HOMEWORK 1

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Atmospheric Flight Control

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2. Given linearized equations of motion for lateral motion $\dot{x} = Ax + Bu + Gd$

(a)

$$\begin{pmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{pmatrix} = \begin{pmatrix} -0.5 & 0 & -1 & 0.02 \\ -150 & -7 & -0.15 & 0 \\ 30 & 0.1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix} + \begin{pmatrix} -0.02 \\ 56 \\ 1 \\ 0 \end{pmatrix} \delta\delta_a + \begin{pmatrix} 0.5 \\ 150 \\ -30 \\ 0 \end{pmatrix} v_g.$$

$u_0 = 272$ m/s, $L_v = 580$ m, $\alpha_v = 10$ m/s.

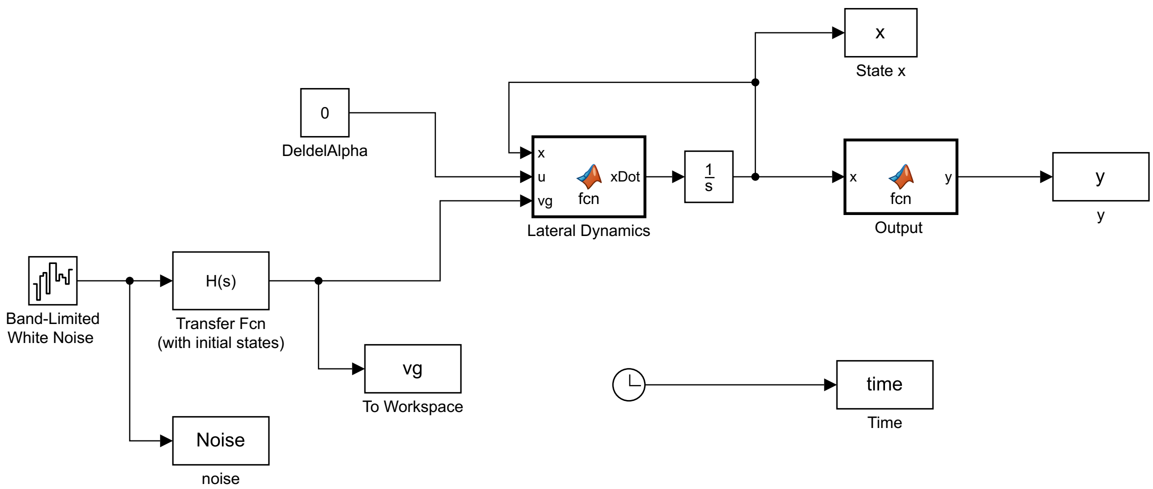

Figure 1: Simulink Block Diagram of lateral gust rejection model

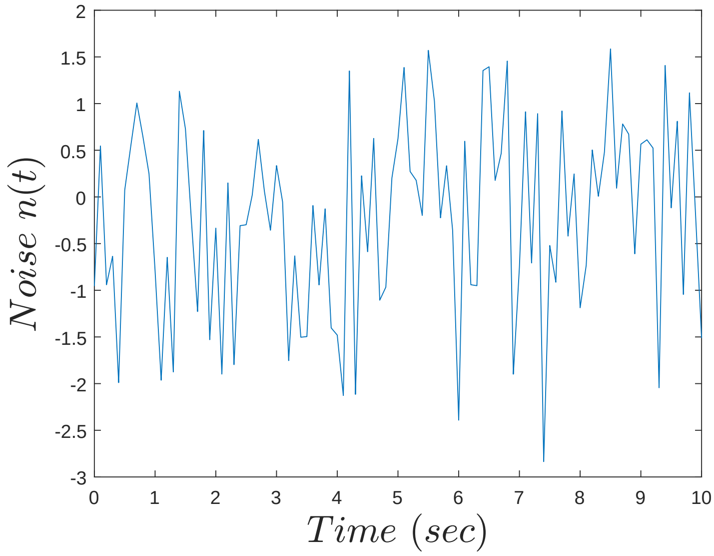

Figure 2: Plot of noise history

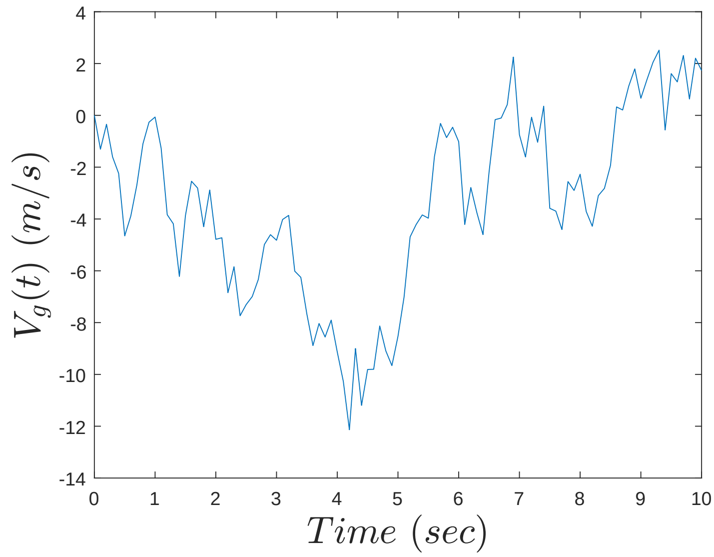

Figure 3: Plot of velocity trajectories.

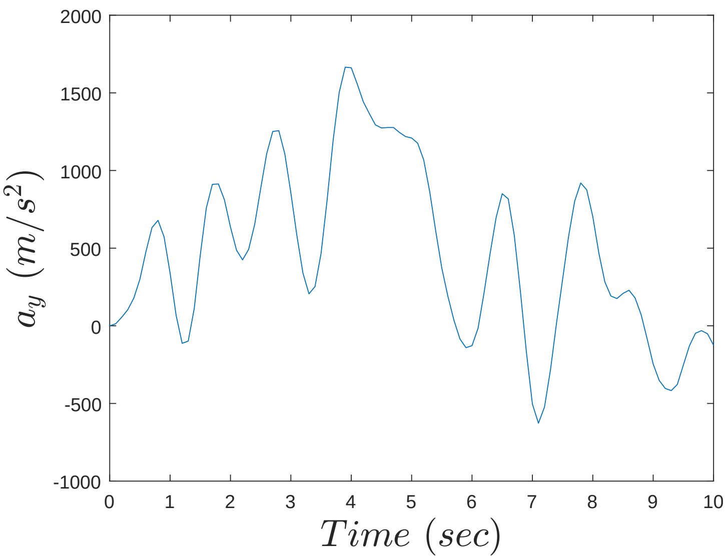

Figure 4: Plot of noise acceleration trajectories.

(b) Converting from Transfer function to state space for solving the Lyapunov Equations

$$\begin{aligned} H_v(s) = \frac{V_g(s)}{n(s)} &= \frac{22.5s^2 + 85.56s + 14.6}{1.492s^3 + 8.982s^2 + 6.388s + 1} \\ \frac{V_g(s}{n(s0} &= \frac{15.120s^2 + 57.346s + 9.786}{s^3 + 6.020s^2 + 4.281s + 0.9702} \end{aligned} \tag{1}$$

Taking the inverse Laplace transform of (1) with zero initial conditions, we get

$$\dddot{v}_g + 6.02\ddot{v}_g + 4.281\dot{v}_g + 0.9702v_g = 15.120\ddot{n} + 57.3246\dot{n} + 9.786n$$

$$\dddot{v}_g + 6.02\ddot{v}_g - 15.120\ddot{n} - 57.3246\dot{n} = 9.786n - 0.9702v_g$$

$$\text{Lets assume, } \dot{x}_3 = \dddot{v}_g + 6.02\ddot{v}_g - 15.120\ddot{n} - 57.3246\dot{n}$$

$$\dot{x}_3 = 9.786n - 0.6702v_g = \dddot{v}_g + 6.02\ddot{v}_g - 15.120\ddot{n} - 57.3246\dot{n} \tag{2}$$

From (2),

$$x_3 = \ddot{v}_g + 6.20\dot{v}_g + 4.281v_g - 15.120\dot{n} - 57.346n \tag{3}$$

$$x_2 = \dot{v}_g + 6.20v_g - 15.120n \tag{4}$$

$$x_1 = v_g \tag{5}$$

Now from (2), (3), (4) and (5),

$$\dot{x}_3 = 9.786n - 0.6702v_g = -0.6702x_1 + 9.786n \tag{6}$$

$$\dot{x}_2 = \ddot{v}_g + 6.20\dot{v}_g - 15.120\dot{n} = x_3 - 4.2181x_1 + 57.346n \tag{7}$$

$$\dot{x}_1 = \dot{v}_g = x_2 - 6.20v_g + 15.120n = x_2 - 6.20x_1 + 15.120n \tag{8}$$

$$y = v_g = x_1 \tag{9}$$

Now writing in state space form we get,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -6.20 & 1 & 0 \\ -4.218 & 0 & 1 \\ -0.6702 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 15.120 \\ 57.346 \\ 9.786 \end{bmatrix} n \tag{10}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{11}$$

which in the form of $\dot{x} = A_g x + B_g n$ and $y = C_g x$ where x be the 3 states for disturbance model. It can be observed that this system is Controllable Canonical form. Using the above matrices and using the *Lyap* function in MATLAB, we get the RMS value of lateral acceleration of $\sqrt{E\{a_y^2\}} = 1.5604 \times 10^3 = 1560.4 \quad m/s^2$.

