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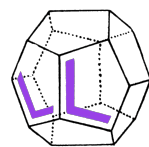
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## Categories and Hilbert Spaces

cat

$\begin{matrix} /- \\ \dots \\ -/ \end{matrix}$

Shanghe Chen and Dean Young

We wish to acknowledge the collaborative efforts of Shanghe Chen and Dean Young. Together the authors are pursuing these plans as a long term project.





# 1. Contents

| Section           | Description                                    |
|-------------------|--|
| Introduction      |  |
| Contents          |  |
| Unicode           |  |
| ...               |  |
| <code>h1lb</code> | Countable strict twocategory of hilbert spaces |

## 2. Introduction

### 3. Unicode

Lean 4 uses unicode, and this entails an extensive catalogue of characters to choose from. Here is a list of the unicode characters we will use:

| Symbol                         | Unicode     | VSCode shortcut   | Use  |
|--------------------------------|-------------|---|--|
| Lean's Kernel                  |             |   |  |
| $\times$                       | 2A2F        | <code>\times</code>   | Product of types                             |
| $\rightarrow$                  | 2192        | <code>\rightarrow</code>                                    | Hom of types                                 |
| $\langle, \rangle$             | 27E8, 27E9  | <code>\langle \rangle</code> , <code>\rangle \langle</code> | Product term introduction                    |
| $\mapsto$                      | 21A6        | <code>\mapsto</code>  | Hom term introduction                        |
| $\wedge$                       | 2227        | <code>\wedge</code>   | Conjunction                                  |
| $\vee$                         | 2228        | <code>\vee</code>   | Disjunction                                  |
| $\forall$                      | 2200        | <code>\forall</code>  | Universal quantification                     |
| $\exists$                      | 2203        | <code>\exists</code>  | Existential quantification                   |
| $\neg$                         | 00AC        | <code>\neg</code>   | Negation                                     |
| Variables and Constants        |             |   |  |
| $\alpha, \omega, A, \Omega$    | 03B1, 03C9  |   | Variables and constants                      |
| $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ | 0030 - 0039 |   | Variables and constants                      |
| $\alpha, \omega, A, \Omega$    | 03B1, 03C9  |   | Variables and constants                      |
| $\alpha, \omega, A, \Omega$    | 03B1, 03C9  |   | Variables and constants                      |
| $\alpha, \omega, A, \Omega$    | 03B1, 03C9  |   | Variables and constants                      |
| $\alpha, \omega, A, \Omega$    | 03B1, 03C9  |   | Variables and constants                      |
| Categories                     |             |   |  |
| $1$                            | 1D7D9       | <code>\b1</code>  | The identity morphism                        |
| $\circ$                        | 2218        | <code>\circ</code>  | Composition                                  |
| Twocategories                  |             |   |  |
| $1$                            | 1D7CF       |   | Horizontal identity map                      |
| $\bullet$                      | 2022        | <code>\smul</code>  | Horizontal composition of objects            |
| $\bullet$                      | 2219        |   | Horizontal composition of morphisms          |
| Adjunctions                    |             |   |  |
| $\rightleftarrows$             | 21C4        | <code>\rightleftarrows</code>                               | Adjunctions                                  |
| $\leftrightarrows$             | 21C6        | <code>\leftrightarrows</code>                               | Adjunctions                                  |
| $\cdot$                        | 1BC94       |   | Right adjoints                               |
| $\cdot$                        | 0971        |   | Left adjoints                                |
| $\dashv$                       | 22A3        | <code>\dashv</code>   | The condition that two Functors are adjoint  |
| Monads and Comonads            |             |   |  |
| $?, \iota$                     | 003F, 00BF  | <code>?, \iota</code>                                       | The corresponding (co)monad of an adjunction |
| $!, j$                         | 0021, 00A1  | <code>!, \!</code>  | The (co)-Eilenberg-(co)-Moore adjunction     |
| $^{\cdot}, _{\cdot}$           | A71D, A71E  |   | The (co)exponential map                      |
| Miscellaneous                  |             |   |  |
| $\sim$                         | 223C        | <code>\sim</code>   | Homotopies                                   |
| $\simeq$                       | 2243        | <code>\equiv</code>   | Equivalences                                 |
| $\cong$                        | 2245        | <code>\cong</code>  | Isomorphisms                                 |
| $\infty$                       | 221E        | <code>\infty</code>   | Infinity categories and infinity groupoids   |
| $\infty$                       | 221E        | <code>\infty</code>   | Infinity categories and infinity groupoids   |

Of these, the characters  $^{\cdot}, _{\cdot}, 1$ , and  $\bullet$  do not have VSCode shortcuts, and so we

provide alternatives for them.

It is not possible to copy the from the pdf to the clipboard while preserving the integrity of the code. To see the official Lean 4 file please click the link on the top right of the front page or click this link.



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Further reading:

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