



# Internal Universes

$\infty_-(\infty\text{-Cat})$	$D(\infty_-(\infty\text{-Cat}))$	$\infty_-(\infty\text{-Cat})/C$	$D(\infty_-(\infty\text{-Cat})/C)$
$\infty_-(\infty\text{-Grpd})$	$D(\infty_-(\infty\text{-Grpd}))$	$\infty_-(\infty\text{-Grpd})/G$	$D(\infty_-(\infty\text{-Grpd})/G)$
$\infty_-(\infty\text{-Grpd}_0)$	$D(\infty_-(\infty\text{-Grpd}_0))$	$\infty_-(\infty\text{-Grpd}_0)/G_0$	$D(\infty_-(\infty\text{-Grpd}_0)/G_0)$

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## 1. Introduction

## 2. Unicode

Here is a list of the unicode characters I will use:

Symbol	Unicode	VSCode shortcut	Use
Lean's Kernel			
$\times$	2A2F	<code>\times</code>	Product of types
$\rightarrow$	2192	<code>\rightarrow</code>	Hom of types
$\langle, \rangle$	27E8, 27E9	<code>\langle, \rangle</code>	Product term introduction
$\rightarrowsto$	21A6	<code>\mapsto</code>	Hom term introduction
$\wedge$	2227	<code>\wedge</code>	Conjunction
$\vee$	2228	<code>\vee</code>	Disjunction
$\forall$	2200	<code>\forall</code>	Universal quantification
$\exists$	2203	<code>\exists</code>	Existential quantification
$\neg$	00AC	<code>\neg</code>	Negation
Variables and Constants			
$a, b, c, \dots, z$	1D52, 1D56		Variables and constants
$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$	1D52, 1D56		Variables and constants
$-$	207B		Variables and constants
$0.1.2.3.4.5.6.7.8.9$	2080 - 2089	<code>\0-\9</code>	Variables and constants
$A, \dots, Z$	1D538	<code>\bbA, \dots, \bbZ</code>	Variables and constants
$a, \dots, z$	1D552	<code>\bba, \dots, \bbz</code>	Variables and constants
$\alpha, \omega, A, \Omega$	03B1-03C9		Variables and constants
Categories and Bicatgories			
$1$	1D7D9	<code>\b1</code>	The identity morphism
$?$	2218		Composition
			Composition
			Composition
Adjunctions			
$.$	1BC94		Right adjoints
$\cdot$	0971		Left adjoints
$\dashv$	22A3	<code>\dashv</code>	The condition that two functors are adjoint
Monads and Comonads			
$?, \complement$	003F, 00BF	<code>?, \complement</code>	The corresponding (co)monad of an adjunction
$!, i$	0021, 00A1	<code>!, \backslash!</code>	The (co)-Eilenberg-(co)-Moore adjunction
$\uparrow, \downarrow$	A71D, A71E		The (co)AdjMon maps
Miscellaneous			
$\simeq$	2243	<code>\equiv</code>	Equivalences
$\cong$	2245	<code>\cong</code>	Isomorphisms
$\perp$	22A5	<code>\bot</code>	The overobject classifier
$\infty$	221E	<code>\infty</code>	Infinity categories and infinity groupoids

Of these, the characters  $\uparrow$ ,  $\downarrow$ , and  $\cdot$  do not have VSCode shortcuts, and so I provide alternatives for them. Possibly they will have to be changed if this work assimilates into a larger project.

It is not possible to copy the from the pdf to the clipboard while preserving the integrity of the code. To see the official Lean 4 file please click the link on the top right of the front page or this.

## Lean 1

```
import Mathlib.CategoryTheory.Bicategory.Basic
import Mathlib.CategoryTheory.Types
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.Combinatorics.Quiver.Basic
import Mathlib.CategoryTheory.Category.Init
import Aesop
import Init
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.CategoryTheory.Bicategory.Strict
import Mathlib.CategoryTheory.ConcreteCategory.Bundled
import Mathlib.CategoryTheory.Functor.Basic
import Init.Core
import Mathlib.CategoryTheory.Category.Cat

import TheWhiteheadTheorem

-- #check
-- #
```



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### 3. Contents

Section	Description
Unfinished	
Contents	
Unicode	
Introduction	
PART I: Internal Universes	
Chapter 1: $\infty_-(\infty\text{-Grpd}_0)$	
$\vec{\chi}.$	
$\vec{\chi}'$	
$D(\vec{\chi}.)$	
$D(\vec{\chi}')$	
$D(\infty_-(\infty\text{-Grpd}_1)^{-})$	
$D([-^{\circ}, \infty_-(\infty\text{-Grpd}_1)])$	
$D(\infty_-(\infty\text{-Grpd}_1)^{-}) \simeq D([-^{\circ}, \infty_-(\infty\text{-Grpd}_1)])$	
Chapter 2: $\infty_-(\infty\text{-Grpd})$	
$\vec{\chi}.$	
$\vec{\chi}'$	
$D(\vec{\chi}.)$	
$D(\vec{\chi}')$	
$D(\infty_-(\infty\text{-Grpd})^{-})$	
Chapter 3: $\infty_-(\infty\text{-Cat})$	
$\chi.$	
$\chi'$	
$D(\chi.)$	
$D(\chi')$	
$D(\infty_-(\infty\text{-Cat})^{-})$	
$D([-^{\circ}, \infty_-(\infty\text{-Cat})])$	
$D(\infty_-(\infty\text{-Cat})^{-}) \simeq D([-^{\circ}, \infty_-(\infty\text{-Cat})])$	
PART II: Internal Universes	
Chapter 4: Monadicity $D(O\dots) : D(\infty\text{-Grpd}_0)$ , and $D(\infty\text{-Grpd}_0/X_0) \rightleftarrows D(\infty\text{-Grpd}_0/Y_0)$	
Chapter 5: Monadicity $D(O\dots)$ , $D(\infty\text{-Grpd})$ , and $D(\infty\text{-Grpd}/X) \rightleftarrows D(\infty\text{-Grpd}/Y)$	
Chapter 6: Monadicity $D(O\dots)$ , $D(\infty\text{-Cat})$ , and $D(\infty\text{-Cat}/C) \rightleftarrows D(\infty\text{-Cat}/D)$	



# PART 1: Constructing Three Internal Universes

$$\infty_-(\infty\text{-Grpd}_0)$$

$$\infty_-(\infty\text{-Grpd})$$

$$\infty_-(\infty\text{-Cat})$$

## PART 2: Monadicity

Monadicity,  $D(\infty\text{-Grpd}_0)$ , and  
 $D(\infty\text{-Grpd}_0/X_0) \rightleftarrows D(\infty\text{-Grpd}_0/Y_0)$

Monadicity,  $D(\infty\text{-Grpd})$ , and  
 $D(\infty\text{-Grpd}/X) \rightleftarrows D(\infty\text{-Grpd}/Y)$

Monadicity,  $D(\infty\text{-Cat})$ , and  
 $D(\infty\text{-Cat}/\mathcal{C}) \rightleftarrows D(\infty\text{-Cat}/\mathcal{D})$



## PART 3: Kan Extensions

#### About the Author

Dean Young is a graduate student at New York University, where he studies mathematics.

