Matlab: diving deeper

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Plots

- $\begin{cases} e^x & -3 < x < 0 \\ cos(x) & 0 \le x < 3 \end{cases}$

- $x = e^{-t/10} * sin(5t)$
  $y = e^{-t/10} * cos(5t)$
  $z = t$

Get R square from polyval

$$R^2 \equiv 1 - \frac{SS_{res}}{SS_{tot}}$$

Solving ODEs

- ODE45: standard solver
- Runge Kutta method

General problem:

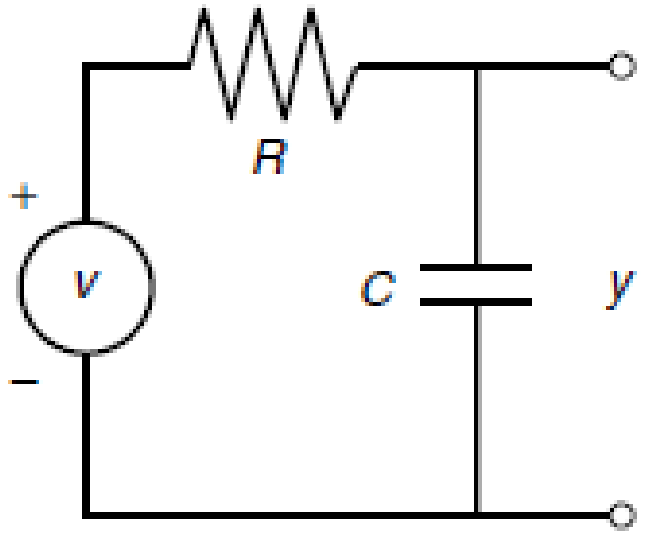
$$\frac{d\boldsymbol{x}}{dt} = f(t, \boldsymbol{x})$$

$$x(\boldsymbol{t_o}) = \boldsymbol{x_o}$$

STEPS:

- **Unknowns (what's $\boldsymbol{x}$)**
- **Input file (what's $\frac{d\boldsymbol{x}}{dt}$)**
- **Solution**

Solving ODEs



$$0.1\dot{y} + y = 0$$
$$y(0) = 2$$

- **Unknowns:**

$$x(1) = y$$

- **Input file:**

$$\frac{dx}{dt}(1) = -10 * y$$

- **Solution:**

$$\mathbf{x} = \begin{bmatrix} y|_{t=0} \\ y|_{t=1} \\ \\ \\ \\ y|_{t=10} \end{bmatrix}$$

Solving ODEs

$$m\ddot{y} + \dot{y}e^{y} - y^{2} = 5, \qquad y_0 = 3, \dot{y}_0 = -1,$$

- **Unknowns:**

$$\begin{aligned} x(1) &= y \\ x(2) &= \dot{y}, \end{aligned}$$

- **Input file:**

$$\begin{aligned} \frac{d}{dt}x(1) &= x(2) \\ \frac{d}{dt}x(2) &= \frac{1}{m}\left(5 - x(2)e^{x(1)} + (x(1))^2\right) \end{aligned}$$

- **Solution:**

$$\mathbf{x} = \begin{bmatrix} y\,|_{t=0} & \dot{y}\,|_{t=0} \\ y\,|_{t=1} & \dot{y}\,|_{t=1} \\ & \\ & \\ & \\ y\,|_{t=10} & \dot{y}\,|_{t=10} \end{bmatrix}$$

Solving ODEs

$$m\ddot{y} + \dot{y}e^{y} - y^{2} = 5, \qquad y_0 = 3, \dot{y}_0 = -1,$$

$$\frac{d^3z}{dt^3} + \frac{d^2z}{dt^2} - \sin(z) = t, \qquad z_0 = 0, \dot{z}_0 = 0, \ddot{z}_0 = 1.$$

- **Unknowns:**

$$\mathbf{x} = [y, \dot{y}, z, \dot{z}, \ddot{z}]$$

- **Input file:**

$$\begin{aligned}
\frac{d}{dt}x(1) &= x(2) \\
\frac{d}{dt}x(2) &= \frac{1}{m}\left(5 - x(2)e^{x(1)} + (x(1))^2\right), \\
\frac{d}{dt}x(3) &= x(4) \\
\frac{d}{dt}x(4) &= x(5) \\
\frac{d}{dt}x(5) &= t - x(5) + \sin\left(x(3)\right),
\end{aligned}$$

Solving ODEs

- **Solution:**

$$\mathbf{x} = \begin{bmatrix} y\,|_{t=0} & \dot{y}\,|_{t=0} & z\,|_{t=0} & \dot{z}\,|_{t=0} & \ddot{z}\,|_{t=0} \\ y\,|_{t=1} & \dot{y}\,|_{t=1} & z\,|_{t=1} & \dot{z}\,|_{t=1} & \ddot{z}\,|_{t=1} \\ & & \cdots & & \\ & & \cdots & & \\ & & \cdots & & \\ y\,|_{t=10} & \dot{y}\,|_{t=10} & z\,|_{t=10} & \dot{z}\,|_{t=10} & \ddot{z}\,|_{t=10} \end{bmatrix}.$$

Symbolic Calculus

Table 8.3–1 Symbolic calculus functions

Command	Description
`diff(E,v,n)`	Returns the *n*th derivative of the expression `E` with respect to the variable `v`. Both `v` and `n` are optional.
`int(E,v,a,b)`	Returns the integral of the expression `E` with respect to the optional variable `v` over the interval $[a, b]$.
`limit(E,v,a)`	Returns the limit of the expression `E` as the variable `v` goes to `a`. Both `v` and `a` are optional. If `a` is omitted, the limit is taken as `v` goes to 0.
`limit(E,v,a,'d')`	Returns the limit of the expression `E` as the variable `v` goes to `a` from the direction specified by `d`, which may be `right` or `left`.
`symsum(E)`	Returns the symbolic summation of the expression `E`.
`taylor(f,n,a)`	Gives the first `n-1` terms in the Taylor series for the function defined in the expression `f`, evaluated at the point $x = a$. If the parameter `a` is omitted, the function returns the series evaluated at $x = 0$.

A Concise introduction to Matlab (Palm, 2008).