Applied Acoustics 132 (2018) 58–81

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Effects of array scaling and advanced beamforming algorithms on the angular resolution of microphone array systems

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ARTICLE INFO

Keywords:
Beamforming
Microphone array
Angular resolution

ABSTRACT

Several advanced beamforming methods have been developed in the past 20 years that have dramatically improved the angular resolution of microphone array systems. Meanwhile, the Rayleigh criterion has long been considered the standard criterion for angular resolution of such systems. In this investigation four microphone arrays were constructed as scaled models of a fifth microphone array. All of the arrays were subjected to a thorough regimen of testing with both broadband and narrowband sources. Using conventional beamforming, the angular resolution of each scaled array was determined as a function of frequency and compared to the Rayleigh criterion. The analysis was repeated with the advanced TIDY, DAMAS, DAMAS2, and CLEAN-SC beamforming algorithms so that the effects of array scaling and advanced beamforming can be determined as a function of frequency and compared to conventional beamforming.

1. Introduction

Microphone array systems have been in use for more than 40 years and have proven useful in a variety of applications [1]. The most traditional form of acoustic beamforming is the Delay-and-Sum (DAS) beamforming algorithm, wherein time delays are applied to the pressure signal from each sensor and the resulting signals are summed to form the beamform map [2]. However, the angular resolution of the DAS beamforming algorithm is poor. Frequency-domain beamforming (FDBF) methods were developed in the 1980s using the Fast Fourier Transform. Researchers have since developed improved frequency-domain beamforming algorithms such as CLEAN, RELAX, and SEM. Many such algorithms are based on the idea of manipulating the Cross Spectral Matrix and the concept of spatial source coherence [3–8]. More recently, beamforming algorithms have been developed based on methods of deconvolution [9,10], methods based on the spatial coherence of point sources and sidelobes in the frequency domain [11], and spatial coherence methods in the time domain [12]. These are the DAMAS, DAMAS2, CLEAN-SC, and TIDY algorithms, respectively. The current investigation will explore the concept of angular resolution utilizing two time-domain beamforming algorithms and three frequency-domain algorithms: the conventional DAS algorithm as well as the more advanced TIDY, DAMAS, DAMAS2, and CLEAN-SC beamforming algorithms.

The concept of angular resolution refers to the ability of a system to differentiate between two closely-spaced sources. A system with inadequate angular resolution will show two closely-spaced sources as a single source. The most widely-used criterion for the angular resolution of a microphone array system is the Rayleigh criterion, which indicates that angular resolution is proportional to array diameter and inversely proportional to the signal's wavelength. This criterion was originally utilized in the field of optical imaging, but is also applicable to acoustics. However, the Rayleigh criterion is a convenient reference point rather than a fixed physical limit. The Rayleigh resolution criterion defines the angular separation between two sources at the point where the maximum of the Airy disk (analogous to a diffraction pattern) of one source is located at the first minimum of the Airy disk of the second source. Using the small angle approximation, the Rayleigh criterion is given by:

$$W = \frac{rD}{\lambda z}, \tag{1}$$

where r is the separation distance between the two sources, D is the diameter of the array, λ is the wavelength of the sources and z is the separation distance between the source and the array [13]. The Rayleigh criterion W is given by the first zero of the first-order Bessel function of the first kind, divided by a factor of pi to convert into radians. This results in a value of W = 1.22 for a circular, continuous

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https://doi.org/10.1016/j.apacoust.2017.11.009
Received 2 January 2017; Received in revised form 15 June 2017; Accepted 13 November 2017
Available online 22 November 2017
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Fig. 1. Expected angular resolution based on Rayleigh criterion for each of the five microphone arrays.

Fig. 2. OptiNav compact microphone array and data acquisition chassis.

Fig. 3. Completed 0.8× Array.

aperture. Re-arranging Eq. (1) to show the minimum resolvable separation angle explicitly gives:

$$\theta = W\frac{\lambda}{D}. \tag{2}$$

Although a great deal of understanding can be gained from mathematical analysis, the effects of array scaling on angular resolution have not been clearly demonstrated in practice. A deeper understanding of the effects of array geometry and advanced beamforming algorithms can be gained by performing carefully controlled experiments. The purpose of this work is to experimentally determine the angular resolution of microphone arrays as the array size is scaled to larger and smaller dimensions, and to explore the effect of five beamforming algorithms on the angular resolution of the array. To accomplish this, four microphone arrays were constructed as scaled models of a fifth commercially-available baseline array. A battery of tests was then performed on each of the five arrays using the DAS, TIDY, DAMAS, DAMAS2, and CLEAN-SC algorithms, and the results were analyzed and compared. In this way the effect of array scaling and the effect of the advanced beamforming algorithms can be determined as a function of frequency and compared to conventional DAS beamforming.

For the experiments carried out in the following sections, the baseline microphone array has a diameter of 0.73 m and is hereafter referred to as the "1×" array. An 80% scale model of this array, called the "0.8×" array was constructed with a diameter of 0.59 m. Scaled-up versions of the baseline array were constructed with diameters that are

Fig. 4. Modular breadboard circuit including microphone (lower left) and amplifier circuit.

Fig. 5. 3.66 m (5×) Microphone array undergoing testing.

Fig. 6. 7.32 m (10×) Microphone array undergoing testing.

five, ten, and fifteen times larger than the baseline array. The 5×, 10×, and 15× microphone arrays have diameters of 3.66 m, 7.32 m, and 10.98 m, respectively. Using the Rayleigh criterion in Eq. (2), the expected angular resolution is plotted in Fig. 1 as a function of frequency for each of the five microphone arrays. The resolvable angular

Fig. 7. 10.98 m (15×) Microphone array undergoing testing.

