11.3.2

$$\rho = \frac{r}{\hbar}\sqrt{8m|\varepsilon|} \qquad \lambda = \frac{Ze^2}{4\pi\varepsilon_0\hbar}\sqrt{\frac{m}{2|\varepsilon|}}$$

$$\frac{d}{dr^2} = \frac{8m\varepsilon}{\hbar^2}\frac{d^2}{d\rho^2} \qquad \frac{Ze^2}{4\pi\varepsilon_0} = \frac{\hbar\lambda}{\sqrt{m/2|\varepsilon|}}$$

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} - \frac{Ze^2}{4\pi\varepsilon_0 r} + \frac{\hbar^2 \ell(\ell+1)}{2mr^2} + \frac{\hat{c}}{r^2} - \varepsilon\right)u(r) = 0$$

$$\Rightarrow \left(-4\varepsilon\frac{d^2}{d\rho^2} - \frac{4\varepsilon\lambda}{\rho} + 4\varepsilon\frac{\ell(\ell+1)}{\rho^2} + 4\varepsilon\frac{c}{\rho} - \varepsilon\right)u(r) = 0$$

$$\Rightarrow \left(-4\varepsilon\frac{d^2}{d\rho^2} - 4\varepsilon\frac{\lambda}{\rho} + 4\varepsilon\frac{\hat{\ell}(\hat{\ell}+1)}{\rho^2} - \varepsilon\right)u(\rho) = 0$$

$$\Rightarrow \left(\frac{d^2}{d\rho^2} + \frac{\lambda}{\rho} - \frac{\hat{\ell}(\hat{\ell}+1)}{\rho^2} + \frac{1}{4}\right)u(\rho) = 0$$

$$u(\rho) = e^{-\rho/2}\rho^{\hat{\ell}+1}f(\rho) \quad , \quad f(\rho) = \sum_{n=0}^{3} A_n\rho^n$$

$$\frac{d}{d\rho}u = -\frac{1}{2}u + (\hat{\ell}+1)\frac{u}{\rho} + e^{-\rho/2}\rho^{\hat{\ell}+1}f'$$

$$\frac{d^2}{d\rho^2}u = \frac{1}{4}u - \frac{1}{2}(\hat{\ell}+1)\frac{u}{\rho} - \frac{1}{2}e^{-\rho/2}\rho^{\hat{\ell}+1}f'$$

$$+ (\hat{\ell}+1)\frac{du}{d\rho}\frac{1}{\rho} - (\hat{\ell}+1)\frac{u}{\rho^2}$$

$$- \frac{1}{2}e^{-\rho/2}\rho^{\hat{\ell}+1}f' + e^{-\rho/2}(\hat{\ell}+1)\rho^{\hat{\ell}}f' + e^{-\rho/2}\rho^{\hat{\ell}+1}f''$$

$$= \dots$$

$$= -\rho f''(\rho) + (-2-2\hat{\ell}+\rho)f'(\rho) + (1+\hat{\ell}-\lambda)f(\rho) = 0$$

$$\Leftrightarrow -\sum_{n=1} \rho^n n(n+1)A_{n+1} - \sum_{n=0} (2(1+\hat{\ell}))\, n A_n \rho^{n-1}$$

$$+ \sum_{n=0} n A_n \rho^n + \sum_{n=0} (1+\hat{\ell}-\lambda) A_n \rho^n = 0$$

$$\Leftrightarrow \sum_{n=1} \rho^n \left(A_{n+1}\left((n+1)(n+2(1+\hat{\ell})\right) + A_n (1+\hat{\ell}-\lambda+n)\right) = 0$$

$$A_{n+1} = -A_n \frac{(1+n+\hat{\ell}-\lambda)}{(n+1)(n+2(1+\hat{\ell}))}$$

$$\hookrightarrow \lambda = \hat{\ell}+k+1 = \hat{\ell}+n' \qquad , n' = k+1$$

Für: $\hat{c} = 0 \quad , k = n-\ell+1 \quad \Rightarrow \quad n' = n-\ell+\hat{\ell}$

$$\varepsilon = -\frac{Z^2}{n'^2} E_R = -\frac{Z^2}{(n-\ell+\hat{\ell})^2} E_R$$

$$\hat{\ell}(\hat{\ell}+1) = \ell(\ell+1) + \hat{c} \qquad , \hat{c} \ll 1$$

$$\Rightarrow (\hat{\ell}+1/2)^2 = (\ell+1/2)^2 + \hat{c}$$

$$\Rightarrow \hat{\ell}+1/2 = (\ell+1/2)\left(1+\frac{1}{2}\frac{\hat{c}}{(\ell+1/2)^2}\right)$$

$$\Rightarrow \hat{\ell} \approx \ell + \frac{\hat{c}}{(2\ell+1)} \quad \Rightarrow \varepsilon = -\frac{Z^2 E_R}{\left(n+\frac{\hat{c}}{2\ell+1}\right)}$$