3. Pitch/alpha Stability Augmentation System (SAS)

Given:

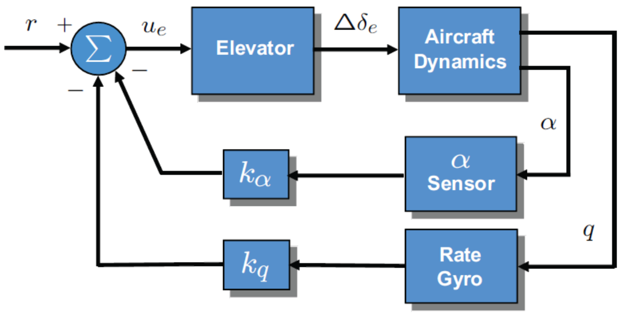

Figure 5: Pitch Alpha SAS

(a) Characteristics equation for the aircraft motion:

$$0.3739S^4 + 1.3343s^3 + 0.5280s_0^2.0381s - 0.0235$$

roots of the characteristics equation:

-3.1299 + 0.0000i

-0.2964 + 0.2061i

-0.2964 - 0.2061i

0.1541 + 0.0000i

It can be seen that one of the pole is in Right Half Plane, so that aircraft is not stable.

(b) Inner Loop Design

$$\frac{\Delta_\alpha}{\Delta\delta_e} = \frac{-(0.0602s^3 + 4.432s^2 + 0.2008s + 0.3103)}{0.3739S^4 + 1.3343s^3 + 0.5280s_0^2.0381s - 0.0235} \tag{12}$$

We made a cascaded system as below and

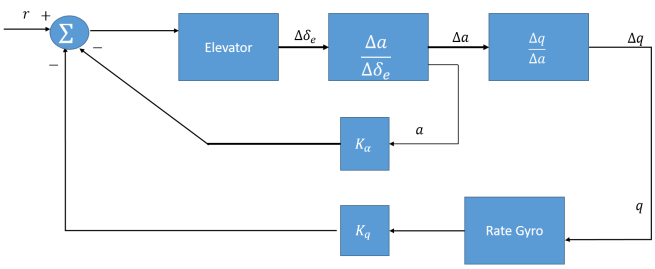

Figure 6: Pitch Alpha SAS

$$\frac{\Delta_q}{\Delta_\alpha} = \frac{\Delta_\alpha}{\Delta\delta_e} \times \frac{\Delta\delta_e}{\Delta_q} = \frac{4.464s^3 + 9.399s^2 + 0.477s}{0.0602s^3 + 4.433s^2 + 0.2008s + 0.3103} \tag{13}$$

Note that system $\frac{\Delta_q}{\Delta_\alpha}$ has 3 poles as :

-0.0222 + 0.2637i

-0.0222 - 0.2637i

-73.5869 + 0.0000i

which will stay the same throughout the design process and are different from aircraft equations of motion. Designing for just the innner loop we get, $K_\alpha = 0.3$

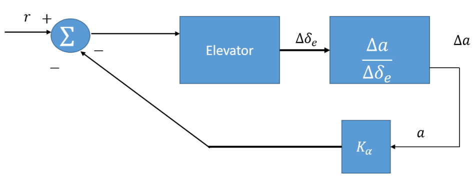

Figure 7: Pitch Alpha SAS

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-1.29e-02 + 1.96e-01i	6.58e-02	1.96e-01	7.75e+01
-1.29e-02 - 1.96e-01i	6.58e-02	1.96e-01	7.75e+01
-1.80e+00 + 1.27e+00i	8.16e-01	2.20e+00	5.57e-01
-1.80e+00 - 1.27e+00i	8.16e-01	2.20e+00	5.57e-01

Figure 8: Pitch Alpha SAS Inner Loop

Poles :

Phugoid: $\lambda_1 = -0.0129 \pm 0.1957i, \zeta = 0.0658, w_n = 0.1961$

Short Period: $\lambda_2 = -1.7955 \pm 1.2702i, \zeta = 0.8164, w_n = 2.1994$.

It matches the damping for short period $\zeta > 0.8$ To tunes the gains $K_\alpha = 0.3$, we use the information from root locus

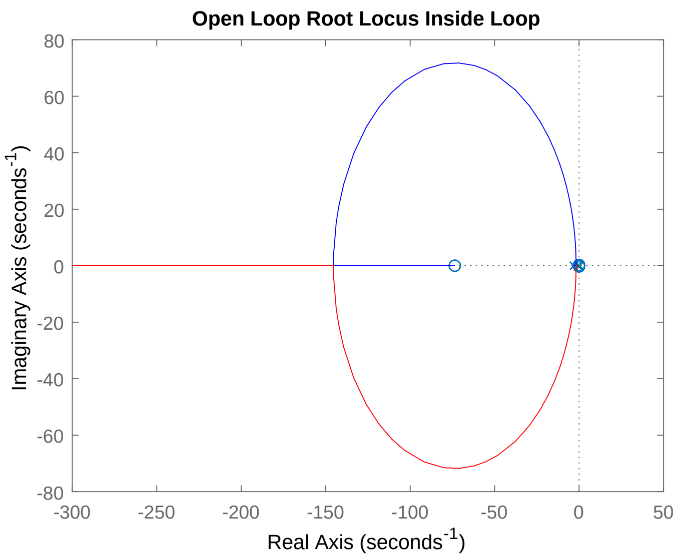

Figure 9: Open Loop Root Locus

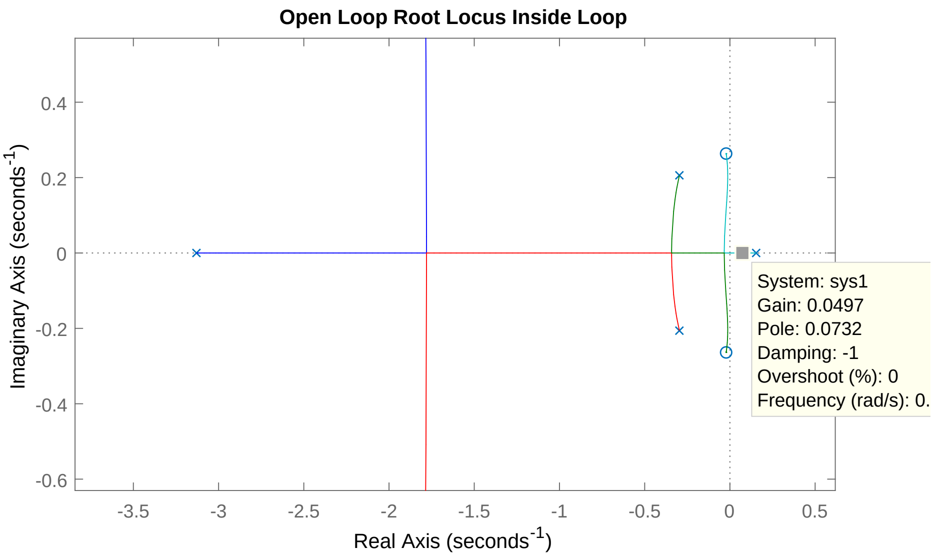

Figure 10: Open Loop Root Locus

So it can be seen that gain has to be greater than 0.07 to make the system stable. By tuning the gains to $K_\alpha = 0.3$ we also meet the other requirement $\zeta_{\lambda_2} > 0.8$

To close the outer loop we choose $K_q = 0.75$ Followings are the poles:

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-2.20e-02 + 1.22e-01i	1.78e-01	1.24e-01	4.54e+01
-2.20e-02 - 1.22e-01i	1.78e-01	1.24e-01	4.54e+01
-2.22e-02 + 2.64e-01i	8.38e-02	2.65e-01	4.51e+01
-2.22e-02 - 2.64e-01i	8.38e-02	2.65e-01	4.51e+01
-2.79e+00	1.00e+00	2.79e+00	3.58e-01
-4.36e+00	1.00e+00	4.36e+00	2.29e-01
-7.36e+01	1.00e+00	7.36e+01	1.36e-02

Figure 11: Pitch Alpha SAS Outer Loop

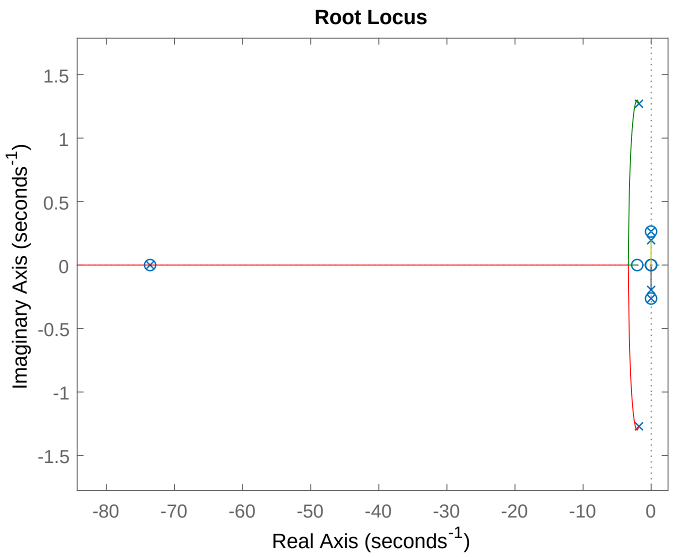

Figure 12: Open Loop Root Locus for Outside Loop

Phugoid:$-0.0220 \pm 0.1216i$, $\zeta = 0.1783$, $w_n = 0.1236$. It does satisfy the design requirements for Level 1 handling qualities for which $\zeta > 0.04$.
For the short period $\zeta = 1.0$, $w_n = 2.79, 4.36$