Fig. 8. Speaker holder for angular resolution testing.

Table 1
Angular resolution testing regimen.

Microphone array testing regimen	
Test frequencies [Hz]	Speaker separation [in.]
(200)[a]	3.5
250	4
315	5
400	6
500	7
630	8
800	9
1000	10
1250	11
1600	12
2000	13
2500	14
3150	15
4000	16
5000	17
6300	18
8000	19
(10,000)[b]	20
White noise	Single speaker

[a] 200 Hz tested on 5×, 10×, and 15× arrays only.
[b] 10,000 Hz tested on 0.8× and 1× arrays only.

resolution between two acoustic sources (measured in degrees on the horizontal axis) is expected to decrease as either the diameter of the array is increased or the frequency of the signal is increased.

2. Five scaled microphone arrays

2.1. Baseline array

The baseline array on which each of four scaled microphone arrays are based is the 24-channel OptiNav microphone array produced by OptiNav, Inc. It consists of 24 electret condenser microphones arranged to approximate a spiral pattern with a diameter of 0.73 m. The microphone pressure signals pass through an amplifier and the data is acquired with a MOTU 24 I/O 24-channel chassis. Each channel has 24-

Fig. 9. (a) Low Frequency Unresolved, (b) Inconsistently Resolved, (c) Resolved, and (d) High Frequency Unresolved acoustic sources.

Fig. 10. (a) Low Frequency Unresolved, (b) Inconsistently Resolved, (c) Resolved, and (d) High Frequency Unresolved acoustic sources without images.

bit resolution, and the data is acquired at 44.1 kHz. The MOTU 24 I/O interfaces with a laptop computer equipped with recording software via a PCIe-424 MAGMA Express Box. The data is saved on the laptop for later processing using OptiNav's Beamform Interactive plugin to the ImageJ image processing software. In addition to the microphones, the microphone array contains a video camera in the center of the array. The array's video camera interfaces directly with the laptop computer via USB connection. The OptiNav compact microphone array is shown in Fig. 2.

2.2. Four new scaled arrays

For comparison against the baseline OptiNav array, four additional microphone arrays were constructed. The microphone placement within each array is a linearly scaled (blown-up or down) version of the OptiNav array geometry. Each array was constructed with 6 mm electret condenser microphones, an amplifier circuit with a gain factor of 26, and DB25 parallel-port pinout cards for connection to the same MOTU 24 I/O data acquisition system used by the OptiNav array. An image of the completed 0.8× array is shown in Fig. 3.

For the scaled-up arrays, a modular construction approach was necessitated by the size of the arrays. Each of the 24 microphones was integrated into a combined breadboard containing the microphone itself and the amplifier circuit. An image of one of the 24 boards is shown in Fig. 4.

Due to the size of the scaled-up microphone arrays, the arrays were installed on the roof of the Illinois Institute of Technology Research Institute (IITRI) building. IITRI is a 21-story building on the southern edge of the campus of the Illinois Institute of Technology (IIT) in Chicago, IL. To lay out the geometry of the microphone arrays on the roof, the microphone positions were meticulously measured, checked, and verified. The individual microphone locations were marked with two intersecting pieces of exterior-grade tape. When the array was undergoing testing, the microphone boards shown in Fig. 4 were carefully placed at the intersection of the two pieces of tape, and the connection circuitry was laid out and connected. When not in use, the array was packed up and stored inside. In this way, the entire array circuitry was protected from environmental degradation. Testing was performed only in calm conditions with no precipitation.

The rooftop location offered a large unobstructed space on which the microphone arrays could be installed. In addition, the rooftop was open to the environment, and thus the air through which the sound waves propagate can be approximated as an infinite medium. Therefore, free space propagation of the sound signals is assumed. Images of the fully-assembled 5×, 10×, and 15× arrays are given in Figs. 5–7, respectively. In Figs. 6 and 7, the locations of the microphone circuit boards are circled in red for clarity.

2.3. Angular resolution testing

Each of the five arrays was subjected to testing at a variety of frequencies, source angular separation values, and both broadband and narrowband sources. To control the angular separation values, a speaker holder was used to hold two test speakers at separation distances ranging from 8.9 cm (3.5 in.) to 50.8 cm (20 in.). An image of the speaker holder with two speakers located at 50.8 cm (20 in.) separation distance is given in Fig. 8. By varying the speaker separation on the speaker holder and the height of the test stand, a wide range of angular separation values can be achieved. The angular separation of the speakers can be calculated using a simple trigonometric relationship. The test regimen included testing each array at 18 acoustic frequencies spaced at one-third octave intervals plus a broadband signal. The signals supplied to the two speakers were generated by independent function generators, and the broadband signals were Gaussian random white noise signals. This sequence was repeated for 19 speaker separation values. Thus, the testing sequence involved testing five different microphone arrays at 18 different frequencies for 19 different speaker separation distances. This gave a total of 1710 test cases. The testing regimen is summarized in Table 1. For the present study, only the broadband data will be used for analysis.

Fig. 11. Beamforming results with DAS algorithm: (a) 0.8×, (b) 1×, (c) 5×, (d) 10×, (e) 15× arrays.

Fig. 12. Beamforming results with TIDY algorithm: (a) 0.8×, (b) 1×, (c) 5×, (d) 10×, (e) 15× arrays.

3. Results

For clarity, the results will be divided into time-domain algorithm results (DAS and TIDY) and frequency-domain algorithm results (DAMAS, DAMAS2, and CLEAN-SC). The results will be combined and compared after they are discussed separately.

