OKLAHOMA STATE UNIVERSITY
School of Mechanical and Aerospace Engineering

MAE 5010 - Atmospheric Flight Control

Homework #1 (Assigned: 2/1, Due: 2/20)
Gust Modeling and SISO Flight Control Design

1. Thoroughly review the handouts posted on blackboard, including (a) longitudinal and lateral flight dynamics; (b) handling qualities; and (c) AIAA gust spectra paper.

2. At a particular steady wings-level flight condition, a high-speed reconnaissance aircraft has the following linearized equations of lateral motion $\dot{x} = Ax + Bu + Gd$:

$$\begin{pmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{pmatrix} = \begin{pmatrix} -0.5 & 0 & -1 & 0.02 \\ -150 & -7 & -0.15 & 0 \\ 30 & 0.1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix} + \begin{pmatrix} -0.02 \\ 56 \\ 1 \\ 0 \end{pmatrix} \Delta\delta_a + \begin{pmatrix} 0.5 \\ 150 \\ -30 \\ 0 \end{pmatrix} v_g$$

where all of the states and control surface (aileron) deflection are in rad or rad/s. The forward speed at this reference flight condition is $u_0 = 272$ m/s, and the lateral gust velocity v_g is in m/s.

(a) Generate an augmented state space model assuming lateral gust disturbances v_g can be approximated by pushing a white noise signal $n(t)$ through a shaping filter derived from the Von Karman PSD with length scale $L_v = 580$ m and gust intensity $\sigma_v = 10$ m/s:

$$H_v(s) = \sigma_v \sqrt{\frac{L_v}{u_0}} \frac{1 + 2.7478\frac{L_v}{u_0}s + 0.3398\left(\frac{L_v}{u_0}\right)^2 s^2}{1 + 2.9958\frac{L_v}{u_0}s + 1.9754\left(\frac{L_v}{u_0}\right)^2 s^2 + 0.1539\left(\frac{L_v}{u_0}\right)^3 s^3}$$

Implement this in Simulink using the *Band-Limited White Noise* source for $n(t)$ with noise power 0.1, sample time 0.1 and seed 25533, and generate a plot of $n(t)$, $v_g(t)$ in m/s and the lateral acceleration a_y at the CG in m/s^2 for 10 seconds. Assume the output equation for the lateral acceleration is $a_y = (-0.5u_0 \;\; 0 \;\; 0 \;\; 0)\, x$. *Be sure to set a fixed integration time in Simulink to match the sample time of the white noise block, 0.1 seconds, and make sure all plots submitted from MATLAB have axes labels with units.*

(b) Estimate the RMS value of the lateral acceleration $\sqrt{E\{a_y^2\}}$ using the Lyapunov equation approach discussed in class.

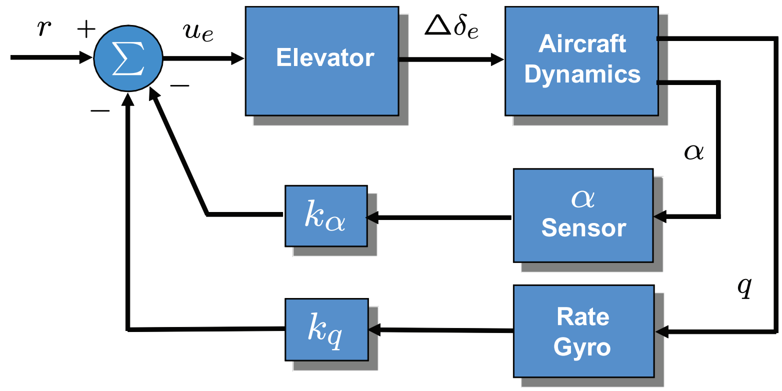

Figure 1: Pitch Alpha SAS

3. In this problem you will use sequential loop closure to design a pitch/alpha stability augmentation system (SAS) for a fixed wing UAV. The short period transfer functions are

$$\begin{aligned}\frac{\Delta\alpha}{\Delta\delta_e} &= \frac{-(0.0602s^3 + 4.4326s^2 + 0.2008s + 0.3103)}{0.3739s^4 + 1.3343s^3 + 0.5280s^2 + 0.0381s - 0.0235} \\ \frac{\Delta q}{\Delta\delta_e} &= \frac{-s(4.4642s^2 + 9.3994s + 0.4775)}{0.3739s^4 + 1.3343s^3 + 0.5280s^2 + 0.0381s - 0.0235}.\end{aligned}$$

You may assume that the sensor and actuator dynamics are unity. Since positive elevator deflection induces a negative pitching moment, you need to include a -1 in the above transfer functions in order to use the positive gain root locus.

(a) Show that the aircraft is not open loop stable.

(b) First close the inner loop by choosing a gain k_α in order to bring the unstable pole into the left half plane, and set the damping on the short period mode to $\zeta \geq 0.8$. The MATLAB command `damp` is useful here.

(c) Next, close the outer loop by choosing a gain k_q that sets the damping of the phugoid mode to be $\zeta \geq 0.1$. Does this new value of the phugoid damping satisfy Level 1 handling qualities? What is the resulting damping and natural frequency of the short period mode?