4. Pitch Attitude Control Augmentation

Vehicle Dynamics

$$G(s) = \frac{\Delta\theta}{\Delta\delta_c} = \frac{r}{Js^2 + bs + mgl} = \frac{0.25}{0.0475s^2 + 0.05s + 0.735} \tag{14}$$

Design specifications:

- Steady state error of less than 1%

  For low frequency approximation, we used the following,

$$|L(0)| > 100$$
$$20\log_{10} L(0) > 20\log_{10}(100)$$
$$|L(0)| > 40dB$$

- Tracking error of less than 5% from 0 to 0.16 Hz (1 rad/s)

$$|\frac{1}{L(jw)}| < 0.05$$
$$20\log_{10} L(jw) > 20\log_{10}(100)$$
$$|L(jw)| > 26dB$$

- Closed loop step response with a maximum overshoot of 20%

$$M_r < 20\%$$
$$\zeta = \left(\frac{ln^2(0.20)}{\pi^2 + ln^2(0.20)}\right)^{1/2}$$
$$\zeta = 0.456$$

  Open loop phase margin $\approx 100\zeta = 45.6°$

- Closed loop frequency response with no more than 3 dB gain at all frequencies

$$|T(jw)| < 3dB$$
$$|T(jw)| = 1.412$$
$$\zeta = \left(\frac{\frac{1}{2} + \sqrt{(1.412)^2 - 1}}{2(1.412)}\right)^{1/2} = 0.383 < \zeta < 0.924 = \text{ Closed loop damping required}$$

  open loop: $0.383 < \zeta < 0.924$

- Disturbance Rejection

$$|T(jw)| < 20\log_{10}(0.10) \text{ for } \omega > 100Hz$$
$$|T(jw)| < -20\ dB$$

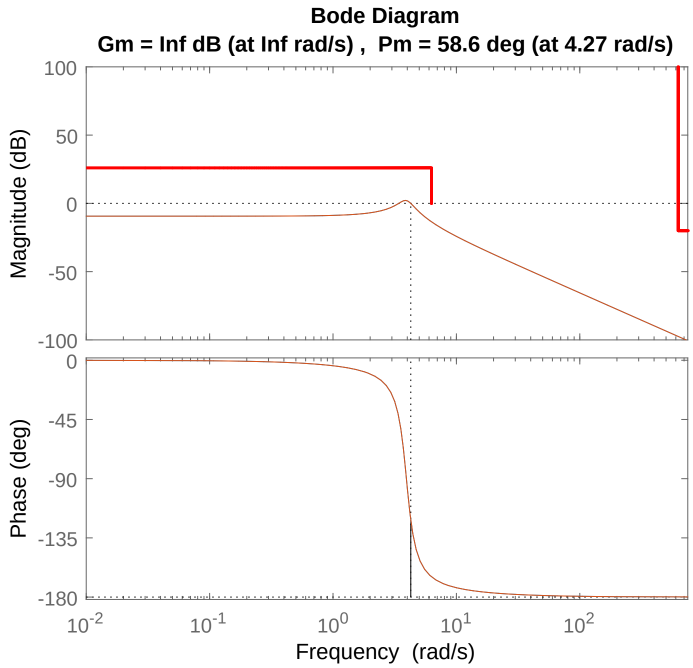

Figure 13: Bode plot of open loop transfer function without compensator.

Based on the above plot we designed a proportional gain to increase the magnitude over 40 dB line.

- **P Controller Design**

  We know that

$$
\begin{aligned}
|L(0)| &> 100 \\
\left|\frac{0.25K_P}{0.0475s^2+0.05s+0.735}\right|\Bigg|_{s=0} &> 100 \\
\frac{0.25K_P}{0.735} &> 100 \\
K_P &> 294
\end{aligned}
$$

When $K_P = 300$, we get the following plot

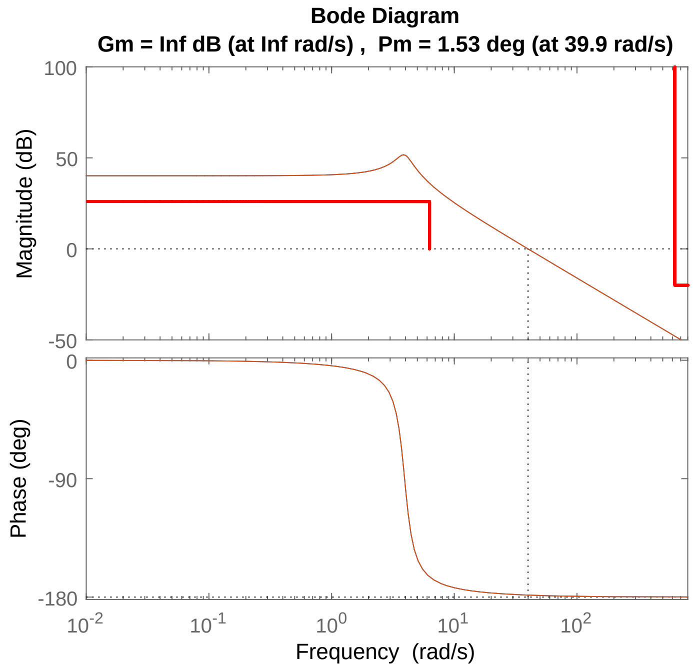

Figure 14: Bode plot of open loop transfer function with a proportional controller.

Since the phase margin is stilling lagging in order to address that, we now introduce a phase lead compensator. We set our gains $K_P = 500$.

- **Lead Compensator Design**

$$C_1(s) = \frac{K(\tau s + 1)}{\alpha \tau s + 1} \tag{15}$$

(a)

$$\Delta\Phi = PM_{des} - PM_{act} + 5^\circ \tag{16}$$

$$\Delta\Phi = 45.6 - 23.89 + 5 = 49.42^\circ \tag{17}$$

$$\alpha = \frac{1 - \sin\Delta\Phi}{1 + \sin\Delta\Phi} = 0.136 \tag{18}$$

$$\omega_c = 51.4 \quad rad/s \tag{19}$$

$$\tau = \frac{1}{w_c\sqrt{\alpha}} = 0.0527 \tag{20}$$

$$K = \frac{1}{|L(jw_c)|} \tag{21}$$

With the compensator the open loop transfer function looks like

$$L_1(s) = G(s)K_P = \frac{0.25K_P}{0.0475s^2 + 0.05s + 0.735} \tag{22}$$

From (21) and (22), we get

$$\begin{aligned}
K &= \frac{1}{|L_1(jw_c)|} \\
&= \frac{1}{\left|\frac{0.25K_P}{0.0475s^2+0.05s+0.735}\right|}\Bigg|_{j\omega_c=51.4j} \\
&= \frac{1}{\left|\frac{0.25\times500}{-0.0475(51.4^2)+0.05(51.4)j+0.735}\right|} \\
&= \left|\frac{-0.0475(51.4^2)+0.05(51.4)j+0.735}{0.25\times500}\right| \\
&= \left|\frac{-125.49+2.57j+0.735}{125}\right| \\
&= \left|\frac{-124.755+2.57j}{125}\right| \\
&= \left|\frac{124.78}{125}\right| \\
&= 0.9982 \approx 1.0
\end{aligned} \tag{23}$$

With the initial sets of gains calculated above we get the following plots

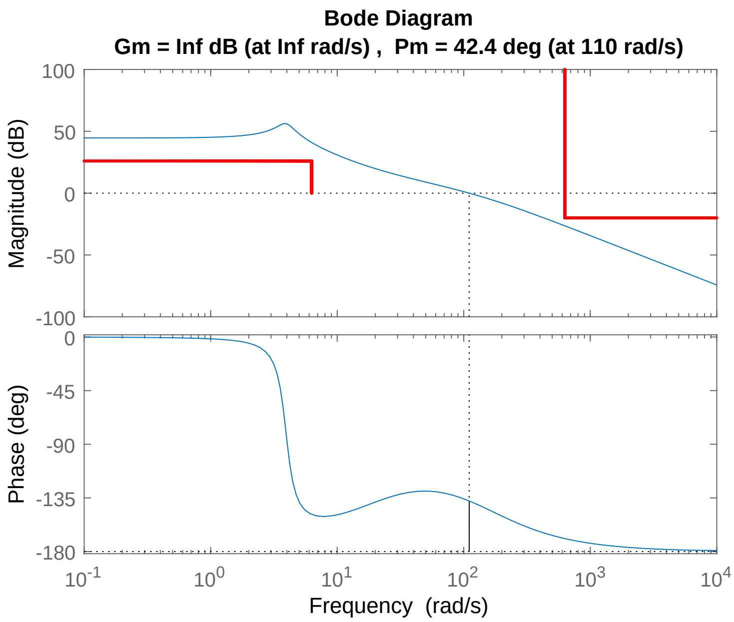

Figure 15: Bode plot of open loop transfer function with a proportional + lead compensator.

