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Two Theorems in Classifying Space The

$\vec{\mathrm{E}}$: Functor (OperadicCategory ∞-Cat) ∞-Cat	$\vec{\mathrm{e}}$: {C : ∞-Cat } $\to$ { D : ∞-Cat } $\to$ (F : ∞-Cat.hom
$\vec{\mathrm{B}}$: Functor (OperadicCategory ∞-Cat) ∞-Cat	$\vec{\mathrm{b}}$: { C : ∞-Cat } $\to$ { D : ∞-Cat } $\to$ (F : ∞-Cat.hon
$\vec{\partial}$: (C : OperadicCategory ∞-Cat) $\to$ ∞-Cat.hom ($\vec{\mathrm{E}}$.obj C) ($\vec{\mathrm{B}}$.obj C)	⍰ : { C : ∞-Cat } $\to$ { D : ∞-Cat } $\to$ (F : ∞-Cat.hon
$\vec{\vec{\mathrm{E}}}$: Functor (OperadicGroupoid ∞-Grpd) ∞-Grpd	$\vec{\vec{\mathrm{e}}}$: { X : ∞-Cat } $\to$ { Y : ∞-Cat } $\to$ (F : ∞-Cat.hom
$\vec{\vec{\mathrm{B}}}$: Functor (OperadicGroupoid ∞-Grpd) ∞-Grpd	$\vec{\vec{\mathrm{b}}}$: { X : ∞-Grpd } $\to$ { Y : ∞-Grpd } $\to$ (F : ∞-Cat.h
$\vec{\vec{\partial}}$: (G : OperadicCategory ∞-Grpd) $\to$ ∞-Grpd.hom ($\vec{\vec{\mathrm{E}}}$.obj G) ($\vec{\vec{\mathrm{B}}}$.obj G)	⍰ : { X : ∞-Grpd } $\to$ { Y : ∞-Grpd } $\to$ (F : ∞-Grp
E : OperadicGroup ∞-Grpd$_{-1}$ $\longrightarrow$ ∞-Grpd$_{-1}$	e : { X$_{-1}$: ∞-Grpd$_{-1}$ } $\to$ { Y$_{-1}$: ∞-Grpd$_{-1}$ } $\to$ (
B : OperadicGroup ∞-Grpd$_{-1}$ $\longrightarrow$ ∞-Grpd$_{-1}$	b : { X$_{-1}$: ∞-Grpd$_{-1}$ } $\to$ { Y$_{-1}$: ∞-Grpd$_{-1}$ } $\to$ (
∂ : (G$_{-1}$: OperadicGroup ∞-Grpd$_{-1}$) $\to$ ∞-Grpd$_{-1}$.hom (E.obj G$_{-1}$) (B.obj G$_{-1}$)	⍰ : { G$_{-1}$: ∞-Grpd$_0$ } $\to$ { Y$_0$: ∞-Grpd$_0$ } $\to$ (F :

E. Dean Young

1. Contents

$\vec{E}$: Functor (OperadicCategory ∞-Cat) ∞-Cat
$\vec{e}$: {C : ∞-Cat } → { D : ∞-Cat } → (F : ∞-Cat.hom C D) → Functor (OperadicPresheaf ($\vec{O}$.obj D)) (∞-Cat⁄D)
$\vec{B}$: Functor (OperadicCategory ∞-Cat) ∞-Cat
$\vec{b}$: { C : ∞-Cat } → { D : ∞-Cat } → (F : ∞-Cat.hom C D) → Functor (OperadicPresheaf ($\vec{O}$.obj D)) (∞-Cat⁄D)
∂ : (C : OperadicCategory ∞-Cat) → ∞-Cat.hom ($\vec{E}$.obj C) ($\vec{B}$.obj C)
⍰ : { C : ∞-Cat } → { D : ∞-Cat } → (F : ∞-Cat.hom C D) → (∞-Cat⁄D).hom ($\vec{e}$.obj F) ($\vec{b}$.obj F)

`Chapter 11:` The Recognition Theorem for ∞-Categories
`Chapter 12:` The Classifying Space Theorem for ∞-Categories

Introduction

In "'TheWhiteheadTheoremandTwoVariations"', I will be defining six "internal" structures based on "'Galois Theories"' by Janelidze and Borceux, as well as six "'operadic"' structures

and "'ThePuppeSequenceandTwoVariations"'. In "'InternalUniverses"', I considered straightening and unstraightening and three variations of it, which were each considered before and after the application of D(-). This made for the six diagrams depicted on page ???. In this repository, we consider the classifying space B.

