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∞-Spaces

E. Dean Young

We wish to acknowledge the collaborative efforts of E. Dean Young and Jiazhen Xia. Dean Young initially formulated the introduction with twelve goals, posting them on the Lean Zulip in August of 2023. Together the authors are pursuing these plans as a long term project.

1. Introduction

In this document I would like to develop a construction of the classifying space functor which can be applied indefinitely, B^1. I will write B^n for the n-fold composition of B^1. In "'TheWhiteheadTheoremandTwoVariations"', we developed two models of ∞-Grpd_(A) and ∞-Grpd_(B). These will produce two models on which B^1 can be defined:

(i) B^1 : OperadicGroup $\bullet$ OperadicGroup ∞-Grpd_(A) $\longrightarrow$ OperadicGroup $\bullet$ OperadicGroup ∞-Grpd_(A)

(ii) B^1 : OperadicGroup $\bullet$ OperadicGroup ∞-Grpd_(B) $\longrightarrow$ OperadicGroup $\bullet$ OperadicGroup ∞-Grpd_(B)

∞-Spaces ...

explore derivations and connections.

In this section, which makes use of the previous section concerning Haar integral, I intend to cover the ordinary versions of Poincare duality, Pontrjagin duality, and Fourier duality, as well as versions of these theorems using language enabled by the previous repositories. This won't culminate until far into the future, so for now I have jotted down some sketches.

2. Contents

The contents below reflect most or all of the contents of each chapter, each of which is adds a fairly minimal .

Section	Description
Unfinished	
Contents	
Unicode	
Introduction	
PART I: ∞-SPACES	
Chapter 1: Abelian Groups	
abelian_group	The type of abelian groups
AbelianGroup	The category of abelian groups
Chapter 2: ∞-Spaces	
∞-space	∞-spaces are (here) algebras for the little squares operad
∞-Space	the category of ∞-spaces is the category of algebras for the little squares operad
Chapter 3: Tensor Product of Abelian Groups	
- ⊗_(AbelianGroups) -	Mathlib's tensor product of abelian groups
[-,-]_(AbelianGroups)	Mathlib's hom of abelian groups
Chapter 4: Tensor Product of ∞-Spaces	
- ⊗_(∞-Space) -	
[-,-]_(∞-Space)	
Chapter 5: Rings and Commutative Rings	
ring	The type of rings
Ring	The category of rings
Chapter 6: A^{∞}-Rings and E^{∞}-Rings	
A^{∞}-ring	The type of A^{∞}-rings
A^{∞}-Ring	The category of A^{∞}-Rings
E^{∞}-ring	The type of E^{∞}-rings
E^{∞}-Ring	The category of E^{∞}-Rings
Chapter 7: Modules and Modules over Commutative Rings	
Chapter 8: A^{∞}-Modules and E^{∞}-Modules	
A∞-Mod	
A∞-Mod	
Chapter 9: Set ⇄ Ab	
$S^{\cdot}$	The free abelian group functor
$S_{\cdot}$	The forgetful functor from abelian groups to sets
Chapter 10: ∞-Grpd ⇄ ∞-Space	
	The free ∞-space given an ∞-groupoid
	The underlying ∞-groupoid of an ∞-space
PART II: DERIVATIONS AND CONNECTIONS	

`Chapter 11:` Lie Algebras	
`Chapter 12:` Derivations	
derivation	Definition of a derivation
`Chapter 13:` L^∞ Algebras	
`Chapter 14:` ∞-Derivations	
∞-derivation	Definition of ???
`Chapter 15:` Tensor Product of Lie Algebras	
- ⊗_(∞-Space) -	
[-,-]_(∞-Space)	
`Chapter 16:` Tensor Product of L^∞-Algebras	
ring	The type of rings
Ring	The category of rings
`Chapter 17:` Connections	
connection	Definition of a connection
`Chapter 18:` ∞-Connections	
∞-connection	Definition of ???
`Chapter 19:` Tensor Product of Lie Algebra Representations	
- ⊗_(∞-Space) -	
[-,-]_(∞-Space)	
`Chapter 20:` Tensor Product of L^∞-Representations	
ring	The type of rings
Ring	The category of rings

after this we develop chain complexes of these.

