Delft Aerospace

Delft University of Technology

Practical assignment AE4-301P: Exercise Automatic Flight Control System Design

Design of some autopilot systems for the Lockheed Martin F-16 model

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October 1, 2018

CONTROL & SIMULATION

DELFT AEROSPACE

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Chapter 1

Introduction

1.1 Introduction

The practical Automatic Flight Control System Design, code AE4-301P, is an additional practical which follows up the course Automatic Flight Control System Design, code AE4-301. This practical is intended for **all** students who attended the course. The objective of this practical is to become familiar with classical flight controllers and their design, and to gain insight in handling qualities of open-loop and controlled aircraft. This exercise is a home-work assignment to be solved using Matlab and Simulink in teams of two students.

1.2 Goal of the practical

The goal of this practical is to design a control system using classical control theory. The control design is in fact a simple version of a F-16 military aircraft flight control system. Only two command systems will be discussed during this practical. These systems consist of a pitch rate command system and a terrain following system.

1.3 Background information

The practical assignment is set up in such a way that it must be possible to solve the complete assignment by means of the material presented in the assignment and by means of the lecture notes and the additional information on Brightspace. However, if the students would like to collect some additional background information, this could be found in ref. [1], [2], [3], [4], [5], [6] and [7].

1.4 Reporting

A written report must be delivered after completion of the practical assignment. Please make sure to explain the chosen procedure as well as the numerical results, but don't make your report too detailed. Matlab code used for answering the questions must be included in a separate zip-file. Make sure that all figures have clear and readable labels. For scientific reporting, the use of LaTeXis highly preferred, but you are allowed to use other programs, as long as you convert everything to a pdf file in the end. **The deadline for handing in the practical report is: 17:00pm, Friday December 21st, 2018.** All reports and code must be submitted through Brightspace. Submission instructions can be found under the Assignments section of the AE4301 Brightspace page.

Chapter 2

F-16 model

The aircraft model to be used for this practical is chosen to be an agile fighter model, namely the Lockheed F-16, of which a picture and a threeview can be found in fig. 2.1 and 2.2.

Figure 2.1: A pair of Lockheed F-16's in the Wild Weasel role, armed with pairs of AMRAAMs, Sidewinders and HARM anti-radar missiles together with a pair of additional underwing fuel tanks and a jamming pod on the centerline.

This non-linear model is obtained from University of Minnesota and it simulates the dynamics of the real aircraft. This plant can simulate the response of an actual F-16 using one of two models as described by ref. [7] and ref. [2]. From this point forward the model described by ref. [7] will be referred to as the low fidelity model and the model described in ref. [2] will be referred to as the high fidelity model. The main difference between both models is the use of different aerodynamic data and in the low fidelity model, a complete decoupling between longitudinal and lateral equations of motions is applied. Finally, the aerodynamic properties of the slats control surfaces at the wing leading edges are included in the high fidelity model only. More detailed information concerning the model structure and an introduction in the use of this airplane model can be found in ref. [6]. It is highly recommended to study the manual in ref. [6] first before proceeding with the practical assignment, in order to understand the working principle of the program and to be capable to use the model properly.

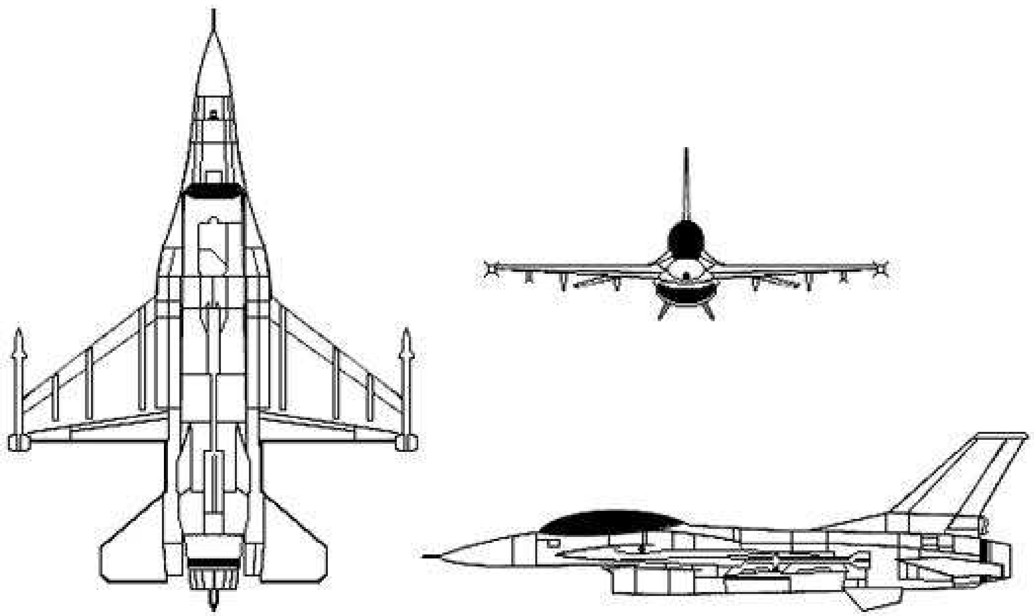


Figure 2.2: Threeview of the Lockheed F-16 Fighting Falcon

Chapter 3

Assignment set up

Roughly speaking, the set up of the assignment is as follows:

1. choice of flight condition
2. trim and linearisation
3. open loop analysis
4. design of a pitch rate command system satisfying CAP/Gibson Mil-specs
5. design of a terrain following mode
6. reporting

The first three topics serve as a preparatory part of the assignment. During the elaboration of these topics, the material is prepared and some practice is built up which both serve as a basis for the subsequent topics. After trimming and linearizing the non-linear model for the chosen flight condition, one obtains the state space model which will serve as the basis to design some flight control system components, to be discussed later. The open loop analysis investigates the inherent behaviour of the aircraft. In a real world flight control system, this analysis has to be performed for **all** flight conditions in the flight envelope and the results obtained by this analysis have to be compared with the flying and handling quality requirements, in this case MIL-F-8587L, and if any requirement is violated a stability augmentation system (SAS) has to be designed such that the modified behaviour complies with all requirements. However, this part of the job has been skipped here. Instead, the second part consists of the remaining two topics where some components of the flight control system have to be designed. The requirements the closed loop system has to comply (considerably reduced with reference to the MIL-F-8587L requirements) are mentioned in each topic. First a pitch rate command system needs to be designed. This type of command system is used for tracking tasks at low velocities. (For high velocity tracking tasks, a g-command is commonly used instead.) Finally, an automatic terrain following controller based on LQR control is designed.

