NIGHT OF CHANCES
PORTFOLIO OPTIMIZATION WORKSHOP

BRNO, OCTOBER 2018

STEFAN LINNER, MATEJ SLAVIK

CAPCO

THE FUTURE. NOW.

https://github.com/linner-stefan/portfolio-optimization-workshop

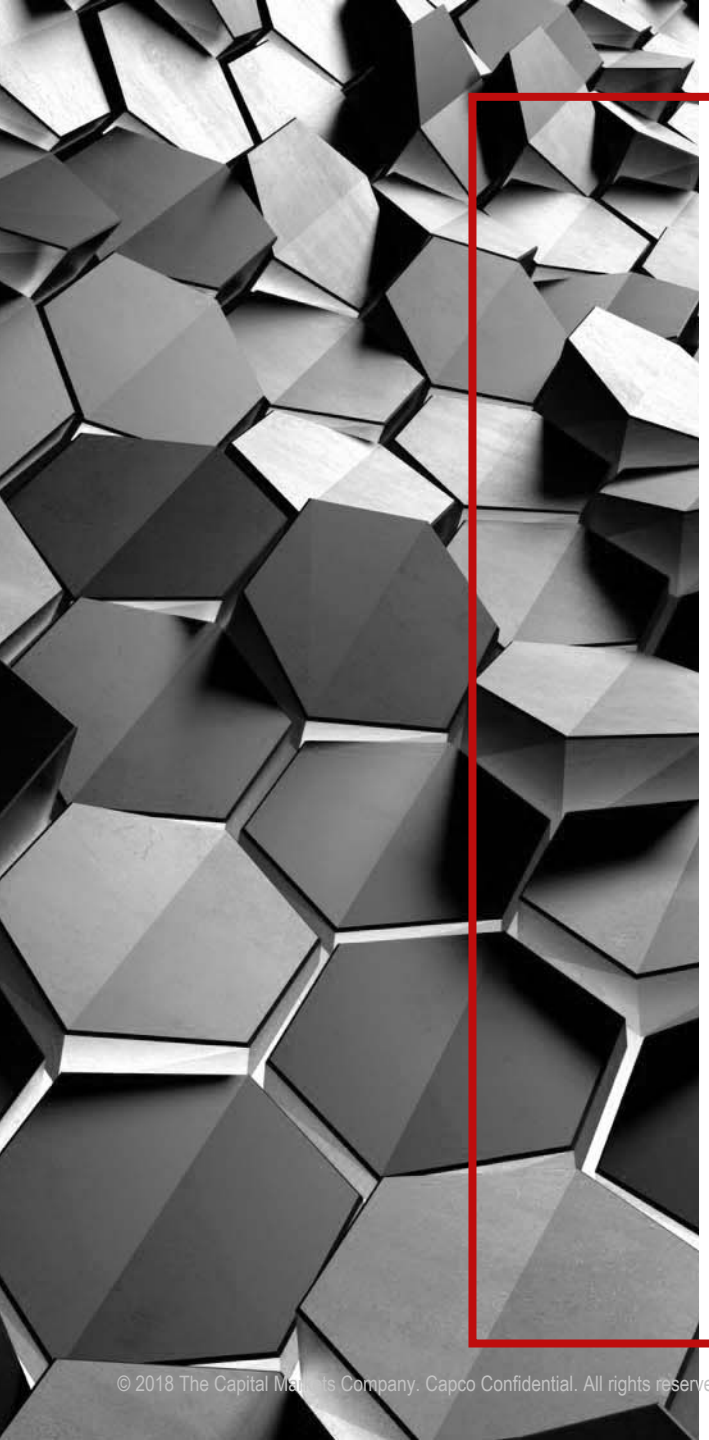
PREREQUISITES

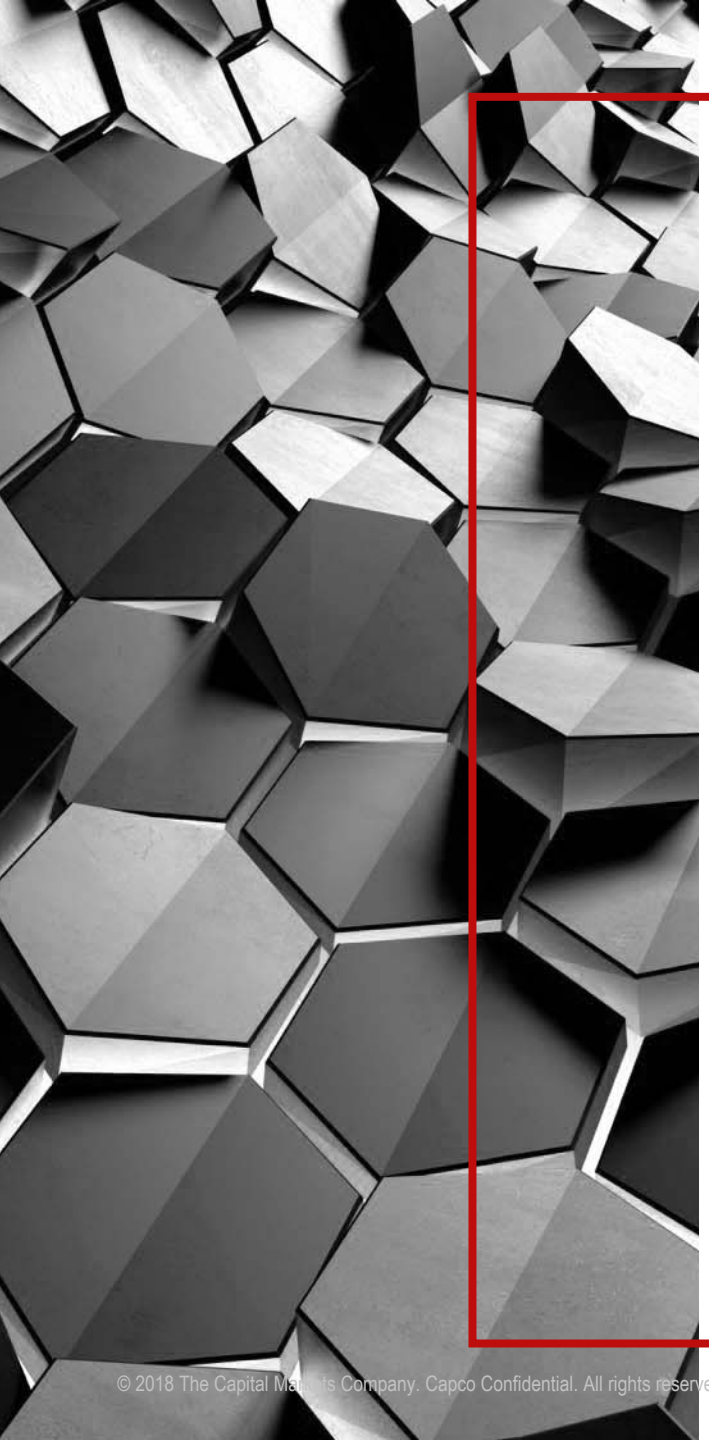
MAKE SURE YOU ARE READY

MAKE SURE YOU ARE READY & YOUR LAPTOP IS SET UP FOR THE TASK!

THE PREREQUISITES AVAILABLE AT:

HTTPS://GITHUB.COM/LINNER-STEFAN/PORTFOLIO-OPTIMIZATION-WORKSHOP





FINANCIAL MARKETS

CLASSIFICATION & FINANCIAL ASSETS

FINANCIAL MARKETS

WHAT DOES IT MEAN?