The table of contents below reflects the tentative long-term goals of the authors, with the main goal the pursuit of the Whitehead theorem for a point-set model involving Mathlib's predefined homotopy groups.

1. cup and cap product

Ideas for future applications:

1. `https://arxiv.org/pdf/2206.13563.pdf`

1. One of the basic things I wanted out of this was homotopy colimit preserving maps $(E^{\mathtt{inf}}\text{-Alg } A)^{o} \longrightarrow \infty\text{-Grpd}$

PART 1: PART I: ABELIAN GROUPS AND ∞-SPACES

Eight Structures			
Strict		Lax	
Unitial	Actional	Unitial	Actional
InternalMonoid	InternalMonoidAction	A^{∞}	OperadicMonoidAction
InternalCommutative Monoid	InternalCommutativeMonoidAction	OperadicMonoid	OperadicMonoidAction

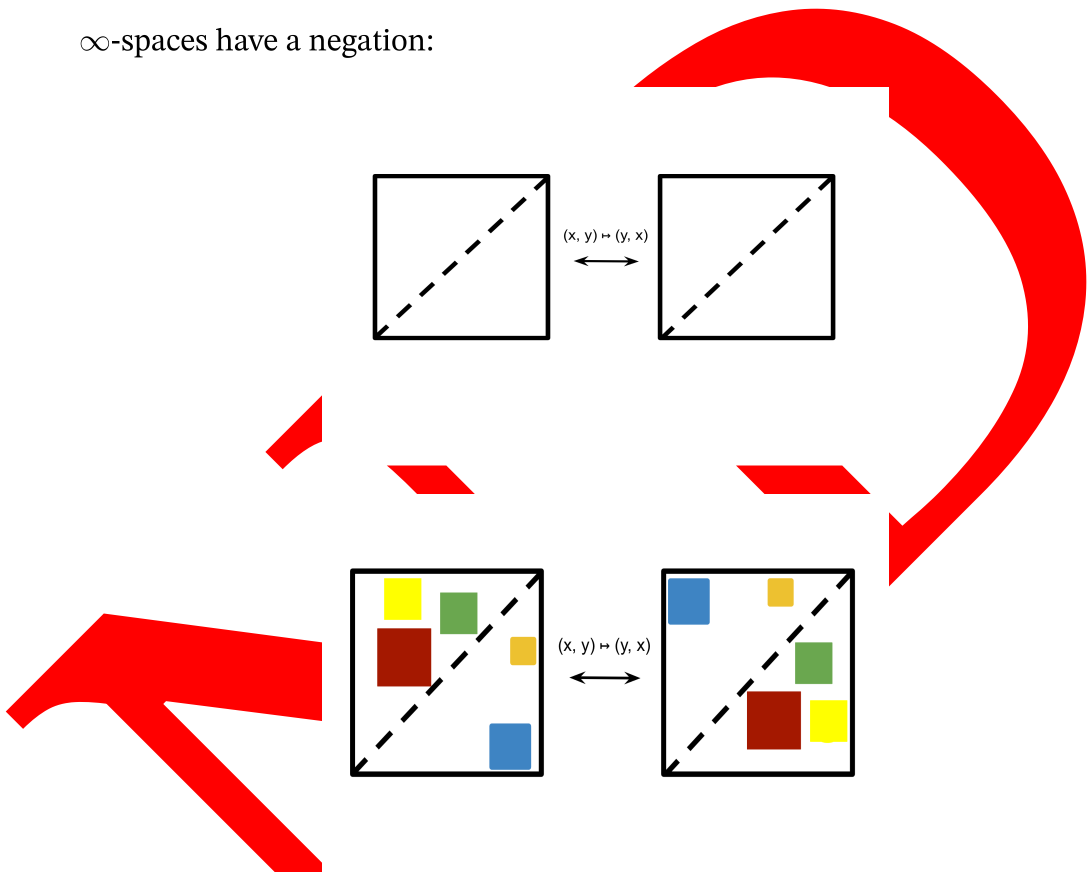
Abelian Groups

In this chapter I cover the type of abelian groups as well as the category of abelian groups.

