Matrix Manual

Linlin Ge

1 Symbols

Symbols	Descriptions
$\equiv$	is equivalent to
$:=$	equal by definition
$\triangleq$	
$\approx$	is approximately
$\sim$	similar or weak approximately

2 Norm on Vector Space

Definition 2.1. The size of a vector $v = (a_1, \cdots, a_n)^T$ is

$$\|v\| = \|v\|_2 = \sqrt{a_1^2 + \cdots + a_n^2} \tag{2.1}$$

3 Matrix Norms

Definition 3.1. 1-norm is maximal column sum

$$\|A\|_1 = \max_{j=1}^{n} \sum_{i=1}^{n} |a_{ij}| \tag{3.1}$$

Definition 3.2. ∞-norm is maximal row sum

$$\|A\|_1 = \max_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}| \tag{3.2}$$

Definition 3.3. 2-norm

$$\|A\|_2 = \max_{i=1}^{n} \sqrt{\lambda_i(A^T A)} \tag{3.3}$$

Theorem 3.1 *For $A, B \in M_n(R)$*

(I) $\|A\| \geq 0$, with equality if and only if $A = O$

(II) $\|A + B\| \leq \|A\| + \|B\|$

(III) $\|cA\| = |c|\|A\|$

(IV) $\|AB\| \leq \|A\|\|B\|$

(V) $\|A\| = \|A^T\|$

(VI) $\|AA^T\| = \|A^TA\| = \|A\|^2$. Thus $\|A\| = \sqrt{\|AA^T\|} = \sqrt{\|A^TA\|}$

(VII) $|(Av, w)| \leq \|A\|\|v\|\|w\|$

(VIII) $\|A\|_{sup} \leq \|A\| \leq n\sqrt{n}\|A\|_{sup}$

References