.py file
.tex file
.pdf file
.lean file

∞-Spaces

Mon	D(∞-Cat)	$\vec{\Sigma}$	$\vec{\Omega}$	$\vec{\mathrm{P}}$	InfPreShf	D(∞-Cat/C)	$\vec{\sigma}$	$\vec{\omega}$	$\vec{\mathrm{p}}$
ComMon	D(∞-Grpd)	$\overleftrightarrow{\Sigma}$	$\overleftrightarrow{\Omega}$	$\overleftrightarrow{\mathrm{P}}$	IntAct	D(∞-Grpd/G)	$\overleftrightarrow{\sigma}$	$\overleftrightarrow{\omega}$	$\overleftrightarrow{\mathrm{p}}$
IntGrp	D(∞-Grpd$_0$)	Σ	Ω	P	IntAct$_0$	D(∞-Grpd$_0$/G$_0$)	σ	ω	p

E. Dean Young

Plans to prove three variations of the Whitehead theorem of homotopy groups in Lean 4, with extensive use of Mathlib 4

We wish to acknowledge the collaborative efforts of E. Dean Young and Jiazhen Xia. Dean Young initially formulated the introduction with twelve goals, posting them on the Lean Zulip in August of 2023. Together the authors are pursuing these plans as a long term project.

1. Introduction

In this document I would like to develop a construction of the classifying space functor which can be applied indefinitely.

Segal ∞-Spaces and their relationship with
explore derivations and connections.

In this section, which makes use of the previous section concerning Haar integral, I intend to cover the ordinary versions of Poincare duality, Pontrjagin duality, and Fourier duality, as well as versions of these theorems using language enabled by the previous repositories. This won't culminate until far into the future, so for now I have jotted down some sketches.

2. Contents

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Unfinished	
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after this we develop chain complexes of these.

The table of contents below reflects the tentative long-term goals of the authors, with the main goal the pursuit of the Whitehead theorem for a point-set model involving Mathlib's predefined homotopy groups.

1. cup and cap product

Ideas for future applications:

1. `https://arxiv.org/pdf/2206.13563.pdf`

1. One of the basic things I wanted out of this was homotopy colimit preserving maps $(E^{\texttt{inf}}\text{-Alg } A)^{o} \longrightarrow \infty\text{-Grpd}$

PART 1: PART I: ABELIAN GROUPS AND ∞-SPACES

Eight Structures			
Strict		Lax	
Unitial	Actional	Unitial	Actional
InternalMonoid	InternalMonoidAction	OperadicMonoid	OperadicMonoidAction
InternalCommutative Monoid	InternalCommutativeMonoidAction	OperadicMonoid	OperadicMonoidAction

Abelian Groups

Abelian groups are internal groups in internal groups.

Tensor Product of Abelian Groups

Rings and Modules

make sure to include Alg

∞-Spaces

The little squares operad is operadic groups in operadic groups in .
In the repository concerning classifying spaces, I OperadicGroup^2 ∞-Grpd.

1. OperadicCategory^2, OperadicGroupoid^2, OperadicGroup^2

Could ∞-spaces be operadic groups in operadic groups?

Tensor Product of ∞-Spaces

∞-Rings and ∞-Modules

make sure to include ∞-Alg...

∞-Grpd $\rightleftarrows$ ∞-Space

Set $\rightleftarrows$ AbelianGroup
∞-Grpd $\rightleftarrows$ ∞-Space
Ring $\rightleftarrows$ ∞-Ring
Mod R $\rightleftarrows$ ∞-Mod R

1. π_n of an ∞-space arising from an ∞-groupoid vs. H_n ...
2.
3.
4.

$$\texttt{Ring} \rightleftarrows \infty\text{-}\texttt{Ring}$$

$$\mathtt{Mod\ R} \rightleftarrows \infty\text{-}\mathtt{Mod\ R}$$

PART 2: PART II: DERIVATIONS AND CONNECTIONS

(Alg R)/B $\rightleftarrows$ Mod B	(∞-Alg R)/B $\rightleftarrows$ ∞-Mod B
??? $\rightleftarrows$???	??? $\rightleftarrows$???

Lie Algebras

Lie Algebra Representations

Derivations

1. I would like to first construct the lie-algebra of derivations using the spectrum $\Omega^{\mathtt{inf}}$`.obj X`. It seems related to coalgebra endomorphisms from $\Omega^{\mathtt{inf}}$`.obj X` to itself.

2. Lie algebras and Der⍰(A,A)

Connections

1. I would like to first construct the lie-algebra representation of flat connections using the spectrum $\omega^{\mathtt{inf}}$`.obj X).obj V`.
2.
3. Lie algebra representations and ...
4.
5.
6.
7. Somehow connections are the dual of the free abelian group action

Some goals:

1.

1. smooth (etale locally ???)
2. analytic (etale locally ???)

$\mathbb{C}P^1$,$\mathbb{C}P^1$ $\cong$ $\mathbb{C}$(x)

3. Our $\mathbb{R}$ is ∂.obj $\mathbb{Z}$ regarded as an object in [$\vec{\gamma}$,∞_(∞-Grpd)]
4. B.obj det : B.obj U(n) $\longrightarrow$ B.obj U(1)
5. Which E_∞ Space have a Chern class?

Cohomology with coefficients in `[-,B`$\mathbb{C}^{\mathtt{x}}$`]` plays a .

1.
2. d2 is wedge with (representable...)

L^{∞} Algebras

L^∞ Algebra Representations

∞-Derivations

∞-Connections

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