AMS 572, Fall 2016

balanced design

Example: Dear hunting

Solution 1. Solution: (a) This is one-way ANOVA with 5 "samples" (K=5), and 6 observations per sample ($n_i \equiv n = 6$), and thus the total sample size is N=30. The grand mean is

$$\bar{\bar{y}} = \frac{341.7 + 412.5 + \cdots + 430.0}{5} = 411$$

We are testing $H_0 : \mu_1 = \mu_2 = \cdots = \mu_5$ versus H_a: at least one of these equalities is not true. The test statistic is

$$F_0 = \frac{MSA}{MSE}$$

	DF	MS	F
Among	4	24067.12	14.42
Within	25	1668.588	
total	29		

where

$$MSA = \frac{\sum_{i=1}^{a} n_i(\bar{y}_i - \bar{\bar{y}})^2}{a-1} = \frac{6}{4}[(341.7 - 411)^2 + \ldots + (430.0 - 411)^2] = 24067.17$$

and

$$MSE = s^2 = \frac{\sum_{i=1}^{a}(n_i - 1)s_i^2}{\sum_{i=1}^{a}(n_i - 1)} = \frac{1}{5}[40.8^2 + \ldots + 38.1^2] = 1668.588$$

Therefore

$$F_0 = \frac{24067.17}{1668.588} \approx 14.42$$

Since $F_0 \approx 14.42 > F_{4,25,0.05} = 2.67$, we reject the ANOVA hypothesis H_0 and claim that the rifles are not equally good

(b) Now we will do the pairwise comparison using Tukey's method. The Tukey method will reject any pairwise null hypothesis $H_{0ij} : \mu_i = \mu_j$ at FWE=α if

$$|t_{ij}| = \frac{|\bar{y}_i - \bar{y}_j|}{\sqrt{MSE/2/n}} > \frac{q_{K,N-K,\alpha}}{\sqrt{2}}$$

$$\frac{|\bar{y}_i - \bar{y}_j|}{s/\sqrt{n}} > q_{K,N-K,\alpha}$$

In our case, K=5, n=6, $s = \sqrt{1668.588} \approx 40.85$, N-K=25, α=0.05, and $q_{a,N-a,\alpha} = q_{5,25,0.05} \approx$ 4.17. Therefore, we would reject H_{0ij} if

$$|\bar{y}_i - \bar{y}_j| > s * q_{a,N-a,\alpha}/\sqrt{n} \approx 69.54$$

The conclusion is that at the familywise error rate of 0.05, we declare that the following rifle pairs are significantly different: 4/1, 4/2, 4/3, 4/5, 5/1, 2/1.