A marketplace where buyers and sellers participate in the trade of various assets, typically defined by tools leading to efficient markets - transparent pricing, basic regulations, costs and fees, market forces determining the prices. Two major components:

- Money market
- Capital market

Market efficiency indicates how much do prices reflect available and relevant information. You can't beat the efficient market as all information is already included in the price! Components to market efficiency:

- Information arbitrage
- Fundamental valuation
- Full insurance
- Operational efficiency

Although markets are deemed efficient in the long run, short-term inefficiencies allow investors to capitalize on anomalies and reap higher rewards that may be out of proportion to the level of risk. Those anomalies are exactly what investors in capital markets try to uncover.

PROPERTIES	MONEY MARKETS	CAPITAL MARKETS
Horizon	< 1 year	> 1 year
Aspects	High liquidity & low return, "safe place" to deposit money. Raising money, parking the money in the short-term.	Long term liquidity & risk management, capital raising & investments

Market classification (Securities classification)	• "Cash Investments"
	  • Deposits
	  • Certificates of deposits (CDs)
	  • US Treasury bills (T-Bills)
	  • Commercial paper
	  • Municipal notes
	  • Eurodollars
	  • Repurchase agreements (REPOs)
	  • Collateral loans
	  • Acceptances
	  • Bill of exchanges
		• Fixed-income
		  • Bonds
		  • Convertible / Callable / Puttable Bonds
		• Equity
		  • Preffered stock
		  • Common stock
		  • Stock Market Indices
		• Derivatives
		  • Options (Stocks, Fx, interest rates, commodities, …)
		  • Futures / Forwards (FX, Interest rate,…)
		• Alternative investments
		  • Forex / Cryptocurrencies / Commodities
		  • Hedge / Mutual Funds / Venture Capital
		  • Real Estate (REITs, CREFs, Ownership)

Market classification (security life cycle)	• **Primary "New Issues" Market** – 1st time an instrument is issued on an exchange (e.g. a stock via IPO). Used by entities to obtain funds via debt or equity issues • **Secondary Market** – trading newly issued securities on an exchange, where the securities were issued (e.g. NASDAQ, NYSE, AMEX). Represents the largest portion of all trades. • **OTC "Dealer" Market** – type of Secondary Market with fewer regulation, securities traded outside of exchanges (e.g. OTCBB, Pink Sheets). Typical market instruments penny stocks. • **Third / Fourth Markets** – OTC transactions between broker-dealers and large institutions / Transactions amongst large institutions	

GREAT, NOW I KNOW THERE ARE LOADS OF ASSET TYPES, AM I A MILLIONAIRE NOW?

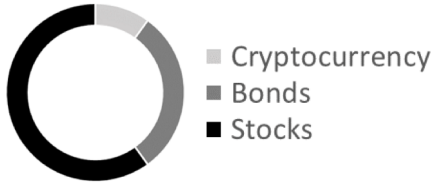
PORTFOLIO MANAGEMENT

PORTFOLIO MANAGEMENT PROCESS IN A NUTSHELL

HOW DO I CONSTRUCT A PORTFOLIO?

KEY ASPECTS

Portfolio is a set of investments held by an individual or an organization. Each of the included financial instruments represents such portion of portfolio value (**weight**), which matches desired risk-return portfolio allocation.



Portfolio Management is a scientific alchemy of making decisions about strategy considering aspects not limited to:

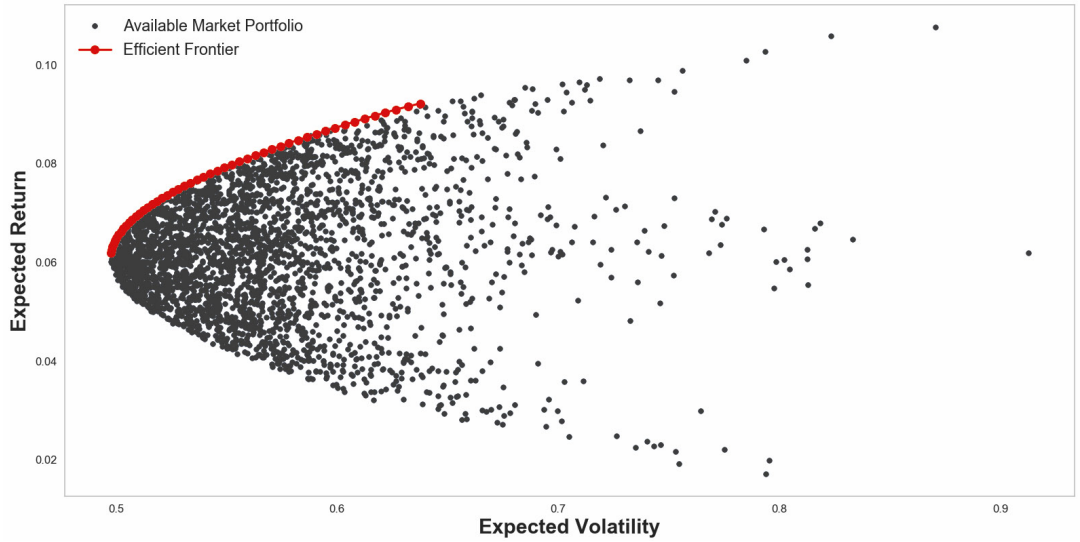
- performance targets & risk appetite (Return, Variance, Correlation, StdErr, Sharpe Ratio, …)
- constraints (diversification, liquidity, max / min weight per asset class, ESG, regulation …)
- horizon (short- / mid- / long-term)
- purpose (e.g. retirement plan, financing school, investment, funding, hedging, …)

Investment Portfolio Statement (IPS) is constructed which defines documents the specifics listed above and concludes how will be the portfolio constructed (e.g. expert opinion, following index, **quantitative optimization** or combination) at inception and managed further (e.g. Buy & Hold, Regular rebalancing, Maintaining certain indicator constant).

Frontier is a hyperbolic curve plotted on graph with Portfolio's Risk (X-axis) and Expected Return (Y-axis). The curve surrounds all possible combinations of selected assets packaged into a portfolio (available market portfolios).

Efficient Frontier is the upward sloping hyperbola, which reveals the optimal portfolios offering highest (max.) expected return for a defined level of risk or conversely, the lowest (min.) risk for a given level of expected return. All portfolios below this curve are **sub-optimal.**

EFFICIENT FRONTIER & AVAILABLE PORTFOLIOS EXMAPLE



Desired **Optimal Portfolios** lay on the **Efficient Frontier** as they provide maximum expected return at a given level of risk measure (expected volatility). Rational and risk-averse investor will select one of these portfolios in order to support his investment strategy (e.g. higher risk acceptance rings higher expected returns).

HOW DO I DETERMINE THE OPTIMAL WEIGHTS OF THE RESPECTIVE ASSETS?

MODERN PORTFOLIO THEORY (HARRY MARKOWITZ)

INTEGRAL METHODOLOGY IN THE PORTFOLIO OPTIMIZATION JOURNEY

HOW DO I CONSTRUCT AN OPTIMAL PORTFOLIO?

KEY VARIABLES & METHODOLOGY

VARIABLES

$P_{i,t}$ - price of i-th asset in time t
$\mu_{i,t}$ - return of i-th asset during period t
$\bar{\mu}_i$ - mean return of i-th asset
σ_i^2 - variance of i-th asset

ω_i - weight of i-th asset in Portfolio p
μ_p - expected portfolio return
σ_p^2 - expected portfolio variance

INPUT

$$\mu_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

VARIABLES

$\bar{\mu}_i = E[\mu_i]$

$\sigma_i^2 = E[(\mu_i - E[\mu_i])^2]$

$\mu_i = [\mu_{i,1}, \mu_{i,2}, \dots, \mu_{i,t}]$

IDEA BEHIND

The portfolio composed of diversified individual stocks will be less than the risk inherent in holding any one of the individual stocks.

