LIFTED HINGE-LOSS MARKOV RANDOM FIELD

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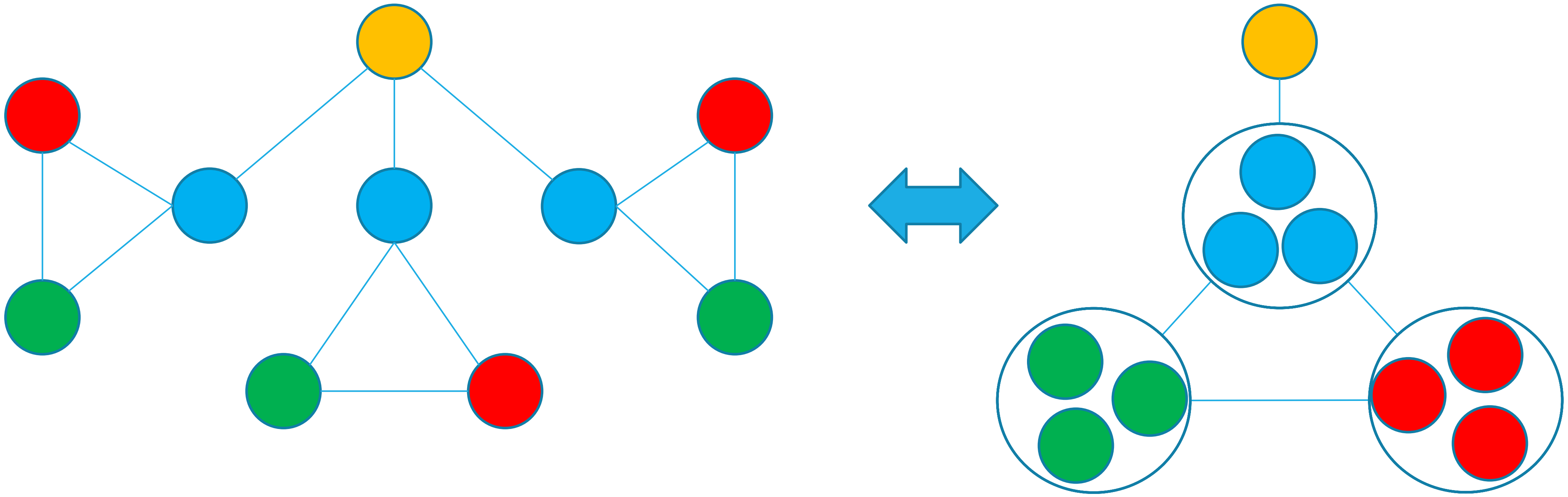
LIFTED INFERENCE

- Exploit symmetry and shrink the problem
- Smaller problem is potentially faster to solve



