CLEMSON AUTOMOTIVE ENGINEERING

AuE 8930- Robust Predictive Control **Fall 2020**

Homework 4 –Robust MPC

Assigned: 10-22-2020 Due: 11-5-2020 5:00PM **Submit PDF on CANVAS**

Instructions:

A) Please submit your work as a well-organized report showing work steps with results and discussions as a single PDF (file name: YOUR LASTNAME_HWK1). You may either type or scan-in any handwritten items but merge and label those at the correct locations in your document. Include plot outputs within the same document at the corresponding locations; MATLAB codes may be included as Appendix, if relevant. Please annotate figures/plots clearly with legible font sizes and legends as necessary.

B) Explain your steps and results. Work submitted without any effort to explain your steps and results will be graded poorly (up to 50% for the score assigned for that part).

Problem Set Complete the following exercises in the text by Rawlings et al, 2nd edition

Exercise 3.6 Simulating a Robust MPC Controller

Additional Information and expanded problem details:

- Note that the notations for the nominal system state and constraints are different from the lecture/text. Use the ones given in the problem statement.
- Part (a). Doesn't need software for this one-dimensional problem with A_K stable.
- Part (b). You will need to use the outer RPI approximation of the mRPI (outer-bounding tube) as the latter is difficult to compute.

  Proceed as follows to compute the tightening parameters for the state constraints:
  - Write the constraints as:

    $\mathbb{X} = \{x \in \mathbb{R}^n | c_j x \leq d_j, \forall j \in J\}, and\ \mathbb{U} = \{u \in \mathbb{R}^m | a_j u \leq b_j, \forall j \in J\}$ J is the index for list of constraints (E.g., In the present case, the control constraint has $J = 2$ (two inequalities).
  - Compute the following for a given N.

    $\phi_N^j = \max\{c'_j \sum_{i=0}^{N-1} A_K^i w_i | w_i \in \mathbb{W}\}$ and $\theta_N^j = \max\{a'_j K \sum_{i=0}^{N-1} A_K^i w_i | w_i \in \mathbb{W}\}$
  - The tightened constraints are:

    $$\mathbb{Z} = \{z \in \mathbb{R}^n | c'_j z \leq d_j - (1-\alpha)^{-1}\phi_N^j, \forall j \in J\}, and\ \mathbb{V} = \{v \in \mathbb{R}^m | a_j' v \leq b_j - (1-\alpha)^{-1}\theta_N^j, \forall j \in J\}$$
  - Select N=50 and $\alpha = 10^{-4}$ for part (b) and give the final tightened constraint sets (numerical values of their bounds must be computed). Depict each set on a 2D sketch along with the original constraints.
- Part (c). Design a nominal MPC. Formulate a quadratic cost function with stage cost parameters $Q = eye(2); R = 1$ and the LQR terminal cost. The nominal MPC would use the tightened constraints from part (b). Simulate it on its own and check that it performs as expected (with no disturbance, and with a non-zero realized disturbance *within* the assumed bounds). You can use the initial state $x_o = [-1; -1]$.
- Part (d). Simulate the tube-based Robust MPC for the four initial states given in the problem statement, considering a *uniformly* distributed disturbance with $|w| \leq 0.1$. You will need to generate a random disturbance sequence for this. Plot the phase portrait as requested.
  - Include the time-history response of the closed loop for the initial state $x_o = [-1; -1]$. Both the control action and the realized disturbance sequence should also be plotted on separate axes.