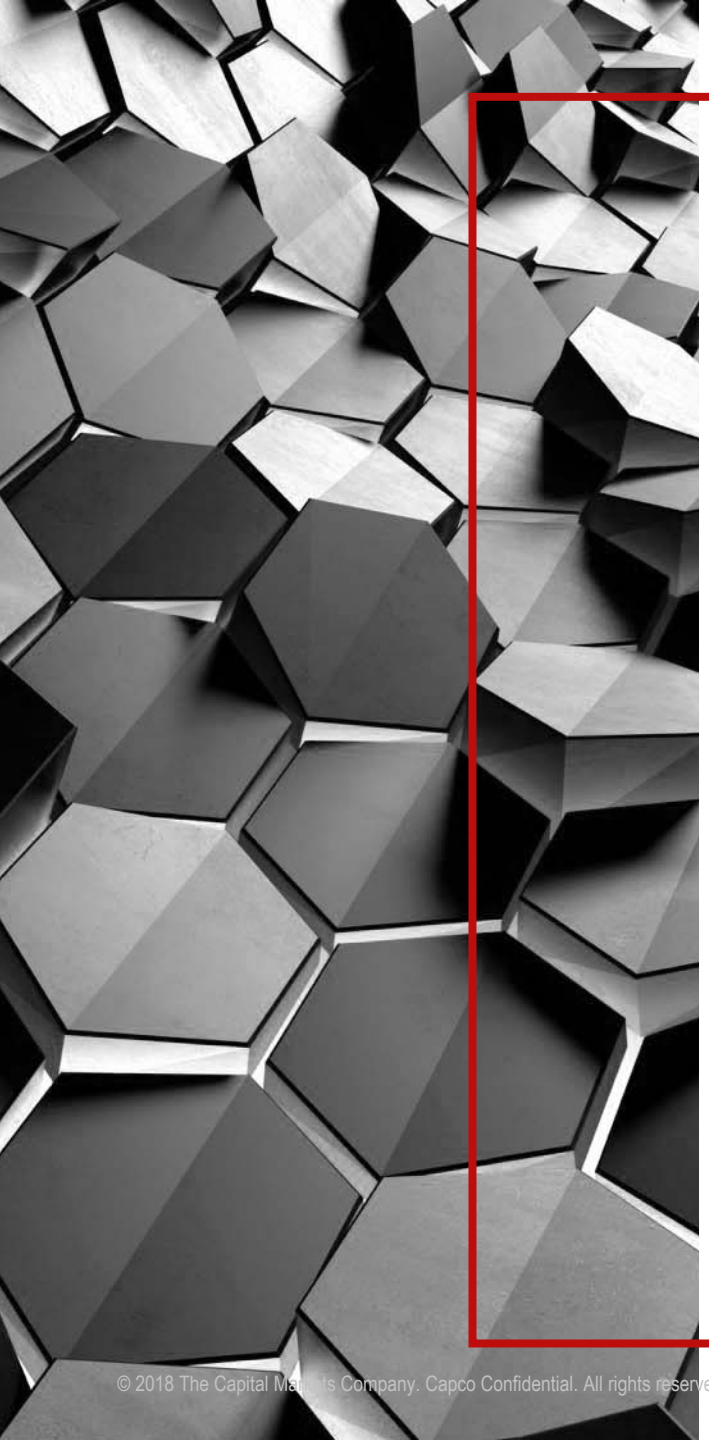
By undertaking investment in a basket of the assets, rater than choosing only one, an investor can leverage the benefits of diversification - a reduction in the riskiness of the portfolio. MPT quantifies the benefits of diversification, also known as not putting all of your eggs in one basket.

OPTIMALIZATION PROBLEM

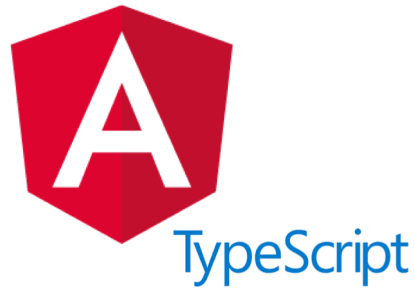
$$\min \frac{1}{2} \sum_{i,j=1}^{n} \omega_i \omega \sigma_{ij}$$

SUBJECT TO

$$\sum_{i=1}^{n} \omega_i \bar{\mu}_i = \bar{\mu}_p \qquad \sum_{i=1}^{n} \omega_i = 1$$



TECHNOLOGIES



WORKSHOP IMPLEMENTATION STEPS

1. Construct optimization **inputs**
2. Calculate **min risk** portfolio
3. Calculate **max return** portfolio
4. Calculate other **efficient** portfolios
5. **Constraint** the optimization

INPUTS & MODEL CONSTRUCTION

HISTORIC ASSET RETURNS USED TO DERIVE MEAN, COVARIANCE & CONSTRUCT OPTIMIZATION MODEL CONSTRIANTS

INPUT: HISTORICAL RETURNS							
	Week 1	Week 2	Week 3	…	Week 150	Week 151	Week 152
Asset 1	-1.0856306	0.99734545	0.2829785	…	-0.90932702	0.47026375	-1.11143045
Asset 2	-0.74882747	0.56759473	0.71815054	…	-0.35929672	-1.60969508	0.01357006
Asset 3	-1.77422355	-1.20137731	1.09625679	…	2.04043199	1.01591697	-1.63378817
Asset 4	-0.45059869	0.60959032	1.17374438	…	-0.1227329	1.79448602	-0.32050951

INPUTS & MODEL CONSTRUCTION

HISTORIC ASSET RETURNS USED TO DERIVE MEAN, COVARIANCE & CONSTRUCT OPTIMIZATION MODEL CONSTRIANTS

INPUT: HISTORICAL RETURNS

	Week 1	Week 2	Week 3	…	Week 150	Week 151	Week 152
Asset 1	-1.0856306	0.99734545	0.2829785	…	-0.90932702	0.47026375	-1.11143045
Asset 2	-0.74882747	0.56759473	0.71815054	…	-0.35929672	-1.60969508	0.01357006
Asset 3	-1.77422355	-1.20137731	1.09625679	…	2.04043199	1.01591697	-1.63378817
Asset 4	-0.45059869	0.60959032	1.17374438	…	-0.1227329	1.79448602	-0.32050951

CALCULATE

1. Mean returns (**f**)
2. Covariance (**H**)

CONSTRUCT

1. Inequality coefficients (**A**)
2. Equality coefficients (**Aeq**)
3. Inequality constraints (**b**)
4. Equality constraints (**beq**)

INPUTS & MODEL CONSTRUCTION

HISTORIC ASSET RETURNS USED TO DERIVE MEAN, COVARIANCE & CONSTRUCT OPTIMIZATION MODEL CONSTRIANTS

INPUT: HISTORICAL RETURNS

	Week 1	Week 2	Week 3	…	Week 150	Week 151	Week 152
Asset 1	-1.0856306	0.99734545	0.2829785	…	-0.90932702	0.47026375	-1.11143045
Asset 2	-0.74882747	0.56759473	0.71815054	…	-0.35929672	-1.60969508	0.01357006
Asset 3	-1.77422355	-1.20137731	1.09625679	…	2.04043199	1.01591697	-1.63378817
Asset 4	-0.45059869	0.60959032	1.17374438	…	-0.1227329	1.79448602	-0.32050951