RELATED WORK

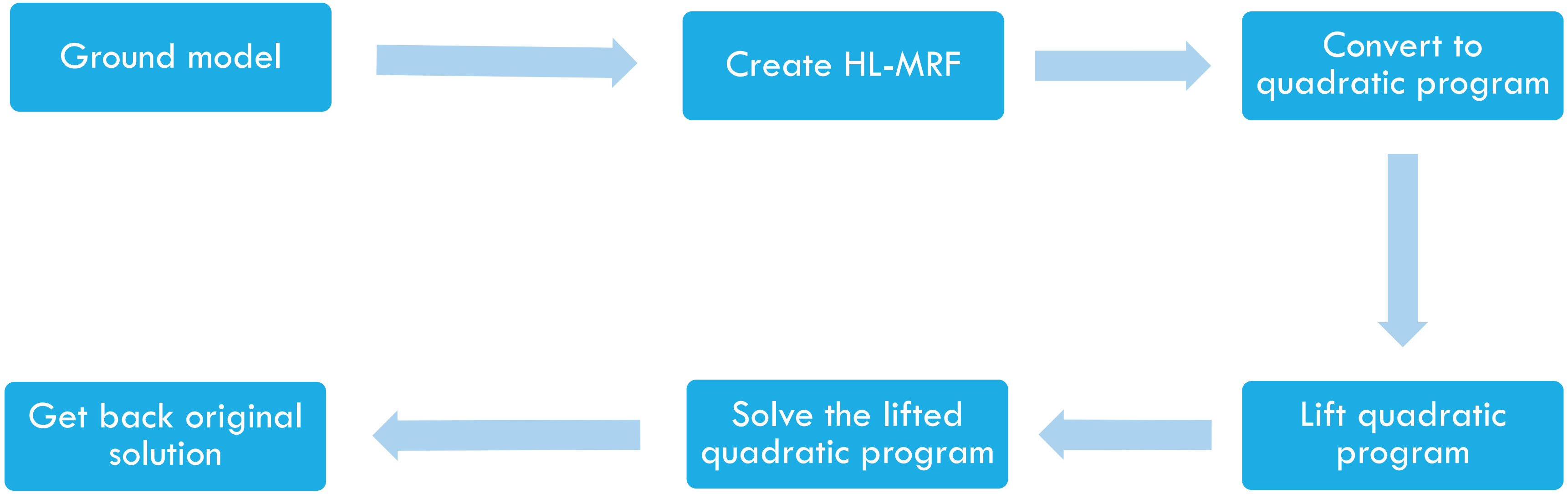
Binary variables only

- Lifted belief propagation: Singla and Domingos, AAAI 08; Kersting et al., UAI 09; Kersting et al., AAAI 10; Ahmadi et al., IJCAI 11
- Lifted variable elimination: Poole, IJCAI 03; de Salvo Braz et al., IJCAI 05
- Lifted variational inference: Bui et al., UAI 13, UAI 14; Mladenov and Kersting, UAI 15
- Search based approach: Gogate and Domingos, StarAI 10; den Broeck et al., IJCAI 11

- Lifted linear and quadratic programs: Mladenov et al., AISTATS 12; Mladenov et al., AAAI 17

Most relevant: do not make Boolean assumption

LIFTED INFERENCE THROUGH TRANSFORMATION



Refer to this approach as LHL-MRF(Gurobi)

Issue: Solving QP is much slower than solving HL-MRF objective directly

COLOR REFINEMENT ALGORITHM

- Graph isomorphism detection
- Efficient
- Iterative algorithm

Initialisation All vertices get the same colour.

Refinement Step Two vertices v, w get different colours if there is some colour c such that v and w have different numbers of neighbours of colour c.

Refinement is repeated until colouring stays stable.

https://materials.dagstuhl.de/files/15/15401/15401.MartinGrohe.Slides.pdf

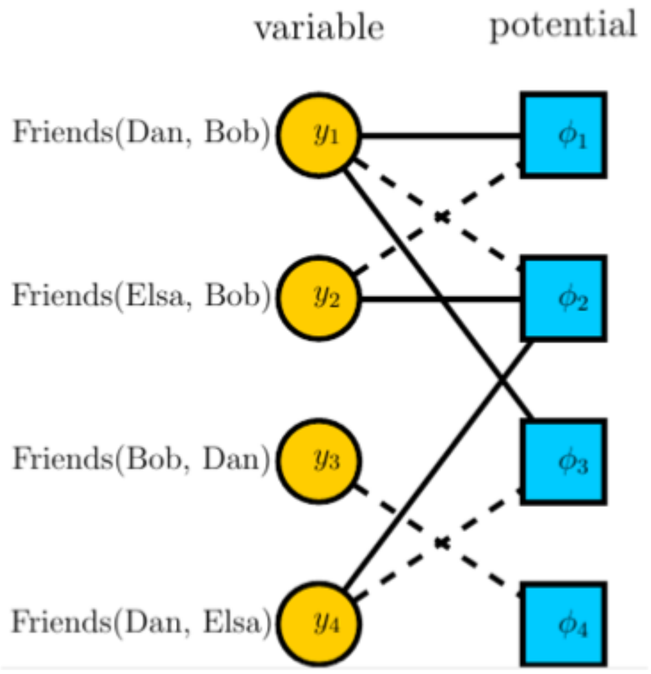
LIFTED HL-MRF APPROACH

- Construct factor graph representation G
- Initialize color such that
  - All unknown random variables y_i get same color
  - Two factors ϕ_j ϕ_m get same color if their weights, constants and power are the same
- Perform color refinement to obtains set of colors C^u and C^v
- Construct graph G' such that
  - Random variables y_k represents y_i \in C_k^u
  - Factors ϕ_l represents $\phi_j \in C_l^v$
  - Create edge between ϕ_l and y_k if $\exists\, e_{ij}\ i \in C_k^u\ and\ j \in$ \$C_l^v$
  - Value of edge $\frac{\sum_i x_{ki}}{|C_l^v|}$
  - Weights to the potential are summed