Let $F : C \longrightarrow D : \infty$-Cat.hom C D be an ∞-functor. Given either the C-infinity presheaf in ∞-Cat/C arising from $F : \infty$-Cat/D or the C-infinity presheaf in ∞-Cat/D arising from $\mathrm{Id}_C : \infty$-Cat/C, we obtain in both cases an internal presheaf in the corresponding derived category. However, not all internal categories D : InternalCategory D(∞-Cat/C) arise from and not all internal presheaves S : InternalPresheaf D D(∞-Cat/C) arise from C-infinity presheaves over some C-infinity category in ∞-Cat/C.

In "'InternalUniverses"', we showed the straightening/unstraightening categorical equivalence and three variations using the six Ω-functors and six E-functors, treating the situations before and after the application of D(-) seperately for a total of six goals.

In this section, we consider classifying spaces as well as a perspective about remembering information concerning a right or left adjoint applied to a particular functor or object in the following way: E and Ω and their respective five variations give "'remembrant"' functors E-infinity and Ω-infinity, which each produce internal presheaves in respective derived categories.

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Plans to prove three variations of the
Whitehead theorem of homotopy groups in
Lean 4, with extensive use of Mathlib 4
```

$\vec{\mathrm{E}} : \infty\text{-Cat} \longrightarrow \infty\text{-Cat}$	$\vec{\mathrm{B}} : \infty\text{-Cat} \longrightarrow \infty\text{-Cat}$	$\vec{\partial} : \infty\text{-Cat} \longrightarrow \infty\text{-Cat}$	$\vec{\mathrm{e}} : (\mathrm{C} : \infty\text{-Cat}) \to (\mathrm{D} : \infty\text{-Cat}) \to$ Adjunc
$\overleftrightarrow{\mathrm{E}} : \infty\text{-Grpd} \longrightarrow \infty\text{-Grpd}$	$\overleftrightarrow{\mathrm{B}} : \infty\text{-Grpd} \longrightarrow \infty\text{-Grpd}$	$\overleftrightarrow{\partial} : \infty\text{-Grpd} \longrightarrow \infty\text{-Grpd}$	$\vec{\mathrm{e}} : (\mathrm{X} : \infty\text{-Grpd}) \to (\mathrm{Y} : \infty\text{-Grpd}) \to$ Adju
$\mathrm{E} : \infty\text{-Grpd}_0 \longrightarrow \infty\text{-Grpd}_0$	$\mathrm{B} : \infty\text{-Grpd}_0 \longrightarrow \infty\text{-Grpd}_0$	$\partial : \infty\text{-Grpd}_0 \longrightarrow \infty\text{-Grpd}_0$	$\mathrm{e} : (\mathrm{X} : \infty\text{-Grpd}_0) \to (\mathrm{Y} : \infty\text{-Grpd}_0) \to$ Ad

By the time this repository is seen to in 2025, I will have filled out a certain six operadic structures to do with ∞-categories and ∞-groupoids. Each of these structures will be made to work together with $\mathrm{Pow}\,X : \mathrm{Type} : \lambda(n : \mathbb{N}), X \to X$. The six operadic structures are endofunctions of one of six mathematical objects, here with an option for 12 based on models A and B.

