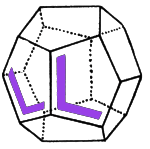


.js file .py file
.java file .tex file
.cpp file .pdf file
.c file .lean file



Internal Universes

$\infty_-(\infty\text{-Cat})$	$D(\infty_-(\infty\text{-Cat}))$	$\infty_-(\infty\text{-Cat})/C$	$D(\infty_-(\infty\text{-Cat})/C)$
$\infty_-(\infty\text{-Grpd})$	$D(\infty_-(\infty\text{-Grpd}))$	$\infty_-(\infty\text{-Grpd})/G$	$D(\infty_-(\infty\text{-Grpd})/G)$
$\infty_-(\infty\text{-Grpd}_0)$	$D(\infty_-(\infty\text{-Grpd}_0))$	$\infty_-(\infty\text{-Grpd}_0)/G_0$	$D(\infty_-(\infty\text{-Grpd}_0)/G_0)$

E. Dean Young

1. Introduction

2. Unicode

Here is a list of the unicode characters I will use:

Symbol	Unicode	VSCode shortcut	Use
Lean's Kernel			
\times	2A2F	<code>\times</code>	Product of types
\rightarrow	2192	<code>\rightarrow</code>	Hom of types
\langle, \rangle	27E8, 27E9	<code>\langle, \rangle</code>	Product term introduction
\mapsto	21A6	<code>\mapsto</code>	Hom term introduction
\wedge	2227	<code>\wedge</code>	Conjunction
\vee	2228	<code>\vee</code>	Disjunction
\forall	2200	<code>\forall</code>	Universal quantification
\exists	2203	<code>\exists</code>	Existential quantification
\neg	00AC	<code>\neg</code>	Negation
Variables and Constants			
a, b, c, \dots, z	1D52, 1D56		Variables and constants
$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$	1D52, 1D56		Variables and constants
$-$	207B		Variables and constants
$0.1.2.3.4.5.6.7.8.9$	2080 - 2089	<code>\0-\9</code>	Variables and constants
$\mathbb{A}, \dots, \mathbb{Z}$	1D538	<code>\bba, ..., \bbz</code>	Variables and constants
$\mathfrak{a}, \dots, \mathfrak{z}$	1D552	<code>\bba, ..., \bbz</code>	Variables and constants
$\alpha, \omega, \mathbb{A}, \Omega$	03B1-03C9		Variables and constants
Categories and Bicatgories			
1	1D7D9	<code>\b1</code>	The identity morphism
\circ	2218		Composition
			Composition
			Composition
Adjunctions			
\cdot	1BC94		Right adjoints
\cdot	0971		Left adjoints
\dashv	22A3	<code>\dashv</code>	The condition that two functors are adjoint
Monads and Comonads			
$?, \mathfrak{c}$	003F, 00BF	<code>?, \mathfrak{c}</code>	The corresponding (co)monad of an adjunction
$!, \mathfrak{i}$	0021, 00A1	<code>!, \mathfrak{i}</code>	The (co)-Eilenberg-(co)-Moore adjunction
$\mathfrak{i}, \mathfrak{c}$	A71D, A71E		The (co)AdjMon maps
Miscellaneous			
\simeq	2243	<code>\equiv</code>	Equivalences
\cong	2245	<code>\cong</code>	Isomorphisms
\perp	22A5	<code>\bot</code>	The overobject classifier
∞	221E	<code>\infty</code>	Infinity categories and infinity groupoids

Of these, the characters \mathfrak{i} , \mathfrak{c} , and \mathfrak{c} do not have VSCode shortcuts, and so I provide alternatives for them. Possibly they will have to be changed if this work assimilates into a larger project.

It is not possible to copy the from the pdf to the clipboard while preserving the integrity of the code. To see the official Lean 4 file please click the link on the top right of the front page or this.

Lean 1

```
import Mathlib.CategoryTheory.Bicategory.Basic
import Mathlib.CategoryTheory.Types
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.Combinatorics.Quiver.Basic
import Mathlib.CategoryTheory.Category.Init
import Aesop
import Init
import Mathlib.CategoryTheory.DiscreteCategory
import Mathlib.CategoryTheory.Bicategory.Strict
import Mathlib.CategoryTheory.ConcreteCategory.Bundled
import Mathlib.CategoryTheory.Functor.Basic
import Init.Core
import Mathlib.CategoryTheory.Category.Cat

import TheWhiteheadTheorem

-- #check
-- #
```


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PART 1: Constructing Three Internal Universes

$$\infty_-(\infty\text{-Grpd}_0)$$

$$\infty_-(\infty\text{-Grpd})$$

$$\infty_-(\infty\text{-Cat})$$

PART 2: Monadicity

Monadicity, $D(\infty\text{-Grpd}_0)$, and
 $D(\infty\text{-Grpd}_0/X_0) \rightleftarrows D(\infty\text{-Grpd}_0/Y_0)$

Monadicity, $D(\infty\text{-Grpd})$, and
 $D(\infty\text{-Grpd}/X) \rightleftarrows D(\infty\text{-Grpd}/Y)$

Monadicity, $D(\infty\text{-Cat})$, and
 $D(\infty\text{-Cat}/C) \rightleftarrows D(\infty\text{-Cat}/D)$

PART 3: Kan Extensions

About the Author

Dean Young is a graduate student at New York University, where he studies mathematics.

