Internal Universes

$\infty_-(\infty\text{-Cat})$	$D(\infty_-(\infty\text{-Cat}))$	$\infty_-(\infty\text{-Cat})/C$	$D(\infty_-(\infty\text{-Cat})/C)$
$\infty_-(\infty\text{-Grpd})$	$D(\infty_-(\infty\text{-Grpd}))$	$\infty_-(\infty\text{-Grpd})/G$	$D(\infty_-(\infty\text{-Grpd})/G)$
$\infty_-(\infty\text{-Grpd}_0)$	$D(\infty_-(\infty\text{-Grpd}_0))$	$\infty_-(\infty\text{-Grpd}_0)/G_0$	$D(\infty_-(\infty\text{-Grpd}_0)/G_0)$

1. Introduction

In this repository I would like to consider Kan extensions and homotopy Kan extensions, obtaining a thorough account of Lurie's straightening and unstraightening concepts.

1. Complexes of abelian groups and ∞-spaces are the same as ∞-presheaves on E.obj $\mathbb{N}$.

2.

2. Unicode

Here is a list of the unicode characters I will use:

Symbol	Unicode	VSCode shortcut	Use
		Lean's Kernel	
$\times$	2A2F	\times	Product of types
$\rightarrow$	2192	\rightarrow	Hom of types
$\langle,\rangle$	27E8,27E9	\langle,\rangle	Product term introduction
$->sto$	21A6	\mapsto	Hom term introduction
$\wedge$	2227	\wedge	Conjunction
$\vee$	2228	\vee	Disjunction
$\forall$	2200	\forall	Universal quantification
$\exists$	2203	\exists	Existential quantification
$\neg$	00AC	\neg	Negation
		Variables and Constants	
$^a,^b,^c,...,^z$	1D52,1D56		Variables and constants
$^0,^1,^2,^3,^4,^5,^6,^7,^8,^9$	1D52,1D56		Variables and constants
$^-$	207B		Variables and constants
$_0,_1,_2,_3,_4,_5,_6,_7,_8,_9$	2080 - 2089	\0-\9	Variables and constants
$\mathbb{A},...,\mathbb{Z}$	1D538	\bbA,...,\bbZ	Variables and constants
$\mathbb{a},...,\mathbb{z}$	1D552	\bba,...,\bbz	Variables and constants
α-ω,$\Alpha$-Ω	03B1-03C9		Variables and constants
		Categories and Bicategories	
$\mathbb{1}$	1D7D9	\b1	The identity morphism
$\circ$	2218		Composition
			Composition
			Composition
		Adjunctions	
$.$	1BC94		Right adjoints
$.$	0971		Left adjoints
$\dashv$	22A3	\dashv	The condition that two functors are adjoint
		Monads and Comonads	
?,¿	003F, 00BF	?,\?	The corresponding (co)monad of an adjunction
!,¡	0021, 00A1	!, \!	The (co)-Eilenberg-(co)-Moore adjunction
ꜝ,ꜞ	A71D, A71E		The (co)AdjMon maps
		Miscellaneous	
$\simeq$	2243	\equiv	Equivalences
$\cong$	2245	\cong	Isomorphisms
$\bot$	22A5	\bot	The overobject classifier
∞	221E	\infty	Infinity categories and infinity groupoids

Of these, the characters ꜝ, ꜞ, . and . do not have VSCode shortcuts, and so I provide alternatives for them. Possibly they will have to be changed if this work assimilates into a larger project.

It is not possible to copy the from the pdf to the clipboard while preserving the integrity of the code. To see the official Lean 4 file please click the link on the top right of the front page or this.

```
Lean 1

import Mathlib.CategoryTheory.Bicategory.Basic
import Mathlib.CategoryTheory.Types
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.Combinatorics.Quiver.Basic
import Mathlib.CategoryTheory.Category.Init
import Aesop
import Init
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.CategoryTheory.Bicategory.Strict
import Mathlib.CategoryTheory.ConcreteCategory.Bundled
import Mathlib.CategoryTheory.Functor.Basic
import Init.Core
import Mathlib.CategoryTheory.Category.Cat

import TheWhiteheadTheorem

-- #check
-- #
```

3. Contents

Implementation Progress

Writing Progress

$$\infty_{-}(\infty\text{-}\mathrm{Grpd}_0)$$

$\infty\_(\infty\text{-Grpd})$

$\infty\text{-}(\infty\text{-Cat})$

$\infty_{\_}(\infty\text{-Spc})$

4. Bibliography

1.

About the Author

Dean Young is a graduate student at New York University, where he studies mathematics.