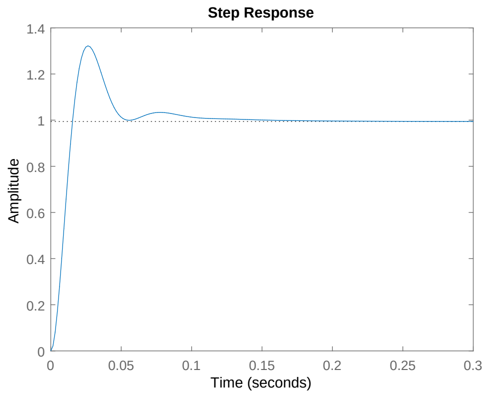

Figure 16: Bode plot of Closed loop transfer function with a proportional + lead compensator.

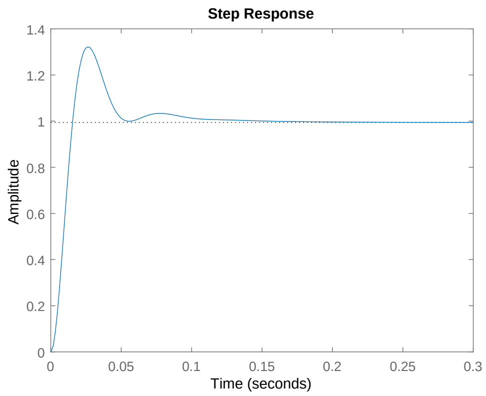

Figure 17: Step response of Closed loop transfer function with a proportional + lead compensator.

RiseTime: 0.0105

SettlingTime: 0.0987

SettlingMin: 0.8964

SettlingMax: 1.3221

Overshoot: 32.9832

Undershoot: 0

Peak: 1.3221

PeakTime: 0.0264

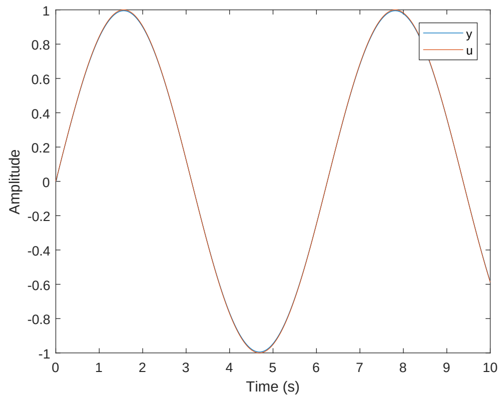

Figure 18: Tracking response of Closed loop transfer function with a proportional + lead compensator.

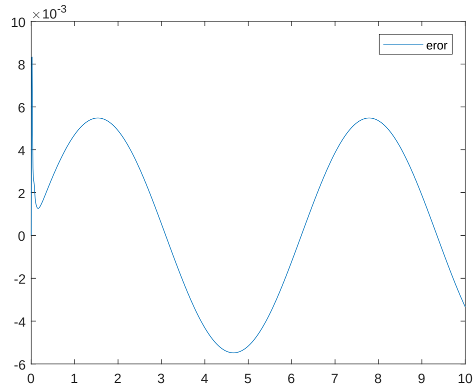

Figure 19: Error of Closed loop transfer function with a proportional + lead compensator.

Now we tune our gains to match all the constraint including overshoot and phase margin. Following is final gains used for both controllers with the plots.

$$\textbf{P Controller} \implies K_{PI} = 500,$$
$$\textbf{Lead Compensator} \implies \alpha = 0.07, \quad K_{lead} = 0.65, \quad \tau_{lead} = 0.0527$$

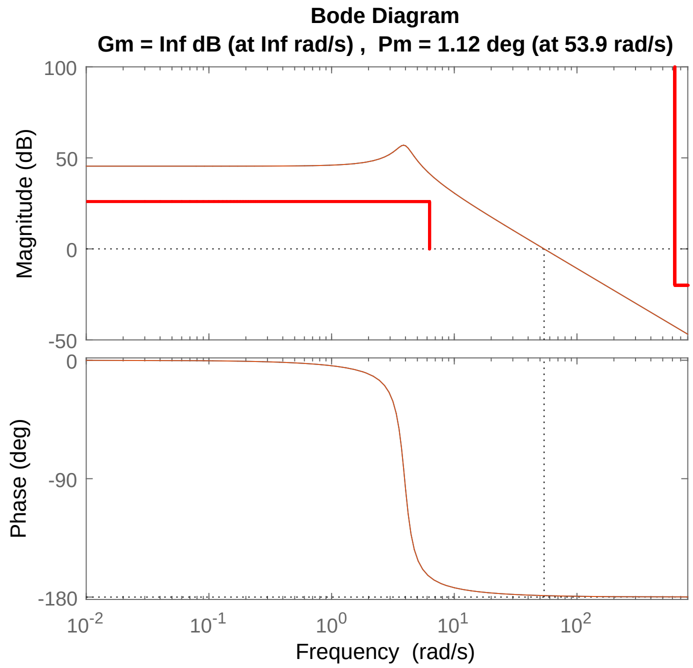

Figure 20: Bode plot of open loop transfer function with a proportional controller $K_P = 550$.

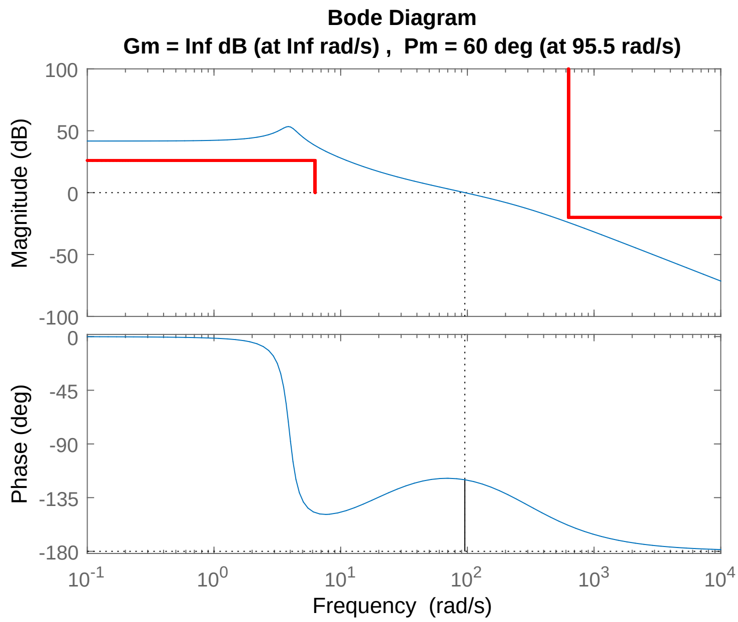

Figure 21: Final bode plot of open loop transfer function with a proportional + lead compensator.

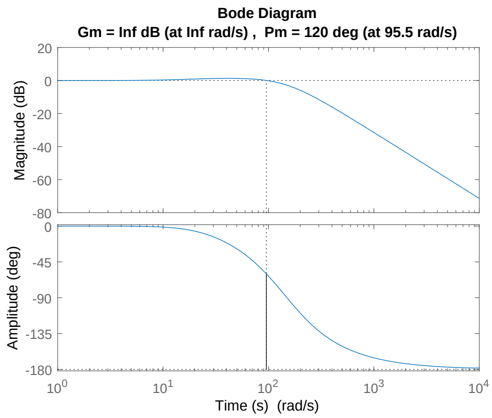

Figure 22: Final bode plot of Closed loop transfer function with a proportional + lead compensator.

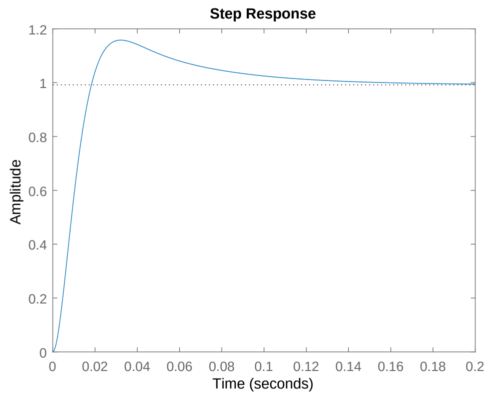

Figure 23: Step response of Closed loop transfer function with a proportional + lead compensator.