In the following pages, a detailed elaboration of the assignment set up will be given.

Chapter 4

Choice of flight condition

First, a flight condition in the simulation model envelope needs to be chosen. The command system design will be based upon this point in the envelope.
In real flight controller design, a complete and extensive set of flight conditions needs to be defined in order to design the controllers properly. However, in this practical, it will be sufficient to consider **only one flight condition**. This condition can be chosen out of the following twelve different flight conditions, which have been chosen inside the model envelope. **Select your flight condition based on the rules in table** 4.1 and use it throughout the report, unless is stated otherwise. Mind that in real life, this design needs to be performed in each flight condition and thereafter gain scheduling needs to be performed over all the results, such that the complete flight envelope region is covered in which the flight control system is operational.

- altitude levels: 10000ft, 20000ft, 30000ft, 40000ft
- velocity levels: 300ft/s, 600ft/s, 900ft/s

4.1 Individual flight condition for accelerometer position analysis

For the task of the accelerometer position analysis in section 5.1, the following flight condition is defined for everybody:

- altitude level: 15000ft
- velocity level: 500ft/s

Last digit student number	First letter NetID	Altitude	Velocity
0-2	a-h	$10000ft$	$300ft/s$
	i-p	$10000ft$	$600ft/s$
	q-z	$10000ft$	$900ft/s$
3-5	a-h	$20000ft$	$300ft/s$
	i-p	$20000ft$	$600ft/s$
	q-z	$20000ft$	$900ft/s$
6-7	a-h	$30000ft$	$300ft/s$
	i-p	$30000ft$	$600ft/s$
	q-z	$30000ft$	$900ft/s$
8-9	a-h	$40000ft$	$300ft/s$
	i-p	$40000ft$	$600ft/s$
	q-z	$40000ft$	$900ft/s$

Table 4.1: Rules for selecting your flight condition. If you work together with somebody else, the rules apply to the person whose name appears first on the cover (you can decide who that will be).

Chapter 5

Trim and linearisation

First the F-16 model needs to be trimmed and linearised in the two flight conditions from the previous chapter. The trimmed LTI model for the individual flight condition serves for the analysis concerning the influence of an accelerometer position in section 5.1. The other trimmed LTI-model for the self-chosen flight condition will serve as the basis in the remainder of the practical where the analysis and command system will be based upon.

Explain why a trim procedure is absolutely necessary before a linearisation procedure can be started, in order to obtain a meaningful result.

Trim and linearize the nonlinear F-16 model in the two flight conditions using the m-file `findf16dynamics` . After completing the linearization procedure for the flight conditions (using high and low fidelity models), give the orders of the results obtained after the iterations of the cost function. These results serve as a measure of the achieved accuracy. Give comments on the reliability of the results, and choose another flight condition if necessary, i.e. if the accuracy is insufficient.

From this point onward, only the low fidelity model needs to be taken into account.

5.1 Influence of an accelerometer position

In a fighter aircraft, if an accelerometer is placed close to the pilot's station, aligned along the body Z_b-axis, and used as the feedback sensor for control of the elevator, the pilot has precise control over his Z_b-axis g-load during high-g manoeuvres. If $1g$ is subtracted from the accelerometer output, the control system will hold the aircraft approximately in level flight with no control input from the pilot. If the pilot blacks out from the g-load, and relaxes any force on the control stick, the aircraft will return to $1g$ flight. Other useful features of this system are that the accelerometer output contains a component proportional to angle of attack α and can inherently stabilize an unstable short-period mode, and the accelerometer is an internal sensor that is less noisy and more reliable than an angle of attack sensor. (This matter has been discussed in the tenth lecture, with the subject static stability augmentation systems, static SAS)
The normal acceleration a_n at a point P, fixed in the aircraft body, is defined to be the component of acceleration at P in the negative Z_b direction of the body axes. The contributing component of the gravity acceleration g can be expressed in terms of pitch and roll angles (ignoring the oblateness of the earth). The accelerometer output is then proportional to the specific force:

$$f_n = a_n + |g| \cdot \cos\theta \cdot \cos\phi \tag{5.1}$$

If the measurement is expressed in g units, the ratio $|g|/g_D$ (g_D is the standard gravity $9.80665 m/s^2$ or the local value) is very close to unity near to the earth's surface, so that:

$$f_n \approx a_n + \cos\theta \cdot \cos\phi \quad [g \text{ units}] \tag{5.2}$$

In level flight, at small pitch attitude angles, the feedback signal for the control system is:

$$(f_n - 1) \approx a_n \quad [g \text{ units}] \tag{5.3}$$

This normal acceleration is approximately zero in steady level flight; it is often called the "incremental" normal acceleration, and given the symbol n_z when in g units. Note that the component of acceleration along the lift axis, in steady-state flight with α, β and ϕ small, can be written in terms of load factor as $(n - \cos\theta)$ g-units. At small angle of attack α the lift direction is nearly coincident with the body negative Z_b-axis, and,

$$a_n \approx n - \cos\theta \quad [g \text{ units}] \tag{5.4}$$

or,

$$a_n \approx n \quad [g \text{ units}] \tag{5.5}$$

Therefore, the accelerometer measurement is an approximate measurement of load factor, under the above conditions.

If the accelerometer is on the body X_b-axis, at a distance x_a forward of the aircraft cg, and the aircraft is not rolling and yawing, the transport acceleration at that point is,

$$a_n = \frac{-\left(a_z - \dot{q}x_a\right)}{g_D} = n_z + \frac{\dot{q}x_a}{g_D} \quad [g \text{ units}] \tag{5.6}$$

where $a_z[m/s^2]$ is the vertical acceleration in the center of gravity and $n_z = -\frac{a_z}{g_D}$. If this equation is included in the nonlinear aircraft model then, in steady level-flight, the normal acceleration is close to zero and numerical linearization will yield a linear equation for a_n. A linear equation can also be obtained algebraically by finding the increment in the aerodynamic and thrust forces due to perturbations in the state and control variables, and this involves the Z-derivatives. However, this is beyond the scope of this practical.