CALCULATE

1. Mean returns (**f**)
2. Covariance (**H**)

CONSTRUCT

1. Inequality coefficients (**A**)
2. Equality coefficients (**Aeq**)
3. Inequality constraints (**b**)
4. Equality constraints (**beq**)

CONVEX OPTIMIZATION MODEL

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Variables:

x – portfolio weights (asset weights)

PORTFOLIO
IN TERMS OF OPTIMIZATION

$$x_1 + x_2 + x_3 + x_4$$

x – *portfolio weight (asset weight)*,
- percentage from the total investment amount

0.1 + 0.3 + 0.4 + 0.2

10% + 30% + 40% + 20%

100.000 € + 300.000 € + 400.000 € + 200.000 €
(*if the investment amount is 1.000.000 €*)

INPUTS & MODEL CONSTRUCTION

HISTORIC ASSET RETURNS USED TO DERIVE MEAN, COVARIANCE & CONSTRUCT OPTIMIZATION MODEL CONSTRIANTS

INPUT: HISTORICAL RETURNS

	Week 1	Week 2	Week 3	…	Week 150	Week 151	Week 152
Asset 1	-1.0856306	0.99734545	0.2829785	…	-0.90932702	0.47026375	-1.11143045
Asset 2	-0.74882747	0.56759473	0.71815054	…	-0.35929672	-1.60969508	0.01357006
Asset 3	-1.77422355	-1.20137731	1.09625679	…	2.04043199	1.01591697	-1.63378817
Asset 4	-0.45059869	0.60959032	1.17374438	…	-0.1227329	1.79448602	-0.32050951

CALCULATE

1. Mean returns (**f**)
2. Covariance (**H**)

CONSTRUCT

1. Inequality coefficients (**A**)
2. Equality coefficients (**Aeq**)
3. Inequality constraints (**b**)
4. Equality constraints (**beq**)

CONVEX OPTIMIZATION MODEL

$$\min_{x} \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Variables:

x – portfolio weights (asset weights)

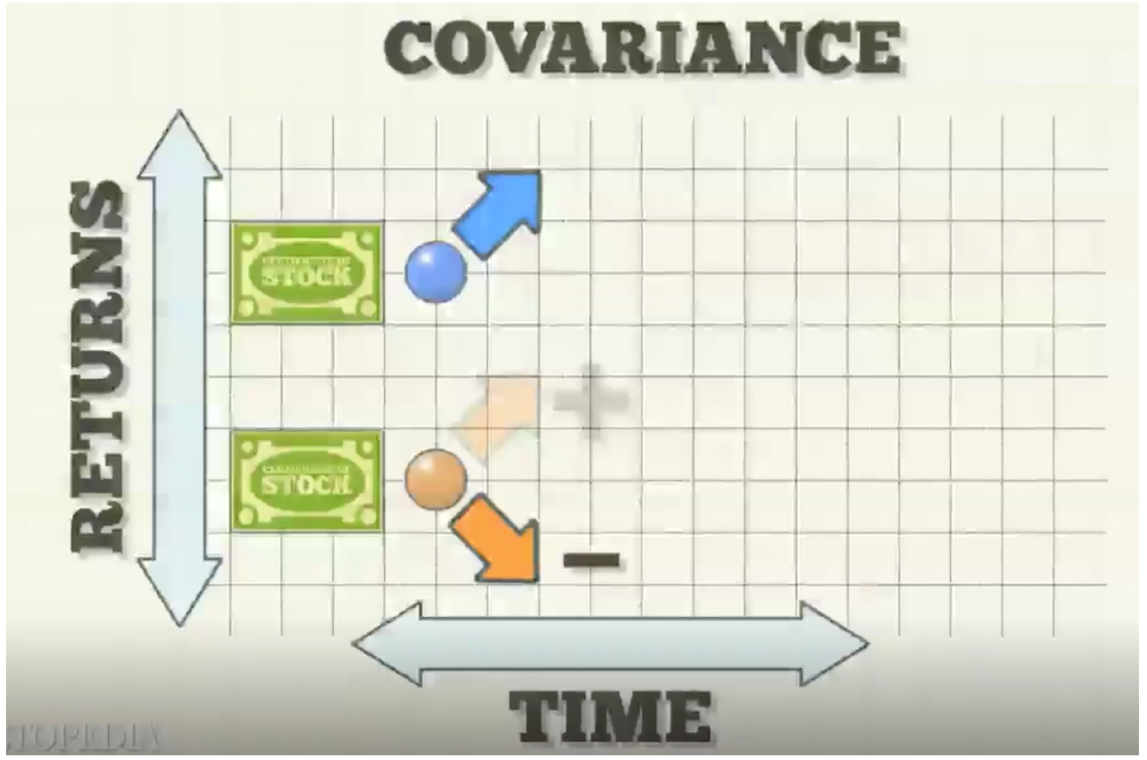
MODEL DECOMPOSITION: COVARIANCE

A MEASURE OF THE DIRECTIONAL RELATIONSHIP BETWEEN THE RETURNS ON TWO RISKY ASSETS

- positive covariance
  - asset returns move together
- negative covariance
  - returns move inversely

$$C = \begin{bmatrix} cov(x,x) & cov(x,y) & cov(x,z) \\ cov(y,x) & cov(y,y) & cov(y,z) \\ cov(z,x) & cov(z,y) & cov(z,z) \end{bmatrix}$$

Variances



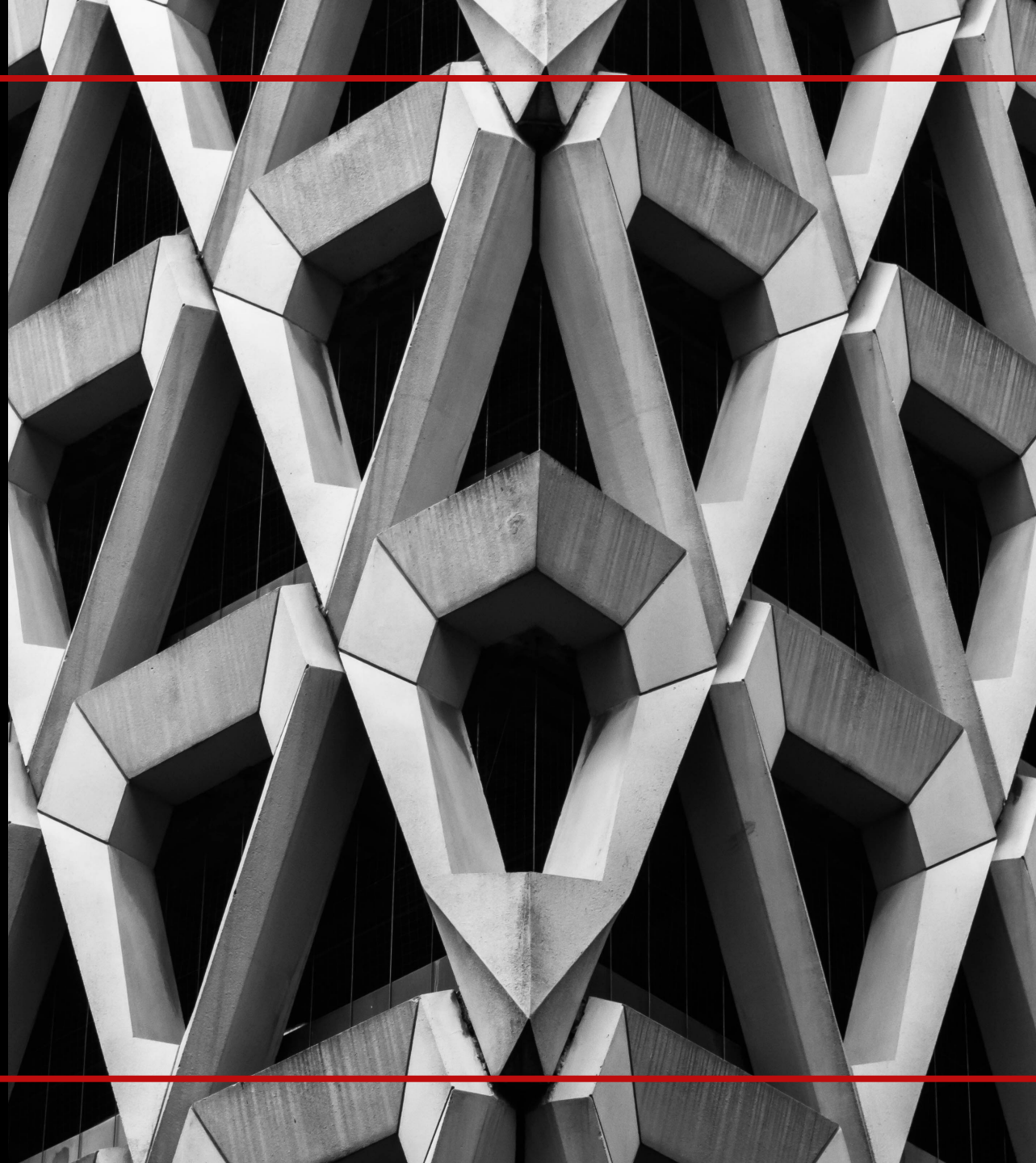
https://www.investopedia.com/terms/c/covariance.asp

