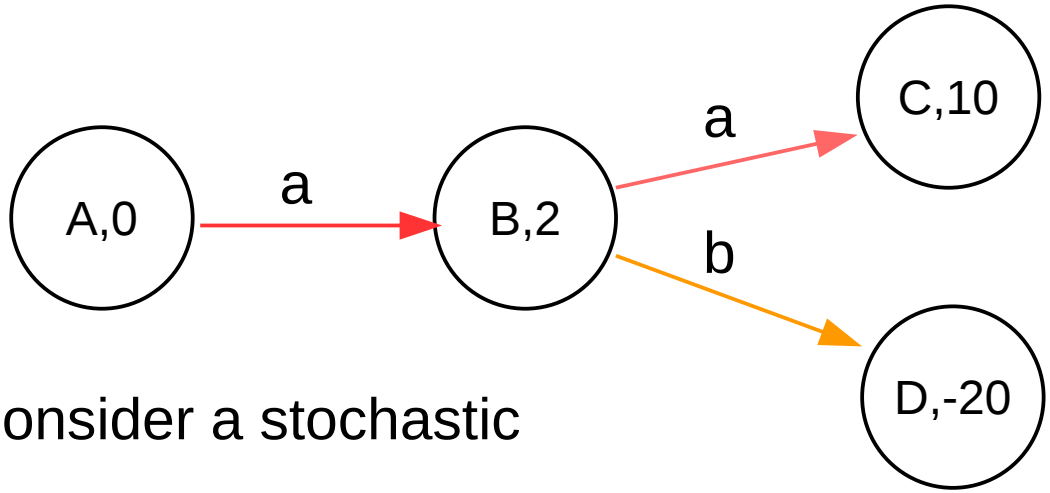

Now let's consider a stochastic policy:

$\gamma = 0.5$

$\pi(a|B) = 0.8$

$\pi(b|B) = 0.2$

$v_\pi(B) = 0.8 \cdot 10 + 0.2 \cdot (-20) = 4$

$v_\pi(A) = 2 + \gamma\, v_\pi(B) = 4$

A and B are less valuable under this policy, because the agent may go to the bad state.

Understanding the value

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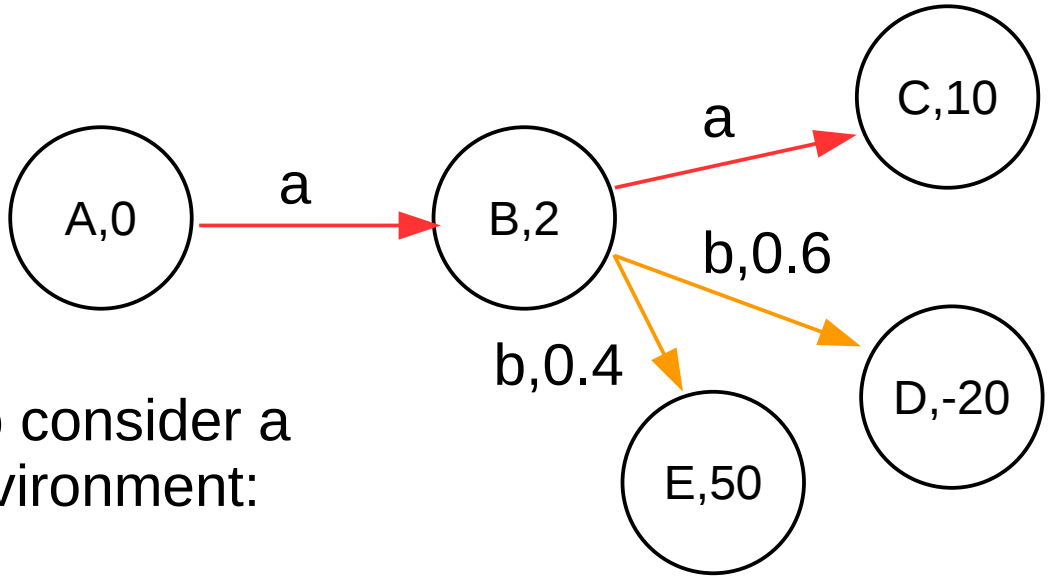

Now let's also consider a stochastic environment:

$\gamma = 0.5$

$\pi(a|B) = 0.8$

$\pi(b|B) = 0.2$

$T(B,b,D) = 0.6$

$T(B,b,E) = 0.4$

$$v_\pi(B) = 0.8 \cdot 10 + 0.2 \cdot (0.6 \cdot (-20) + 0.4 \cdot 50) = 9.6$$

$$v_\pi(A) = 2 + \gamma v_\pi(B) = 6.8$$

Taking b in B may end up in a good state, and the values reflect that.

