

Homework 3 –MPC with Time Delays**Assigned: 09-28-2020****Due: 10-9-2020****5:00PM****Submit PDF on CANVAS****Instructions:**

- A) Please submit your work as a well-organized report showing work steps with results and discussions as a single PDF (file name: YOUR LASTNAME_HWK1). You may either type or scan-in any handwritten items but merge and label those at the correct locations in your document. Include plot outputs within the same document at the corresponding locations; MATLAB codes may be included as Appendix, if relevant. Please annotate figures/plots clearly with legible font sizes and legends as necessary.
- B) Explain your steps and results. Work submitted without any effort to explain your steps and results will be graded poorly (up to 50% for the score assigned for that part).

Problem Set Complete the following exercises in the text by Rawlings et al, 2nd edition

1. **Exercise 2.6 Terminal Constraint and Region of Attraction**

Additional Information for part c. You may omit the computation of the region of attraction as it involves a bit of coding, unless you find a ready-made projection algorithm. However, please write down the expression for the region of attraction/feasibility \mathcal{X}_N in terms of notations discussed in class and in this specific problem.

2. **Exercise 1.61. MPC, PID and time delay.**

Additional information/settings. A reasonable sample time is about $1/5^{\text{th}}$ of the long delay. Choose an output disturbance model with $B_d = 0$ and $C_d = I$. For the MPC, set $N = 15$, output regulation weight $Q_y = 1$; the infinite horizon cost-to-go for the terminal cost. Set the penalty on the input rate of change Δu as $S=1$. Process noise covariance $Q_\omega = 0.005$; $Rv = 0.01$.

Tune the PID controller using a Ziegler-Nichols tuning rule (look it up). When your design is done, compare the integral square error achieved by the MPC vs. the PID design.

Notes:

a) Output regulation simply means the output y is desired to be regulated to zero, instead of the state x (in the objective function).

b) For a very practical example that illustrates the importance of time delays in automotive systems and how to design MPC that deals with time-delays, please refer to the following paper. You may find the time delay modeling ideas useful (state augmentation).

S. Di Cairano, D. Yanakiev, A. Bemporad, I. V. Kolmanovsky and D. Hrovat, "An MPC design flow for automotive control and applications to idle speed regulation," 2008 47th IEEE Conference on Decision and Control, Cancun, 2008, pp. 5686-5691, doi: 10.1109/CDC.2008.4738865.

b) While you can pursue the MATLAB MPC toolbox example on "MPC for plants with time delay", you should solve this problem with your own code, similar to the previous homework and examples provided. That is, set up the objective function, list constraints, and implement the closed-loop simulation with the MPC controller solved "on-line".