IMPLEMENTATION

COVARIANCE AND MEAN RETURNS

- **PCA.covarianceMatrix()**
- **meanReturns(historicReturns)**

- **Nd4j.create(doube[])**
- **Nd4j.create(double[][])**



MODEL DECOMPOSITION: CONSTRAINTS

INEQUALITY & EQUALITY CONSTRAINTS FURTHER DEFYNING THE OPTIMISATION MODEL

1. **Inequality**
   - each asset weight <= 1
   - each asset weight > 0
2. **Equality**
   - sum of all asset weights = 1

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

MODEL DECOMPOSITION: CONSTRAINTS

INEQUALITY – UPPER BOUNDARY

1. **Inequality**
   - each asset weight <= 1
   - each asset weight > 0
2. **Equality**
   - sum of all asset weights = 1

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Inequality Coefficients (A)

	Asset 1	Asset 2	Asset 3
Constraint 1	1	0	0
Constraint 2	0	1	0
Constraint 3	0	0	1

Inequality Constants (b)

Constraint 1	1	*x1 <= 1*
Constraint 2	1	*x2 <= 1*
Constraint 3	1	*x3 <= 1*

A1.x1 + A2.x2 + A3.x3 <= *b*

MODEL DECOMPOSITION: CONSTRAINTS

INEQUALITY – LOWER BOUNDARY

1. **Inequality**
   - each asset weight <= 1
   - each asset weight > 0
2. **Equality**
   - sum of all asset weights = 1

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Inequality Coefficients (A)

	Asset 1	Asset 2	Asset 3
Constraint 1	?	?	?
Constraint 2	?	?	?
Constraint 3	?	?	?

Inequality Constants (b)

Constraint 1	?
Constraint 2	?
Constraint 3	?

A1.x1 + *A2.x2* + *A3.x3* <= *b*

MODEL DECOMPOSITION: CONSTRAINTS

INEQUALITY – LOWER BOUNDARY

1. **Inequality**
   - each asset weight <= 1
   - each asset weight > 0
2. **Equality**
   - sum of all asset weights = 1

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Inequality Coefficients (A)

	Asset 1	Asset 2	Asset 3
Constraint 1	-1	0	0
Constraint 2	0	-1	0
Constraint 3	0	0	-1

Inequality Constants (b)

Constraint 1	0	*x1 > 0*
Constraint 2	0	*x2 > 0*
Constraint 3	0	*x3 > 0*

A1.x1 + *A2.x2* + *A3.x3* <= *b*

MODEL DECOMPOSITION: CONSTRAINTS

INEQUALITY

Inequality Coefficients (A)

	Asset 1	Asset 2	Asset 3
Constraint 1	1	0	0
Constraint 2	0	1	0
Constraint 3	0	0	1
Constraint 4	-1	0	0
Constraint 5	0	-1	0
Constraint 6	0	0	-1

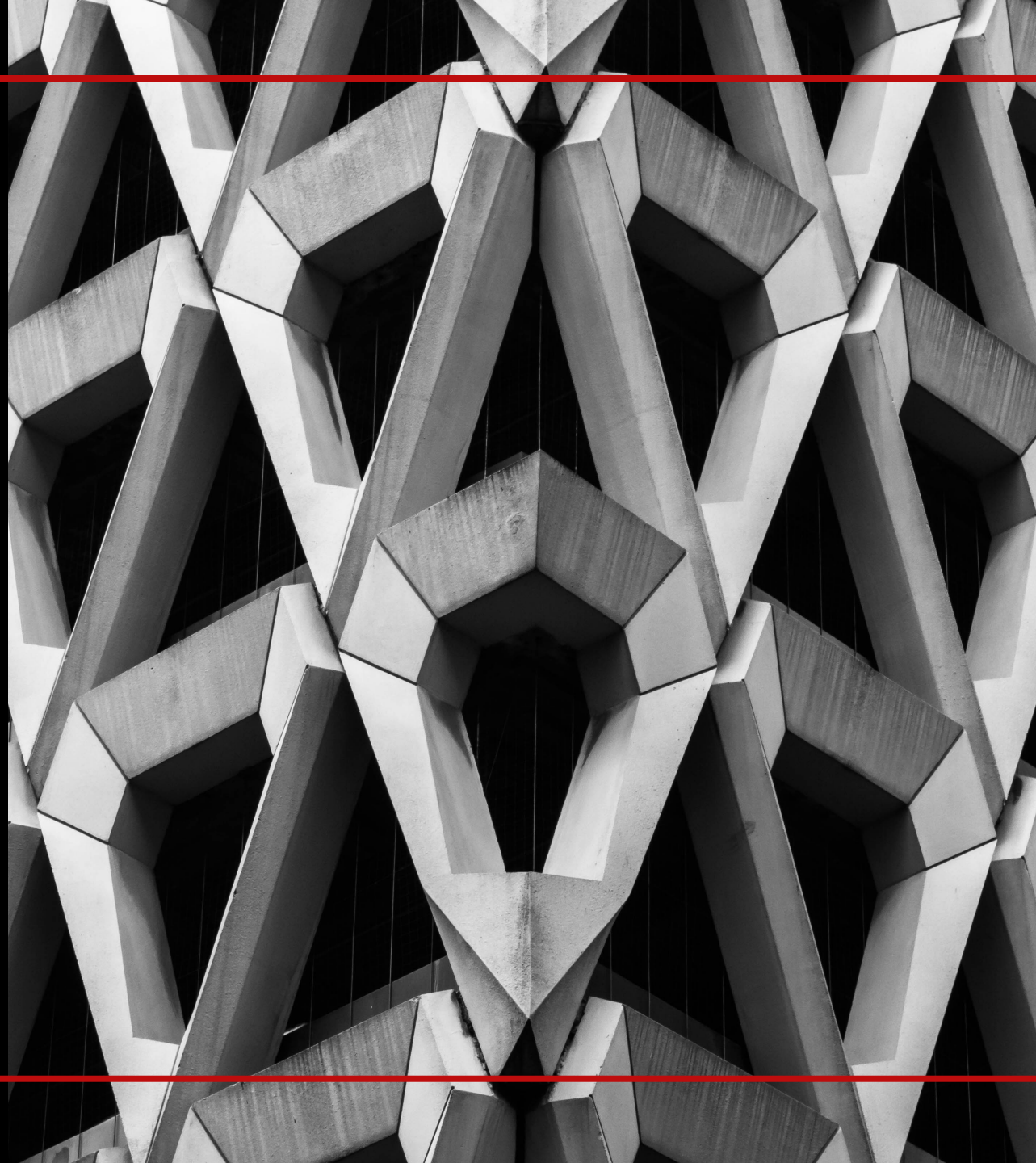
Inequality Constants (b)

