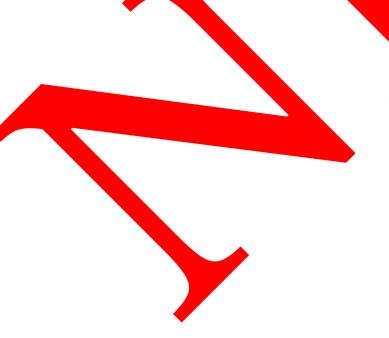
.py file
.tex file
.pdf file
.lean file



Internal Universes

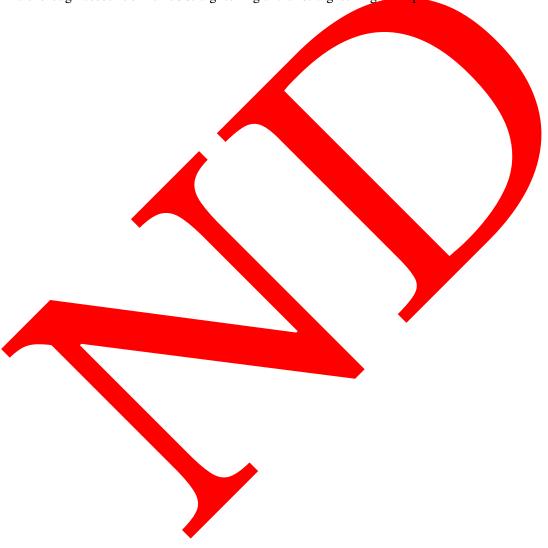
∞ _(∞ -Cat)	$\mathtt{D}(\infty_{-}(\infty\mathtt{-Cat}))$	∞_(∞-Cat)/C	$D(\infty_{}(\infty-Cat)/C)$
∞ _(∞ -Grpd)	$\mathtt{D}(\infty_{-}(\infty\mathtt{-Grpd}))$	∞ _(∞ -Grpd)/G	$D(\infty_{(\infty-Grpd)/G)}$
$\infty_{-}(\infty\text{-Grpd}_{0})$	$D(\infty_{-}(\infty-Grpd_0))$	$\infty_{-}(\infty\text{-Grpd}_{0})/G_{0}$	$D(\infty_{-}(\infty-Grpd_{0})/G_{0})$

E. Dean Young



1. Introduction

In this repository I would like to consider Kan extensions and homotopy Kan extensions, obtaining a thorough account of Lurie's straightening and unstraightening concepts.



2. Unicode

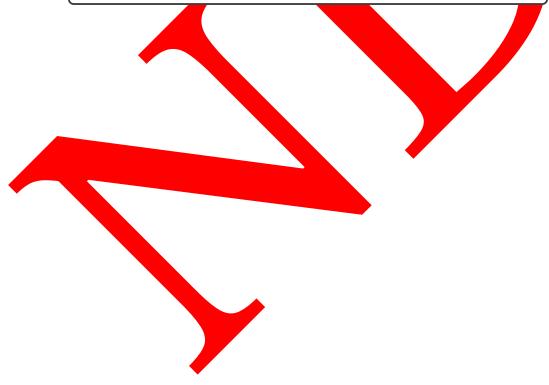
Here is a list of the unicode characters I will use:

Symbol	Unicode	VSCode shortcut	Use		
		Lean's Kern	el		
×	2A2F	\times	Product of types		
\rightarrow	2192	\rightarrow	Hom of types		
ζ,>	27E8,27E9	\langle,\rangle	Product term introduction		
-> sto	21A6	\mapsto	Hom term introduction		
٨	2227	\wedge	Conjunction		
V	2228	vee	Disjunction		
A	2200	\forall	Universal quantification		
3	2203	\exists	Existential quantification		
_	00AC	\neg	Negation		
Variables and Constants					
a,b,c,,z	1D52,1D56		Variables and constants		
0,1,2,3,4,5,6,7,8,9	1D52,1D56		Variables and constants		
-	207B		Variables and constants		
0,1,2,3,4,5,6,7,8,9	2080 - 2089	\0-\9	Variables and constants		
A,,Z	1D538	bbA,,\bbZ	Variables and constants		
0,,ℤ	1D552	\bba,,\bbz	Variables and constants		
α - ω ,A- Ω	03B1-03C9		Variables and constants		
Categories and Bicategories					
1	1D7D9	\b1	The identity morphism		
?	2218		Composition		
			Composition		
			Composition		
Adjunctions					
	1BC94		Right adjoints		
	0971		Left adjoints		
-	22A3	\dashv	The condition that two functors are adjoint		
Monads and Comonads					
?,¿	003F, 00BF	?,\?	The corresponding (co)monad of an adjunction		
!,j	0021, 00A1	!, \!	The (co)-Eilenberg-(co)-Moore adjunction		
!,	A71D, A71E		The (co)AdjMon maps		
Miscellaneous					
~	2243	equiv	Equivalences		
≅	2245	cong	Isomorphisms		
1	22A5	\bot	The overobject classifier		
∞	221E	\infty	Infinity categories and infinity groupoids		

