AMS 597 Spring 2018
Homework 4 Due 04/24/2018 4:00 PM

Instruction: Submit your homework via Blackboard. If you have difficulty submitting via Blackboard, email the TA directly. No late homework will be accepted, based on the time the email is sent out. If you are submitting via email (not recommended), put this as the subject of your email

AMS597 2018: Homework/Exam #4. ID: XXXXXXXXX. Name: XXX XXX

1. Compute a Monte Carlo estimate of

$$\int_0^{\pi/3} sinx\mathrm{d}x.$$

   Compare your estimate with the exact value of the integral.

2. We will estimate ω of

$$\omega = \int_0^{0.5} e^{-x}\mathrm{d}x$$

   using two different approaches:

   (a) Compute a Monte Carlo estimate ($\hat{\omega}$) of ω by sampling from Uniform(0, 0.5), and estimate the variance of $\hat{\omega}$.

   (b) Compute a Monte Carlo estimate (ω^*) of ω by sampling from the exponential distribution, and estimate the variance of ω^*.

   (c) Compare the two variances. Which one is smaller?

3. (a) Generate $X_1, \ldots, X_{20}$ from $N(0, 1)$. Consider testing $H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$. Compute the p-value from (1) one sample t-test and (2) exact wilcoxon signed rank test. Repeat this process 1000 times. Estimate the empirical Type I error for both tests at $\alpha = 0.05$.

   (b) Now generate $X_1, \ldots, X_{20}$ from $N(0.5, 1)$. Consider testing $H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$. Compute the p-value from (1) one sample t-test and (2) exact wilcoxon signed rank test. Repeat this process 1000 times. Estimate the empirical power for both tests at $\alpha = 0.05$.

4. Implement the bivariate Spearman rank correlation test as a permutation test. Compare the achieved significance level of the permutation test with the p-value reported by `cor.test` on the following samples:

```
> set.seed(123)
> x <- rnorm(50)
> y <- 0.2*x+rnorm(50)
```