Understanding the value

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$$v_\pi(B) = 0.8 \cdot 10 + 0.2 \cdot \boxed{(0.6 \cdot (-20) + 0.4 \cdot 50)} = 9.6$$

Probability of taking an action

consequences of action

Breaking down action consequences:

$$(0.6 \cdot (-20 + 0.5 \cdot 0) + 0.4 \cdot (50 + 0.5 \cdot 0))$$

$$(p(D|B,b) \cdot (r(B,b,D) + \gamma v_\pi(D)) + p(E|B,b) \cdot r(B,b,E) + \gamma v_\pi(D))$$

Having chosen an action (b, in this case), the value is the probability of ending up in each possible state, times the value of that state.

Understanding the value

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Probability of taking an action

consequences of action

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s'} p(s'|s,a)\left(r(s,a,s') + \gamma v_\pi(s')\right)$$

probability of each possible state

value of each possible state

Example: going to work

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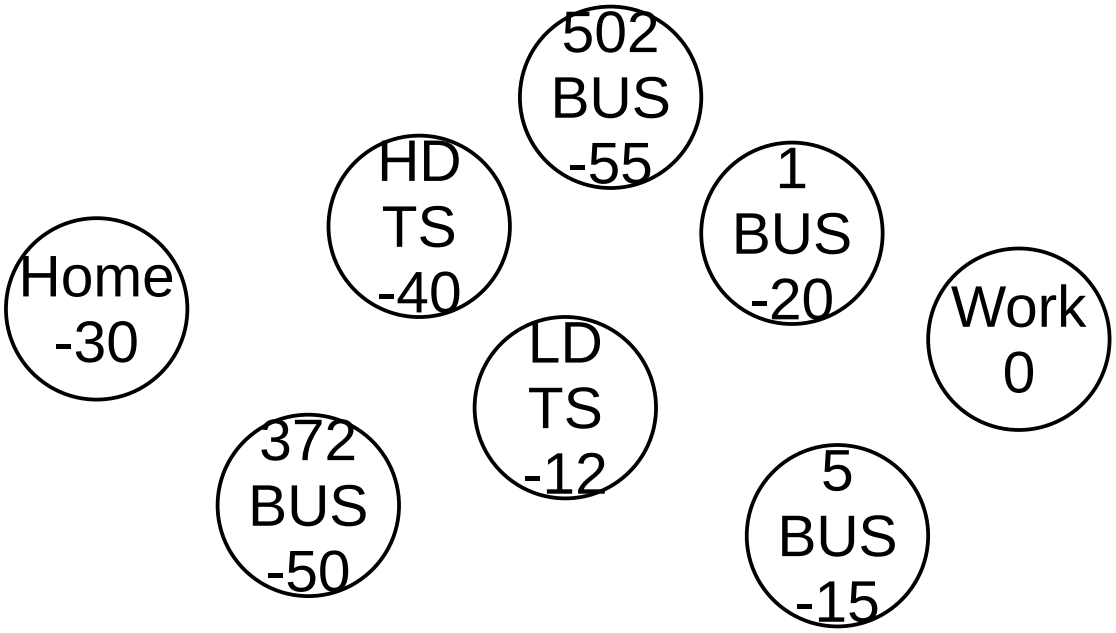

Actions: walk, take bus, take train, drive

How do I retrieve a policy if I only have the values?