The data analysis begins by considering the most ubiquitous type of acoustic source – broadband – and the most conventional of the beamforming algorithms: Delay-and-Sum (DAS). The DAS algorithm has been in use for many decades and serves as a baseline analysis. The DAS algorithm was utilized to construct acoustic source maps for each array, angular separation value, and one-third octave band using the white noise signal. Following analysis, the acoustic source map for each set of conditions was judged to (a) consistently resolve the two acoustic sources, (b) inconsistently resolve the two acoustic sources, or (c) not resolve the two acoustic sources. The second category, where the algorithm inconsistently resolves the two sources, was necessary because there were numerous instances where the algorithm could resolve the sources for certain time steps while it was unable to resolve the sources in all time steps. This typically occurred near the algorithm's resolution limit for a given array geometry and frequency band. In addition, when the range of frequencies became high enough, the beamform map became contaminated by sidelobes. When the sidelobes contaminated the beamform map to the point that the acoustic sources were no longer distinguishable from the sidelobes, the acoustic sources could no longer be considered to be resolved. Examples of unresolved, inconsistently resolved, and resolved acoustic sources are given in Fig. 9. The acoustic source maps are overlaid on the image taken with the array's video camera. Although the term "inconsistently resolved" refers to inconsistency in the time domain, it is apparent in the static image shown in Fig. 9(b) that the algorithm is very near the resolution limit for this particular set of parameters. Fig. 9 shows the results from the 5× array with 25.4 cm (10 in.) of separation

Fig. 13. Beamforming map for 15× array at 20.3 cm (8in.) speaker separation with DAS algorithm at 1122–1413 Hz using integration times of (a) 0.1 s, (b) 0.2 s, (c) 0.4 s, (d) 0.8 s, (e) 2.0 s, and (f) 4.0 s.

Fig. 14. Z-corrected rooftop results with DAS algorithm: (a) 5×, (b) 10×, (c) 15× arrays.

between the speakers, corresponding to an angular separation of 3.1°. The differences between Fig. 9(a)–(d) are due entirely to the frequency band utilized by the DAS algorithm. Fig. 9(a) uses a frequency band with a center frequency of 1000 Hz, Fig. 9(b) uses a frequency band with a center frequency of 1600 Hz, Fig. 9(c) uses frequencies with a center frequency of 2500 Hz, and Fig. 9(d) utilizes frequencies with a center frequency of 10,000 Hz.

The discernment between unresolved, inconsistently resolved, and resolved sources is made more obvious when the acoustic source map is shown by itself against a solid black background rather than overlaid over an image. Fig. 10 shows the same acoustic source maps without the image of the test apparatus.

3.1. DAS results

This analysis was carried out using the DAS algorithm for each array, each frequency band, and each speaker separation distance. The results for the five arrays are given in Fig. 11. In each case, the discretized Rayleigh criterion curves are plotted as "R"s along with the results. Where the algorithm consistently resolves the two sources, the bin is colored green. Where the algorithm inconsistently resolves the sources the bin is colored yellow, and where the algorithm cannot resolve the sources – or the sources cannot be identified due to sidelobe contamination – the bin is colored red.

The lower operating frequency limit of the arrays, denoted in Fig. 11 by the uppermost green and yellow bins, is limited by the ability of the algorithm to resolve two acoustic sources at low frequencies. It is apparent that the Rayleigh criterion can indeed be exceeded under certain conditions. It appears that the Rayleigh limit becomes increasingly difficult to exceed as the array size increases, even though the calculation of the Rayleigh limit already incorporates the array diameter. This effect will be addressed in Section 3.3.

The upper operating frequency limit, denoted in Fig. 11 by the lowermost green and yellow bins, is limited by the appearance and subsequent dominance of sidelobes and grating lobes in the acoustic source map. As the frequency increases, the inter-element spacing becomes larger relative to the wavelength, which will eventually lead to grating lobes. At the upper frequency limit, the actual acoustic sources in the acoustic source map are no longer distinguishable from the grating lobes. Because each array in this experiment has an identical number of microphones, the inter-element spacing becomes larger as the array size is increased. Thus, the useful upper frequency limit is reduced as the array size is increased.

Fig. 15. Z-corrected rooftop results with TIDY algorithm: (a) 5×, (b) 10×, (c) 15× arrays.

3.2. TIDY results

The analysis described above was repeated using the TIDY beamforming algorithm. The TIDY algorithm is an advanced time-domain beamforming algorithm useful for improved beamforming of broadband sources. The results are given in Fig. 12.

Compared to the DAS results in Fig. 11 the TIDY results show both superior angular resolution at low frequencies and superior mitigation of grating lobes at high frequencies. This results in an expanded operating range for the TIDY algorithm as compared to the DAS algorithm, which appears as an expanded green region in the results tables. The reason that the TIDY algorithm outperforms the DAS algorithm is because the TIDY algorithm effectively seeks to remove the Point Spread Function from the beamform map, leaving only a cleaner true acoustic source map. The conventional DAS algorithm makes no such accommodations for removing the inherent Point Spread Function of the acoustic source. Each one of the five arrays shows a significant increase in performance with the TIDY algorithm as compared to the DAS algorithm. However, the smaller arrays (0.8×, 1×, 5×) exhibit particularly large gains in performance, with the gains occurring at both the low and high frequency edges of the useful operating region.

