

Varmelikningen i 2-dim.

$$u_t = -(u_{xx} + u_{yy})$$

$$\gamma = \frac{k}{h^2}$$

Ser på hver enkelt del

$$\sigma = \frac{k}{d^2}$$

$$u_t = \frac{u(t+k) - u(t)}{k} = \frac{u_t - u_{t-1}}{k}$$

$$u_{xx} = \frac{u(x+h) - u(x)}{h} \quad u_{xx} = \frac{u(x+2h) - u(x+h) - u(x+h) + u(x)}{h^2}$$

$$u_{xx} = \frac{u(x+2h) - 2u(x+h) + u(x)}{h^2}$$

$$u_{xx} = \frac{u_{x+1} - 2u_x + u_{x-1}}{h^2}$$

$$u_{yy} = \frac{u_{y+1} - 2u_y + u_{y-1}}{d^2}$$

$$\frac{u_t - u_{t-1}}{k} = \frac{u_{x+1} - 2u_x + u_{x-1}}{h^2} + \frac{u_{y+1} - 2u_y + u_{y-1}}{d^2}$$

Samlet:

$$\frac{u_{t,x,y} - u_{t-1,x,y}}{k} = \frac{u_{t,x+1,y} - 2u_{t,x,y} + u_{t,x-1,y}}{h^2} + \frac{u_{t,x,y+1} - 2u_{t,x,y} + u_{t,x,y-1}}{d^2}$$

$$u_{t+1,x,y} = (u_{t,x+1,y} - 2u_{t,x,y} + u_{t,x-1,y}) \cdot \frac{k}{h^2} + (u_{t,x,y+1} - 2u_{t,x,y} + u_{t,x,y-1}) \cdot \frac{k}{d^2} + u_t$$

Parallelliserer:

$$B = \begin{bmatrix} 1 & -2\gamma \end{bmatrix}$$