Constraint 1	1	$x1 <= 1$
Constraint 2	1	$x2 <= 1$
Constraint 3	1	$x3 <= 1$
Constraint 4	0	$x1 > 0$
Constraint 5	0	$x2 > 0$
Constraint 6	0	$x3 > 0$

IMPLEMENTATION

INEQAULITY CONSTRAINTS

- Nd4j.eye(size)
- Nd4j.concat(0, topMatrix, bottomMatrix)
- Nd4j.zeros(rows, columns)
- Nd4j.ones(rows, columns)



MODEL DECOMPOSITION: CONSTRAINTS
EQUALITY

1. Inequality
   - each asset weight <= 1
   - each asset weight > 0
2. **Equality**
   - sum of all asset weights = 1

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Equality Coefficients (Aeq)

?

Equality Constants (beq)

?

MODEL DECOMPOSITION: CONSTRAINTS

EQUALITY

1. Inequality
   - each asset weight <= 1
   - each asset weight > 0
2. **Equality**
   - sum of all asset weights = 1

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq. \end{cases}$$

Equality Coefficients (Aeq)

	Asset 1	Asset 2	Asset 3
Constraint 1	1	1	1

Equality Constants (beq)

Constraint 1	1

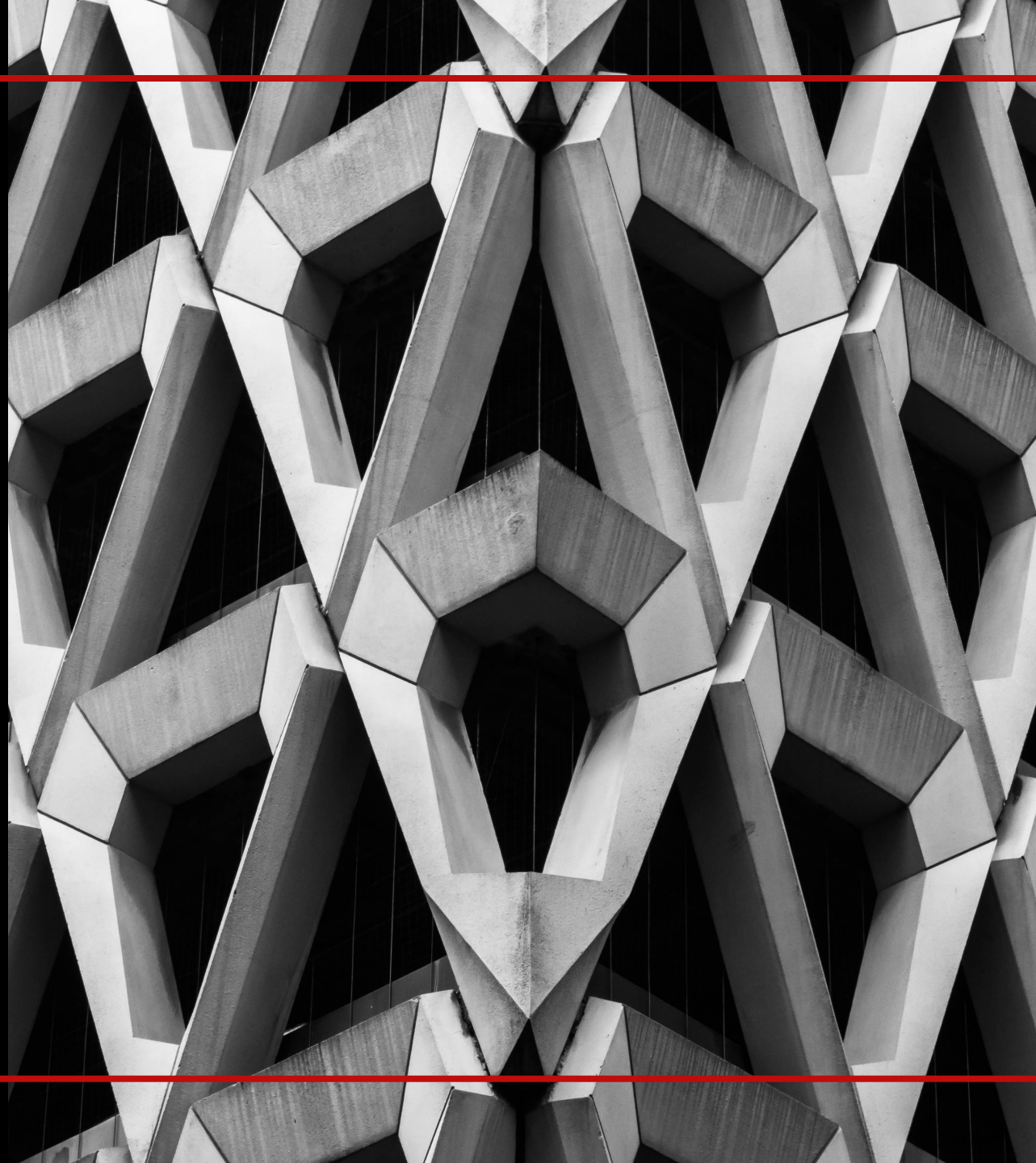
x1+x2+x3 = 1

A1.x1 + A2.x2 + A3.x3 > b

IMPLEMENTATION

EQULITY CONSTRAINTS

- **Nd4j.ones(rows, columns)**

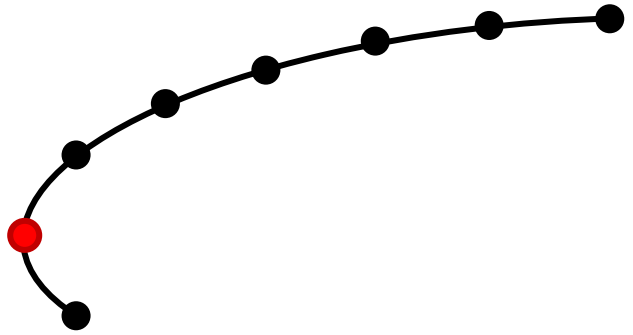


WORKSHOP IMPLEMENTATION STEPS

1. Construct optimization **inputs**
2. Calculate **min risk** portfolio
3. Calculate **max return** portfolio
4. Calculate other **efficient** portfolios
5. **Constraint** the optimization

WORKSHOP IMPLEMENTATION STEPS

1. Construct optimization **inputs**
2. Calculate **min risk** portfolio
   - solve **quadratic** optimization problem (uses the covariance)
3. Calculate **max return** portfolio
4. Calculate other **efficient** portfolios
5. **Constraint** the optimization