RiseTime: 0.0125

SettlingTime: 0.1209

SettlingMin: 0.9235

SettlingMax: 1.1580

Overshoot: 16.7547

Undershoot: 0

Peak: 1.1580

PeakTime: 0.0320

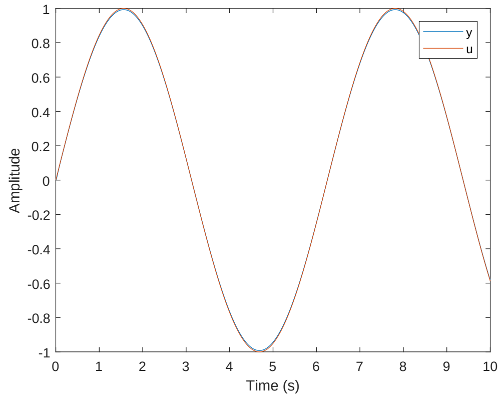

Figure 24: Tracking response of Closed loop transfer function with a proportional + lead compensator.

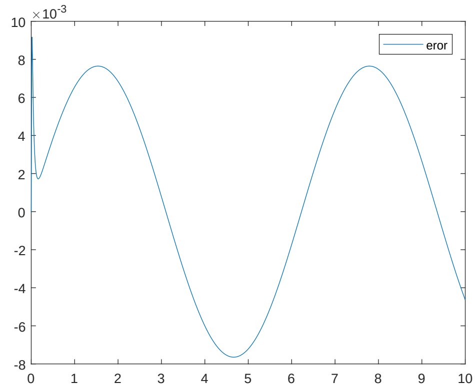

Figure 25: Error of Closed loop transfer function with a proportional + lead compensator.

5. Problem 5: Velocity Hold Control System Design

(a) Given:

Throttle Servo Dynamics:

$$G_{\text{servo}}(s) = \frac{1}{0.1s+1} \tag{24}$$

Engine Dynamics:

$$G_{\text{engine}}(s) = \frac{10}{10s+1} \tag{25}$$

Aircraft Dynamics:

$$G_{\text{A/C}}(s) = \frac{\Delta u}{\Delta \delta_T} = \frac{0.038s}{s^2+0.039s+0.053} \tag{26}$$

Sensor Dynamics:

$$S_{\text{sen}}(s) = 10s+1 \tag{27}$$

Open Loop Transfer function

$$L(s) = C_s(s)G_{\text{servo}}(s)G_{\text{engine}}(s)G_{\text{A/C}}(s)S_{\text{sen}}(s) \tag{28}$$

Design specifications:

- Steady state error of less than 5%
  For low frequency approximation, we used the following,

$$|L(0)| > 100$$
$$20\log_{10} L(0) > 20\log_{10}(100)$$
$$|L(0)| > 26dB$$

- Tracking error of less than 5% from 0 to 0.16 Hz (1 rad/s)

$$|\frac{1}{L(jw)}| < 0.05$$
$$20\log_{10} L(jw) > 20\log_{10}(20)$$
$$|L(jw)| > 26dB$$

- Closed loop step response with a maximum overshoot of 25%

$$M_r < 25\%$$
$$\zeta = \left(\frac{ln^2(0.25)}{\pi^2 + ln^2(0.25)}\right)^{1/2}$$
$$\zeta = 0.4037$$

  Open loop phase margin $\approx 100\zeta = 40.37 \approx 40.4^o$

Plot of open loop transfer function without the compensator

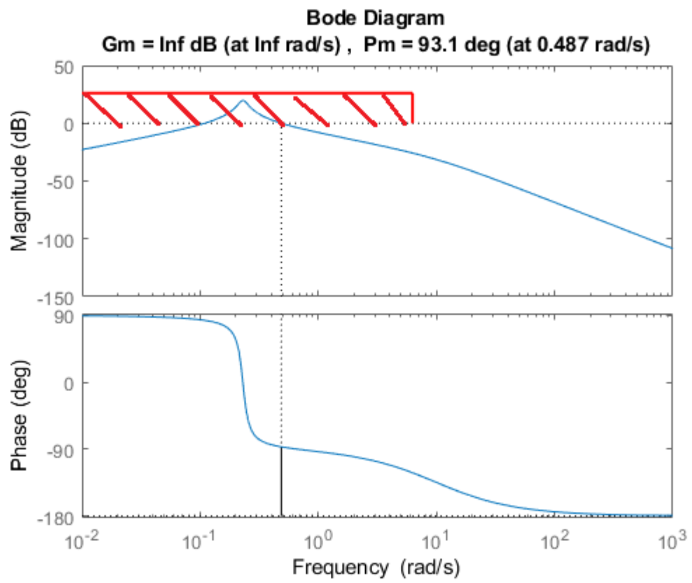

Figure 26: Bode plot of the open loop transfer function without the compensator.

As specified in the design above, we want the bode plot to be above the rectangular region in Fig 26. We want to increase the gain at low frequency and increase the slope of the system in the beginning. So, we introduced a PI controller. The Intgral gain in the PI controller increases the initial slope and the proportional gain increases the magnitude of the bode plot so that it will be always higher than 26 dB and meet those requirements.

(b) **PI Control Design:**

$$C_1(s) = K_P + \frac{K_I}{s} = \frac{K}{\tau s}(\tau s + 1) = K + \frac{K}{\tau}s \tag{29}$$

Take $\omega = 4$ rad/s, that gives $\tau = 1/\omega = 0.25$.

To find the initial estimate of the gains. From Fig. 26, we want increase the gains by about 30 dB. That gives us, $20\log_{10}(K) \approx 30$ dB, $\implies K = 31.6$.

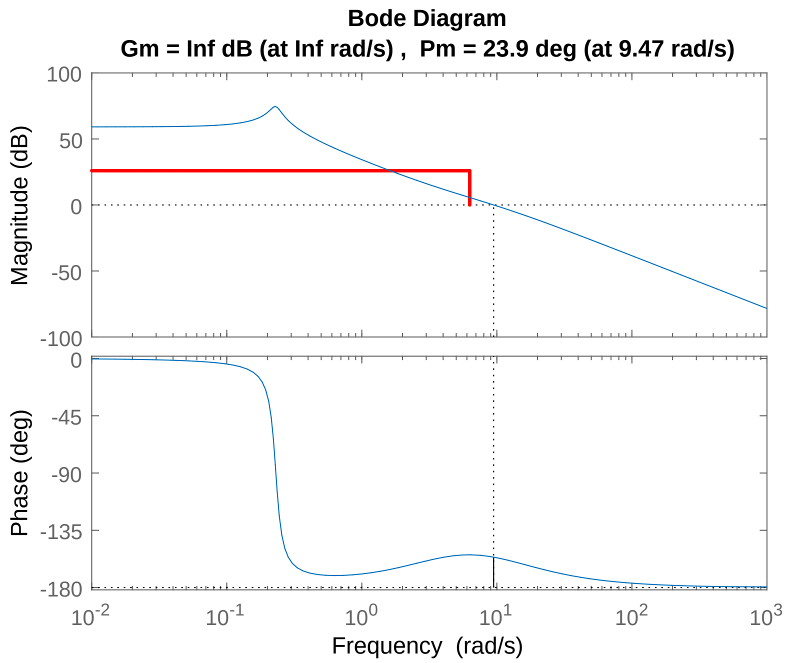

Figure 27: Initial bode plot of the open loop transfer function with PI compensator.

Since the phase margin is stilling lagging in order to address that, we now introduce a phase lead compensator.

(c) **Lead Compensator Design**

$$C_1(s) = \frac{K(\tau s + 1)}{\alpha \tau s + 1} \tag{30}$$

i.

$$\Delta\Phi = PM_{des} - PM_{act} + 5^\circ \tag{31}$$

$$\Delta\Phi = 40.4 - 23.89 + 5 = 21.50^\circ \tag{32}$$

$$\alpha = \frac{1 - \sin\Delta\Phi}{1 + \sin\Delta\Phi} = 0.7318 \tag{33}$$

$$\omega_c = 9.46 \quad rad/s \tag{34}$$

$$\tau = \frac{1}{w_c\sqrt{\alpha}} = 0.1235 \tag{35}$$

$$K = \frac{1}{|L(jw_c)|} \tag{36}$$

With the PI compensator the open loop transfer function looks like

$$L_1(s) = \frac{30.02s^3 + 123.1s^2 + 12.01s}{0.25s^5 + 2.535s^4 + 0.3617s^3 + 0.1436s^2 + 0.01325s} \tag{37}$$

From (37) and (30), we get

$$
\begin{aligned}
K &= \frac{1}{|L_1(j w_c)|} \\
&= \frac{1}{\left|\frac{30.02s^3+123.1s^2+12.01s}{0.25s^5+2.535s^4+0.3617s^3+0.1436s^2+0.01325s}\right|} \\
&= \left|\frac{(0.25s^5+2.535s^4+0.3617s^3+0.1436s^2+0.01325s)}{(30.02s^3+123.1s^2+12.01s)}\right|\Bigg|_{j\omega_c=9.46j} \\
&= \left|\frac{(19840.6854j+20302.17-306.211j-12.85+0.1253j)}{(-25414.64j-11016.41+113.52j)}\right| \\
&= \left|\frac{19534.6j+20289.32}{25301.12j-11016.41}\right| \\
&= 1.020633
\end{aligned} \tag{38}
$$

With the above parameters we get the following bode plot for PI + Lead Compensator