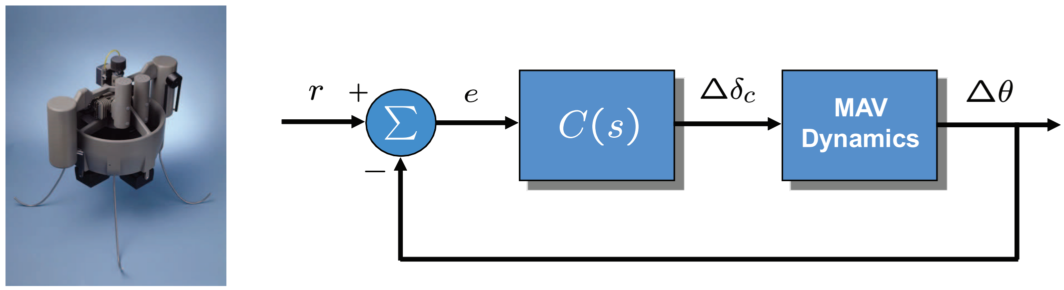

Figure 2: Ducted Fan MAV

4. In this problem you will design a pitch attitude control augmentation system (CAS) for a ducted fan MAV. Use the following transfer function to represent the vehicle dynamics:

$$\frac{\Delta\theta}{\Delta\delta_c} = \frac{r}{Js^2 + bs + mgl},$$

where $g = 9.8$ m/s^2, $m = 1.5$ kg, $b = 0.05$ kg/s, $l = 0.05$ m, $J = 0.0475$ kg/m^3, and $r = 0.25$ m. You are provided with the following specifications for the controller design:

- Steady state error of less than 1%
- Tracking error of less than 5% from 0 to 1 Hz
- Closed loop step response with a maximum overshoot of 20%
- Closed loop frequency response with no more than 3 dB gain at all frequencies
- Disturbance rejection from reference to output of at least 10X above 100 Hz

If you cannot meet all of the specifications, you should prioritize them in the order listed.

(a) Convert the closed loop performance requirements above into open loop constraints and generate a plot of the open loop (uncontrolled) plant versus the constraints.

(b) Design a dynamic compensator $C(s)$ for the system that satisfies the specifications. Generate a bode plot containing $L(s)$, along with all of the constraints to verify your design (it is often useful to visualize $P(s)$, $C(s)$, and $L(s)$ on the same plot). You should include additional plots (at least the closed loop step response and bode plot, along with $e(t)$ for a reference input of $r = \sin(2\pi\omega t)$ for $\omega = 1$ Hz) to show that all specs have been met. Note that the MATLAB command `margin` will be helpful here.

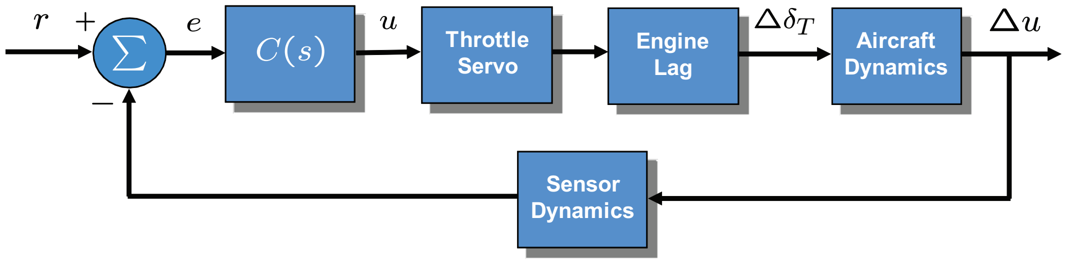

Figure 3: Velocity Hold Control System

5. In this problem you will design a Velocity Hold Control System for a transport aircraft. You may assume that the engine is modeled with a first order throttle servo time constant of $\tau_T = 0.1$, and engine dynamics $10/(10s+1)$. You may also assume that the accelerometer and pitot tube sensors are represented by the combined transfer function $10s + 1$ in the feedback loop. You can assume the transfer function from $\Delta\delta_T$ to Δu is given by

$$\frac{\Delta u}{\Delta\delta_T} = \frac{0.038s}{s^2 + 0.039s + 0.053}.$$

You are provided with the following specifications for the controller design. Note that $L(s)$ includes the transfer functions for the compensator, the engine and throttle dynamics, the aircraft dynamics, *and* the sensor dynamics.

- Steady state error of less than 5%
- Tracking error of less than 5% from 0 to 0.16 Hz (1 rad/sec)
- Closed loop step response with a maximum overshoot of 25%

(a) Convert the closed loop performance requirements above into open loop constraints and generate a plot of the open loop (uncontrolled) plant versus the constraints.

(b) Design a dynamic compensator $C(s)$ to meet all the constraints you derived above. Generate a bode plot of $L(s)$ that includes all the constraints so that you can visually verify that your design meets all performance requirements. What do you estimate the closed loop bandwidth of your design to be?

(c) Generate a closed loop step response to verify that your design meets the constraints on steady state error and overshoot, and plot the time response $e(t)$ for a reference input of $r = \sin(2\pi\omega t)$ for $\omega = 0.16$ Hz to verify the tracking constraint. Was the second order assumption used to derive the constraints helpful in this case? Did you have to go back and adjust the compensator to meet the requirements?