PORTFOLIO RETURN

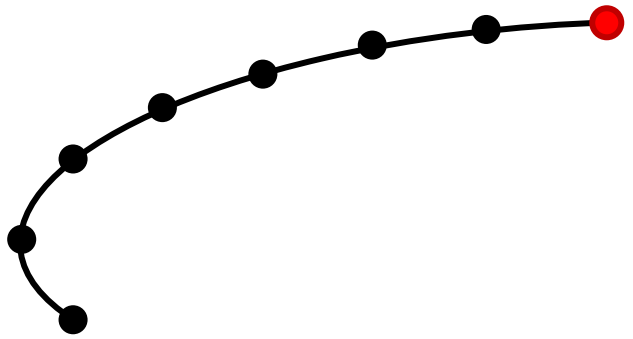
$$weights * meanReturns^T$$

PORTFOLIO RISK

$$\sqrt{weights * Covariance * weights^T}$$

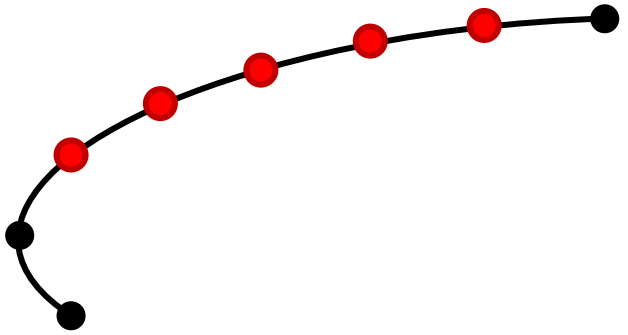
WORKSHOP IMPLEMENTATION STEPS

1. Construct optimization **inputs**
2. Calculate **min risk** portfolio
   - solve **quadratic** optimization problem (uses the covariance)
3. Calculate **max return** portfolio
   - solve **linear** optimization problem
     (no covariance, mean returns not negated)
4. Calculate other **efficient** portfolios
5. **Constraint** the optimization



WORKSHOP IMPLEMENTATION STEPS

1. Construct optimization **inputs**
2. Calculate **min risk** portfolio
   - solve **quadratic** optimization problem (uses the covariance)
3. Calculate **max return** portfolio
   - solve **linear** optimization problem
     (no covariance, mean returns not negated)
4. Calculate other **efficient** portfolios
   - solve **quadratic** optimization problem with additional equality constraint for a specific **portfolio return**
5. **Constraint** the optimization



CALCULATE OTHER EFFICIENT PORTFOLIOS

- for each intermediate from efReturn[1] to efReturn[nPortfolios – 2]
  – solveConvex(cov, f, A, b, **efAeq, efBeq**)

Equality Coefficients (efAeq)

	Asset 1	Asset 2	Asset 3
Constraint 1	1	1	1
Constraint 2	meanReturn[1]	meanReturns[2]	meanReturns[3]

Equality Constants (efBeq)

Constraint 1	1
Constraint 2	efReturn[i]

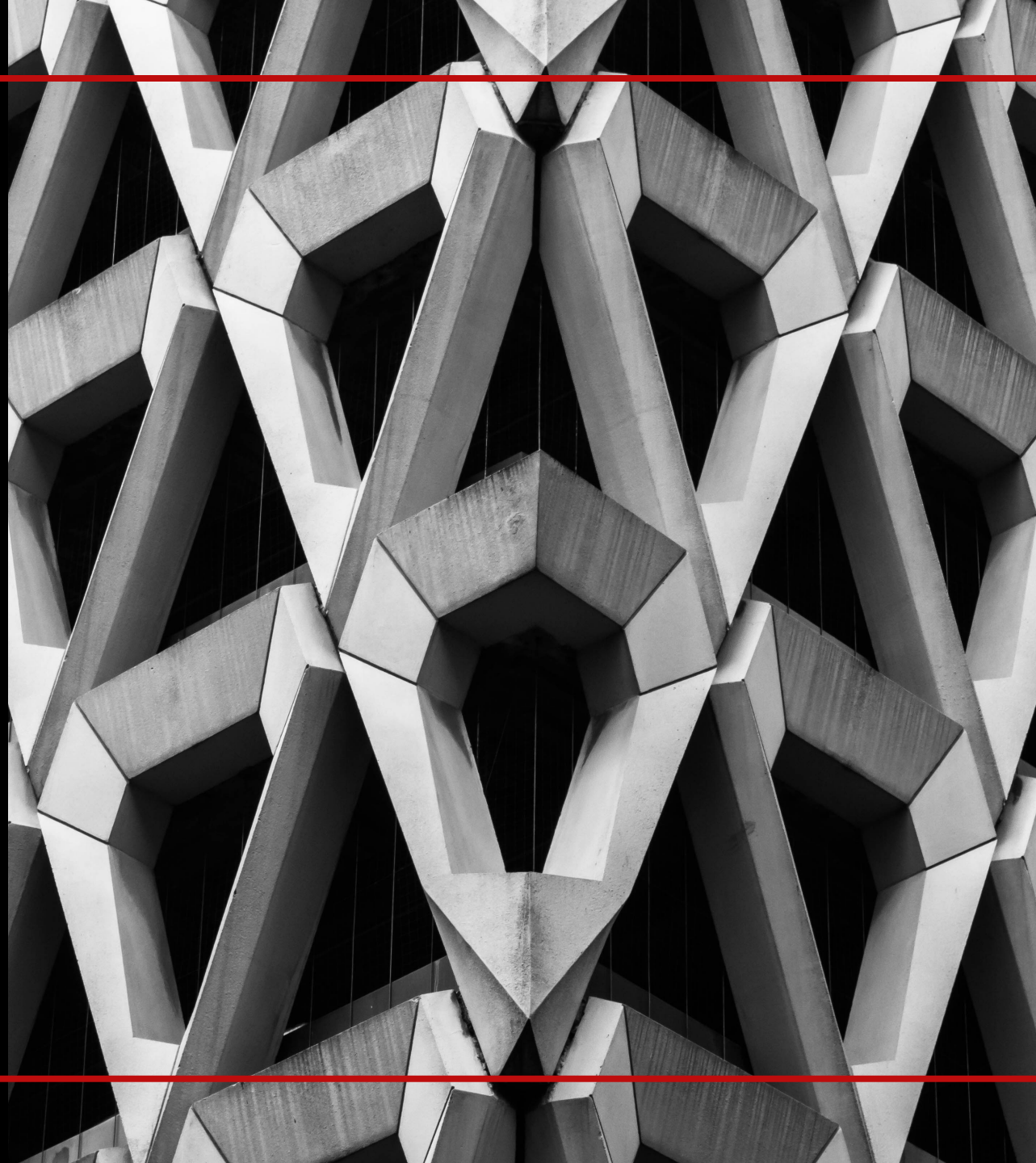
IMPLEMENTATION

OTHER EFFICIENT PORTFOLIOS

- interpolate missing efReturns[1] – efReturns[n-2]
- call solveConvex() 'nPortfolios - 2' times for each intermediate efReturn,
  with additional equality constraint with meanReturns as coefficients and efReturn as a constant
- add to efPortfolios and efRisks