The following steps are required for the individual flight condition trimmed LTI model (h = 15000 ft and V = 500 ft/s):

1. Include equation 5.6 in the nonlinear aircraft model in the Simulink file `lin_f16block` , by generating an additional output making use of the states coming out of the c-code block. The Simulink file `lin_f16block` is linearized in `findf16dynamics` by means of the `linmod` operation.*Hint 1: mind that the Matlab command 'linmod' cannot handle the simulink block* $\frac{dy}{dt}$. *Hint 2: mind that* n_z *is one of the available states.*
2. Trim and linearize this model numerically in the selected flight condition and you will notice the consequence of the additional output on the state space matrices.
3. Give the linearized output equation ($y = Cx + Du$) for normal acceleration a_n at the cg ($x_a = 0$).
4. List the states the normal acceleration a_n depends on.
5. Determine the elevator-to-normal-acceleration transfer function.
6. Draw the normal acceleration response to a negative step elevator command (aircraft nose up), focus on the initial response during the first few seconds and adapt the time step in order to obtain a smooth time response.
7. Which transfer function component forces the step response to be initially in the opposite direction of the reference signal?
8. Give a physical explanation of this non-minimum-phase behaviour.
9. Calculate and analyse the transfer function zero values for different accelerometer positions: $x_a = 0ft$, $x_a = 5ft$, $x_a = 5.9ft$, $x_a = 6ft$, $x_a = 7ft$ and $x_a = 15ft$. Draw and compare the initial normal acceleration responses to a negative step elevator command (aircraft nose up) for the different accelerometer positions.
10. At what position is the "instantaneous center of rotation" located? Explain your answer.
11. Where should the pilot's station preferably be located: before, in or after the instantaneous center of rotation? Explain your answer.
12. Explain why it is important to place the accelerometer close to a node of the most important fuselage bending mode.

Chapter 6

Open loop analysis

The open loop analysis consists of two major parts. First, the obtained LTI state space model from the previous chapter must be reduced to the familiar flight dynamics form of the fourth order. Thereafter, the motion characteristics of the open loop model must be calculated. These characteristics show the inherent flying qualities of the aircraft.

As already announced, first of all, for eigenmotion identification, the linearized matrices can be reduced considerably. Concretely, the matrices can be reduced to the following states of interest:

- longitudinal states of interest: V_t, α, θ, q;
- lateral states of interest: β, ϕ, p, r

Mind that that an ordinary reduction procedure (by selecting the appropriate rows and columns in the state space model matrices) results in input matrices $B = 0$, since the actuator dynamics have been ignored there is no direct connection anymore between inputs and states. Therefore, an alternative approach is necessary. This approach will be shown here for the longitudinal model. The procedure for the lateral model is analogous.
First of all, reduce the linearized state space model for the flight condition chosen in chapter 5, but with the actuator dynamics included. This means that you have to find a 6×6 matrix A_{OL} and a 6×2 matrix B_{OL}, for the LTI model containing the following states: velocity v_T, angle of attack α, pitch attitude angle θ, pitch rate q, elevator deflection δ_{el} and throttle setting δ_{th} and as inputs the electrical signals for the elevator actuator u_{el} and for the engine u_{eng}. So far, this can be done by the ordinary reduction procedure, which is the selection of the appropriate rows and columns in the complete state space model matrices.
Subsequently, remove the actuator and engine dynamics out of this reduced model. This can be done according to the following principle, found in ref. [7]. When one has an interconnection of an aircraft model, such as the F-16, which has the states v_T, α, θ, q and input δ_{el} and which is represented by the state space matrices $A_{a/c}$, $B_{a/c}$, $C_{a/c}$ and $D_{a/c}$, and an actuator model represented by the transfer function $H_{servo} = \frac{a}{s+a}$, the open loop interconnection state space structure is constructed as follows:

$$\dot{x} = \left[\begin{array}{cccc|c} & & & & \\ & A_{a/c} & & & B_{a/c} \\ & & & & \\ \hline 0 & 0 & 0 & 0 & -a \end{array}\right] \cdot \left[\begin{array}{c} v_T \\ \alpha \\ \theta \\ q \\ \hline \delta_{el} \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \hline a \end{array}\right] \cdot u_{el} \tag{6.1}$$

where the additional state δ_{el} is the elevator deflection, serving as an actuator state.

By means of this principle, the LTI model between the elevator deflection δ_e and the states can be deduced from the LTI model between the actuator input u_{el} and those same states. It is fairly straightforward to verify the applicability of this principle by recalculating the model with actuator dynamics by putting the model without actuator dynamics, obtained here, and the actuator dynamics themselves, with $a = 20.2$, in series. Mind that there exist more non-unique LTI models to represent the same dynamic system, where its transfer functions as well as the eigenvalues of its state matrix A are unique.

The state space matrices $A_{a/c}$, $B_{a/c}$, $C_{a/c}$ and $D_{a/c}$ obtained above will be used in section 6.1 and chapter 7.

6.1 Calculation of the inherent motion characteristics

In order to analyse the inherent behaviour of the F-16 model, the following motion characteristics need to be calculated for the flight condition concerned.

- For periodic eigenmotions, namely short period, phugoid and Dutch roll:
  natural frequency ω_n, damping ratio ζ, period P and time to damp to half amplitude $T_{1/2}$
- For anti-periodic eigenmotions, namely aperiodic roll and spiral:
  natural frequency ω_n, time constant τ and time to damp to half amplitude $T_{1/2}$

Draw the different eigenmotion time responses to support the numbers calculated above.

