STEP 1: Fixing first detection j, testing $v = m_j$ (+ redoing some of Ryan's calculations)

$$V^{lo}_{BS} = \max_{k : (\Gamma y)_k \geq 0} \left\{ \frac{-(\Gamma y)_k}{(\Gamma v)_k} \|v\|_2^2 + v^T y \right\}$$

- each k'th row of Γ corresponds to some i.
- in fact, let's substitute in (i, s) for k

$$V^{lo}_{BS} = \max_{\substack{i \neq j \\ s \in \{\pm 1\}}} \left\{ \frac{-(b_j + s b_i)^T y}{(b_j + s b_i)^T m_j} \|v\|_2^2 + v^T y \right\}$$

$$m_j^T m_j = \|m_j\|_2 = \left(\tfrac{1}{j} \cdots \tfrac{1}{j}\ \tfrac{1}{n-j} \cdots \tfrac{1}{n-j}\right)^2 = \left(\tfrac{1}{j}\right)^2 j + \left(\tfrac{1}{n-j}\right)^2 (n-j)$$
$$= \frac{1}{j} + \frac{1}{n-j}$$
$$= \frac{n}{j(n-j)}$$

- what can we simplify?

① $b_j^T m_j = \sqrt{\dfrac{j(n-j)}{n}}\, m_j^T m_j = \sqrt{\dfrac{n}{j(n-j)}}$

② $b_i^T m_j = \sqrt{\dfrac{i(n-i)}{n}}\, m_i^T m_j = \sqrt{\dfrac{i(n-i)}{n}}\, (\cdots) = \begin{cases} \sqrt{\dfrac{n}{i(n-i)}}\, \dfrac{i}{j} & \text{if } i > j \\[2ex] \sqrt{\dfrac{n}{i(n-i)}}\, \dfrac{n-i}{n-j} & \text{if } i < j \end{cases}$

$$\begin{cases} \text{if } i < j \to \begin{array}{cccccc} (\circ \cdots & \circ & \circ & \circ \cdots & \circ) \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ 1 & i & & j & n \end{array} \to m_i^T m_j = \left(\dfrac{1}{j} + \dfrac{j-i}{(n-i)j} + \dfrac{1}{n-i}\right) \\[3ex] \text{if } i > j \to \begin{array}{cccccc} (\circ \cdots & \circ & \circ & \circ \cdots & \circ) \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ 1 & j & & i & n \end{array} \to m_i^T m_j = \left(\dfrac{1}{i} + \dfrac{i-j}{(n-j)i} + \dfrac{1}{n-j}\right) \end{cases}$$

if $i < j$

$$\begin{cases} m_i = \left(\tfrac{1}{i} \cdots \tfrac{1}{i}\,\tfrac{1}{i}\,\tfrac{1}{i}\ \boxed{\tfrac{1}{n-i} \cdots \tfrac{1}{n-i}}\ \tfrac{1}{n-i}\,\tfrac{1}{n-i} \cdots \tfrac{1}{n-i}\right) \\[1ex] \qquad\qquad\qquad\quad\ \ i \qquad\qquad\ \ j \\[1ex] m_j = \underbrace{\left(\tfrac{1}{j} \cdots \tfrac{1}{j}\,\tfrac{1}{j}\,\tfrac{1}{j}\right.}\ \boxed{\tfrac{1}{j} \cdots \tfrac{1}{j}}\ \left.\tfrac{1}{n-j}\,\tfrac{1}{n-j} \cdots \tfrac{1}{n-j}\right) \\[2ex] m_i^T m_j = \cancel{i}\left(\dfrac{1}{\cancel{i}}\,\dfrac{1}{j}\right) + (j-i)\dfrac{1}{n-i}\,\dfrac{1}{j} + (\cancel{n-j})\left(\dfrac{1}{n-i}\right)\left(\dfrac{1}{\cancel{n-j}}\right) \\[2ex] m_i^T m_j = \dfrac{1}{j} + \dfrac{j-i}{(n-i)j} + \dfrac{1}{n-i} \end{cases}$$

$$V^{lo}_{BS} = \max\left[\max_{\substack{i<j \\ s=\pm 1}} \left\{ \frac{-(b_j + s b_i)^T y}{\sqrt{\frac{n}{j(n-j)}} + s\sqrt{\frac{n}{i(n-i)}}\,\frac{i}{j}} \right\},\ \max_{\substack{i>j \\ s=\pm 1}} \left\{ \frac{-(b_j + s b_i)^T y}{\sqrt{\frac{n}{j(n-j)}} + s\sqrt{\frac{n}{i(n-i)}}\,\frac{n-i}{n-j}} \right\} \right]$$

vs

$$V^{lo}_{FL} = \max\left[\max_{\substack{i<j \\ s=\pm 1}} \left\{ \frac{-(a_j + s a_i)^T y}{1 + s\, i/j} \right\},\ \max_{\substack{i>j \\ s=\pm 1}} \left\{ \frac{-(a_j + s a_i)^T y}{1 + s\frac{n-i}{n-j}} \right\} \right]$$

STEP 2: Simplifying it even further to match Ryan's original notation:

$$V^{do}_{BS} = \max\left[\max_{\substack{i<j\\ s=\pm1}} \left\{ \frac{-\sqrt{\frac{n}{j(n-j)}}\, j}{\sqrt{\frac{n}{j(n-j)}}\, j + \sqrt{\frac{n}{i(n-i)}}\, si} a_j^T y + \frac{-s\sqrt{\frac{n}{i(n-i)}}\, j}{\sqrt{\frac{n}{j(n-j)}}\, j + \sqrt{\frac{n}{i(n-i)}}\, si} a_i^T y \right\},\right.$$

$$\left. \max_{\substack{i>j\\ s=\pm1}} \left\{ \frac{-\sqrt{\frac{n}{j(n-j)}}\,(n-j)}{\sqrt{\frac{n}{j(n-j)}}\,(n-j) - \sqrt{\frac{n}{i(n-i)}}\, s(n-i)} b_j^T y + \frac{-\sqrt{\frac{n}{i(n-i)}}\, s(n-j)}{\sqrt{\frac{n}{j(n-j)}}\,(n-j) - \sqrt{\frac{n}{i(n-i)}}\, s(n-i)} b_i^T y \right\} \right]$$

$$= \max\left[\max_{\substack{i<j\\ s=\pm1}} \left\{ \overbrace{\frac{-j}{j + \sqrt{\frac{j(n-j)}{i(n-i)}}\, si}}^{(*)} a_j^T y + \frac{-sj}{j + \sqrt{\frac{j(n-j)}{i(n-i)}}\, si} a_i^T y \right\},\right.$$

$$\left. \max_{\substack{i>j\\ s=\pm1}} \left\{ \frac{-(n-j)}{(n-j) - \sqrt{\frac{j(n-j)}{i(n-i)}}\, s(n-i)} a_j^T y + \frac{s(n-j)}{(n-j) - \sqrt{\frac{j(n-j)}{i(n-i)}}\, s(n-i)} a_i^T y \right\} \right]$$

" " is how they differ!

$$V^{do}_{FL} = \max\left[\max_{\substack{i<j\\ s=\pm1}} \left(\frac{-j}{j+si} a_j^T y + \frac{-sj}{j+si} a_i^T y \right), \max_{\substack{i>j\\ s=\pm1}} \left\{ \frac{-(n-j)}{(n-j)+s(n-i)} a_j^T y + \frac{-s(n-j)}{(n-j)+(n-i)s} a_i^T y \right\} \right]$$

Ryan's notation