1. Mathlib.Algebra.Group.Defs

∞-Spaces

∞-spaces have a negation:



For example, ... is operadic groups in operadic groups in O^2.

The commutative nature of composition in the little squares operad is interesting.

OperadicGroup^2 ∞_(∞-Grpd)

Could ∞-spaces be operadic groups in operadic groups?
Abelian groups are internal groups in internal groups.

Tensor Product

1.

Smash Product of ∞-Spaces

Rings and Commutative Rings

A^{∞}-Rings and E^{∞}-Rings

make sure to include Alg
make sure to include ∞-Alg...

Modules over Rings and Commutative Rings

A^{∞}-Modules and E^{∞}-Modules

$\mathtt{Set}_1 \rightleftarrows \mathtt{AbelianGroup}$

$\boxed{\text{Set} \rightleftarrows \text{AbelianGroup}}$

Abelian groups are internal groups in internal groups in sets.

$\infty\text{-}\mathtt{Grpd}_{1} \rightleftarrows \mathtt{OperadicGroup}^2$

∞-spaces are algebras for the little squares operad.

$\boxed{\infty\text{-Grpd}_{-1} \rightleftarrows \text{OperadicGroup}^2}$

1. π_n of an ∞-space arising from an ∞-groupoid
2. H_n of an
3. Dold-Thom theorem
4.
5. B is iterable on this
6. Chn ...
7. $\mu : \mathrm{Chn} \times \mathrm{Chn} \longrightarrow \mathrm{Chn}$

PART 2: PART II: DERIVATIONS AND CONNECTIONS

Derivations

Let `A` be a ring and suppose that `B : Alg A`. `B.dom`.

$$(\mathrm{Alg}\ \mathrm{A})/\mathrm{B} \rightleftarrows \mathrm{Mod}\ \mathrm{B}$$

1. I would like to first construct the lie-algebra of derivations using the spectrum $\Omega^{\mathtt{inf}}$`.obj X`. It seems related to coalgebra endomorphisms from $\Omega^{\mathtt{inf}}$`.obj X` to itself.
2. Lie algebras and Der▯(A,A)

∞-Derivations

$$\Omega : (\mathrm{E}^{\infty}\text{-Alg A})/\mathrm{B} \rightleftarrows \mathrm{E}^{\infty}\text{-Mod B} : \Lambda$$

$$\Omega : (\mathrm{A}^{\infty}\text{-Alg A})/\mathrm{B} \rightleftarrows \mathrm{A}^{\infty}\text{-Mod B} : \Lambda$$

The more typical concept of a free abelian group .

Given an E^{∞}-algebra...., we can form an

Connections

∞-Connections

1. I would like to first construct the lie-algebra representation of flat connections using the spectrum $\omega^{\texttt{inf}}$`.obj X).obj V`.
2.
3. Lie algebra representations and ...
4.
5.
6.
7. Somehow connections are the dual of the free abelian group action

Some goals:

1.

1. smooth (etale locally ???)
2. analytic (etale locally ???)

$\mathbb{C}\mathrm{P}^1$,$\mathbb{C}\mathrm{P}^1$ $\cong \mathbb{C}$(x)

3. Our $\mathbb{R}$ is ∂.obj $\mathbb{Z}$ regarded as an object in [$\vec{\gamma}$,∞_(∞-Grpd)]
4. B.obj det : B.obj U(n) $\longrightarrow$ B.obj U(1)
5. Which E_∞ Space have a Chern class?

Cohomology with coefficients in `[-,B`$\mathbb{C}$`^X]` plays a .

1.
2. d2 is wedge with (representable...)

Given an E^∞-algebra...., we can form an

Bibliography

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About the Author

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