6.2 Some background information for the interested reader...

THIS SECTION CONTAINS NO TASKS. It contains just some background information about expression (6.1). This operation can be justified as found in the Matlab help files. When one has a series interconnection of two LTI models, where sys2 with data A_2, B_2, C_2, D_2 is put in front of sys1 with data A_1, B_1, C_1, D_1, like shown in figure 6.1, then the state space matrices of the complete series interconnection are as follows:

$$A_{series} = \begin{bmatrix} A_1 & B_1C_2 \\ 0 & A_2 \end{bmatrix} \quad B_{series} = \begin{bmatrix} B_1D_2 \\ B_2 \end{bmatrix}$$
$$C_{series} = \begin{bmatrix} C_1 & D_1C_2 \end{bmatrix} \quad D_{series} = \begin{bmatrix} D_1D_2 \end{bmatrix}$$

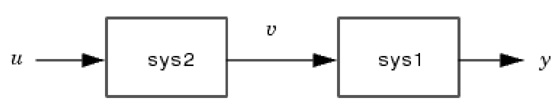


Figure 6.1: Series interconnection

Filling in the aircraft state space matrices $A_{a/c}$, $B_{a/c}$, $C_{a/c}$ and $D_{a/c}$ for sys1 is obvious. The state space model for sys2 is obtained as follows:

$$H_{servo}(s) = \frac{a}{s+a} = (s+a)^{-1}a = C_2(sI - A_2)B_2 + D_2 \tag{6.2}$$

which leads to the observation: $A_2 = -a$, $B_2 = a$, $C_2 = 1$, $D_2 = 0$.
Filling in all these results for A_{series}, B_{series}, C_{series} and D_{series} leads to eq. (6.1).

Chapter 7

Design of a pitch rate command system satisfying CAP/Gibson Mil-specs

In this section a pitch rate command system has to be designed. A pitch rate command system is very effective for tracking tasks at low velocities.

The pitch rate controller has to fulfill the following requirements, applied on the military specifications:

- the CAP (Control Anticipation Parameter) criterion

- the Gibson criterion

Both requirements will be elaborated in depth below, in order to define accurate requirements.

7.1 CAP criterion

For highly augmented airplanes, like the Lockheed F-16, a requirement involving the Control Anticipation Parameter (CAP) has been defined and has in fact replaced the conventional short period undamped natural frequency and damping ratio requirements. This new requirement defines that the airplane must stay within a minimum and maximum range of values of the so-called CAP over a range of allowable short period damping ratios.
Physically, the Control Anticipation Parameter is a measure of manoeuvrability. If the CAP requirement is satisfied, then one can say that the aircraft's "nose follows the stick". This parameter can be defined physically as the ratio of the initial pitch acceleration over the steady state load factor, alternatively expressed in an equation:

$$CAP = \frac{\dot{q}\,(t=0)}{n_z\,(t=\infty)} \tag{7.1}$$

The equation to estimate the control anticipation parameter (CAP), based upon the available data, is defined as follows:

$$CAP = \frac{\omega_{n_{sp}}^2}{n_\alpha} \tag{7.2}$$

where:

$\omega_{n_{sp}}$ is the undamped natural frequency of the short period mode

n_α $= \dfrac{\partial n}{\partial \alpha}$ which is also referred to as the gust- or load-factor-sensitivity of an airplane

Another but more preferable way to calculate the CAP because of the available data is as follows:

$$CAP = \frac{g\omega_{n_{sp}}^2 T_{\theta_2}}{V} \tag{7.3}$$

where the parameter T_{θ_2} is right away available from the short period reduced pitch rate transfer function:

$$\frac{q(s)}{\delta_{el}(s)} = \frac{k_q\left(1 + T_{\theta_2}s\right)}{s^2 + 2\zeta_{sp}\omega_{n_{sp}}s + \omega_{n_{sp}}^2} \tag{7.4}$$

The requirements concerning Control Anticipation Parameter and short period damping ratio are illustrated by the allowable regions shown in fig. 7.1.

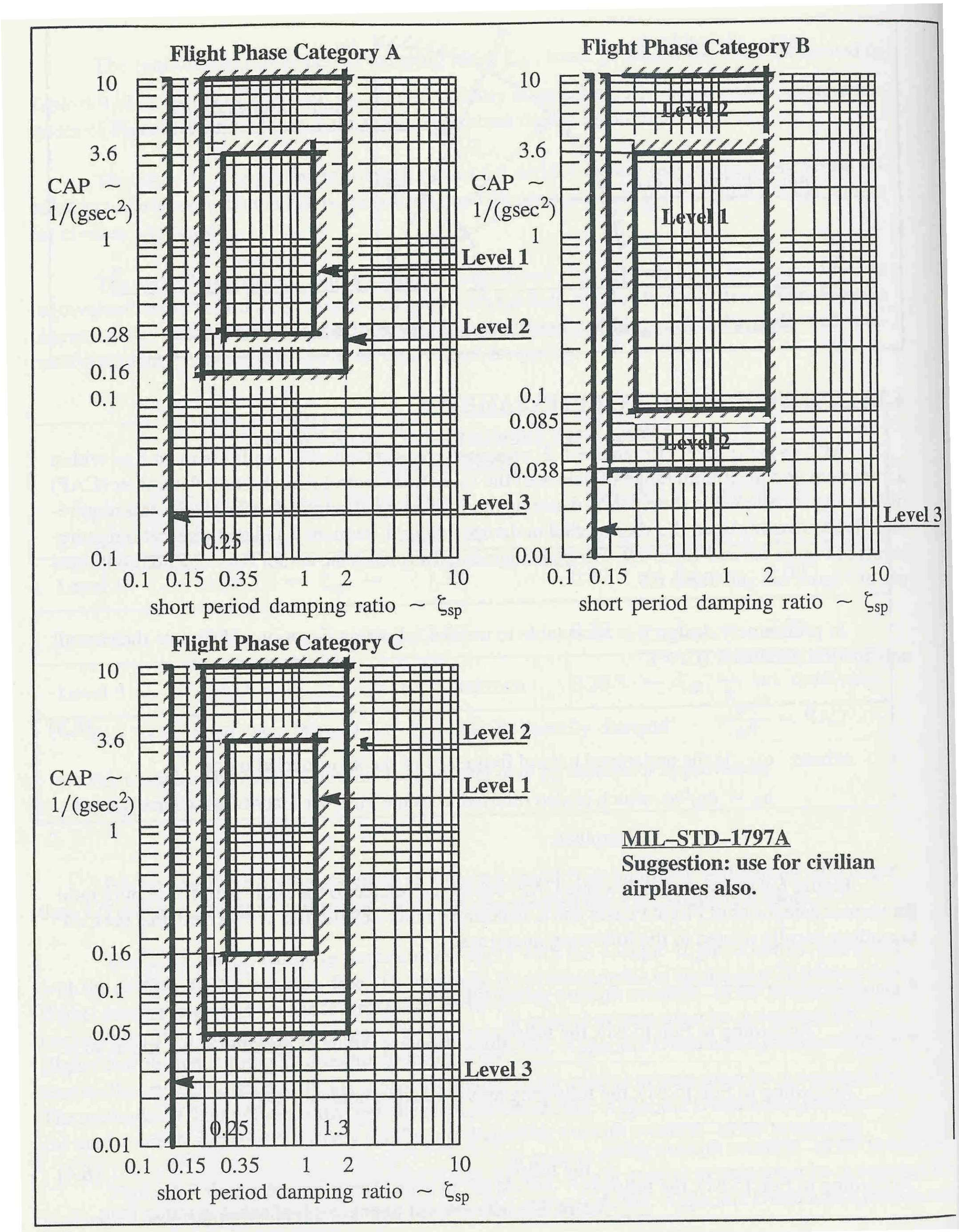


Figure 7.1: Control Anticipation Parameter and short period damping ratio requirements, source: ref. [4]