Example: going to work

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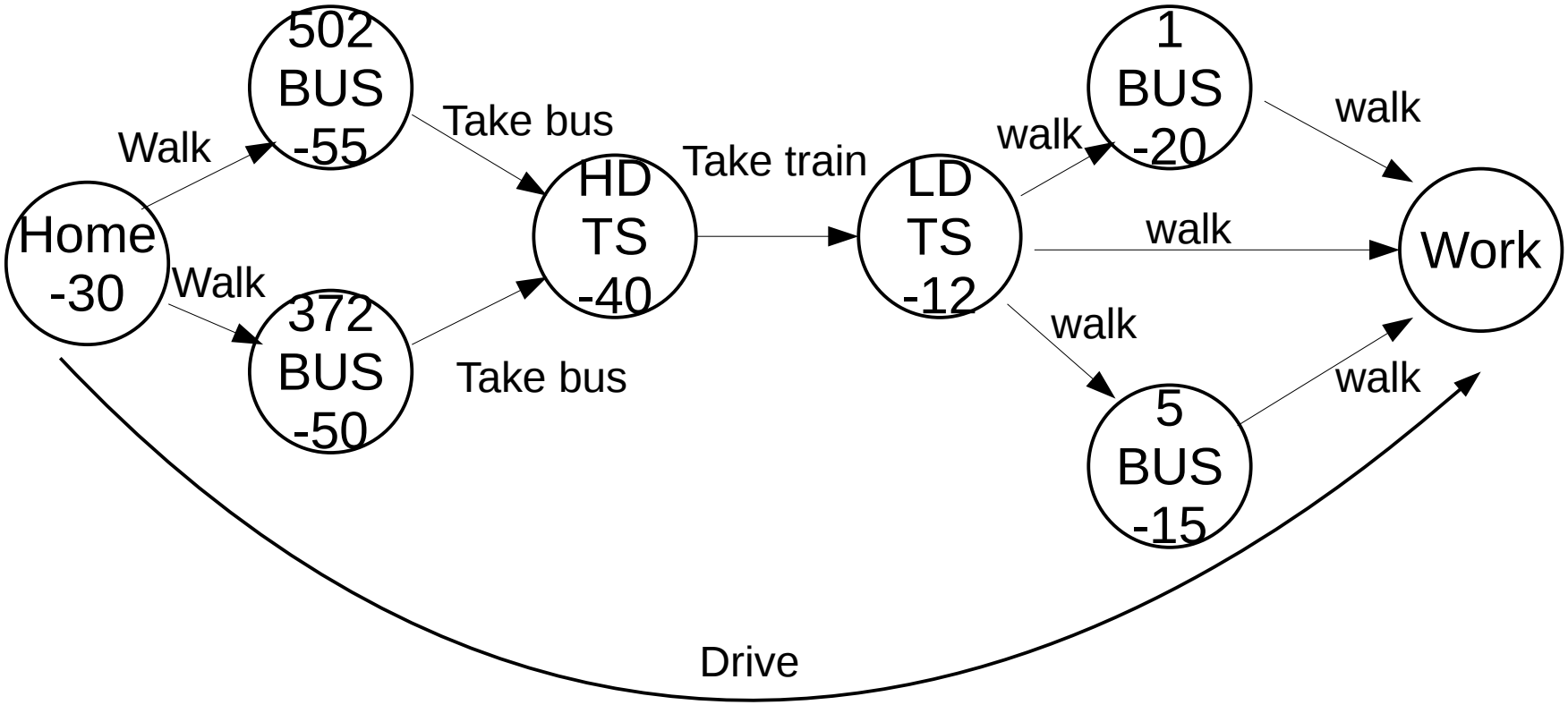

Action value

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We can also specify the value of a particular action in a given state:

$$q_\pi(s,a)=\sum_{s'} p(s'|s,a)\left(r(s,a,s')+\gamma v_\pi(s')\right)$$

By taking the action out of:

$$v_\pi(s)=\sum_a \pi(a|s)\sum_{s'} p(s'|s,a)\left(r(s,a,s')+\gamma v_\pi(s')\right)$$

From functions to policies

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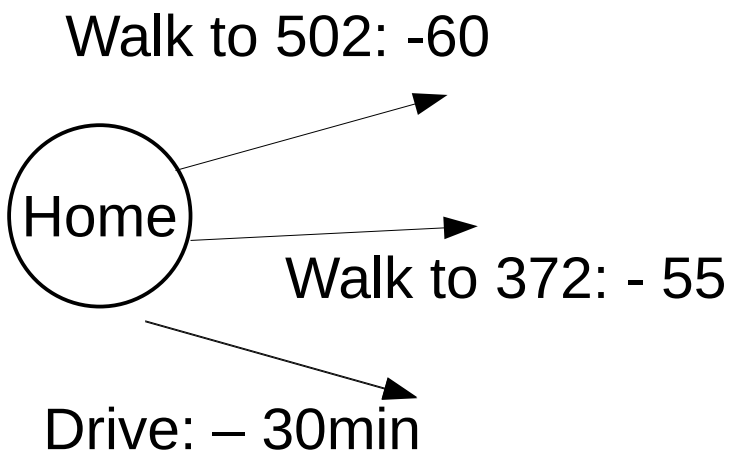
If we have $q^*(s,a)$



$$\pi^*(s) = argmax_a[q^*(s,a)]$$

Does not require the transition model T!