As the size of the array increased, the angular resolution performance of the array improved in absolute terms. That is, for a given frequency, a larger array consistently resolved the sources at a smaller separation angle. However, as the array size increased the angular resolution performance of the array tended to decrease relative to the Rayleigh criterion. The 0.8× array greatly exceeded the Rayleigh criterion over its entire useful operating region, while the performance of the 1× array relative to the Rayleigh criterion was only slightly lower. The 5× array consistently outperformed the Rayleigh criterion over its operating range, but its performance relative to the Rayleigh criterion was slightly lower. The 10× array consistently met the Rayleigh criterion over the higher-frequency portion of its operating range, but as the frequency decreased the 10× array was not able to consistently meet the Rayleigh criterion. However, the 10× array was able to inconsistently match the Rayleigh criterion over most of the low-frequency portion of its operating range. Finally, the 15× array came close to meeting the Rayleigh criterion over the high-frequency portion of its range, but was not able to meet the Rayleigh criterion – even on an inconsistent basis – at the lowest frequencies. The cause of this relative decrease in performance is not immediately clear, but several possible explanations are explored in Section 3.3.

Fig. 16. Angular resolution frontier curve for uncorrected and z-corrected analysis of 5×, 10×, and 15× arrays with the DAS algorithm.

When the array size increased, the effective upper frequency limit decreased. This is expected, and is due to the increasing inter-element spacing as the array size grows larger. Larger inter-element spacing results in the onset and subsequent dominance of grating lobes at lower frequencies because the spacing becomes larger relative to the signal wavelength. The exception to this pattern was between the 0.8× array and the 1× array, where the 1× array had a higher upper frequency limit than the 0.8× array. This was probably due to the manufacturing precision of the 1× array as compared to the 0.8× array. The 1× array was built by OptiNav using precision machining of sheet metal. The 0.8× array was custom-built by the author, and while extreme care was taken in order to be as precise as possible, the placement of the microphones is probably not as precise as the OptiNav array. At very high frequencies, small microphone placement errors may lead to decreased performance.

3.3. Addressing potential sources of error

The DAS and TIDY results presented in the previous sections exhibit profiles that approximate the shape of the Rayleigh curve. However, the relative performance of the arrays compared to each other is not thoroughly explained by the Rayleigh criterion alone. For example, the smaller 0.8× and 1× arrays outperform the Rayleigh criterion by significant margins. Meanwhile, the larger 10× and 15× arrays are only barely capable of meeting the Rayleigh criterion, and in some cases cannot meet the Rayleigh criterion. There are several possible reasons for this:

i. Signal-to-Noise Ratio (SNR): It is possible that the larger arrays – especially at the outside edges – are suffering from poor SNR. This may reduce the effective diameter of the array, thereby reducing the low-frequency performance.
ii. Signal reflection effects: The arrays are assumed to be operating in a free field environment, but of course this is not entirely realistic. The roof has a concrete safety barrier approximately 1 meter tall extending around the perimeter of the roof. As the array size increases, the microphones must be positioned closer to the barrier. Reflection of the test signal off the concrete perimeter barrier may adversely affect the performance of the larger arrays.
iii. Z-axis correction: The initial testing and analysis described above was performed with the assumption that the rooftop under the arrays is a perfectly flat horizontal plane. Accurately measuring and re-analyzing the data with corrected z-axis measurements may improve the results.

3.3.1. Signal-to-Noise Ratio (SNR) effects

Potential SNR issues could be adversely affecting the output of the larger arrays. The SNR may be acceptable for the smaller arrays because the individual microphone elements are closer to the acoustic test sources. But as the radius of the array increases and the array elements become farther away from the test sources, the SNR could decrease. This could reduce the effective diameter of the larger arrays, which would impair the ability of the array to resolve sources at low frequencies. Fortunately, there is a relatively simple test to check if poor SNR is affecting the beamforming output. To check for SNR impacts, the integration time for the beamforming map can be varied. If SNR is adversely affecting the output, then the results will improve with longer integration times. If SNR is not significantly affecting the output, then the beamforming results will vary little as the integration time is changed. This check was performed for both DAS and TIDY algorithms across numerous different angular separation distances and for different initial beamforming results (i.e. green[1], yellow, and red cells in Figs. 11 and 12). For brevity, only one such example case will be given here. Because it may offer the best chance to show improvement, a marginal or "inconsistently resolved" (yellow) case will be shown below. The inconsistently resolved case for the 15× array at 20.3 cm (8 in.) speaker separation with the DAS algorithm at 1122–1413 Hz is given in Fig. 13.

As can be seen in Fig. 13, the beamforming map does not significantly change as the integration time is varied from 0.1 s to 4.0 s. This effect is consistent across both algorithms, all source separation distances, and regardless of initial beamforming result. This indicates that the SNR is not significantly affecting the beamforming output.

3.3.2. Signal reflection effects at boundaries

Another potential source of error is the possibility of signal reflection effects at boundaries near the edges of the larger arrays. The arrays are assumed to be operating in a free field environment, but of course

[1] For interpretation of color in Figs. 11, 12, 14, 15, 27–29, the reader is referred to the web version of this article.

Fig. 17. Angular resolution frontier curve for uncorrected and z-corrected analysis of 5×, 10×, and 15× arrays with the TIDY algorithm.

this is not entirely realistic. In addition to various ventilation equipment located on the rooftop, the roof has a concrete safety barrier approximately 1 meter tall extending around the perimeter of the roof. As the array size increases, the microphones must necessarily be located closer to the concrete barrier. In the largest (15×) array, the closest microphone is located approximately one meter from the wall. Reflection of the test signal off the concrete perimeter barrier may adversely affect the performance of the larger arrays.