7.2 Gibson criterion

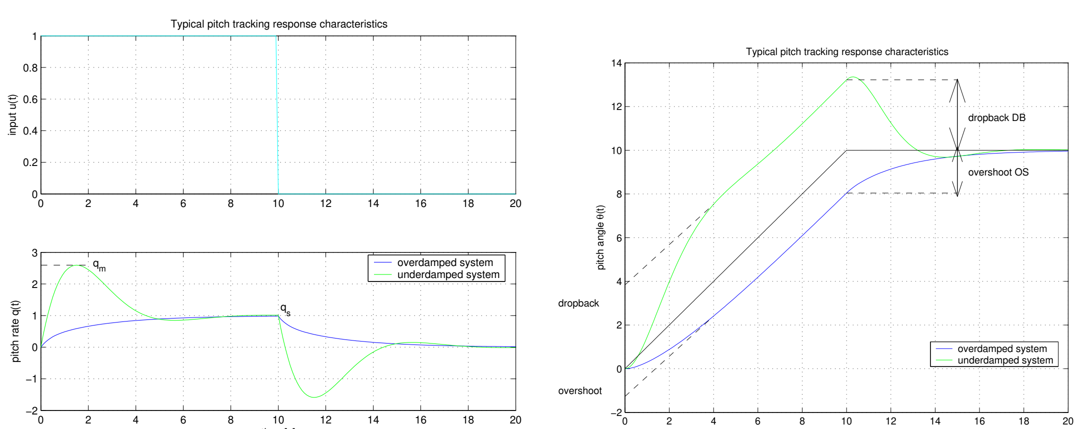
The Gibson Criterion grew out of the need to know how to design command and stability augmentation systems that would result in an aircraft with acceptable flying and handling qualities, a problem that is becoming increasingly urgent as fly-by-wire control systems become more and more common. This criterion consists of two separate criteria, namely the dropback criterion and the phase rate criterion. The phase rate criterion considers the robustness properties of the aircraft. Both will be given below.

7.2.1 Dropback criterion

The criterion was originally defined in terms of limiting values on pitch rate overshoot ratio $\frac{q_m}{q_s}$ and on the ratio of attitude dropback (or overshoot, dependent on the direction of the transition when the step input is removed, see below) to steady state pitch rate. These parameters will be explained first.

All the parameters enumerated and explained below are illustrated in fig. 7.2.

- q_m: maximum pitch rate
- q_s: steady state value of pitch rate
- $\frac{q_m}{q_s}$: pitch rate overshoot ratio
- DB: dropback, amount of negative transition towards final value after the step input has been removed
- OS: overshoot, amount of positive transition towards final value after the step input has been removed



(a) input and pitch rate (b) pitch attitude

Figure 7.2: Typical pitch tracking response characteristics

The requirements imposed on these parameters are shown in fig. 7.3 below.
Note that:

1. if the pitch rate overshoot ratio $\frac{q_m}{q_s} \leq 1$ then dropback is not possible and the lower part of the "satisfactory" region cannot be attained.