Optimal value functions

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We have now the idea of an *optimal* value function, that is a value function that is the highest it can be on every state, and corresponds to optimal policies.

$$v^*(s) \quad q*(s,a)$$

We now see how to compute the optimal value function

Example: optimal policy

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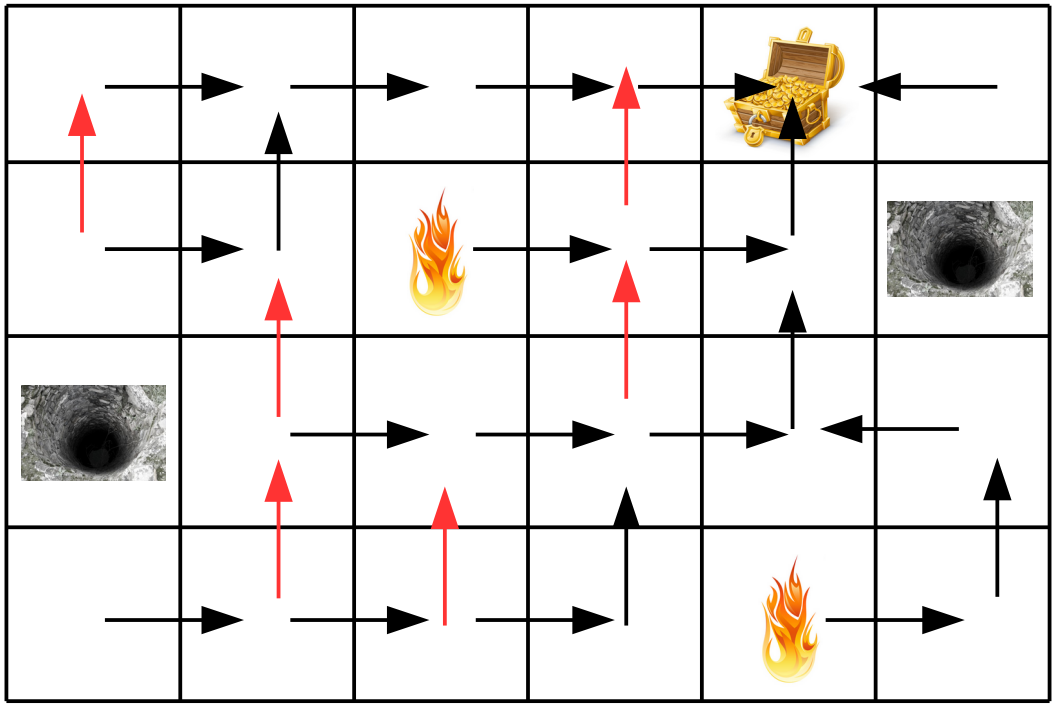

Why is the optimal policy not unique? You can brake ties between equally valuable actions in many ways (other possible optimal actions in red)

Is this a good model?

