模式识别与机器学习第四章作业

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1 求S_w和S_b

设有如下三类模式样本集ω_1, ω_2, ω_3, 其先验概率相等, 求S_w和S_b:

$$\omega_1 : \{(1,0)^T, (2,0)^T, (1,1)^T\}$$
$$\omega_2 : \{(-1,0)^T, (0,1)^T, (-1,1)^T\}$$
$$\omega_3 : \{(-1,-1)^T, (0,-1)^T, (0,-2)^T\}$$

S_w:

$$S_w = \sum_{i=1}^{C} P(w_i) E\{(x-m_i)(x-m_i)^T / w_i\} = \sum_{i=1}^{C} P(w_i) C_i$$

solution:

$$m_1 = \frac{1}{3}(4,1) \Longrightarrow (\omega_{11}-m_1) = \frac{1}{3}(-1,-1), (\omega_{12}-m_1) = \frac{1}{3}(2,-1), (\omega_{13}-m_1) = \frac{1}{3}(-1,2)$$
$$m_2 = \frac{1}{3}(-2,2) \Longrightarrow (\omega_{21}-m_2) = \frac{1}{3}(-1,-2), (\omega_{22}-m_2) = \frac{1}{3}(2,1), (\omega_{23}-m_2) = \frac{1}{3}(-1,1)$$
$$m_3 = \frac{1}{3}(-1,-4) \Longrightarrow (\omega_{31}-m_3) = \frac{1}{3}(-2,1), (\omega_{32}-m_3) = \frac{1}{3}(1,1), (\omega_{33}-m_3) = \frac{1}{3}(1,-2)$$

$\omega_1 : E\{(\omega_1-m_1)(\omega_1-m_1)^T\}$

$$= \frac{1}{3}\{\frac{1}{3}(-1,-1)^T\frac{1}{3}(-1,-1) + \frac{1}{3}(2,-1)^T\frac{1}{3}(2,-1) + \frac{1}{3}(-1,2)^T\frac{1}{3}(-1,2)\}$$
$$= \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$\omega_2 : E\{(\omega_2-m_2)(\omega_2-m_2)^T\}$

$$= \frac{1}{3}\{\frac{1}{3}(-1,-2)^T\frac{1}{3}(-1,-2) + \frac{1}{3}(2,1)^T\frac{1}{3}(2,1) + \frac{1}{3}(-1,1)^T\frac{1}{3}(-1,1)\}$$
$$= \frac{1}{9}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$\omega_3 : E\{(\omega_3-m_3)(\omega_3-m_3)^T\}$

$$= \frac{1}{3}\{\frac{1}{3}(-2,1)^T\frac{1}{3}(-2,1) + \frac{1}{3}(1,1)^T\frac{1}{3}(1,1) + \frac{1}{3}(1,-2)^T\frac{1}{3}(1,-2)\}$$

$$= \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$S_w = \frac{1}{3} \times \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \frac{1}{3} \times \frac{1}{9}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + \frac{1}{3} \times \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

m_0:

$$m_0 = E(x) = \sum_{i=1}^{C} P(\omega_i) m_i = \frac{1}{3}m_1 + \frac{1}{3}m_2 + \frac{1}{3}m_3 = \frac{1}{3}(\frac{1}{3}, -\frac{1}{3})$$

$$w_1 : m_1 - m_0 = \frac{1}{3}(\frac{11}{3}, \frac{4}{3})$$

$$w_2 : m_2 - m_0 = \frac{1}{3}(-\frac{7}{3}, \frac{7}{3})$$

$$w_3 : m_3 - m_0 = \frac{1}{3}(-\frac{4}{3}, -\frac{11}{3})$$

S_b:

$$S_b = \frac{1}{3}\{\frac{1}{3}(\frac{11}{3}, \frac{4}{3})^T \frac{1}{3}(\frac{11}{3}, \frac{4}{3}) + \frac{1}{3}(-\frac{7}{3}, \frac{7}{3})^T \frac{1}{3}(-\frac{7}{3}, \frac{7}{3}) + \frac{1}{3}(-\frac{4}{3}, -\frac{11}{3})^T \frac{1}{3}(-\frac{4}{3}, -\frac{11}{3})\}$$

$$= \frac{1}{27}\begin{bmatrix} \frac{62}{3} & \frac{13}{3} \\ \frac{13}{3} & \frac{62}{3} \end{bmatrix}$$