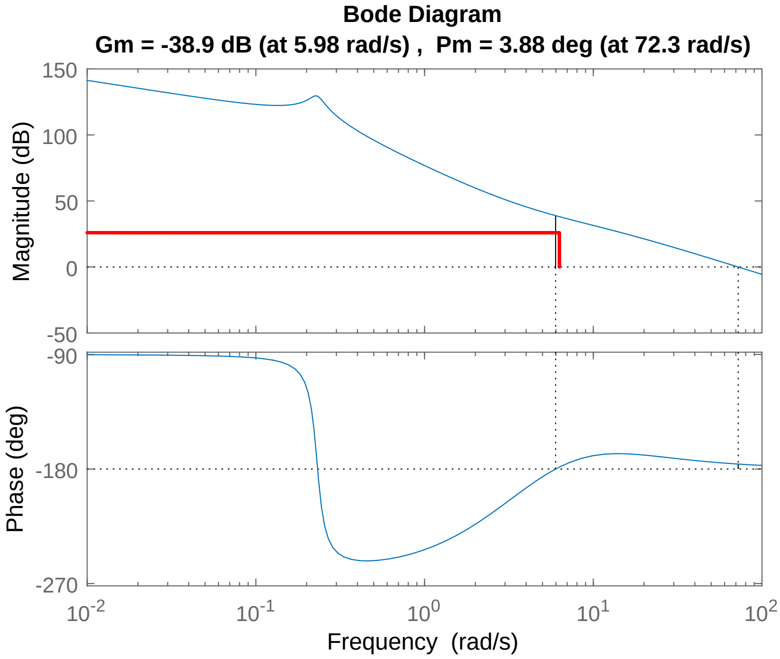

Figure 28: Initial bode plot of the open loop transfer function with PI + Lead compensator.

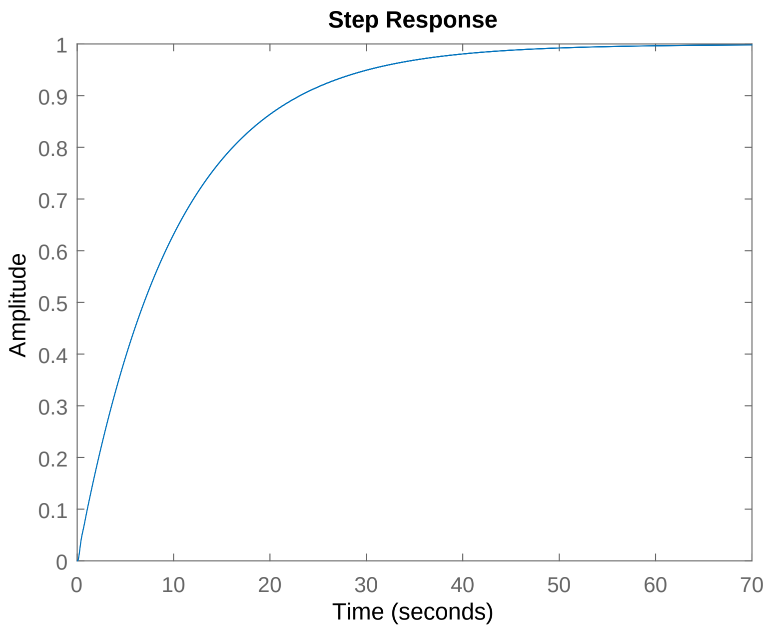

Figure 29: Initial step response of closed loop transfer function.

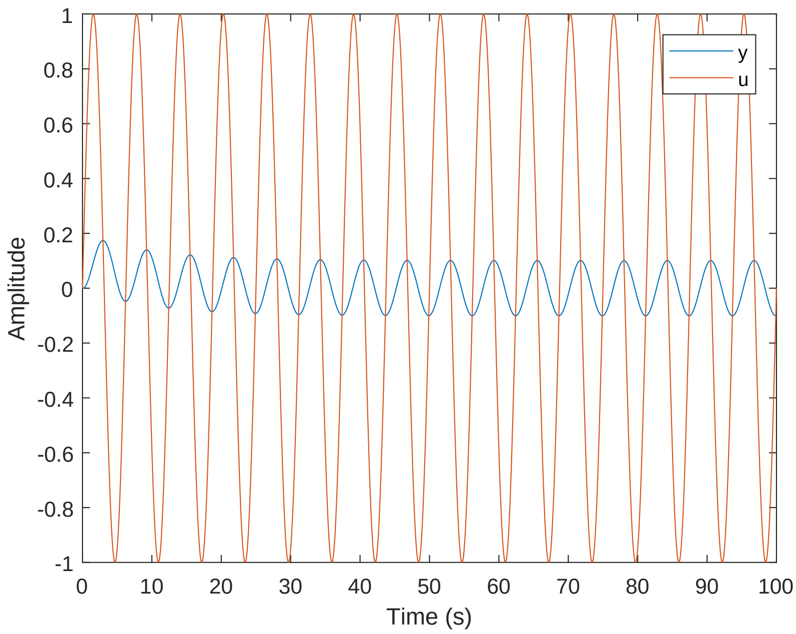

Figure 30: Initial tracking error tracking error plot.

To better tune the system to match the desired phase margin, we tune the gains as follows:

$$\textbf{PI Controller} \implies K_{PI} = 400, \quad \omega_{PI} = 4 \quad rad/s \quad \tau = 0.25$$

$$\textbf{Lead Compensator} \implies \alpha = 0.067, \quad K_{lead} = 0.01, \quad \tau_{lead} = 0.082$$

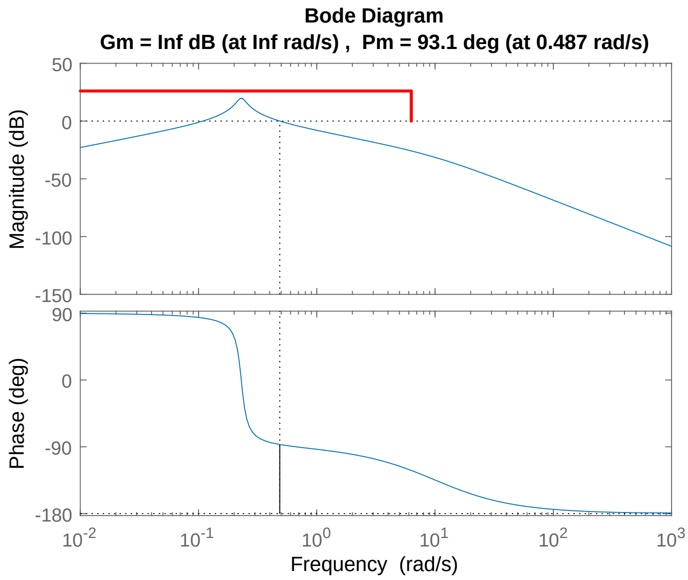

Figure 31: Final bode plot of the open loop transfer function without the compensator.

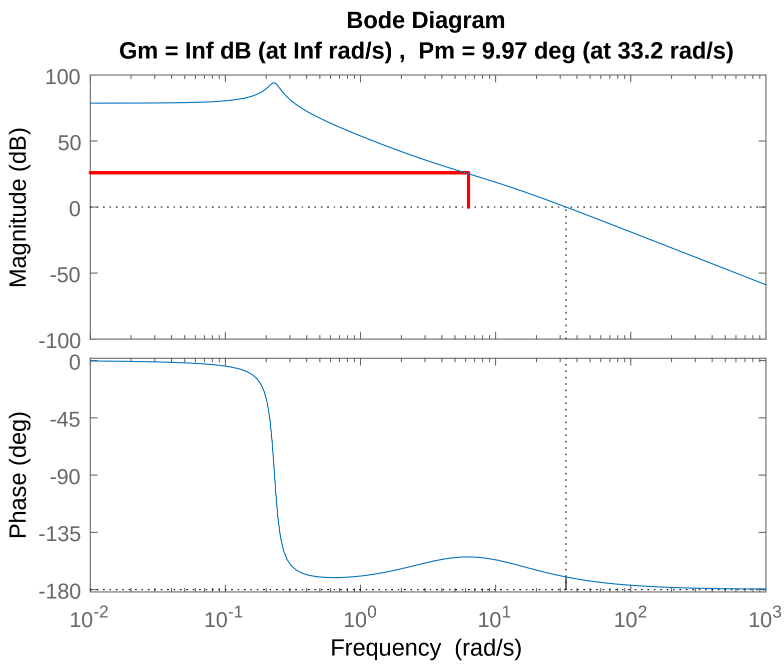

Figure 32: Final bode plot of the open loop transfer function with PI compensator.

