H 10.1 a) $E_{p,\sigma} = E_p + E_\sigma = \frac{p}{2m} + g\mu_B \sigma B$

$$Z_\alpha = \sum_{\substack{n_{\alpha_i}=0,1 \\ i=1,2,\dots}} \exp\left(-\beta\left(E(\{n_{\alpha_i}\}) - \mu N\right)\right) \qquad , \alpha = p,\sigma$$

$$E(\{n_{\alpha_i}\}) = \sum_i n_{\alpha_i} E_{\alpha_i} \quad , N = \sum_i n_{\alpha_i}$$

$$\Downarrow = \prod_i^\infty \left(\sum_{n_{\alpha_i}=0,1} \exp\left(-\beta n_{\alpha_i}(E_{\alpha_i} - \mu)\right)\right)$$

$$= \prod_i^\infty 1 + \exp\left(-\beta(E_{\alpha_i} - \mu)\right)$$

b) $\Omega = -k_B T \ln Z_\alpha$

$$= -k_B T \sum_i^\infty \ln\left(1 + \exp(-\beta(E_{\alpha_i} - \mu))\right)$$

Skript Formel S. 61

$$= -k_B T \sum_\sigma \int_{-\infty}^{\infty} dE \; D_\sigma(E) \ln\left(1 + \exp(-\beta(E - \mu))\right)$$

part. Int. (grenzen fallen weg) $\int_{-\infty}^{E} d\varepsilon \; D_\sigma(\varepsilon) = a_\sigma(\varepsilon) \longrightarrow$ Anzahl belegter Zustände bis Energie E

$$= +k_B T \sum_\sigma \int_{-\infty}^{\infty} dE \; a_\sigma(E) \frac{-\beta \exp(-\beta(E-\mu))}{\exp(-\beta(E-\mu))+1}$$

$$= -\sum_\sigma \int_{-\infty}^{\infty} dE \; a_\sigma(E) \frac{1}{\exp(\beta(E-\mu))+1}$$

$$= -\sum_\sigma \int_{-\infty}^{\infty} dE \; a_\sigma(E) \; f(E)$$

erneute partielle Integration mit $b_\sigma(E) = \int_{-\infty}^{E} d\varepsilon \; a_\sigma(\varepsilon) = \int_{-\infty}^{\infty} d\varepsilon \int_{-\infty}^{\varepsilon} d\varepsilon' \; D_\sigma(\varepsilon')$

$$I = \int_{-\infty}^{\infty} dE \; a_\sigma(E) f(E) = \underbrace{\left[b_\sigma(E) f(E) \right]_{-\infty}^{\infty}}_{0} - \int_{-\infty}^{\infty} dE \; b_\sigma(E) f'(E)$$

$f'(E)$ nur um μ von 0 verschieden, daher Taylor von $b_\sigma(E)$ um $E = \mu$:

$$b_\sigma(E) = b_\sigma(\mu) + \sum_{n=1}^{\infty} \frac{(E-\mu)^n}{n!} \left.\frac{d^n b_\sigma(E)}{dE^n}\right|_{E=\mu}$$

Erster Term im Integral:

$$I_0 \quad - b_\sigma(\mu) \underbrace{\int_{-\infty}^{\infty} dE \; f'(E)}_{-1} = b_\sigma(E_F) = \int_{-\infty}^{\mu} dE \; a_\sigma(E)$$

mit
$$-\frac{f'(E)}{\beta} = \frac{\exp(\beta(E-\mu))}{(\exp(\beta(E-\mu))+1)^2}$$

$$= \frac{1}{\exp(\beta(E-\mu)) + \exp(-\beta(E-\mu)) + 2}$$

$$= \frac{1}{\exp(\beta(E-\mu)) + 2\frac{\exp(\beta(E-\mu))}{\exp(\beta(E-\mu))} + \exp(-\beta(E-\mu))}$$

$$= \frac{1}{(\exp(\beta(E-\mu)) + \exp(-\beta(E-\mu)))^2}$$

$$= \frac{1}{4\cosh^2(\beta(E-\mu))}$$

folgt für die weiteren Glieder des Integrals

$$I_1 = \beta a_0(\mu)\int_{-\infty}^{\infty} dE \frac{(E-\mu)}{4\cosh^2(\beta(E-\mu))} = \beta a_0(\mu)\left(\frac{(E^2-E\mu)}{4\cosh^2(\beta(E-\mu))}\Bigg|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} dE \frac{1}{4\cosh^2(\beta(E-\mu))}\right)$$

$$= \beta a_0(\mu)\left[\frac{E^2-E\mu}{4\cosh^2(\beta(E-\mu))} + \frac{\tanh(\beta(E-\mu))}{4\beta}\right]_{-\infty}^{\infty}$$