Test

Changyuan Lin

January 30, 2018

Contents

1 Chapter 1

1.1 Equation Generated by Python

Consider the differential equation given below.

$$\frac{dy(t)}{dt} - 1.5y(t) = 1.5u(t), y(0) = 3.6 \tag{1}$$

1.2 Table

Consider the singal given below.

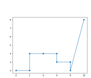


Figure 1: Reconstruct signal $y(t)$.

1.3 Inline Equation

Consider the differential equation given below.

$$\frac{dy(t)}{dt} - 1.5y(t) = 1.5u(t), y(0) = 3.6 \tag{2}$$

- Find an integral form solution for Eq.2.
- Find an analytic solution for Eq.2 when the input $u(t)$ is given as , for $t \geq 0$. Is this input able to stabilize state $y(t)$ when $t \to \infty$?
- Find the difference equation when sampleo time is $\triangle t = 0.3$ by exact discretization.
- Find the difference equation with time derivative approximated at $\triangle t = 0.3$ by finite difference method.
- Find a solution of difference equation and exact diference equation using MATLAB or any other software for $k = 20$. Plot the sequence of numbers obtained by the exact difference equation and by finite difference method and applied piecewise constant input given as follows

$$\begin{aligned} u(0) &= -2 \\ u(1\triangle t) &= -2e^{-2*1\triangle t} \\ u(2\triangle t) &= -2e^{-2*2\triangle t} \\ \cdots &= \cdots \\ u(k\triangle t) &= -2e^{-2*k\triangle t} \end{aligned}$$

  with k=20

  Solution:

- $$y(t) = 5e^{2t}+1.5e^{2t}\int_0^t (-2)e^{-2s}e^{-2s}ds = 5e^{2t}-3e^{2t}\int_0^t e^{-4s}ds = 5e^{2t}-3\int_0^t e^{-4s}\frac{d(-4s)}{-4}$$

$$y(t) = 5e^{2t} + \frac{3}{4}\left[e^{-4t} - 1\right]$$

- Exact discretization $\Delta t = 0.3$:

$$\begin{aligned} y_{k+1} &= e^{a\Delta t}y_k + a^{-1}(e^{a\Delta t} - 1)bu_k \\ &= 1.822y_k + 0.6166u_k \end{aligned}$$

- Finite difference method $\Delta t = 0.3$:

$$
\begin{aligned}
\frac{dy}{dt} \approx \frac{y_{k+1} - y_k}{\Delta t} &= 2y_k + 1.5u_k \\
y_{k+1} &= (1 + 2\Delta t)y_k + 1.5(\Delta t)u_k \\
&= 1.6y_k + 0.45u_k
\end{aligned}
$$

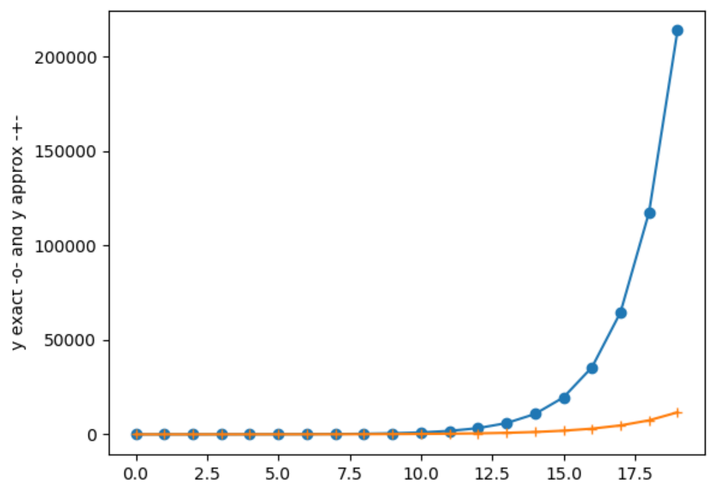
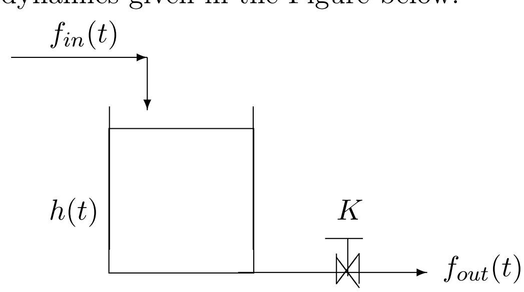


Figure 2: Finite difference and exact discretization $y(k)$.

1.4 ditaa & LaTeX Graph

For the physical process of tank dynamics given in the Figure below:



Solution:

- $\frac{dh}{dt} = -\frac{1}{KA}h + \frac{1}{A}f_{in}$

- Assume that the tank height is measured, $y(t) = h(t)$

$$\Sigma(A, B, C, D) = \Sigma\left(-\frac{1}{K\overline{A}}, \frac{1}{\overline{A}}, 1, 0\right)$$

- The state-space realization is given as:

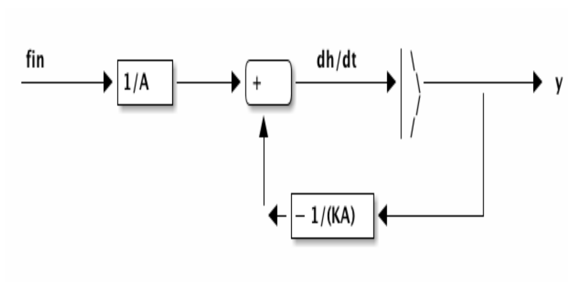


Figure 3: Block diagram elements.

- $$\Phi = e^{-\frac{0.2}{K\overline{A}}}, \quad \Gamma = -K\left(e^{-\frac{0.2}{K\overline{A}}} - 1\right), \quad \theta = 1$$

$$\Sigma(A_d, B_d, C_d, D_d) = \Sigma\left(e^{-\frac{0.2}{K\overline{A}}}, -K\left(e^{-\frac{0.2}{K\overline{A}}} - 1\right), 1, 0\right)$$

- Assume that $h(0) = 0$. Laplace transform:

$$\frac{Y(s)}{U(s)} = \frac{5}{s+1}$$

$$\boxed{\tau = 1, \quad \tau_d = 0 \Rightarrow \Delta t = (0.1 \sim 0.2)1\tau = 0.1 \sim 0.2}$$