To test this hypothesis, a follow-on experiment will be conducted separately in a future work. The test will be conducted in a controlled laboratory environment. Several array geometries will be laid out on a horizontal surface in an open laboratory environment approximating a free field. A complete battery of angular resolution testing will be performed using the same test apparatus that was used to test the arrays on the roof. Next, a reflective barrier will be constructed around the perimeter of the array. The angular resolution testing will be repeated. The results from these test conditions will be compared, and the comparison will provide empirical insight into the effect of perimeter boundary reflection.

3.3.3. Z-axis correction

When the 5×, 10× and 15× microphone arrays were laid out on the rooftop, it was assumed that the roof was a flat horizontal plane. Even if this was not true, the error was assumed to be small. However, errors in z position could be affecting the results. To address this potential issue, z-axis measurements were taken and the results were re-analyzed with the corrected microphone position geometry.

To measure the z-axis position of the microphone array elements, a rotary laser level was placed at the center of each array and leveled. A measuring stick was then placed at every microphone position, and the height of the laser level plane above the microphone position was recorded. By subtracting the average z measurement from each individual measurement, the z-axis coordinate of each microphone was obtained. The average magnitude (absolute value) of the z correction for individual elements in the 5× array was 1.0 cm (0.41 in.), while for the 10× and 15× arrays the average magnitude of the z corrections were 2.8 cm (1.12 in.) and 3.3 cm (1.31 in.), respectively. The array geometry was updated with the corrected z-axis coordinates and the beamforming analysis was re-performed for each case. The 0.8× and 1× arrays were not laid out on the rooftop surface because they are self-contained units; therefore there is no z-correction for the two smallest arrays. Results obtained with the corrected z-axis positions for the 5×, 10×, and 15× arrays are given in Fig. 14 for the DAS algorithm and Fig. 15 for the TIDY algorithm.

The z-axis correction could potentially impact the beamforming results in two distinct ways. It could impact the arrays' ability to resolve

Fig. 18. Average Maximum Frequency for uncorrected and z-corrected geometry with 5×, 10×, and 15× arrays using DAS algorithm.

Fig. 19. Average Maximum Frequency for uncorrected and z-corrected geometry with 5×, 10×, and 15× arrays using TIDY algorithm.

Fig. 20. Minimum resolvable angular resolution data for all rooftop arrays using DAS algorithm.

low frequency sources, i.e. it may affect the angular resolution performance relative to the Rayleigh criterion. Alternatively or additionally, the correction may impact the upper frequency limit of the array, i.e. the maximum frequency at which the array can effectively operate before the onset and dominance of grating lobes. Each of these possibilities will be tested separately.

The effect of z-axis correction on the performance of the array relative to the Rayleigh criterion will be discussed first. To show this potential effect, the minimum frequency at which the beamforming map was able to consistently resolve sources at each angular separation distance was recorded. This is analogous to the upper-most green cell in each column of Figs. 14 and 15. This may be considered the "angular resolution frontier." It is the curve representing the lowest frequency at which the algorithm can consistently resolve two acoustic sources at a given angular separation. The angular resolution frontier curves for uncorrected and z-corrected analysis of each array are presented in Fig. 16 (DAS) and Fig. 17 (TIDY).

A number of conclusions can be drawn from inspection of Figs. 16 and 17. First, it is apparent that for a given angular separation, the TIDY algorithm is able to achieve resolution at a lower frequency band than the DAS algorithm. This is consistent with earlier discussion in Section 3.2. Second, it is also apparent that a larger array is usually able to achieve resolution for a given angular separation at a lower frequency band than a smaller array. Finally, it can be seen that the uncorrected and z-corrected curves for a particular array generally lie on top of each other. That is, the uncorrected and z-corrected angular resolution

Fig. 21. Minimum resolvable angular resolution data for all rooftop arrays using TIDY algorithm.

Fig. 22. Normalized performance of each array with DAS algorithm.

frontier curves are nearly identical. This means that the z-axis correction did not significantly impact the angular resolution performance of the arrays relative to the Rayleigh criterion. This is logical, as the signal wavelengths in this region are on the order of 10–35 cm while the average z errors are on the order of 1–3 cm.

The effect of z-axis correction on the upper frequency limit of the array will be analyzed next. The upper frequency limit is the maximum frequency that the beamforming system can achieve before grating lobes dominate the beamform map. In contrast to the low-frequency resolution, the upper frequency limit of the array does not depend on angular separation of the sources. This can be seen in Figs. 14 and 15, where the bottom edge of the green region is a roughly horizontal line segment. Because it does not depend on angular separation of the sources, an average can be taken across all angular separation values for each array and algorithm. By averaging the maximum frequency achieved by each array and each algorithm with both uncorrected and z-corrected geometries, the effect of z correction on the upper frequency limit can be determined. The average upper frequency limits are shown in Fig. 18 (DAS) and Fig. 19 (TIDY).

By inspection of Figs. 18 and 19 it is apparent that z-coordinate

Fig. 23. Normalized performance of each array with TIDY algorithm.

Fig. 24. Normalized performance of each array with both time-domain algorithms.

correction has a significant impact on the upper frequency limit. In every case, z correction improved the upper frequency limit of the array system. The effect was most pronounced with the 5× array. This is likely because the smaller inter-element spacing of the 5× array meant that it had greater potential to operate at higher frequencies, but it could not do so without precise microphone placement geometry. Even the 15× array showed significant improvement, although it showed less improvement than the 5× and 10× arrays. This is likely because the larger inter-element spacing of the 15× array means that it is not capable of operating at very high frequencies even with very precise microphone placement.

