Assignment 1

Question 1

Determine if each set below is convex.

1. $\{(x, y) \in R^2_{++} | x/y \leq 1\}$
2. $\{(x, y) \in R^2_{++} | x/y \geq 1\}$
3. $\{(x, y) \in R^2_{++} | xy \geq 1\}$
4. $\{(x, y) \in R^2_{++} | xy \leq 1\}$
5. $S = \{x \in R^n | x^T y \leq 1 \text{ for all } x \in C\}$, where C is a set (may not be convex).
6. The ellipsoid $\{x | x^T P^{-1} x \leq 1\}$ where $P \in S^n_{++}$.

Question 2

Give an example of two closed convex sets that are disjoint but cannot be strictly separated.

Question 3

Supporting hyperplanes.

1. Express the closed convex set $\{x \in \mathbb{R}^2_+ | x_1 x_2 \geq 1\}$ as an intersection of halfspaces.
2. Let $C = \{x \in \mathbb{R}^n | \|x\|_\infty \leq 1\}$, the l_∞-norm unit ball in $\mathbb{R}^n$, and let $\hat{x}$ be a point in the boundary of C. Identify the supporting hyperplanes of C at $\hat{x}$ explicitly.
3. Express the ball $B = \{x \in R^n | \|x\|_2 \leq 1\}$ as an intersection of halfspaces.

Question 4

Perspective function $P : R^{n+1} \to R^n$ is defined as:

$$P(x, t) = x/t, \quad \operatorname{dom} P = \{(x, t) \mid t > 0\}$$

where $x \in R^n, t > 0$ (i.e. $dom(P) = \{(x, t) | x \in R^n, t > 0\}$.) Prove:

- If $C \subset dom(P)$ is convex, then $P(C)$ is convex.
- If $D \subset R^n$ is convex, then $P^{-1}(D)$ is convex.