$$\mathtt{B}^{1}\ :\ \mathtt{Functor\ (pow\ OperadicGroup\ 2)\ (pow\ OperadicGroup\ 2)}$$

$$\mathtt{B}^{n}\ :\ \mathtt{Functor\ (pow\ OperadicGroup\ 2)\ (pow\ OperadicGroup\ 2)}$$

In this repository I construct six categorical equivalences:

1.
2.
3.
4.
5.
6.

PART 1: BASED CONNECTED ∞-GROUPOIDS

Operadic Groups and Operadic Group Actions

The Classifying Space and the Total Space

PART 2: ∞-GROUPOIDS

Operadic Groupoids and Operadic Groupoid Actions

The Recognition Theorem for ∞-Groupoids

The Classifying Space Theorem for ∞-Groupoids

PART 3: ∞-CATEGORIES

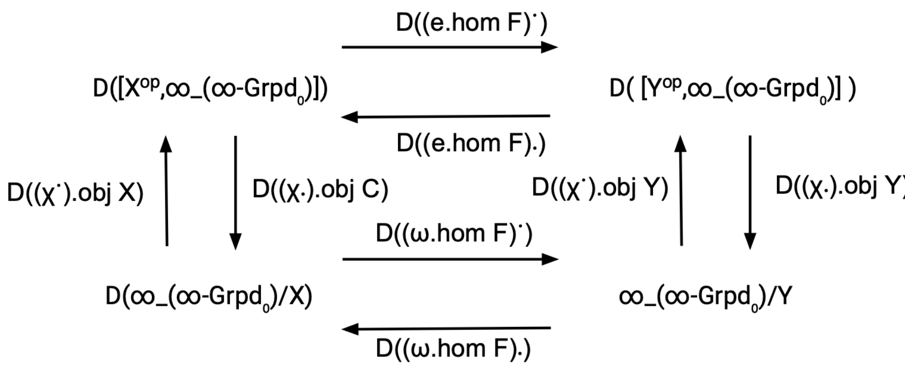
Operadic Categories and Operadic Presheaves

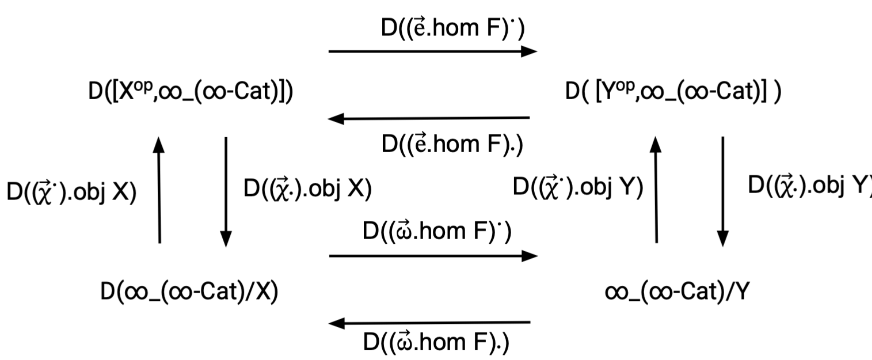
The Recognition Theorem for ∞-Categories

The Classifying Space Theorem for ∞-Categories

Implementation	2024 ∞-category project identity		Lax		Strict	
	ND	Filling Number	Unitial	Actional	Unitial	Actional
First	ND00	Filling number (n : Nat)	C-infinity categories	C-infinity presheaves	Internal categories	Internal presheaves
	ND01	Filling number (n : Nat)	∞-Cat	∞-Cat ∕ C	D(∞-Cat)	D(∞-Cat/C)
Second	ND10	Filling number (n : Nat)	The lax lifted Whitehead theorem	The lax lifted Puppe sequence	The strict Whitehead theorem	The strict Puppe sequence
	ND11	Filling number (n : Nat)	The lax lifted B-Ω equivalence	The lax lifted b-ω equivalence	The strict strict B-Ω equivalence	The strict b-ω equivalence

In "'Internal Universes"' I thought about the six variations of straightening and unstraightening featured in the diagrams below:





$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\mathrm{D}([X^{op},\infty\_(\infty\text{-Grpd})]) \arrow[r, shift left, "\mathrm{D}((\overleftrightarrow{e}.\mathrm{hom}\ F)^{\cdot})"] \arrow[d, shift left, "\mathrm{D}((\overleftrightarrow{\chi}_{\cdot}).\mathrm{obj}\ X)"] & \mathrm{D}([Y^{op},\infty\_(\infty\text{-Grpd})]) \arrow[l, shift left, "\mathrm{D}((\overleftrightarrow{e}.\mathrm{hom}\ F)_{\cdot})"] \arrow[d, shift left, "\mathrm{D}((\overleftrightarrow{\chi}_{\cdot}).\mathrm{obj}\ Y)"] \\
\mathrm{D}(\infty\_(\infty\text{-Grpd})/X) \arrow[u, shift left, "\mathrm{D}((\overleftrightarrow{\chi}^{\cdot}).\mathrm{obj}\ X)"] \arrow[r, shift left, "\mathrm{D}((\overleftrightarrow{\omega}.\mathrm{hom}\ F)^{\cdot})"] & \infty\_(\infty\text{-Grpd})/Y \arrow[u, shift left, "\mathrm{D}((\overleftrightarrow{\chi}^{\cdot}).\mathrm{obj}\ Y)"] \arrow[l, shift left, "\mathrm{D}((\overleftrightarrow{\omega}.\mathrm{hom}\ F)_{\cdot})"]
\end{tikzcd}
$$

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