3.4. Time-domain algorithm results and discussion (DAS, TIDY)

With the corrected time-domain results given above, a summary of each array's angular resolution performance can be seen by plotting the minimum consistently resolvable angular separation for each array at each frequency band. This type of plot provides a simple visualization of angular resolution performance of each array and algorithm. Fig. 20 shows the minimum consistently resolvable angular separation for each array using the DAS algorithm, and Fig. 21 shows the minimum consistently resolvable angular separation for each array using the TIDY algorithm.

Several observations can be made from inspection of Figs. 20 and 21. First, the minimum resolvable separation almost always decreases when the array size increases, as expected. Second, the arrays clearly have different operating bands. Thus, a different array may be selected for an application depending on the frequency of the expected signal. Finally, comparison of the DAS and TIDY plots shows that the TIDY algorithm provides superior angular resolution. For a given array, the angular resolution curve in the TIDY plot is generally shifted down (to smaller angular separation values) and/or shifted left (to lower frequencies) as compared to the DAS plot.

Although Figs. 20 and 21 provide a simple visualization of the operating curves for each array and each algorithm, they are not as useful for comparing the performance of one array to another. To compare the performance of one array to another, the performance can be normalized in reference to a common standard: the Rayleigh criterion. For the

Fig. 25. Average maximum frequency for z-corrected geometry with all arrays using DAS algorithm.

Fig. 26. Average maximum frequency for z-corrected geometry with all arrays using TIDY algorithm.

Fig. 27. Z-corrected results with DAMAS algorithm: (a) 0.8×, (b) 1×, (c) 5×, (d) 10×, (e) 15× arrays.

Fig. 28. Z-corrected results with DAMAS2 algorithm: (a) 0.8×, (b) 1×, (c) 5×, (d) 10×, (e) 15× arrays.

Fig. 29. Z-corrected results with CLEAN-SC algorithm: (a) 0.8×, (b) 1×, (c) 5×, (d) 10×, (e) 15× arrays.

purpose of comparison between arrays, the angular separation required for resolution in each case can be normalized by the angular separation suggested by the Rayleigh criterion in each case. The normalization takes the form of:

$$\text{Normalized Performance}(f, D, \text{Algorithm}) = \frac{\text{Angular separation required for resolution}}{\text{Angular separation suggested by Rayleigh criterion}} \quad (3)$$

A system that greatly exceeds the performance of the Rayleigh criterion will have an angular separation required for resolution that is smaller than the separation angle suggested by the Rayleigh criterion. Thus, its normalized performance will be less than one. A system that does not meet the Rayleigh criterion will have a required angular separation for resolution that is greater than the separation angle suggested by the Rayleigh criterion, and will therefore have a normalized performance that is greater than one. This metric is calculated for each array and each algorithm at each frequency band. For clarity, the results are first plotted independently for the DAS and TIDY algorithms. The normalized performance with the DAS algorithm is shown in Fig. 22, and the normalized performance with the TIDY algorithm is shown in Fig. 23. Finally, the results are combined to facilitate comparison between the algorithms. The full time-domain algorithm results plot is shown in Fig. 24. For brevity, the results plots are grouped by algorithm rather than array size.

The lower curves in Fig. 22 through Fig. 24 represent the best angular resolution performance relative to the Rayleigh criterion. Figs. 22 and 23 show that the best normalized performance is achieved by the smaller arrays, as noted in the previous section. This effect may also be observed from Fig. 24. Fig. 24 also shows that for any given array size, the TIDY algorithm tends to outperform DAS. This is logical, because the DAS algorithm does not make any attempt to separate the acoustic source map from the array's point spread function. In contrast, the TIDY algorithm begins with the DAS algorithm and works with the beamform map initially obtained with DAS, iteratively subtracting coherent energy (sidelobes) from the dirty map and building up a clean map. Thus, the TIDY algorithm should produce cleaner beamform maps with increased clarity and performance. This is very similar to the way the CLEAN-SC algorithm works in the frequency domain by beginning with the conventional frequency-domain beamforming algorithm. The lower curves in Fig. 24 (representing the best performance relative to the Rayleigh criterion) are thus dominated by the smaller arrays and the TIDY algorithm, while the upper curves generally represent the larger arrays and the DAS algorithm.

Finally, the upper frequency operating limit may be plotted for all rooftop arrays with the DAS and TIDY algorithms. The TIDY algorithm generally shows improved upper operating frequency limits over the DAS algorithm because the TIDY algorithm separates the acoustic source map from the point spread function. In contrast, the DAS beamform map includes the effects of the point spread function (see Figs. 25 and 26).

3.5. Frequency-domain algorithm results and discussion (DAMAS, DAMAS2, CLEAN-SC)

The frequency-domain algorithms DAMAS, DAMAS2, and CLEAN-SC were used to process the same rooftop data that was used by the time-domain algorithms discussed in the previous section. The corrected z-axis microphone locations were used for all frequency-domain analysis. As before, beamform maps were produced for each array, each one-third octave frequency band, and each algorithm. The results are given in Fig. 27 (DAMAS), Fig. 28 (DAMAS2), and Fig. 29 (CLEAN-SC).

All three algorithms show relatively good performance with the smaller 0.8×, 1×, and 5× arrays. However, like the DAS and TIDY algorithms, the frequency-domain algorithms have difficulty in meeting the Rayleigh criterion with the larger 10× and 15× arrays.

