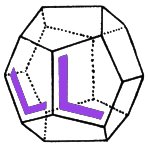
.js file .py file
.java file .tex file
.cpp file .pdf file
.c file .lean file



Internal Universes

∞_(∞-Cat)	D(∞_(∞-Cat))	∞_(∞-Cat)/C	D(∞_(∞-Cat)/C)
∞_(∞-Grpd)	D(∞_(∞-Grpd))	∞_(∞-Grpd)/G	D(∞_(∞-Grpd)/G)
∞_(∞-Grpd$_0$)	D(∞_(∞-Grpd$_0$))	∞_(∞-Grpd$_0$)/G$_0$	D(∞_(∞-Grpd$_0$)/G$_0$)

E. Dean Young

1. Introduction

2. Unicode

Here is a list of the unicode characters I will use:

Symbol	Unicode	VSCode shortcut	Use
		Lean's Kernel	
×	2A2F	\times	Product of types
→	2192	\rightarrow	Hom of types
⟨,⟩	27E8,27E9	\langle,\rangle	Product term introduction
$-> sto$	21A6	\mapsto	Hom term introduction
∧	2227	\wedge	Conjunction
∨	2228	\vee	Disjunction
∀	2200	\forall	Universal quantification
∃	2203	\exists	Existential quantification
¬	00AC	\neg	Negation
		Variables and Constants	
a,b,c,...,z	1D52,1D56		Variables and constants
0,1,2,3,4,5,6,7,8,9	1D52,1D56		Variables and constants
-	207B		Variables and constants
0,1,2,3,4,5,6,7,8,9	2080 - 2089	\0-\9	Variables and constants
𝔸,...,ℤ	1D538	\bbA,...,\bbZ	Variables and constants
𝕒,...,𝕫	1D552	\bba,...,\bbz	Variables and constants
α-ω,Α-Ω	03B1-03C9		Variables and constants
		Categories and Bicategories	
𝟙	1D7D9	\b1	The identity morphism
?	2218		Composition
			Composition
			Composition
		Adjunctions	
.	1BC94		Right adjoints
˙	0971		Left adjoints
⊣	22A3	\dashv	The condition that two functors are adjoint
		Monads and Comonads	
?,¿	003F, 00BF	?,\?	The corresponding (co)monad of an adjunction
!,¡	0021, 00A1	!, \!	The (co)-Eilenberg-(co)-Moore adjunction
',¸	A71D, A71E		The (co)AdjMon maps
		Miscellaneous	
≃	2243	\equiv	Equivalences
≅	2245	\cong	Isomorphisms
⊥	22A5	\bot	The overobject classifier
∞	221E	\infty	Infinity categories and infinity groupoids

Of these, the characters ' , ¸ , . and ˙ do not have VSCode shortcuts, and so I provide alternatives for them. Possibly they will have to be changed if this work assimilates into a larger project.

It is not possible to copy the from the pdf to the clipboard while preserving the integrity of the code. To see the official Lean 4 file please click the link on the top right of the front page or this.

Lean 1

```
import Mathlib.CategoryTheory.Bicategory.Basic
import Mathlib.CategoryTheory.Types
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.Combinatorics.Quiver.Basic
import Mathlib.CategoryTheory.Category.Init
import Aesop
import Init
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.CategoryTheory.Bicategory.Strict
import Mathlib.CategoryTheory.ConcreteCategory.Bundled
import Mathlib.CategoryTheory.Functor.Basic
import Init.Core
import Mathlib.CategoryTheory.Category.Cat

import TheWhiteheadTheorem

-- #check
-- #
```

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$D(\vec{\chi}^{\cdot})$	
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PART 1: Constructing Three Internal Universes

∞_(∞-$\mathtt{Grpd}_0$)

∞_(∞-Grpd)

∞_(∞-Cat)

PART 2: Monadicity

Monadicity, $\mathtt{D}(\infty\text{-}\mathtt{Grpd}_0)$, and $\mathtt{D}(\infty\text{-}\mathtt{Grpd}_0/\mathtt{X}_0) \rightleftarrows \mathtt{D}(\infty\text{-}\mathtt{Grpd}_0/\mathtt{Y}_0)$

Monadicity, $\mathrm{D}(\infty\text{-Grpd})$, and $\mathrm{D}(\infty\text{-Grpd}/X) \rightleftarrows \mathrm{D}(\infty\text{-Grpd}/Y)$

Monadicity, $\mathbb{D}(\infty\text{-Cat})$, and $\mathbb{D}(\infty\text{-Cat}/C) \rightleftarrows \mathbb{D}(\infty\text{-Cat}/D)$

PART 3: Kan Extensions

About the Author

Dean Young is a graduate student at New York University, where he studies mathematics.