{[X^{op},\infty\_(\infty\text{-}\mathrm{Grpd}_0)]} \arrow[r, shift left=2, "(e.\mathrm{hom}\ F)^{\cdot}"] \arrow[d, shift left=2, "(\chi_{\cdot}).\mathrm{obj}\ X"] & {[Y^{op},\infty\_(\infty\text{-}\mathrm{Grpd}_0)]} \arrow[l, shift left=2, "(e.\mathrm{hom}\ F)_{\cdot}"] \arrow[d, shift left=2, "(\chi_{\cdot}).\mathrm{obj}\ Y"] \\
\infty\_(\infty\text{-}\mathrm{Grpd}_0)/X \arrow[u, shift left=2, "(\chi^{\cdot}).\mathrm{obj}\ X"] \arrow[r, shift left=2, "(\omega.\mathrm{hom}\ F)^{\cdot}"] & \infty\_(\infty\text{-}\mathrm{Grpd}_0)/Y \arrow[u, shift left=2, "(\chi^{\cdot}).\mathrm{obj}\ Y"] \arrow[l, shift left=2, "(\omega.\mathrm{hom}\ F)_{\cdot}"]
\end{tikzcd}
$$

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
{[C^{op},\infty\_(\infty\text{-}\mathrm{Cat})]} \arrow[r, shift left=2, "(\vec{e}.\mathrm{hom}\ F)^{\cdot}"] \arrow[d, shift left=2, "(\vec{\chi}_{\cdot}).\mathrm{obj}\ C"] & {[D^{op},\infty\_(\infty\text{-}\mathrm{Cat})]} \arrow[l, shift left=2, "(\vec{e}.\mathrm{hom}\ F)_{\cdot}"] \arrow[d, shift left=2, "(\vec{\chi}_{\cdot}).\mathrm{obj}\ D"] \\
\infty\_(\infty\text{-}\mathrm{Cat})/C \arrow[u, shift left=2, "(\vec{\chi}^{\cdot}).\mathrm{obj}\ C"] \arrow[r, shift left=2, "(\vec{\omega}.\mathrm{hom}\ F)^{\cdot}"] & \infty\_(\infty\text{-}\mathrm{Cat})/D \arrow[u, shift left=2, "(\vec{\chi}^{\cdot}).\mathrm{obj}\ D"] \arrow[l, shift left=2, "(\vec{\omega}.\mathrm{hom}\ F)_{\cdot}"]
\end{tikzcd}
$$

6 goals 6 structures
With these goals I want to create several "'remembrant"' adjunctions:

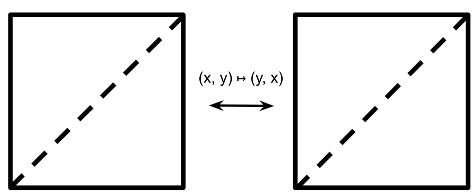
1. $\vec{\gamma}$ $\overleftrightarrow{\gamma}$ γ
2. $\vec{\Sigma}$ $\overleftrightarrow{\Sigma}$ Σ
3. $\vec{\sigma}$ $\overleftrightarrow{\sigma}$ σ
4. Pullback of two homs and a single hom vs. a pushout of two products and a single product

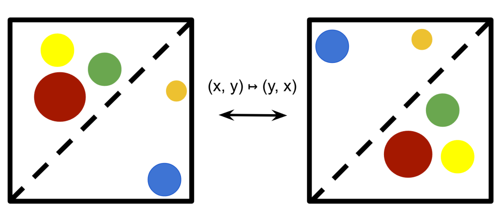
$$\vec{\mathrm{o}} : (\mathrm{C} : \infty\text{-Cat}) \to \infty\text{-Cat}/\mathrm{C} \longrightarrow \mathrm{OperadicPresheaf}\,(\vec{\mathrm{O}}.\mathrm{obj}\ \mathrm{C})$$

defining B

1. It is possible that the B lifts under slightly different conditions than those under which it is an endomorphism.
2. After use of the ∞-box, whose product is difficult, we can invert certain maps to obtain complexes. For this to work we need both biproducts and minus.
3. Not only must these spaces be based; B necessitates that they be A∞ or E∞ (plus some other thing about grouplike, for me).
4. After this we can consider the "free ???", but the product is a bit difficult.

$[\mathbb{N}, \vec{\gamma}\ , X]$





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2.

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