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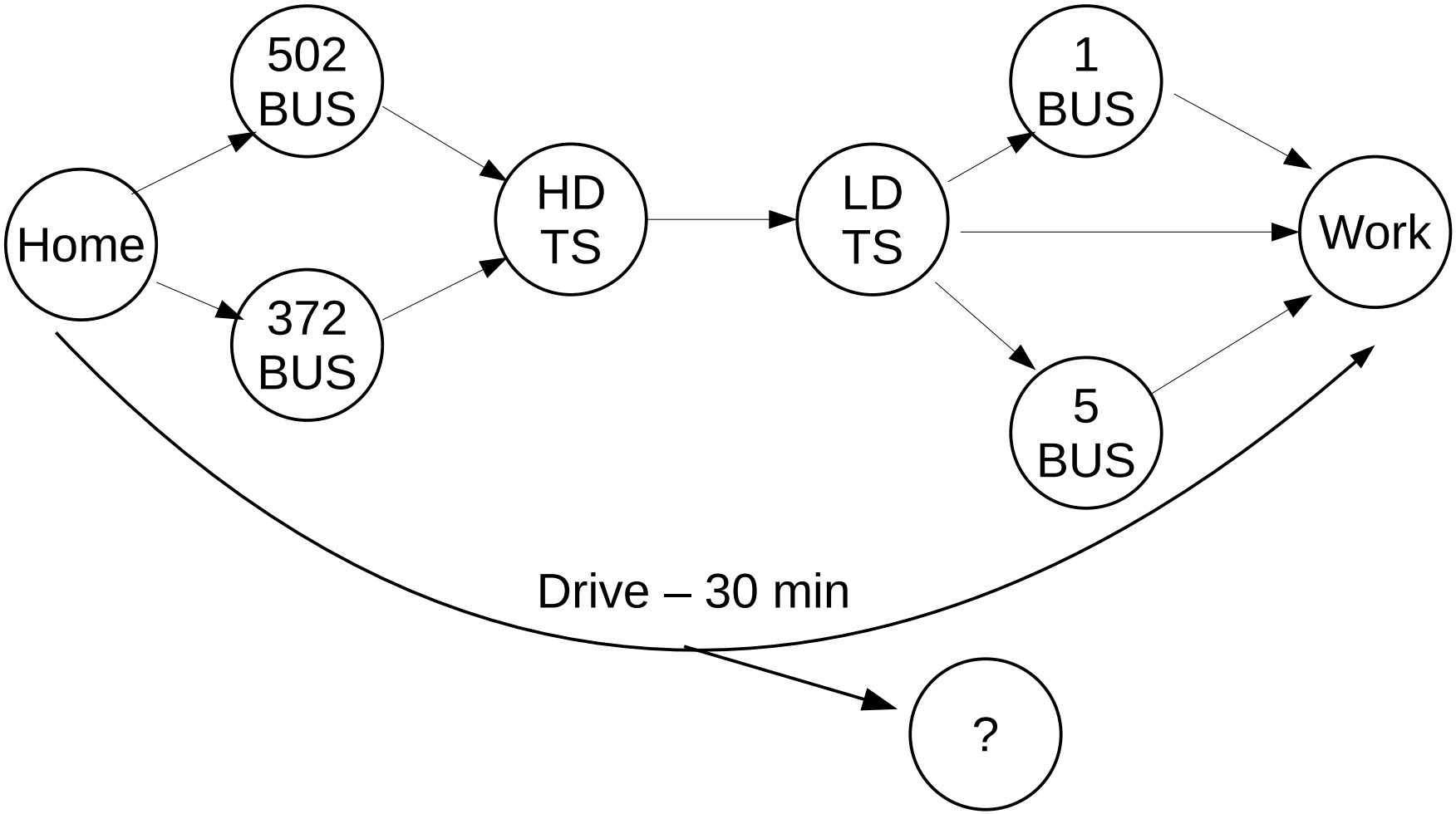

Dynamic Programming

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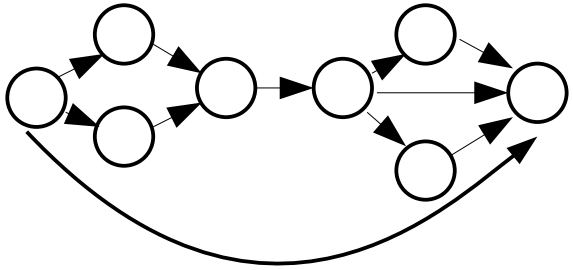
If all elements of the MDP are known:



$$MDP\ M = \langle S, A, T, r \rangle$$

We go from the value of any policy π:

$$q_\pi(s,a) = \sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma\, q_\pi(s', \pi(s')))$$

To the value of the *optimal* policy:

$$q^*(s,a) = \sum_{s'} p(s'|s,a)(r(s,a,s') + max_{a'} \gamma\, q^*(s',a'))$$

To the update rule:

$$q_{k+1}(s,a) = \sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma\, max_{a'} q_k(s',a'))$$

