

# Test

Changyuan Lin

February 6, 2018

## Contents

<b>1</b>	<b>Chapter 1</b>	<b>1</b>
1.1	Equation Generated by Python . . . . .	1
1.2	Table . . . . .	1
1.3	Inline Equation . . . . .	2
1.4	ditaa & L <sup>A</sup> T <sub>E</sub> X Graph . . . . .	3
<b>2</b>	<b>Chapter 2</b>	<b>4</b>
2.1	JavaScript . . . . .	4
2.2	Answer Sheet via Email . . . . .	4
2.3	Answer Sheet via Post . . . . .	4

## 1 Chapter 1

### 1.1 Equation Generated by Python

Consider the differential equation given below.

$$\frac{dy(t)}{dt} - 1.5y(t) = 1.5u(t), y(0) = 3.6 \quad (1)$$

### 1.2 Table

Consider the singal given below.

### 1.3 Inline Equation

Consider the differential equation given below.

$$\frac{dy(t)}{dt} - 1.5y(t) = 1.5u(t), y(0) = 3.6 \quad (2)$$

- Find an integral form solution for Eq.2.
- Find an analytic solution for Eq.2 when the input  $u(t)$  is given as , for  $t \geq 0$ . Is this input able to stabilize state  $y(t)$  when  $t \rightarrow \infty$ ?
- Find the difference equation when sample time is  $\Delta t = 0.3$  by exact discretization.
- Find the difference equation with time derivative approximated at  $\Delta t = 0.3$  by finite difference method.
- Find a solution of difference equation and exact difference equation using MATLAB or any other software for  $k = 20$ . Plot the sequence of numbers obtained by the exact difference equation and by finite difference method and applied piecewise constant input given as follows

$$\begin{aligned} u(0) &= -2 \\ u(1\Delta t) &= -2e^{-2*1\Delta t} \\ u(2\Delta t) &= -2e^{-2*2\Delta t} \\ &\dots = \dots \\ u(k\Delta t) &= -2e^{-2*k\Delta t} \end{aligned}$$

with k=20

Solution:

•

$$\begin{aligned} y(t) &= 5e^{2t} + 1.5e^{2t} \int_0^t (-2)e^{-2s} e^{-2s} ds = 5e^{2t} - 3e^{2t} \int_0^t e^{-4s} ds = 5e^{2t} - 3 \int_0^t e^{-4s} \frac{d(-4s)}{-4} \\ y(t) &= 5e^{2t} + \frac{3}{4} [e^{-4t} - 1] \end{aligned}$$

- Exact discretization  $\Delta t = 0.3$ :

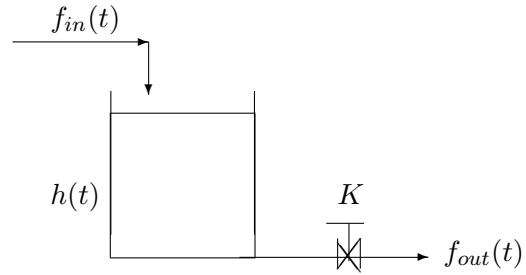
$$\begin{aligned} y_{k+1} &= e^{a\Delta t} y_k + a^{-1}(e^{a\Delta t} - 1)bu_k \\ &= 1.822y_k + 0.6166u_k \end{aligned}$$

- Finite difference method  $\Delta t = 0.3$ :

$$\begin{aligned}\frac{dy}{dt} &\approx \frac{y_{k+1} - y_k}{\Delta t} = 2y_k + 1.5u_k \\ y_{k+1} &= (1 + 2\Delta t)y_k + 1.5(\Delta t)u_k \\ &= 1.6y_k + 0.45u_k\end{aligned}$$

#### 1.4 ditaa & L<sup>A</sup>T<sub>E</sub>X Graph

For the physical process of tank dynamics given in the Figure below:



Solution:

- $\frac{dh}{dt} = -\frac{1}{KA}h + \frac{1}{A}f_{in}$
- Assume that the tank height is measured,  $y(t) = h(t)$

$$\Sigma(A, B, C, D) = \Sigma\left(-\frac{1}{KA}, \frac{1}{A}, 1, 0\right)$$

- The state-space realization is given as:

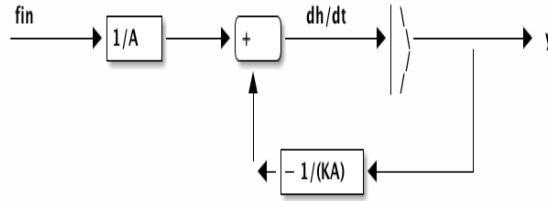


Figure 1: Block diagram elements.

•

$$\Phi = e^{-\frac{0.2}{KA}}, \quad \Gamma = -K \left( e^{-\frac{0.2}{KA}} - 1 \right), \quad \theta = 1$$

$$\Sigma(A_d, B_d, C_d, D_d) = \Sigma \left( e^{-\frac{0.2}{KA}}, -K \left( e^{-\frac{0.2}{KA}} - 1 \right), 1, 0 \right)$$

- Assume that  $h(0) = 0$ . Laplace transform:

$$\frac{Y(s)}{U(s)} = \frac{5}{s+1}$$

$$\tau = 1, \quad \tau_d = 0 \Rightarrow \Delta t = (0.1 \sim 0.2)1\tau = 0.1 \sim 0.2$$

## 2 Chapter 2

### 2.1 JavaScript

### 2.2 Answer Sheet via Email

+RESULTS: AnswerSheet

Student ID:

token:

token:

Question1:

Reset Submit

### 2.3 Answer Sheet via Post

Name:

Gender: male female

E-mail:

Reset Submit Print