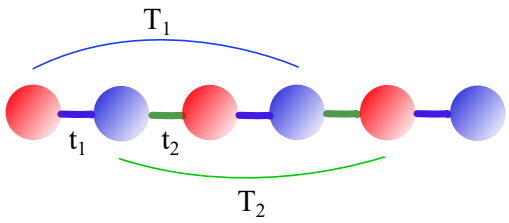

FIG. 1. Schematic set-up of a 1D SSH system. Here t_1 and t_2 are nearest-neighbor hoppings while T_1 and T_2 are second nearest-neighbor hoppings.

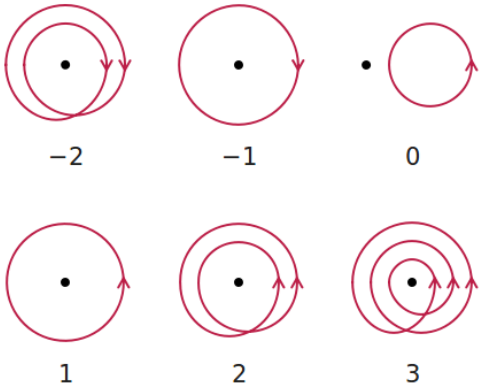

FIG. 2. Winding number. The winding number of a closed, oriented curve with respect to a reference point is a topological invariant that counts how many times the curve winds around the point. Picture credits: Jim Belk, public domain.

SUPPLEMENTARY MATERIAL

The SSH model

The SSH model [**?**] describes the movement of free electrons along a dimerized chain whose basic units consist of two distinct atoms. This movement, usually called "hopping" in the literature, can be made either between atoms in a unit cell or between unit cells, and the allowed hopping rules for a given system completely determine its Hamiltonian. This is because the kinetic energies of the electrons are parameterized by a vector of real numbers $\mathbf{t}$ that also encodes hopping terms, thus allowing for a compact mathematical description of a Hamiltonian in terms of creation/annihilation operators as

$$\hat{H}(\mathbf{t}) = \mathbf{c}^\dagger H(\mathbf{t}) \mathbf{c} \tag{1}$$

where the column vector

$$\mathbf{c} = \left(c_1^A, c_1^B, \cdots, c_{\frac{N}{2}}^A, c_{\frac{N}{2}}^B\right)^T$$

contains annihilation operators $c_p^{A(B)}$ that erase electrons at atom A (B) and lattice site p and similarly the row vector

$$\mathbf{c}^\dagger = \left(c_1^{A\dagger}, c_1^{B\dagger}, \cdots, c_{\frac{N}{2}}^{A\dagger}, c_{\frac{N}{2}}^{B\dagger}\right)$$

contains creation operators $c_p^{A(B)\dagger}$ that produce electrons at atom A (B) and lattice site p. Please note that N is twice the number of unit cells in the chain and therefore an even integer.

The convenience of equation (**??**) is that all information about a system such as its eigenstates and eigenenergies can be recovered from the $N \times N$ matrix $H(\mathbf{t})$. We can thus think of the vectors $\mathbf{t}$ in parameter space as very compact representations of SSH models: each point in $\mathbf{t}$-space can be mapped to a $N \times N$ matrix $H(\mathbf{t})$ whose eigenvectors and eigenvalues can then be computed, as is usually done in quantum mechanics. As an example, a general matrix $H(\mathbf{t})$