Of these, the characters $^{!}$, $^{!}$, and $^{!}$ do not have VSCode shortcuts, and so I provide alternatives for them. Possibly they will have to be changed if this work assimilates into a larger project.

It is not possible to copy the from the pdf to the clipboard while preserving the integrity of the code. To see the official Lean 4 file please click the link on the top right of the front page or this.

Lean 1 import Mathlib.CategoryTheory.Bicategory.Basic import Mathlib.CategoryTheory.Types import Mathlib.CategoryTheory.DiscreteCategory import Mathlib.Combinatorics.Quiver.Basic ${\tt import\ Mathlib.CategoryTheory.Category.Init}$ import Aesop import Init import Mathlib.CategoryTheory.DiscreteCategory import Mathlib.CategoryTheory.Bicategory.Strict ${\tt import\ Mathlib.CategoryTheory.ConcreteCategory.Bundled}$ import Mathlib.CategoryTheory.Functor.Basic import Init.Core import Mathlib.CategoryTheory.Category.Cat import TheWhiteheadTheorem -- #check --





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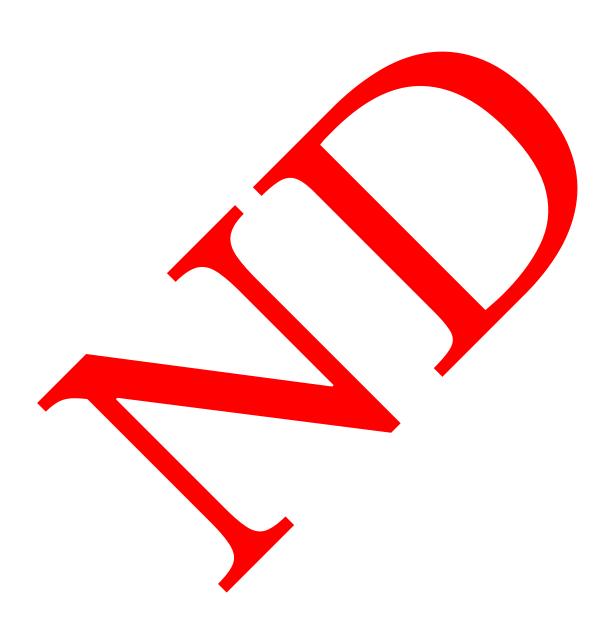


3. Contents

Section	Description				
Unfinished					
Contents					
Unicode					
Introduction					
PART I: Internal Universes					
Chapter 1: $\infty_{-}(\infty$ -Cat)					
$ec{\chi} : \ ec{\chi}$					
$\vec{\chi}$					
$D(\vec{\chi}.)$					
$D(\vec{\chi}^{\cdot})$					
$D(\infty_{-}(\infty - Grpd_{-1})'-)$					
$D([-\circ (\infty (\infty - Grpd_{-1})])$					
$D(\infty_{-}(\infty \operatorname{Grpd}_{-1})') \simeq D([-^{\circ},\infty_{-}(\infty \operatorname{Grpd}_{-1})])$					
Chapter 2: $\infty_{-}(\infty$ -Grpd)					
X.					
$\frac{\vec{\chi}}{\vec{\chi}}$					
$D(\vec{\chi}_{\cdot})$					
$D(\vec{\chi}^{\cdot})$					
$D(\infty_{-}(\infty\text{-Grpd})-)$					
Chapter 3: ∞ (∞ -Grpd ₀)					
χ.					
χ .					
$D(\chi_{\cdot})$					
$D(\chi^{\cdot})$					
$D(\infty_{-}(\infty-Cat)-)$					
$D([-0,\infty_{\infty}(\infty-Cat)])$					
$D(\infty_{-}(\infty-Cat)-) \simeq D([-^{\circ},\infty_{-}(\infty-Cat)])$					



$$\infty_{-}(\infty\text{-Grpd}_{0})$$



$$\infty$$
_(∞ -Grpd)



$$\infty_{-}(\infty$$
-Cat)

In this repository, I would like to think about the relationship between homotopy colimits, directed homotopy colimits, and homotopy colimits over based connected ∞ -groupoids and one object ∞ -groupoids, particularly as it concerns the six "fibrant replace and forget" functors.