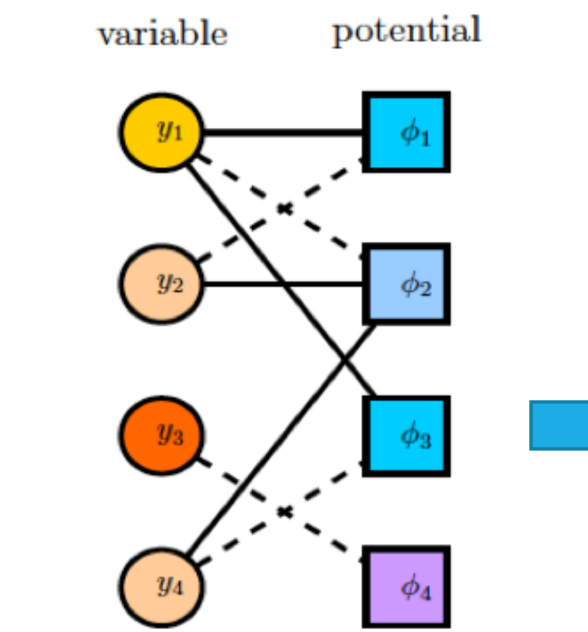
LIFTED HL-MRF

1: $Friend(X,Y) \wedge Friend(Y,Z) \rightarrow Friend(X,Z)$

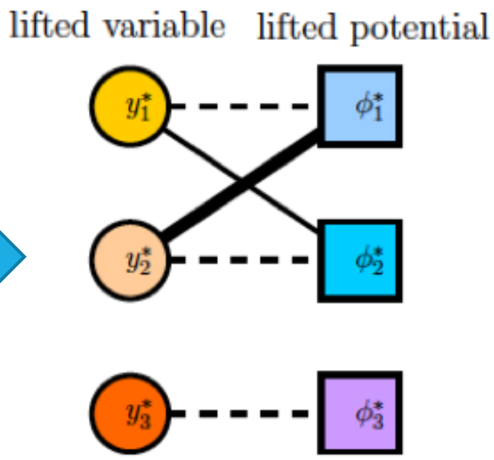
Ground with Bob, Dan, and Elsa



Coloring using
color refinement algorithm



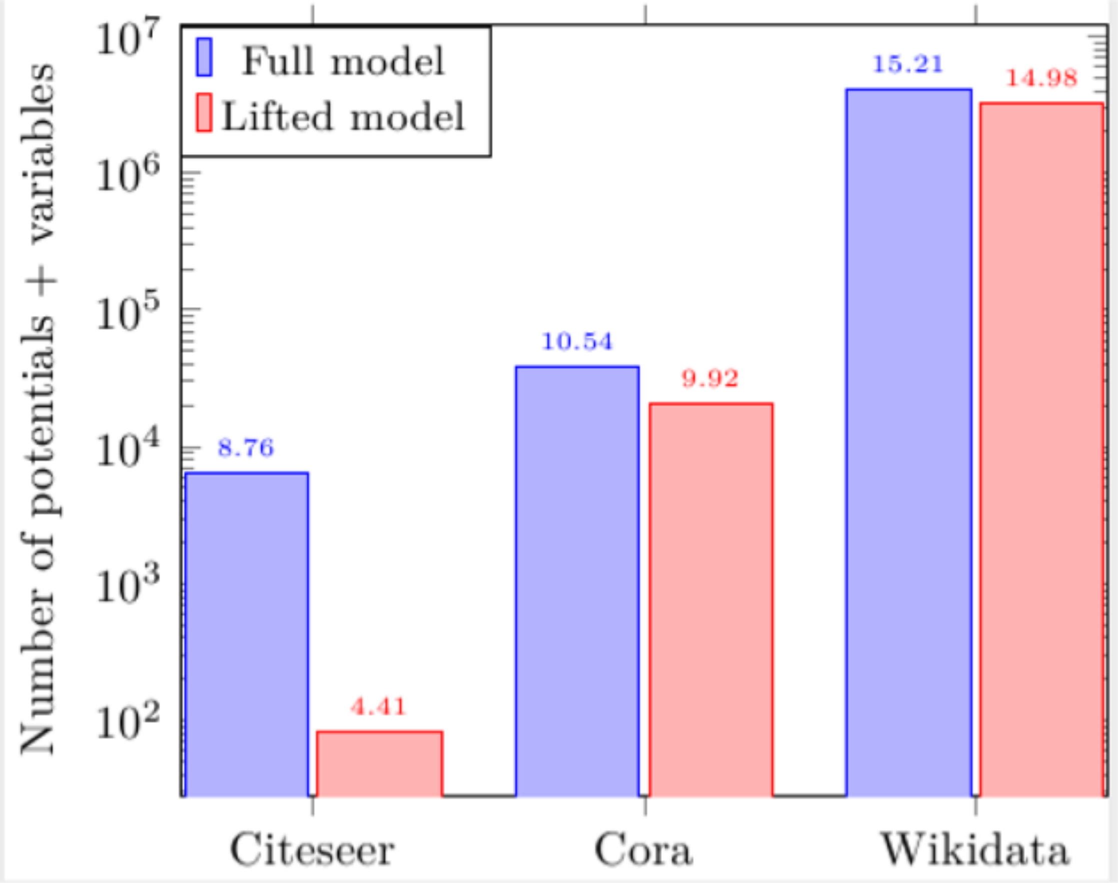
(b) Colored factor graph



(c) Lifted factor graph

EMPIRICAL EVALUATION

- Citeseer: dense graph, large reduction
- Cora: Less dense, lesser reduction
- Wikidata: Continuous predicate, lowest reduction