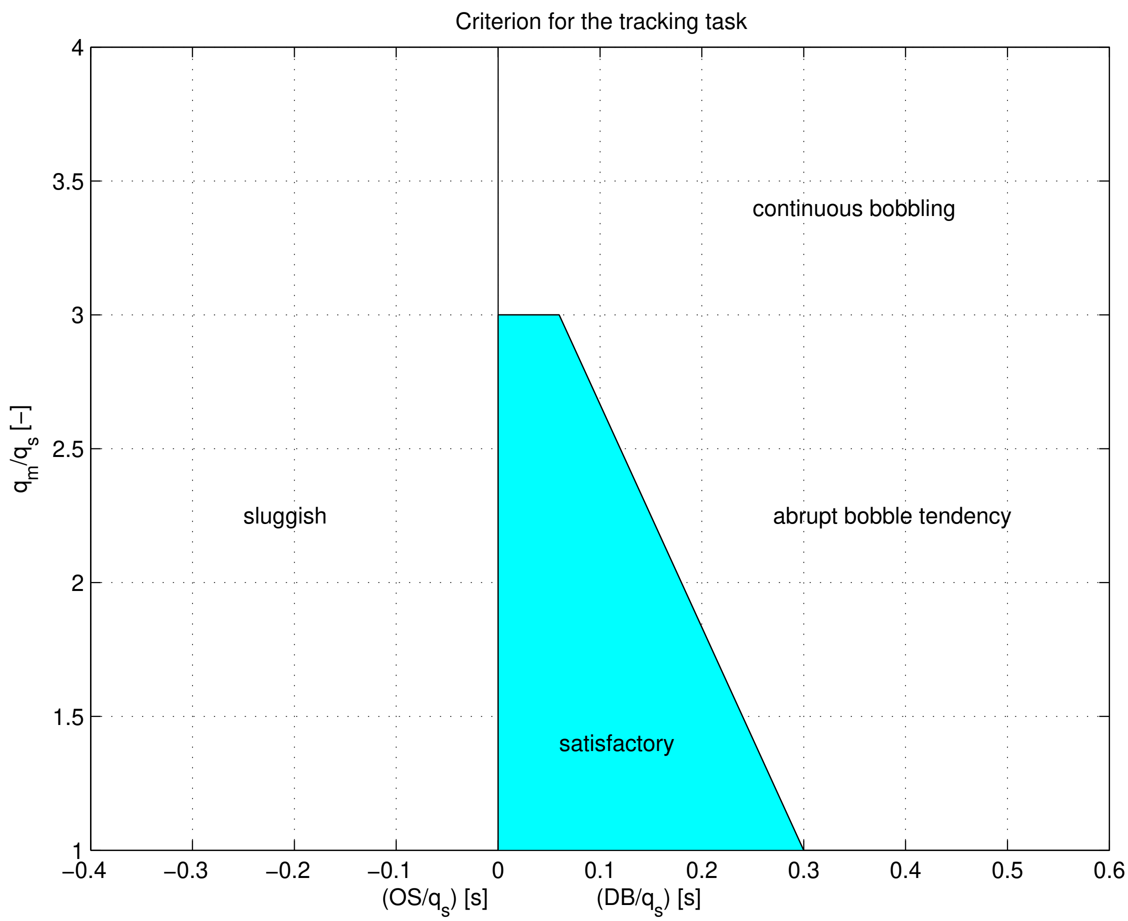


Figure 7.3: The criterion for the tracking task

2. subsequent events have led Gibson to redefine the criterion such that zero dropback only is acceptable. The "satisfactory" region then collapses to the $\frac{q_m}{q_s}$-axis and in the event that this cannot be achieved precisely then it is better to transgress on the side of attitude dropback rather than overshoot.

3. the acceptable values of pitch rate overshoot lies in the range $1.0 \leq \frac{q_m}{q_s} \leq 3.0$

7.2.2 Phase rate criterion

Even when the flying and handling qualities of a high order aircraft are acceptable it is possible that the closed loop gain and phase characteristics may be such that the addition of the pilot leads to the propagation of pilot induced oscillations (PIO) at certain conditions. It is now known that the likelihood of PIO is determined by the degrees of gain and phase compensation instinctively introduced by the pilot in flying the aircraft. The required compensation is in turn determined by the closed loop gain and phase characteristics of the aircraft at frequencies close to the "resonant" frequency of the human pilot. More recently, Gibson has turned his attention to the problem of PIO in otherwise satisfactory aircraft and has identified the desirable gain and phase characteristics for the closed loop high order aircraft if PIO is to be avoided. The findings define a useful adjunct to the criterion described above.

The phase rate criterion is concerned with the closed loop attitude frequency response in the region of $-180°$ phase and is evaluated from a plot of the closed loop attitude frequency response on a Nichols chart as shown in fig. 7.4.

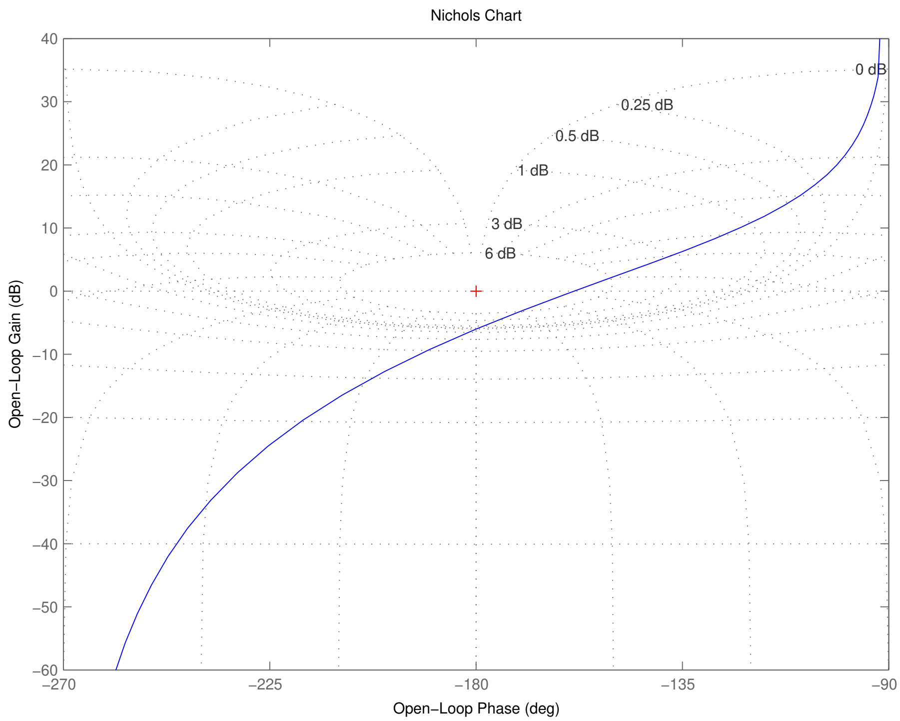


Figure 7.4: Closed loop attitude frequency response

Referring to fig. 7.4 the point of intersect is the "cross over point" where the phase first passes through $-180°$, the frequency corresponding with this point $\omega_{\phi=-180°}$ and the rate of change of phase with frequency at cross over $(\frac{\partial\phi}{\partial\omega})_{\phi=-180°}$ are relevant for the criterion.
Ideally, Gibson has established that:

$$\text{phase rate} = \left(\frac{\partial\phi}{\partial\omega}\right)_{\phi=-180°} \leq 100\frac{°}{Hz} \tag{7.5}$$

if PIO is to be avoided with a reasonable margin of certainty. The phase rate criterion is summarised on fig. 7.5 and generally requires that the cross over point should occur at a frequency of 1 Hz and that the phase rate should be less than 100° per Hz.
When an aircraft fails to meet the criterion suitable gain and phase compensation can be introduced into the command path using a lead-lag or lag-lead filter.

Note that when command path compensation is required it is unlikely that very large phase shift will be required. It may also be necessary to include some high frequency gain compensation in order to maintain the slope of the closed loop attitude frequency response plot to reasonable values at cross over.