From Fig. 27 it can be observed that the DAMAS algorithm shows particularly outstanding low-frequency performance over most of the arrays. In fact, DAMAS offers the best low-frequency performance of any algorithm. This is likely because the DAMAS algorithm numerically deconvolutes the acoustic source map from the point spread function without making significant simplifying assumptions regarding coherent energy sources (sidelobes) like the TIDY and CLEAN-SC algorithms, nor does it assume a simplified shift-invariant point spread function as in the DAMAS2 algorithm.

Comparison of Figs. 28 and 29 does not readily reveal large differences in the operating envelopes between the DAMAS2 and CLEAN-SC algorithms. Both appear to offer better high-frequency performance than the DAMAS algorithm. This is not surprising from the CLEAN-SC

Fig. 30. Minimum resolvable angular resolution data for all rooftop arrays using DAMAS algorithm.

Fig. 31. Minimum resolvable angular resolution data for all rooftop arrays using DAMAS2 algorithm.

algorithm, because it was designed precisely to eliminate coherent energy (sidelobes) from the acoustic source map. This results in a higher upper frequency limit before the onset and dominance of sidelobes and grating lobes. For the DAMAS2 algorithm, the shift-invariant point spread function appears to accomplish a similar task of mitigating sidelobes and allowing the algorithm to operate at higher frequencies than the DAMAS algorithm before the onset and dominance of sidelobes. Neither algorithm can match the low-frequency performance of DAMAS, as the sidelobe mitigation properties of DAMAS2 and CLEAN-SC are not as relevant at low frequencies.

For more detailed comparison between the arrays and algorithms, the minimum resolvable angular separation – also previously referred to as the angular resolution frontier curve – is plotted for each array and each algorithm in Fig. 30 (DAMAS), Fig. 31 (DAMAS2), and Fig. 32 (CLEAN-SC).

A comparison of Fig. 30 through Fig. 32 shows that, as expected, the larger arrays generally provide greater angular resolution than the smaller arrays in absolute terms. In almost every case, the angular resolution frontier curve for the 1× array is shifted down and left (to lower source separation values and lower frequencies, respectively) from the 0.8× curve. Similarly, the 5× array curve is below and to the left of the 1× curve, the 10× curve is below and to the left of the 5×

Fig. 32. Minimum resolvable angular resolution data for all rooftop arrays using CLEAN-SC algorithm.

Fig. 33. Normalized performance of each array with DAMAS algorithm.

curve, and the 15× curve is below and to the left of the 10× curve. The 10× and 15× resolution frontier curves are generally shorter in arc length because the frequency spans of the resolution frontier curve for these arrays are narrower, as can be seen in Fig. 27 through Fig. 29 by the comparatively flat resolution frontier curves. These arrays are capable of operating over a wider band and at lower frequencies, but the resolution in these regions is inconsistent. This can be seen by the large yellow areas in Fig. 27 through Fig. 29.

A close inspection of Fig. 30 through Fig. 32 also reveals that the DAMAS algorithm typically provides superior angular resolution for a given array and frequency band. In addition, it appears that CLEAN-SC often provides superior angular resolution to the DAMAS2 algorithm.

More information may be gained by calculating and plotting the normalized angular resolution for each array, algorithm, and frequency band. For brevity, the normalized angular resolution plots are grouped by algorithm rather than by array. The normalized angular resolution plots grouped by algorithm are shown in Fig. 33 (DAMAS), Fig. 34 (DAMAS2), and Fig. 35 (CLEAN-SC). The plots are combined in Fig. 36 to show the normalized resolution performance of all frequency-domain arrays and algorithms. The lower curves in Fig. 33 through Fig. 35 represent better normalized resolution performance relative to the Rayleigh criterion, while upper curves represent lower performance relative to the Rayleigh criterion.

Although larger arrays offer superior angular resolution in absolute

Fig. 34. Normalized performance of each array with DAMAS2 algorithm.

Fig. 35. Normalized performance of each array with CLEAN-SC algorithm.

terms (shown in Fig. 30 through Fig. 32), inspection of Fig. 33 through Fig. 35 shows that the smaller arrays offer increased normalized performance relative to the Rayleigh criterion. As discussed earlier, this is likely due at least in part to boundary reflection effects. Fig. 36 suggests that the DAMAS2 algorithm may be particularly susceptible to array scaling effects, because the DAMAS2 performance appears to suffer the most significantly as the array size increases. This may be due to the shift-invariant point spread function assumed by DAMAS2.

Inspection of Fig. 36 shows that the best angular resolution performance is usually achieved with the DAMAS algorithm. This is because the DAMAS algorithm makes the least significant computational compromises in the form of simplifying assumptions, as compared to the DAMAS2 and CLEAN-SC algorithms. The CLEAN-SC algorithm typically offers angular resolution performance that approaches – and in a few isolated cases exceeds – the DAMAS algorithm. The DAMAS2 algorithm shows angular resolution that approaches the other algorithms with smaller arrays, but the resolution performance drops off considerably with larger arrays.

While Fig. 33 through Fig. 36 demonstrate the low-frequency performance of each array and algorithm, the high-frequency performance of each algorithm is also of interest. A broad operating envelope is desirable so that the system can operate on a wide range of applications. The maximum consistently resolvable frequency band was averaged over the tests and is plotted in Fig. 37 (DAMAS), Fig. 38

Fig. 36. Normalized performance of each array with all frequency-domain algorithms.

Fig. 37. Average maximum frequency for z-corrected geometry with all arrays using DAMAS algorithm.

Fig. 38. Average maximum frequency for z-corrected geometry with all arrays using DAMAS2 algorithm.

(DAMAS2), and Fig. 39 (CLEAN-SC).