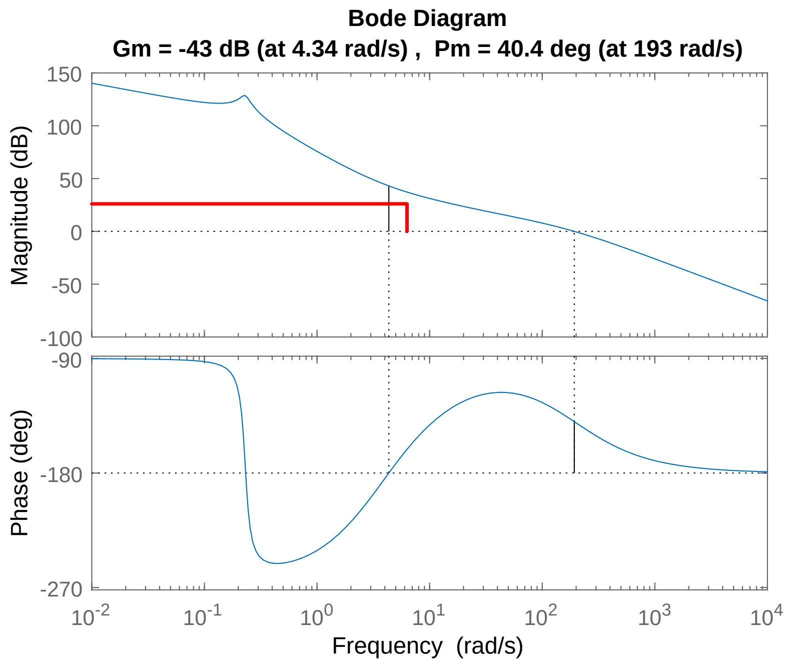

Figure 33: Final bode plot of the open loop transfer function with PI + Lead compensator.

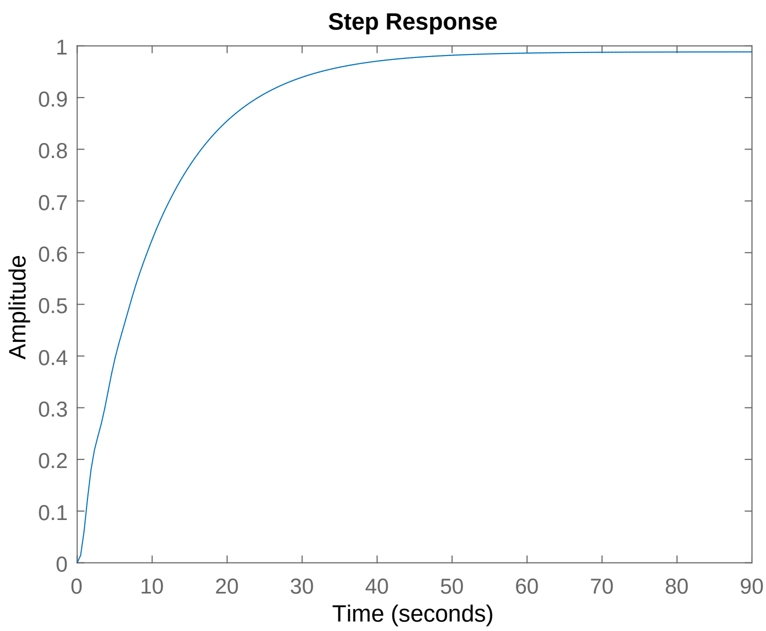

Figure 34: Step response of closed loop transfer function.

RiseTime: 21.8139

SettlingTime: 39.1049

SettlingMin: 0.8898

SettlingMax: 0.9885

Overshoot: 0

Undershoot: 0

Peak: 0.9885

PeakTime: 105.4584

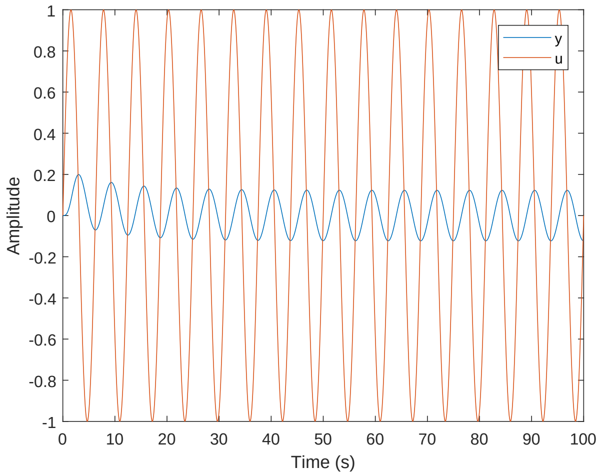

Figure 35: Tracking error.

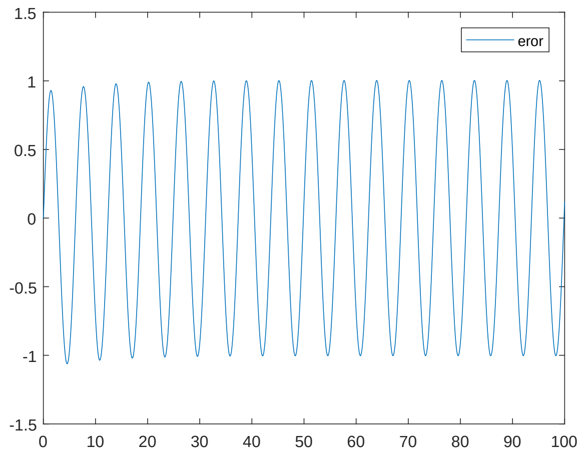

Figure 36: Error plot.

We approximate the closed loop bandwidth of the system as from MATLAB $bandwidth$ we get bandwidth for closed loop system = 0.1002. This can be also analyzed by look at -3 dB bode plot for the closed loop system.

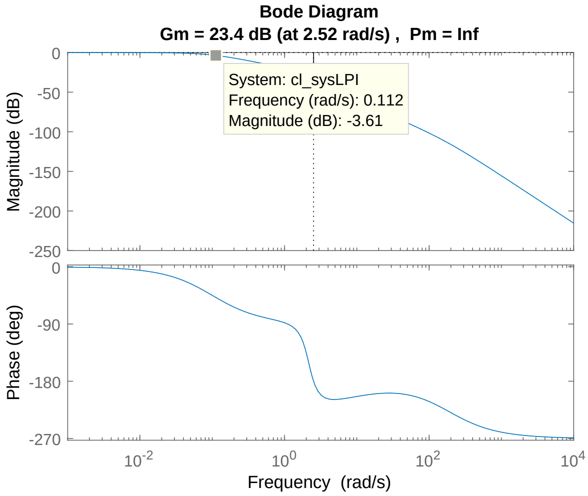

Figure 37: Bandwidth for closed loop system.

From Fig (38) and (39), we can observe that tracking error is very high, it is because our system has very low response and is unable to track the reference signal $u(t)$. I try to change the compensator but it did not change anything, inputting crossover frequency to be really large number did not help either. The approximation for second order system is probably not that good. But if we changed the $\omega = 0.16/100$, we get the following response

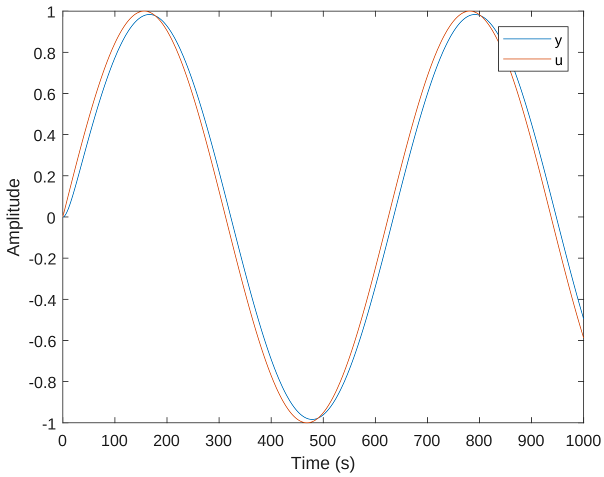

Figure 38: Tracking error with changed ω.

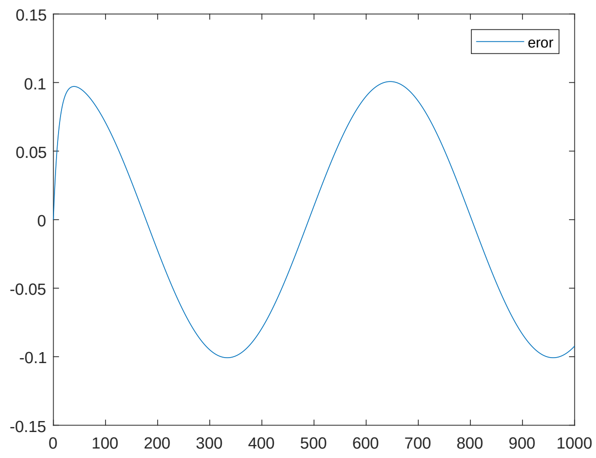

Figure 39: Error plot with changed ω.

