Suppose the characteristic polynomial of a matrix Anxn is factorized as follows, with all the common factors group together:

$$deb(\Im I - A) = (3 - \Im_1)^{\gamma_1} (3 - \Im_2)^{\gamma_2} \cdots (3 - \Im_k)^{\gamma_k}$$

Where Di, Di ... Die are all eigenvalues of Ann, Ti to The are the respective multiplicities of these eigenvalues, with $r_1 + r_2 + \cdots + \gamma_K = r_k$.



Prove that $dim(E_{3i}) \leq \gamma_i$ for all i.



Proof:

Suppose that A E Matrix nxn, and So is an eigenvalue of Anxn, and ? V1, V2, ..., Ve j is a basis of Ezo, and thus, $\dim(E_{20}) = \ell(X)$

Next, extend these V_j 's (j=1,2,...,l) to a basis of IR^n , Which can be denoted by & V1. 192, ..., Ve, Ve+1, ..., Vnf.

Then, We may form a matrix $p = (v_1 \ v_2 \cdots v_k \ v_{k+1} \cdots v_n)$.

(Since 191, 192... In are n linearly independent vectors in IRn, then P is invertible.) Consider $D = P^{-1}AP$, and let $W_j = P^{-1}V_j$, $j = 1, 2, \dots, \ell$.

Claim: Wj is an eigenvector of D associated with the same eigenvalue 7. D. Wi = P-AP. P-Vi = P-AVi $= P^{-1}(Av_i)$ $= P^{-1} \partial v_i$ $= 3.(P^{1}V_{i})$

 $= \partial_1 \omega_j$, $j = 1, 2, \dots, \ell$.

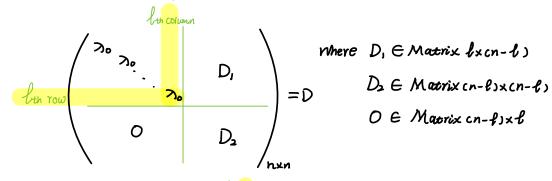
Hence $D \cdot \omega_j = \partial \cdot \omega_j$, $j = 1, 2, \dots, \ell_m$

②. Claim: $(U_j = e_j \text{ for } j = 1, 2, \dots l$, where e_j is the standard basis Vector in $I\mathbb{R}^n$, i.e. $\mathcal{L}_j = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ jth TOOD

Recaul, we have defined $P = (v_1 \ v_2 \ \cdots \ v_n)$, From Linjing https://linn-guo.github.io
and $w_j = P^+v_j = \text{the } j_{th}$ column of the matrix $(P^+v_1 \ P^+v_2 \ \cdots \ P^+v_j \ \cdots \ P^+v_n)$ $= \text{the } j_{th} \text{ column of the matrix } P^+(v_1 \ v_2 \ \cdots \ v_j \ \cdots \ v_n).$ $= \text{the } j_{th} \text{ column of the matrix } I_n \text{ (as } P^-P = I_n \text{ from the above)}.$ $= \ell_j \ , \quad j=1,2,\cdots,\ell. \quad \blacksquare$

From the claim (1) and (2),

 $D \cdot e_j = 3_0 \cdot e_j$ for $j=1,2,3,...,\ell$, and then we can write D as



$$\Rightarrow \Im I - D = \begin{pmatrix} 3 - 3 & & \\ 3 - 3 & & \\ & & & \\ O & \Im I_{n-\ell} - D_2 \end{pmatrix}$$
hun

So, the characteristic polynomial of D is $\det(\Im I - D) = (3 - 3) \cdot \det(\Im I_{n-1} - D_2)$

(3) Claim: det(3I-A) = det(3I-D)

$$det(\Im I - D) = det(\Im I - P^{-1}AP) \qquad \Im I = \Im P^{-1}P = P^{-1}(\Im I) \cdot P$$

$$= det(P^{-1}(\Im I - A) \cdot P) \qquad \Rightarrow \Im I - P^{-1}AP = P^{-1}(\Im I - A) \cdot P$$

$$= det(P^{-1}) \cdot det(\Im I - A) \cdot det(P), \text{ where } P \text{ is non-singular}.$$

$$= \overline{det(P)} \cdot det(\Im I - A) \cdot det(P)$$

$$= det(\Im I - A) \cdot \square$$

Hence, $\det(\Im I - A) = (\Im - \Im_0)^{\ell} \det(\Im I_{n-\ell} - D_2)$

So, we can conclude that

the multiplicity of 30 ≥ l, i.e. ro≥ l. =