Policy iteration

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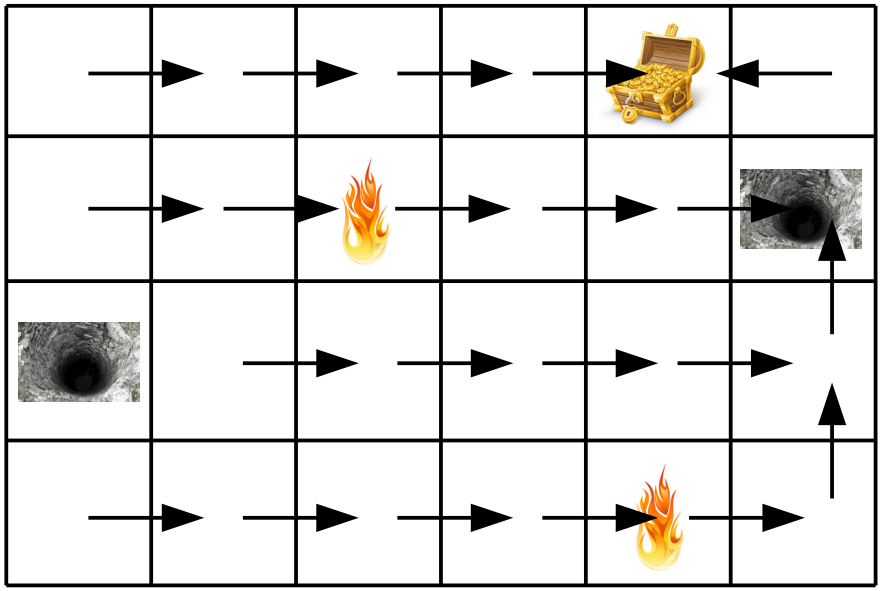
Transition and reward function unknown: $MDP\ M = \langle S, A, ?, ? \rangle$

The agent starts with an arbitrary policy (or, equivalently, action-value function), For instance, this policy:



Policy iteration

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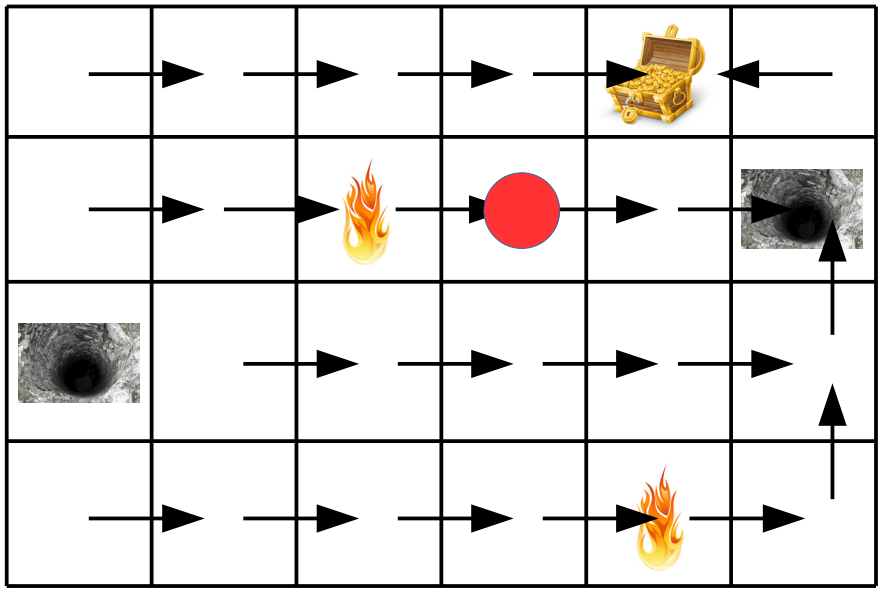
Step 1: policy evaluation

Compute the action-value function of this policy



For simplicity, let's focus on one state:

$\gamma = 0.5$

$$q(\langle 4,2 \rangle, \text{right}) = 0 + \gamma(-100) = -50$$

$$q(\langle 4,2 \rangle, \text{up}) = 0 + \gamma(50) = 25$$

$$q(\langle 4,2 \rangle, \text{left}) = -5 + \gamma^3(-100) = -17.5$$

$$q(\langle 4,2 \rangle, \text{down}) = 0 + \gamma^3(-100) = -12.5$$

Policy iteration

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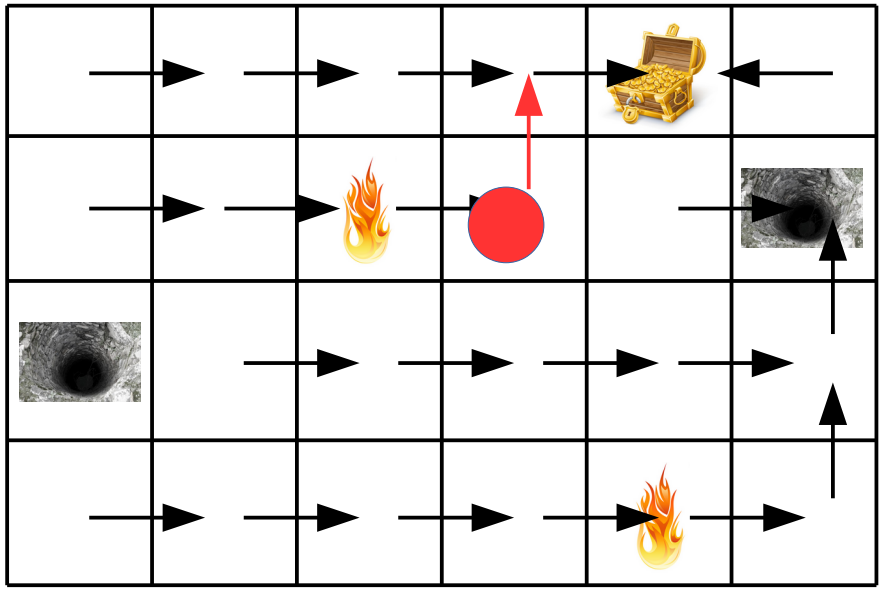
Step 2: policy improvement

Having the value for the current policy, find a state in which a different action would be better than the current one.



$$q(\langle 4,2\rangle, \text{right}) = 0 + \gamma(-100) = -50$$

$$q(\langle 4,2\rangle, \text{up}) = 0 + \gamma(50) = 25$$

$$q(\langle 4,2\rangle, \text{left}) = -5 + \gamma^3(-100) = -17.5$$

$$q(\langle 4,2\rangle, \text{down}) = 0 + \gamma^3(-100) = -12.5$$

The current action is RIGHT, a better action is UP

Generalized Policy iteration

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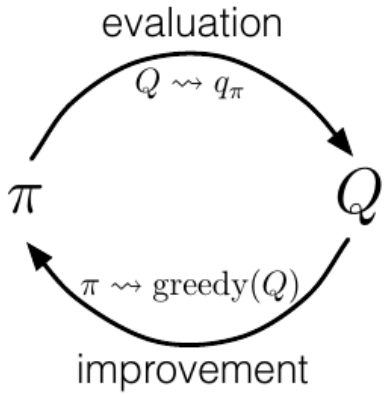

No need to fully evaluate a policy before making an improvement
→ *Generalized* PI

On MDPs this is a *hill-climbing* procedure, every solution is strictly better than previous ones.

How do we evaluate a policy without a model?

Conclusion

Learning outcomes

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- Perform a step of generalized policy iteration

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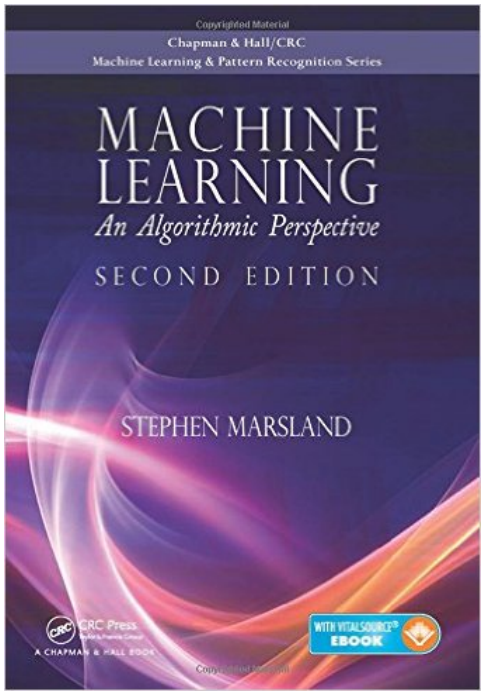

Chapter 11