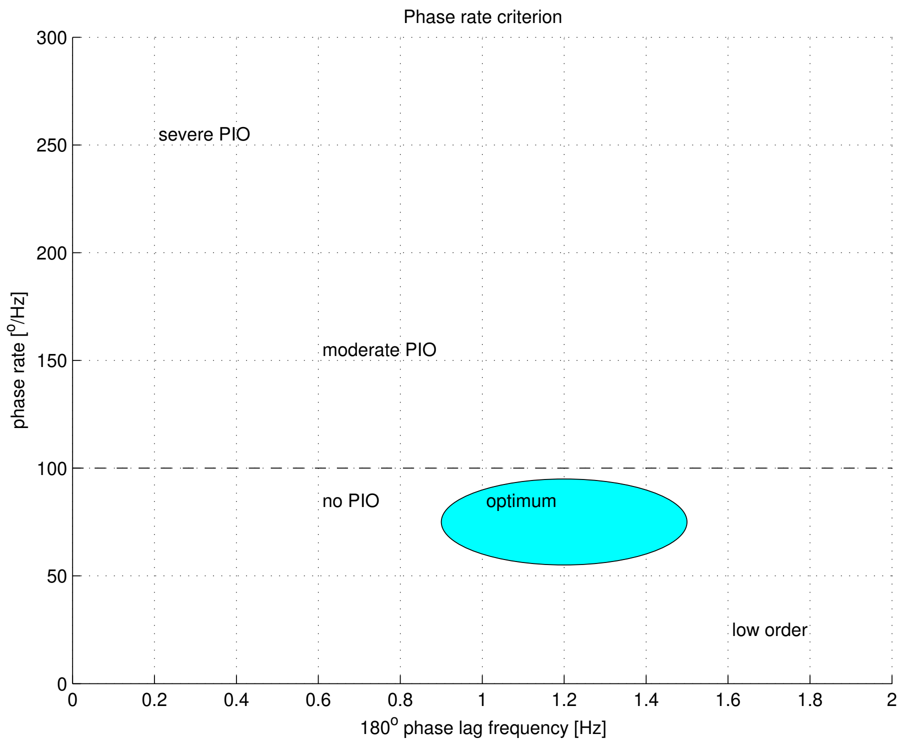


Figure 7.5: The phase rate criterion

7.3 Controller design task

Summarizing, the to-be-designed controller has to fulfill the requirements mentioned above, namely:

- the CAP (Control Anticipation Parameter) criterion
- the Gibson dropback criterion only

The following steps should be included in the design:

1. Construct the short period reduced model from the model without actuator dynamics, calculated in chapter 6. Mind that this short period reduced model involves only two states, namely angle of attack α and pitch rate q.

2. Compare time responses of the pitch rate q on a step input for the 4 state one without actuator dynamics and the reduced 2 state model without actuator dynamics. Analyse these time responses of q and determine which is the dominant mode. As you know that in this case only short term movements are considered, does this cause any problem? Why?

3. The requirements imposed by the CAP and Gibson criteria are not expressed in the usual quantities for controller design... Therefore, these requirements must be converted to the familiar frequency domain (requirements imposed on natural frequency $\omega_{n_{sp}}$, damping ratio ζ and time constant T_{θ_2}). These rewritten criteria can be expressed by the following relationships:

   - $\omega_{n_{sp}}(V,h) = 0.03V(V,h)$ with V in $[m/s]$
   - $1/T_{\theta_2}(V,h) = 0.75\omega_{n_{sp}}(V,h)$
   - $\zeta_{sp}(V,h) = 0.5$

4. What procedure can be used to obtain the required short period frequency $\omega_{n_{sp}}$ and damping ratio ζ?[1] Make a pitch rate command system using such a controller for the selected flight condition such that the short period frequency $\omega_{n_{sp}}$ and damping ratio ζ have the required value. Are the obtained levels of feedback gains acceptable with reference to possible gust? Mind that K_α and K_q are expressed in the units $^\circ/rad$ and $^\circ/(rad/s)$ respectively. Consider severe gust (design vertical gust: 4.572m/s) according to MIL-F-8785C (see lecture 10 about static stability augmentation systems).

5. The T_{θ_2} time constant cannot be modified by pole placement or another control loop structure. This needs to be done by means of pole-zero cancellation and by zero placement by means of a lead-lag prefilter. Explain why this prefilter must be located outside the loop.

6. Give drawings of CAP and Gibson criteria allowable regions together with positions of the design point and the current parameter value. Include time responses of pitch attitude angle and pitch rate. Verify if the requirements are met by implementing the obtained values into the following definitions:

   - $CAP = \frac{\omega_{n_{sp}}^2}{\frac{V}{g}\frac{1}{T_{\theta_2}}}$
   - $\frac{DB}{q_{ss}} = T_{\theta_2} - \frac{2\zeta_{sp}}{\omega_{n_{sp}}}$

[1] Mind that a known value of desired closed loop short period frequency $\omega_{n_{sp}}$ and damping ratio ζ actually mean that the location of the desired closed loop poles are known. In fact, this means that you can place the poles!

Chapter 8

Design of a terrain following system

The last chapter of the practical assignment deals with the design of a terrain following control system for the F-16. This system is very useful for flying at extreme low altitude, since the reaction time of the pilot is too long to cope with the very quick reactions needed at this altitude level and speed conditions. This control system is to be designed by an optimal controller, more precisely a so-called LQR-controller. First the concept of optimal control will be introduced, after which the terrain following system design task is described step by step.

8.1 Optimal control

Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. A control problem includes a cost function that is a function of state and control variables.

Given a dynamical system with time-varying input $u(t)$, time-varying output $y(t)$ and time-varying state $x(t)$, define a cost function to be minimized. The cost function is the sum of the path costs, which usually take the form of an integral over time, and the terminal costs, which is a function only of the terminal (i.e., final) state, $x(T)$. Thus, this cost function typically takes the form:

$$J = \phi\left(x\left(T\right)\right) + \int_0^T L\left(x, u, t\right) dt \tag{8.1}$$

where T is the terminal time of the system. It is common, but not required, to have the initial (i.e., starting) time of the system be 0 as shown. The minimization of a function of this nature is related to the minimization of action in Lagrangian mechanics, in which case $L(x, u, t)$ is called the Lagrangian.

8.1.1 Linear Quadratic Control

Considering LTI models as we are used to:

$$\dot{x} = Ax + Bu \tag{8.2}$$

$$y = Cx \tag{8.3}$$

One common cost function used together with this system description is:

$$J = \int_0^\infty \left(x^T\left(t\right) Q x\left(t\right) + u^T\left(t\right) R u\left(t\right)\right) dt \tag{8.4}$$

where the matrices Q and R are positive-semidefinite and positive-definite, respectively. Note that this cost function is thought in terms of penalizing the control energy (measured as a quadratic form) and the time it takes the system to reach zero-state. These Q and R matrices are chosen by the designer of the LQR controller. They represent the relative importance that is given to minimizing certain states are controls.