S_t:

$$S_t = S_w + S_b = \frac{1}{27}\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} + \frac{1}{27}\begin{bmatrix} \frac{62}{3} & \frac{13}{3} \\ \frac{13}{3} & \frac{62}{3} \end{bmatrix}$$

2 用K-L变换，分别把特征空间维数降到二维和一维

设有如下两类样本集，其出现的概率相等:

$$\omega_1 : \{(0,0,0)^T, (2,0,0)^T, \{(2,0,1)^T, (1,2,0)^T\}$$

$$\omega_2 : \{(0,0,1)^T, (0,1,0)^T, \{(0,-2,1)^T, (1,1,-2)^T\}$$

用K-L变换，分别把特征空间维数降到二维和一维，并画出样本在该空间中的位置（可用matlab计算）

solution:

$$m = \frac{1}{4}\sum_{j=1}^{4}\omega_{1j} + \frac{1}{4}\sum_{j=1}^{4}\omega_{2j} = \frac{1}{4}(6,2,1) \neq 0$$

因此得到的是“次最佳”的结果。

step 1: R

$$R = \sum_{i=1}^{2} P(\omega_i) E(xx^T) = \frac{1}{2}[\frac{1}{4}\sum_{j=1}^{4}\omega_{1j}\omega_{1j}^T] + \frac{1}{2}[\frac{1}{4}\sum_{j=1}^{4}\omega_{2j}\omega_{2j}^T] = \frac{1}{8}\begin{bmatrix} 10 & 3 & 0 \\ 3 & 10 & -4 \\ 0 & -4 & 7 \end{bmatrix}$$

step 2: Φ

解特征值方程$|R-\lambda I|=0 \Longrightarrow$

$$\frac{1}{8}\begin{bmatrix} 10-\lambda & 3 & 0 \\ 3 & 10-\lambda & -4 \\ 0 & -4 & 7-\lambda \end{bmatrix}$$

求R的特征值.由$(10-\lambda)^2(7-\lambda)-16(10-\lambda)-9(7-\lambda)=0$ 得特征值

$$\lambda_1=-0.618, \lambda_2=1.618, \lambda_3=1$$

对应的特征向量为:

$$[0,-0.53,-0.85]$$

$$[0,-0.85,0.53]$$

$$[1,0,0]$$

step 1: y

选λ_2对应的变换向量作为变换矩阵，由$y=\Phi^T x$得变换后的一维模式特征为:

$$\omega_1:\{0,0,0.52,-1.70\}$$

$$\omega_2:\{0.52,-.085,2.22,-1.89\}$$

选λ_2,λ_3对应的变换向量作为变换矩阵，由$y=\Phi^T x$得变换后的二维模式特征为:

