AMS 572 Notes 7

1. Example 2 (Cereal)
   $H_0 : \mu \leq 16$ vs $H_a : \mu > 16$

   (a) Test Statistic : $Z_0 = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} \overset{H_0}{\sim} N(0,1)$

   We reject H_0 at $\alpha = 0.05$ if $Z_0 = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} \geq z_\alpha \Rightarrow \overline{X} \geq \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} = 16 + 1.645 \times \frac{0.4}{\sqrt{25}} =$ 16.1316(oz)

   (b) Power = P(Reject $H_0|H_a$)

$$
\begin{aligned}
&= P(Z_0 \geq z_\alpha | \mu = \mu_a) \\
&= P(\tfrac{\overline{X}-\mu_a}{\sigma/\sqrt{n}} \geq z_\alpha - \tfrac{\mu_a-\mu_0}{\sigma/\sqrt{n}} | \mu = \mu_a) \\
&= P(Z \geq 1.645 - \tfrac{16.13-16}{0.4/\sqrt{25}}) = P(Z \geq 0.02) \doteq 0.49
\end{aligned}
$$

   (c) $\alpha = 0.05, \beta = 0.2, \mu_0 = 16, \mu_a = 16.13, \sigma = 0.4$

$$
n = \frac{(Z_\alpha + Z_\beta)^2 \sigma^2}{(\mu_a - \mu_0)^2} = \frac{(1.645 + 0.845)^2 (0.4)^2}{(16.13 - 16)^2} \doteq 59
$$

2. Example 3 (ALUMCO)
   (a) $H_0 : \mu = 165$ vs $H_a : \mu > 165$ t-test
   $u \leq 165$
   Assume the distribution is normal.

$$
T_0 = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} \overset{H_0}{\sim} t_{n-1}
$$

   At the significance level of $\alpha = 0.05$, we reject H_0 in favor of H_a if $T_0 \geq t_{n-1,\alpha}$
   $T_0 = \frac{87.7}{27.6/\sqrt{7}} \doteq 8.4 > 1.943$, reject H_0 in favor of H_a that the average axial load is greater than 165 pounds.

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