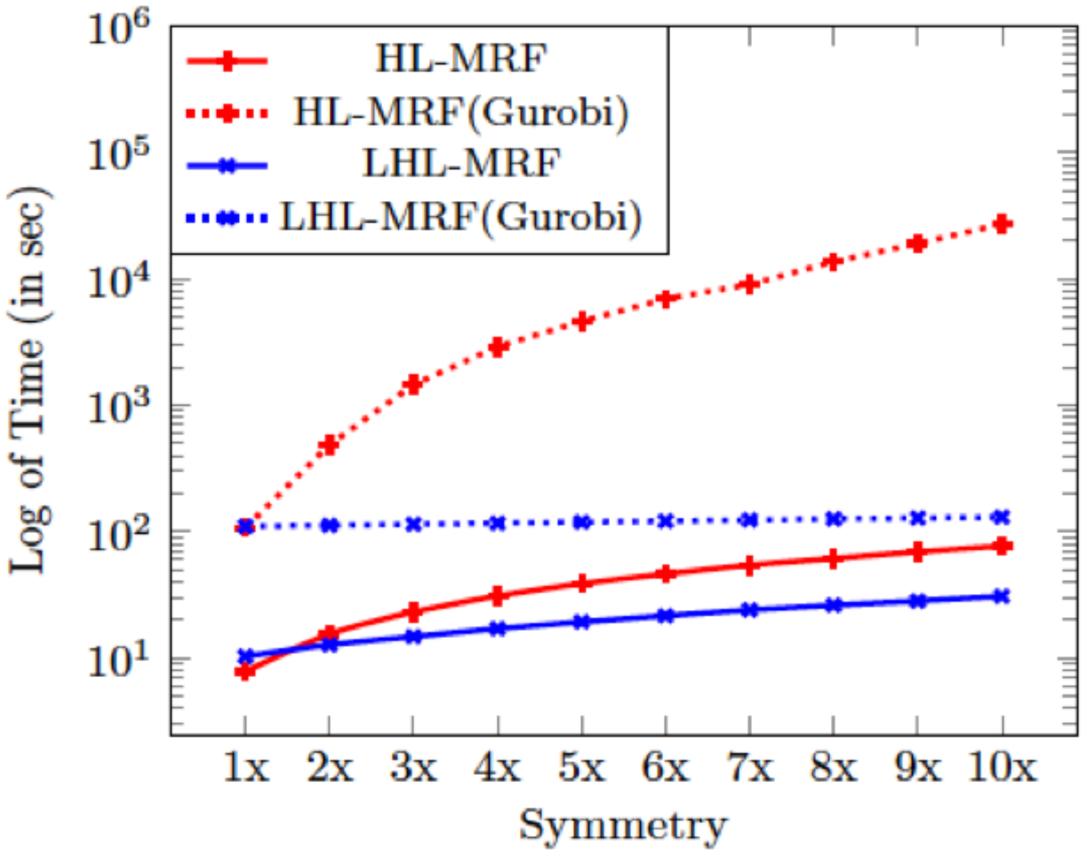
EMPIRICAL EVALUATION

Datasets	HL-MRF (in sec)	LHL-MRF (solving) (in sec)	LHL-MRF (lifting) (in sec)	LHL-MRF (total) (in sec)
Wikidata	636.0	463.7	112.7	576.4
Cora	47.7	17.5	0.53	18.03
Citeseer	57.4	19.8	0.39	20.19

Table 1: Time taken to perform inference on different datasets.

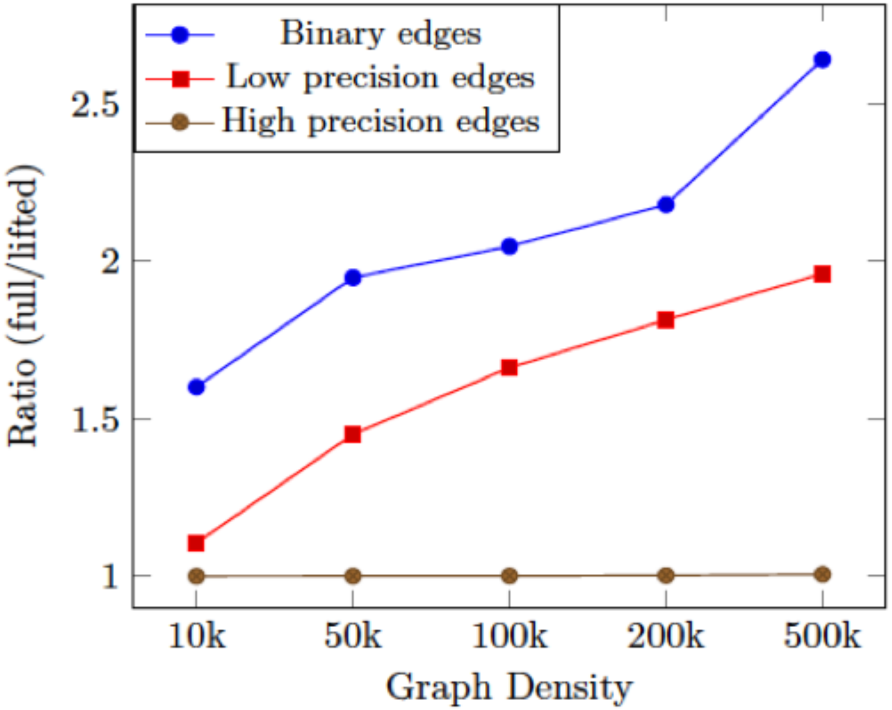
EMPIRICAL EVALUATION

- HL-MRF faster than solving QP (HL-MRF(Gurobi))
- LHL-MRF takes least time as symmetry increases



EMPIRICAL EVALUATION

- Synthetic dataset: Link prediction
- Observed edges: Binary or one decimal or four decimal
- Higher density implies more lifting
- Higher precision less lifting



CONTRIBUTIONS

- First lifted inference approach for HL-MRFs
- Show correctness of the approach
- Effectiveness on realworld datasets
- Show effectiveness compared to other lifted inference approaches
- Experiments on synthetic datasets to analyze effectiveness of approach