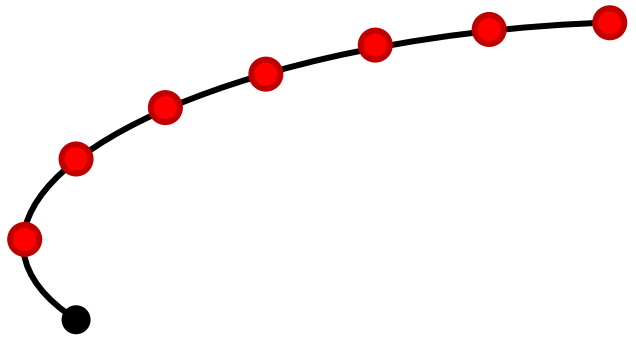
Useful

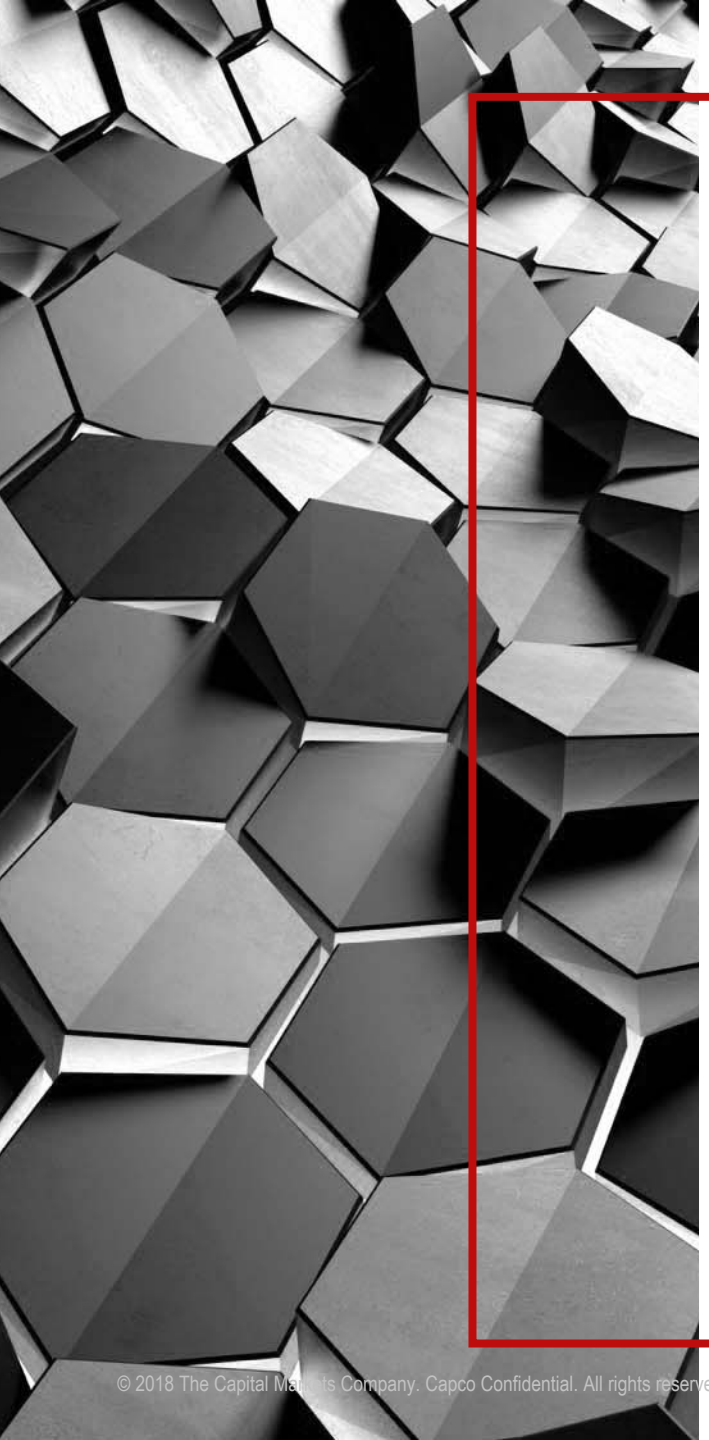
- ArrayUtils.add(array, elementToAdd)
- Nd4j.concat(0, topMatrix, bottomMatrix)



WORKSHOP IMPLEMENTATION STEPS

1. Construct optimization **inputs**
2. Calculate **min risk** portfolio
   - solve **quadratic** optimization problem (uses the covariance)
3. Calculate **max return** portfolio
   - solve **linear** optimization problem
     (no covariance, mean returns not negated)
4. Calculate other **efficient** portfolios
   - solve **quadratic** optimization problem with additional equality constraint for a specific **portfolio return**
5. **Constraint** the optimization
   - run all the previous steps with additional inequality constraints





01	FINANCIAL BACKGROUND
02	TECHNICAL BACKGROUND + IMPLEMENTATION
04	**APPENDIX**

SOURCES

- https://blog.quantopian.com/markowitz-portfolio-optimization-2#Backtesting-on-real-market-data
- https://en.wikipedia.org/wiki/Quadratic_programming
- https://www.mathworks.com/help/optim/ug/quadprog.html
- https://stats.stackexchange.com/questions/241449/matrix-and-regression-model

FAST FACTS

FOUNDED IN **1998**

20 YEARS INDUSTRY EXPERIENCE

28 WORLDWIDE OFFICES ACROSS 4 CONTINENTS

5000+ EMPLOYEES

90+ GLOBAL CLIENTS

INDUSTRIES

- BANKING & PAYMENTS
- CAPITAL MARKETS
- WEALTH & ASSET MANAGEMENT
- INSURANCE
- ENERGY MARKETS

DIGITAL TRANSFORMATION

INNOVATION AND THOUGHT LEADERSHIP

EXECUTION EXCELLENCE

UNIQUE CULTURE

INDUSTRY EXPERIENCE

ABOUT CAPCO

Capco is a global business and technology consultancy focused on financial services.

INNOVATORS: We are innovators who combine a disruptive, design-thinking approach with unrivalled, first-hand industry knowledge and technology acumen.

FACILITATORS: We facilitate the handshake between the business and technology organizations to deliver end-to-end consulting services and solutions.

TRANSFORMATION SPECIALISTS: We help our clients transform and advance their businesses, increase revenue, manage risk and regulatory change.

We consistently deliver these core tenets as part of our brand:

- DIGITAL TRANSFORMATION
- INNOVATION AND THOUGHT LEADERSHIP
- EXECUTION EXCELLENCE
- UNIQUE CULTURE
- INDUSTRY EXPERIENCE

FAST FACTS

FOUNDED IN 1998

20 YEARS INDUSTRY EXPERIENCE

28 WORLDWIDE OFFICES ACROSS 4 CONTINENTS

5000+ EMPLOYEES

90+ GLOBAL CLIENTS

INDUSTRIES

BANKING & PAYMENTS • CAPITAL MARKETS
WEALTH & ASSET MANAGEMENT/INSURANCE
ENERGY MARKETS

ABOUT CAPCO

Capco is a global technology and management consultancy dedicated to the financial services industry. Our professionals combine innovative thinking with unrivalled industry knowledge to offer our clients consulting expertise, complex technology and package integration, transformation delivery, and managed services, to move their organizations forward. Through our collaborative and efficient approach, we help our clients successfully innovate, increase revenue, manage risk and regulatory change, reduce costs, and enhance controls. We specialize primarily in banking, capital markets, wealth and investment management, and finance, risk & compliance. We also have an energy consulting practice. We serve our clients from offices in leading financial centers across the Americas, Europe, and Asia Pacific.

To learn more, visit our website at www.capco.com or follow us on Twitter, Facebook, YouTube, LinkedIn and Instagram.

WORLDWIDE OFFICES

Bangalore • Bangkok • Bratislava • Brussels • Charlotte • Chicago • Dallas • Dusseldorf • Edinburgh • Frankfurt • Geneva • Hong Kong • Houston • Kuala Lumpur • London • New York • Orlando • Paris • Pune • Sao Paulo • Singapore • Stockholm • Toronto • Tysons Corner• Vienna • Warsaw • Washington, D.C. • Zurich

CAPCO.COM