As shown in Fig. 37 through Fig. 39, the relative performance of each algorithm is nearly flipped when considering the high frequency performance. The DAMAS algorithm typically fares worst, because it does not make use of any assumptions that would serve to mitigate sidelobes or grating lobes and thereby increase the upper range of its operating envelope. The DAMAS algorithm shows the lowest high-frequency limit of any of the time-domain or frequency-domain algorithms.

The DAMAS2 and CLEAN-SC algorithms both offer improved high-frequency performance compared to the DAMAS algorithm. This is because both methods make assumptions about the acoustic source map that mitigate sidelobes and grating lobes. For the CLEAN-SC algorithm, the energy that is coherent with the highest sources is explicitly removed from the dirty map and only the highest sources themselves are added to the new clean map. For the DAMAS2 algorithm, a shift-invariant point spread function is utilized. Both methods mitigate sidelobes and grating lobes, thereby extending the useful operating range of the algorithm to higher frequencies.

3.6. Angular resolution comparison: all array sizes and algorithms

For the purpose of clarity the preceding sections have been divided into time-domain and frequency-domain algorithms. However, there are many situations in which both a time-domain and a frequency-domain algorithm could be applicable, and the user may need to choose between them. It is therefore convenient to compare all algorithms side-by-side. To avoid plots with excessive amounts of data, the most convenient method to compare the algorithms is to group the algorithms by array size, and compare all algorithms for a given array size. These

Fig. 39. Average maximum frequency for z-corrected geometry with all arrays using CLEAN-SC algorithm.

combined angular resolution results are given in Fig. 40.

The plots contained in Fig. 40 show the normalized relative performance of the DAS, TIDY, DAMAS, DAMAS2, and CLEAN-SC algorithms for each of the five arrays. Better performance relative to the Rayleigh criterion is represented by the lower curves in the plots, while decreased performance relative to the Rayleigh criterion is represented by the upper curves in the plots. The poorest performance in each plot is shown by the DAS algorithm. This is to be expected, because the DAS algorithm makes no attempt to separate the true acoustic source map from the point spread function of the source.

The best angular resolution performance in almost every case is achieved by the DAMAS algorithm. This is because the DAMAS algorithm makes the least significant computational simplifications or assumptions. Rather, the DAMAS algorithm uses an iterative solver to computationally deconvolute the point spread function from the true acoustic source map. The acoustic source map so calculated is able to resolve adjacent acoustic sources with separation angles smaller than any other algorithm described in this analysis. However, the costs associated with this performance are (i) computation time, and (ii) decreased high-frequency performance.

The remaining algorithms – TIDY, DAMAS2, and CLEAN-SC – achieve intermediate angular resolution performance. Each of these algorithms makes significant simplifying assumptions: the DAMAS2 algorithm assumes a shift-invariant point spread function, while both the TIDY and CLEAN-SC algorithms build up a “clean” beamform map by iteratively removing the peak sources and their coherent energy from the dirty map. In most cases, it appears that the CLEAN-SC algorithm is able to achieve the best resolution among these three algorithms. The performance offered by the TIDY and DAMAS2 algorithms is similar in many cases. However, the performance of the TIDY algorithm appears to surpass the performance of the DAMAS2 algorithm as the size of the array increases. Because the relative performance of the DAMAS2 algorithm drops with each successive increase in array size, this effect is likely due to array scaling and the increasingly poor assumption of a shift-invariant point spread function. With increasing array size, the point spread function is the result of microphone geometry that is spread across a physically larger area compared to the scanning grid, and therefore the shift-invariant point spread function is no longer a good approximation. As a result, the DAMAS2 algorithm is the most negatively impacted by microphone array scaling.

4. Conclusions

A series of experiments was conducted to determine the operating characteristics of five scaled microphone arrays with diameters ranging from 0.59 m to 10.98 m. It is apparent that each array has a different

Fig. 40. Normalized performance of each rooftop array with both time-domain and frequency-domain algorithms: (a) 0.8×, (b) 1×, (c) 5×, (d) 10×, and (e) 15× arrays.

operating range in terms of frequency and angular resolution, as expected based on the Rayleigh criterion. This reinforces the fact that the important performance characteristics of an array must be specified at an early stage of development so that the important parameters of the array can be designed accordingly. For improved angular resolution, the array should have a large diameter and/or it should operate at high frequencies. However, to increase the upper frequency limit of the useful operating range, the inter-element spacing should be kept small compared to the wavelength of interest.

The results of the experiments show the angular resolution of each array using the DAS, TIDY, DAMAS, DAMAS2, and CLEAN-SC beamforming algorithms compared against the Rayleigh criterion curve. The DAS algorithm exhibits the lowest angular resolution because it makes no attempt to remove the point spread function from the acoustic source map. In contrast, the DAMAS algorithm shows the greatest angular resolution performance because it numerically deconvolutes the point spread function from the acoustic source map, providing high resolution acoustic source maps that significantly exceed the Rayleigh criterion. The TIDY, DAMAS2, and CLEAN-SC algorithms show intermediate performance because they each make assumptions or simplifications that improve the angular resolution relative to the DAS algorithm. In most cases, the CLEAN-SC algorithm shows angular resolution that exceeds the angular resolution of the TIDY and DAMAS2 algorithms. The TIDY and DAMAS2 algorithms show similar performance over much of their operating ranges, but the DAMAS2 algorithm appears to be the most negatively impacted by the scaling of the array geometry to larger sizes.

Future work will be focused on the issue of boundary reflection as it pertains to microphone arrays, and how it may impact the performance of the arrays. An investigation will be performed in a controlled laboratory environment to explore the effect of perimeter boundary reflection on the angular resolution performance of microphone arrays.

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