$$\omega_1:\{(0,0),(0,2),(0.52,2),(-1.70,1)\}$$

$$\omega_2:\{(0.52,0),(-0.85,0),(2.22,0),(-1.89,1)\}$$

样本在该空间中的位置（用python计算），一维模式特征如图1 。二维模式特征如图2 。

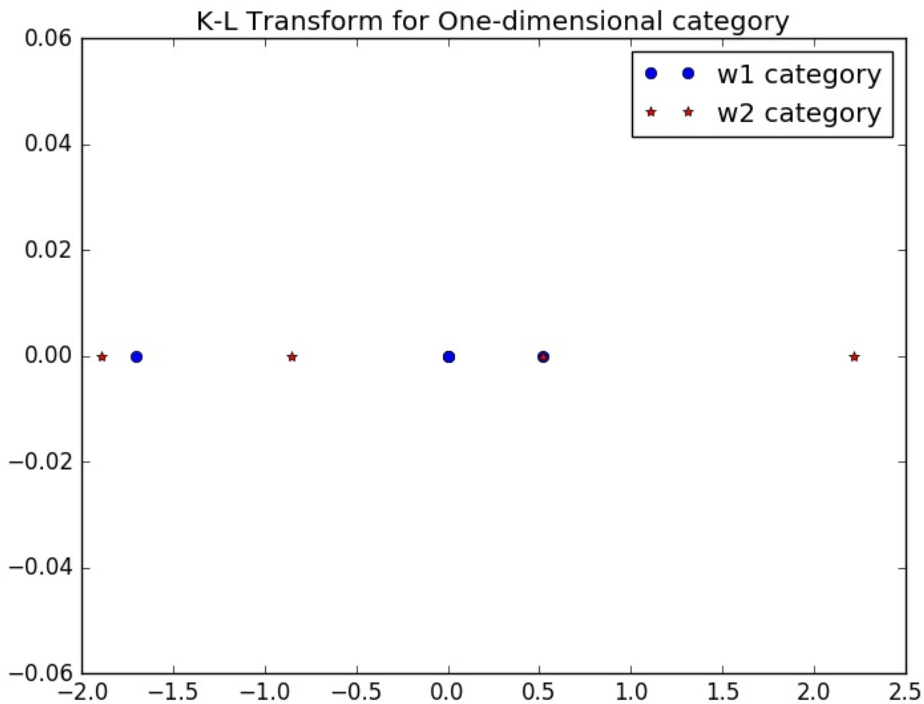

Figure 1: K-L Transform for One-dimensional category

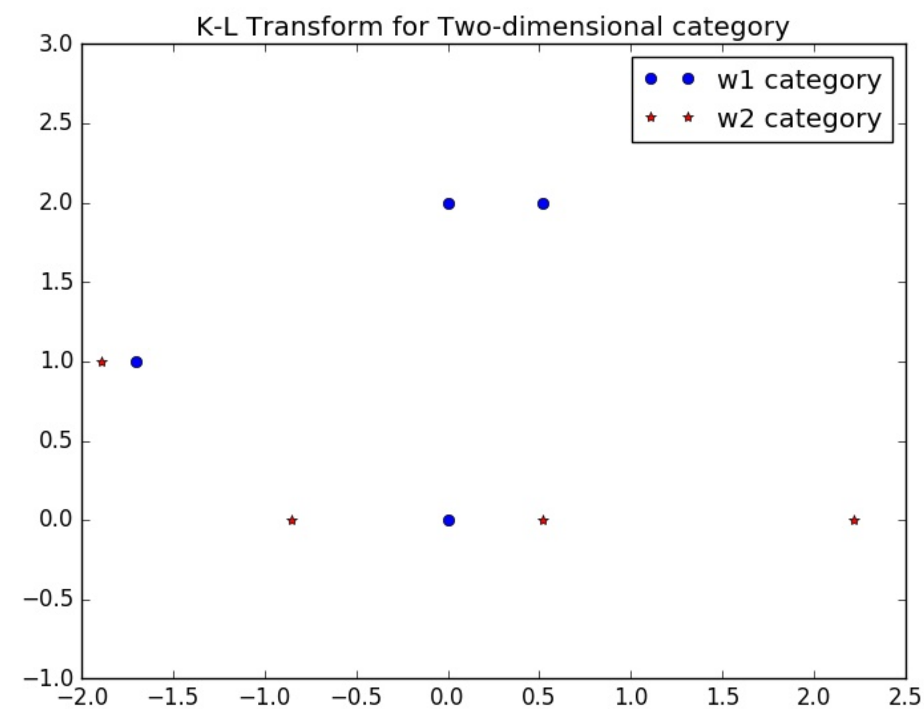

Figure 2: K-L Transform for Two-dimensional category