The optimal control problem defined with the previous function is usually called the state regulator problem and its solution the linear quadratic regulator (LQR) which is no more than a feedback matrix gain of the form

$$u = -Kx \tag{8.5}$$

where K is a properly dimensioned matrix and solution $K = R^{-1}B^T S$ of the continuous time algebraic Riccati equation:

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{8.6}$$

There is a built-in function in Matlab (lqr) that will solve the Riccati equation for you and give you the feedback gains for a given A, B, Q, R.

8.2 Terrain following system design task

For this design, the following steps need to be performed:

1. Trim and linearize the F-16 model for the flight condition altitude $h = 5000ft$ and velocity $V = 300ft/s$[1].

2. Construct the reduced model for this flight condition, containing the **five** states **altitude** h, true airspeed V_t, angle of attack α, pitch attitude angle θ, and pitch rate q.

3. The canyon-like altitude profile as given in picture 8.1 is generated by the Simulink model `canyongenerator.mdl`, which is available on blackboard. The output of the model is the local altitude in meter, making use of the horizontal longitudinal position x $[m]$ as input. As you can see in this picture, the reference altitude for the aircraft is 40m above ground surface. The reference altitude will serve as reference signal for the controlled system. **Mind the conversions between meter and feet!**

4. After trimming at $h = 5000ft$, initialise the altitude of your aircraft in the simulation at the reference altitude from the canyongenerator ($h = 1500m$), so when the simulation starts, you will be on the reference. Although your aircraft is not trimmed exactly for this altitude, it is close enough for your controller to cope with it.

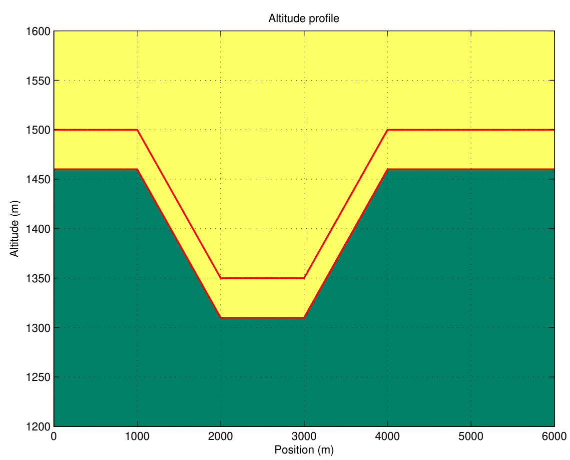


Figure 8.1: reference altitude profile

5. Using a state space model block, define the aircraft model in Simulink. Subsequently, make a series connection at the input with a subsystem block containing the representation of the dynamics (using a transfer function block) and saturation limits (using a saturation block) for engine and actuator as follows:

   - engine: dynamics $H_{engine} = \frac{1}{s+1}$, saturation limits: [1000lb ; 19000lb]
   - elevator actuator: dynamics $H_{elevator} = \frac{20.2}{s+20.2}$, saturation limits: $[-25°; 25°]$

   Mind that when you trimmed the nonlinear model, the engine and the elevator were already set to a constant setting. The saturation blocks should be defined taking these trimmed values into account. Don't forget that δ_{el} and δ_{th} are deviations from the trimmed values and not the total values of elevator deflection and engine power.

[1]Mind that this flight condition is the lowest and slowest possible where the simulation results are still valid.

6. Design an LQR controller for this system using the Matlab command `LQR`. Incorporate this controller in the Simulink block diagram, where the controller gains for all states except for altitude serve as an inner loop stability augmentation state feedback. The controller gains for the altitude are not applied on the geometric altitude itself, but on the error with respect to the reference altitude above ground surface, and work in the outer state feedback loop, as illustrated by the block diagram in figure 8.2.

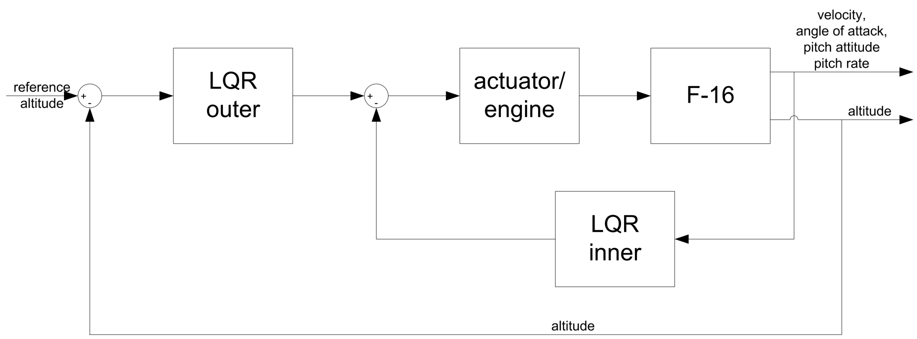


Figure 8.2: Block diagram of the closed loop terrain following structure

7. Optimize your LQR controller for pilot comfort by choosing appropriate values on the diagonal entries of the Q and R matrices, such that altitude overshoot for constant reference does not exceed 1 m, and the (vertical) altitude above ground surface is always larger than 20m, while the engine and actuator limits are not violated. Mind that by increasing a diagonal element of Q or R you are requesting the optimization algorithm to make an increased effort to minimize the corresponding variable. Eg, by increasing the diagonal element of R related to the elevator actuator, you are asking the optimization algorithm to minimize the usage of the elevator. A good policy is to first use identity matrices for Q and R. Then, to satisfy all the requirements, you should increase or decrease the relevant diagonal elements of Q and R by factors of 10.

Finally, some useful hints for solving the terrain following control system design task:

- Don't forget that the states of the system are deviations from the trimmed values. Therefore, the total altitude and the total velocity are given by:
  - $h_{total} = h_{trimmed} + h$
  - $V_{total} = V_{trimmed} + V_T$
- To calculate the position you can use the frequency domain approximation: position$(s) \approx \frac{V_T(s)}{s}$
- You might want to use the following approach:
  1. construct the complete Simulink block diagram of the controlled system
  2. there should be a "scope" block in every signal you want to plot using Matlab. To send data from a "scope" to Matlab, double click each "scope" block, click on "parameters", and in "data history" select "save data to workspace".
  3. run the simulation using the `sim` Matlab command.

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