Note for Probability Theory and Statistics

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1 Function

Welch's Method

$$\rho(x) = \frac{\sigma_r^2}{2}(1 - e^{(\frac{x}{\sigma_r})^2})$$

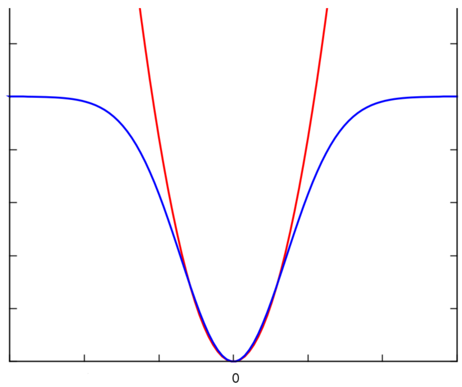


Figure 1: L2 error (red) versus the robust Welsh's function (blue)

Gauss Error Function

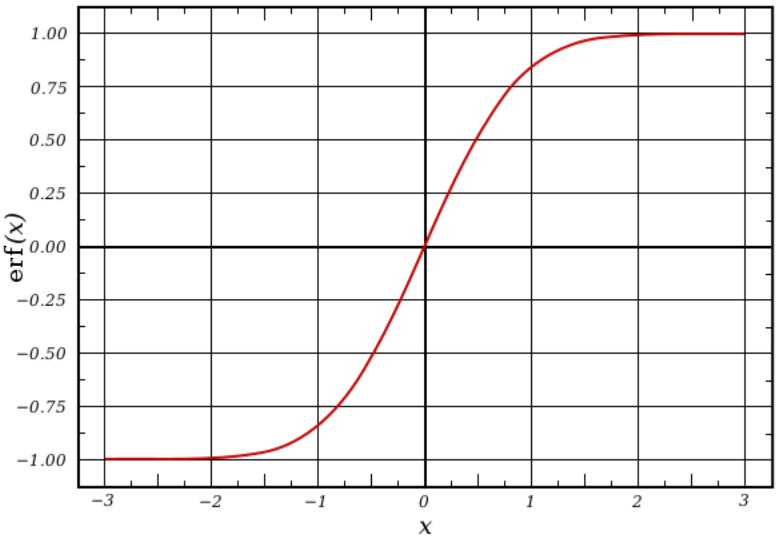
In statistics, for nonnegative values of x, the error function has the following interpretation: for a random variable Y that is normally distributed with mean 0 and variance $1/2$, $erf(x)$ describes the probability of Y falling in the range $[-x, x]$.

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt \tag{1}$$

Approximation gauss error function with elementary functions:

$$erf(x) = 1 - \frac{1}{(1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)^4} \quad x \geq 0 \quad (\textbf{maximum error:} 5 \times 10^{-4}) \tag{2}$$

Where $a_1 = 0.278393, a_2 = 0.230389, a_3 = 0.000972, a_4 = 0.078108$



References