(d)

A Matlab Code

```
% Sandesh Thapa
% Homework 1
% MAE 5010 Flight Controls
% Problem 2

%% Part a
clc;
clear all;
close all;

x0 = [0;0;0;0];

u0 = 272;
Lv = 580;
alv = 10;

num = alv*sqrt(Lv/u0)*[0.3398*(Lv/u0)^2 2.7478*(Lv/u0) 1];
den = [0.1539*(Lv/u0)^3 1.9754*(Lv/u0)^2 2.9958*(Lv/u0) 1];

sys1 = tf(num,den)

sim('Prob2')

```

```
figure
plot(time,Noise);
xlabel('$Time$ $(sec)$','interpreter','latex','FontSize', 20);
ylabel('$Noise$ $n(t)$','interpreter','latex','FontSize', 20);
hold on

figure
plot(time,vg);
xlabel('$Time$ $(sec)$','interpreter','latex','FontSize', 20);
ylabel('$V_g(t)$ $(m/s)$','interpreter','latex','FontSize', 20);
hold on

figure
plot(time,y);
xlabel('$Time$ $(sec)$','interpreter','latex','FontSize', 20);
ylabel('$a_y$ $(m/s^2)$','interpreter','latex','FontSize', 20);

%% Part B
% From numerical caluclations we got from state space
% Controllabe Cannoical form
Ag = [-6.20 1 0;
      -4.218 0 1 ;
      -0.6702 0 0];
Bg = [15.120;57.246;9.786];
Cg = [1 0 0];
Dg = 0;

A = [-0.5,  0, -1, 0.02;
     -150, -7, -0.15, 0;
     30, 0.1, -1, 0;
     0 1 0 0];

B = [-0.02, 56, 1, 0]';

G = [0.5; 150; -30; 0];
u0 = 272; % m/s
C = [-0.5*u0 0 0 0];

M1 = [A ;zeros(3,4)];
M2 = [G*Cg; Ag];
M = [M1,M2];

N = [zeros(4,1); Bg]
```

```
Al = M;
Ql = N*N';

X_cap = lyap(Al,Ql);

F = [C zeros(1,3)]

rms = sqrt(F*X_cap*F')
%%

% Check using matlab
[Agg,Bgg,Cgg,Dgg] = tf2ss(num,den)
% Matlab gives the matrics in observable cannonical form

M1g = [A ;zeros(3,4)];
M2g = [G*Cgg; Agg];
Mgg = [M1g,M2g];

Ngg = [zeros(4,1); Bgg]

Alg = Mgg;
Qg1 = Ngg*Ngg';

X_capg = lyap(Alg,Qg1);

Fg = [C zeros(1,3)]

rms = sqrt(Fg*X_capg*Fg')
```

```
% Sandesh Thapa
% Homework 1
% MAE 5010 Flight Controls
% Problem 3

clc;
close all;
clear all;

%% Part a

syms x real
solx = solve (0.3739*x^4 + 1.3343*x^3+ 0.5280*x^2+ 0.0381*x -0.0235 ==0,x);

```

```
r = roots([0.3739 1.3343 0.5280 0.0381  -0.0235])

%% Inner Loop
num1 = [0.0602 4.4326 0.2008  0.3103];
num2 = [4.4642 9.3994 0.4775 0];

den = [0.3739 1.3343 0.5280 0.0381  -0.0235];

sys1 = tf(num1,den);
figure
rlocus(sys1)
title('Open Loop Root Locus Inside Loop')
% [Wno,Zo,Po]= damp(sys1)
hold on

Ka = 0.3;
% Ka = 0.25;
sys_cl = feedback(Ka*sys1,1)
[Wn,Z,P] = damp(sys_cl)
damp(sys_cl)

figure
rlocus(sys_cl);
title('Close Loop Root Locus Inside Loop')
hold on
figure
step(sys_cl)
hold on
stepinfo(sys_cl)
%% Outer loop
sys2 = tf(num2,num1)

% K_ss = 0.3103/0.0235;
figure
rlocus(sys2)
title('Open Loop Root Locus Outside Loop')
[Wn2o,Z2o,P2o]= damp(sys2)
hold on
% K_ss = 1.1;

%open loop system
Kq = 0.75;
sys_2OL = sys_cl*Kq*sys2
```

```
[Wn2Open,Z2open,P2open] = damp(sys_2OL)
figure
rlocus(sys_2OL);
hold on


Kq = 1;
sys_cl2 = feedback(sys_cl*Kq*sys2,1)
[Wn2,Z2,P2] = damp(sys_cl2)
figure
rlocus(sys_cl2);
hold on
figure
step(sys_cl2);
stepinfo(sys_cl2)
```

```

%Problem 4
% Part a

clc;
close all;
clear all;

% P Controller
Kp = 550;
% Kp = 500
num = [0.25*Kp];
den = [0.0475 0.05 0.735];

sys = tf(num,den)
figure
title('Open Loop Plot with a P Controller')
x1 = .01:.01:2*pi*1;
y1 = 26*ones(size(x1));
y2 = 0:0.01:26;
x2 = 2*pi*1*ones(size(y2));
x3 = 2*pi*100:0.01:10000;
y3 = -20*ones(size(x3));
y4 = -20:0.01:100;
x4 = 2*pi*100*ones(size(y4));

plot(x1,y1,'r',x2,y2,'r','LineWidth',1.5)
hold on
```

```
plot(x3,y3,'r','LineWidth',1.5)
hold on
plot(x4,y4,'r','LineWidth',1.5)
% plot(x3,y3)
hold on
margin(sys)
opts = bodeoptions('cstprefs');
% opts.PhaseVisible = 'off';
% opts.FreqUnits = 'Hz';
opts.Xlim = [0.01 2*pi*100];
% opts.Ylim = [-40 80]
% axis([0:01 2*pi*100], [-40 80])
grid on
hold on
bodeplot(sys,opts)
[Gm,Pm,Wgm,Wpm] = margin(sys)

%% Lead Compensator

alpha = 0.07;
tau = 0.0527;
k = 0.65;

% alpha = 0.136;
% tau = 0.0527;
% k = 1.0;


C_s = tf([k*tau k],[alpha*tau 1])

L_s = C_s*sys
figure
% x1 = .01:.0001:1;
% y1 = 40*ones(size(x1));
plot(x1,y1,'r',x2,y2,'r','LineWidth',1.5)
hold on
plot(x3,y3,'r','LineWidth',1.5)
hold on
plot(x4,y4,'r','LineWidth',1.5)
% plot(x3,y3)
hold on
margin(L_s)
hold on
```

```
%%

figure
cl_sys = feedback(sys*C_s, 1)
margin(cl_sys)
hold on
figure
step(cl_sys)
stepinfo(cl_sys)

%%
t = [0:0.01: 10];
w = 0.16;
u = sin(2*pi*w*t);

y = lsim(cl_sys,u,t);
ylabel('Amplitude');
xlabel('Time (s)')
figure
plot(t,y,t,u);
legend('y', 'u')
ylabel('Amplitude');
xlabel('Time (s)')

hold on

e = (u' - y) ;
figure
plot(t,e);
legend('eror')
```

```
%Problem 5
% Part a
clc; close all; clear all;

num = [0.038 0];
den = [1 0.039 0.053];

Air_dyns = tf(num,den)

Eng_dyns = tf([10],[10 1])

Servo_dyns = tf([1],[0.1 1])

```

```
Sen_dyns = tf([10 1],[1])

sys = Servo_dyns*Eng_dyns*Air_dyns*Sen_dyns
sys1 = Servo_dyns*Eng_dyns*Air_dyns;

figure
hold on
x1 = .0001:.0001:2*pi*1;
y1 = 26*ones(size(x1));
y2 = 0:0.01:26;
x2 = 2*pi*1*ones(size(y2));
% plot(x1,y1,'k')
plot(x1,y1,'r',x2,y2,'r','LineWidth',1.5)
hold on
margin(sys)
hold on

%% PI Controller
% Kp = 400;
% Ki = 50;
K = 300;
% Ki = 1;

w_pi = 4; % rad/s
tau_pi = 1/w_pi;

% Kp = 1;
Ki = K/tau_pi;

C1 = tf([K*tau_pi K],[tau_pi 0])
L_1 = C1*sys

figure
hold on
% x1 = .0001:.0001:2*pi*1;
% y1 = 26*ones(size(x1));
% y2 = 0:0.01:26;
% x2 = 2*pi*1*ones(size(y2));
% plot(x1,y1,'k')
plot(x1,y1,'r',x2,y2,'r','LineWidth',1.5)
hold on
margin(L_1)
```

```
%% Lead Compensator
[Gm,Pm,Wgm,Wpm] = margin(C1*sys)

del_Pm = 40.4 - Pm + 5
% alpha = (1-sind(del_Pm)/(1+ sind(del_Pm)))
alpha = 0.067
% alpha = 0.8
k = 0.01;
% k = 1.02;
% syms k real
Wc = Wpm;
% tau = 1/(Wc*sqrt(alpha))
tau = 0.082
C2 = tf([k*tau k],[alpha*tau 1])

L_s = C1*C2*L_1
figure
hold on
% x1 = .01:.0001:1;
% y1 = 26*ones(size(x1));
plot(x1,y1,'k');
plot(x1,y1,'r',x2,y2,'r','LineWidth',1.5)
hold on
margin(L_s)
figure
cl_sysLPI = feedback(C1*C2*Servo_dyns*Eng_dyns*Air_dyns, Sen_dyns);
margin(cl_sysLPI)
 hold on

%%
figure
step(cl_sysLPI)
stepinfo(cl_sysLPI)
bandwidth(cl_sysLPI)
t = [0:0.1: 1000];
w = 0.16/100;
u = sin(2*pi*w*t);

y1 = lsim(cl_sysLPI,u,t);
figure
plot(t,y1,t,u);
legend('y', 'u')
ylabel('Amplitude');
```

```
xlabel('Time (s)')
hold on

e = (u' - y1) ;
figure
plot(t,e);
legend('eror')
```

References