I would also like to incorporate two notions of the formal addition of an interval object and directed interval object, as well as six theorems concerning monadicity that are related to it.

the left adjoint to precomposition in the case of the derived completion functors:

1. (completion of an ∞ -category with respect to the directed derived category $D(\infty$ -Cat))

$$\vec{\chi}: (X: \infty\text{-Cat}) \to (Y: \infty\text{-Cat}) \to \infty\text{-Cat.hom } X Y \to \text{Adjunction } D([-^{\circ}, \infty\text{-Cat}])$$

 $D([-^{\circ}, \infty\text{-Cat}])$

2. (completion of an ∞ -groupoid with respect to the directed derived category $D(\infty-Grpd)$) $\vec{\chi}: [-\circ, D(\infty-Grpd)] \rightleftharpoons [-\circ, D(\infty-Grpd)]$

$$\vec{\chi}$$
: $(X:\infty\text{-Grpd}) \to (Y:\infty\text{-Grpd}) \to \infty$ -Grpd.hom $X:Y \to Adjunction$ [-0, $D(\infty\text{-Grpd})$] [-0, $D(\infty\text{-Grpd})$]

3. (completion of a based connected ∞ -groupoid with respect to the category of based connected ∞ -groupoids) $\chi: [-^{\circ}, D(\infty\text{-Grpd}_0)] \rightleftharpoons [-^{\circ}, D(\infty\text{-Grpd}_0)]$:

$$\chi: (X: \infty\text{-}\mathrm{Grpd}_0) \to (Y: \infty\text{-}\mathrm{Grpd}_0) \to \infty\text{-}\mathrm{Grpd}_0.\text{hom } X Y \to \text{Adjunction}$$

$$[\xrightarrow{\circ} , D(\infty\text{-}\mathrm{Grpd}_0)] [\xrightarrow{\circ} , D(\infty\text{-}\mathrm{Grpd}_0)]$$

1. E, e and the various 'up to homotopy' structures (six)

2.

I like to use a notation of upper and lower dots for the left and right adjoint in Lean 4, but it requires specific fonts that not all systems have, and the two different characters do not always display like one another.

These three functors produce

1. Links back to the category section concerning the category of elements and how it can be used to express pointed Kan extensions.

2. χ hom : Functor $D(\infty$ -Cat) $\rightleftharpoons D(\infty$ -Cat) : χ .

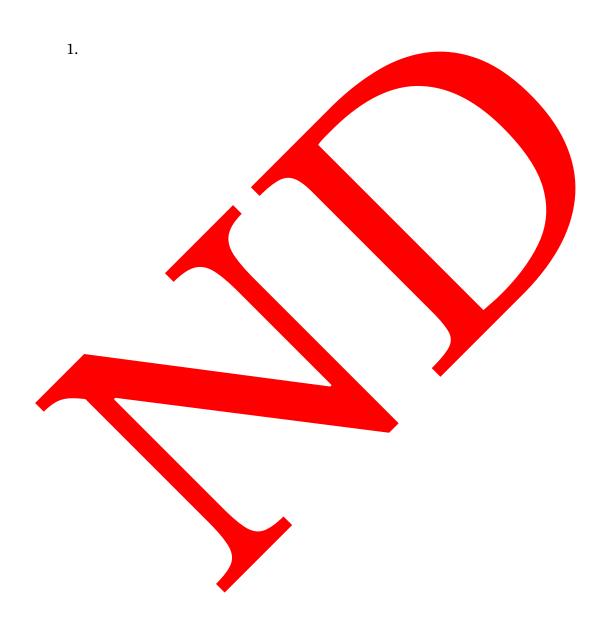
3. *χ*

4.

After establishing several interesting features of Lan $D(\infty\text{-Cat}),\,F^{\text{O}}\,:\,[C^{\text{O}}\,,D(\infty\text{-Cat})]$



4. Bibliography



About the Author

Dean Young is a graduate student at New York University, where he studies mathematics.

