模式识别与机器学习第三章作业

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2016 年12 月10 日

1 将该问题看做一个四类分类满足情况匱的问题，将其中一类划分为一个子类，其余三类划分为第二个子类。第二个子类中用多类情况匳的方法判别$\frac{7\times(7-1)}{2} = 21$ 个判别函数，所以共需$21 + 4 = 25$ 个判别函数。

2 多类情况1 如图1 多类情况2 如图2 多类情况3 如图3.

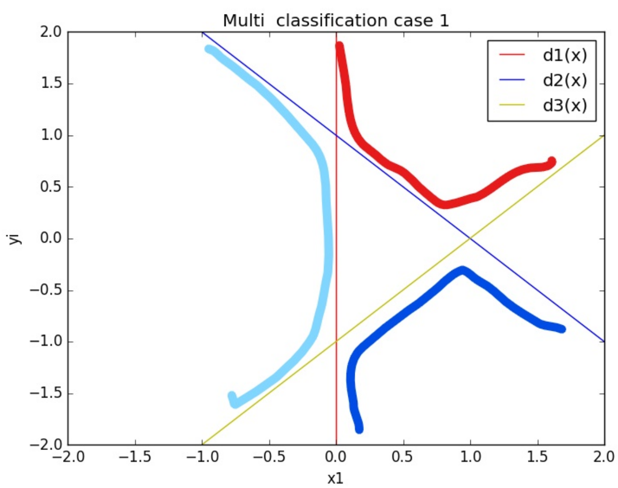

Figure 1: multi model ONE

3 如果这些模式线性可分，则至少需要匵个系数分量。建立二次多项式判别函数，则至少需要$C_5^2 = 10$ 个系数分量.

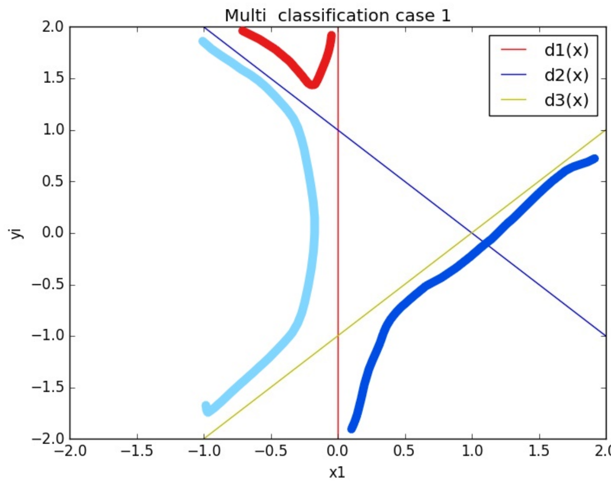

Figure 2: multi model TWO

4 将向量写成增广形式:

$$x_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$$
$$x_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}^T$$
$$x_3 = \begin{pmatrix} 1 & 0 & 1 & 1 \end{pmatrix}^T$$
$$x_4 = \begin{pmatrix} 1 & 1 & 0 & 1 \end{pmatrix}^T$$
$$x_5 = \begin{pmatrix} 0 & 0 & -1 & -1 \end{pmatrix}^T$$
$$x_6 = \begin{pmatrix} 0 & -1 & -1 & -1 \end{pmatrix}^T$$
$$x_7 = \begin{pmatrix} 0 & -1 & 0 & -1 \end{pmatrix}^T$$
$$x_8 = \begin{pmatrix} -1 & -1 & -1 & -1 \end{pmatrix}^T$$

取C=1, $\omega(1) = \left(\begin{smallmatrix} 0 & 0 & 0 \end{smallmatrix}\right)^T$

$$\omega^T(1)x_{(1)} = 0 = 0 \Rightarrow \omega(2) = \omega(1) + x_{(1)} = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$$
$$\omega^T(2)x_{(2)} = 1 > 0 \Rightarrow \omega(3) = \omega(2) = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$$
$$\omega^T(3)x_{(3)} = 1 > 0 \Rightarrow \omega(4) = \omega(3) = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$$
$$\omega^T(4)x_{(4)} = 1 > 0 \Rightarrow \omega(5) = \omega(3) = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$$
$$\omega^T(5)x_{(5)} = -1 < 0 \Rightarrow \omega(6) = \omega(5) + x_{(5)} = \begin{pmatrix} 0 & 0 & -1 & 0 \end{pmatrix}^T$$
$$\omega^T(6)x_{(6)} = 1 > 0 \Rightarrow \omega(7) = \omega(6) = \begin{pmatrix} 0 & 0 & -1 & 0 \end{pmatrix}^T$$
$$\omega^T(7)x_{(7)} = 0 = 0 \Rightarrow \omega(8) = \omega(7) + x_{(7)} = \begin{pmatrix} 0 & -1 & -1 & -1 \end{pmatrix}^T$$
$$\omega^T(8)x_{(8)} = 3 > 0 \Rightarrow \omega(9) = \omega(8) = \begin{pmatrix} 0 & -1 & -1 & -1 \end{pmatrix}^T$$

接下来按照这样一直循环下去，可以得到最后解为$\omega(41) = \left(\begin{smallmatrix} 2 & -2 & -2 & 1 \end{smallmatrix}\right)^T$ 对应的判别函数为

$$d(x) = 2x_1 - 2x_2 - 2x_3 + 1$$

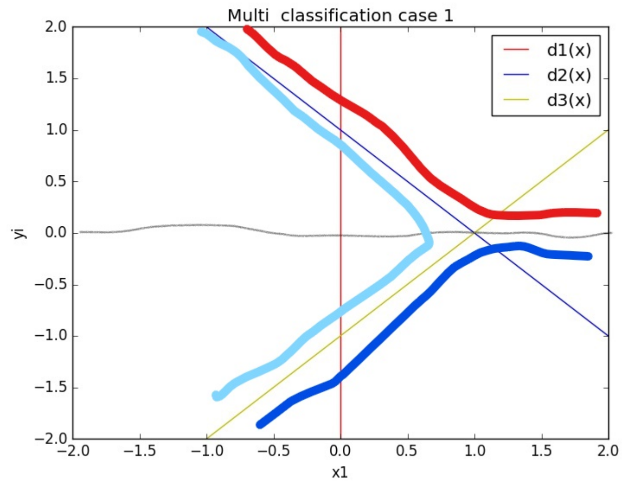

Figure 3: multi model THREE

5 将向量写成增广形式:

$$x_1 = \begin{pmatrix} -1 & -1 & 1 \end{pmatrix}^T$$
$$x_2 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$$
$$x_3 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$$

取初始值$C = 1, \omega_1(1) = \omega_2(1) = \omega_3(1) = (\,0\ 0\ 0\,)^T$
第一轮以$x_1 = (\,-1\ -1\ 1\,)^T$ 为训练样本:

$$d_1(1) = \omega_1^T(1)x_1 = 0 \Rightarrow \omega_1(2) = \omega_1(1) + x_1 = \begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$$
$$d_2(1) = \omega_2^T(1)x_1 = 0 \Rightarrow \omega_2(2) = \omega_2(1) - x_1 = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$$
$$d_3(1) = \omega_3^T(1)x_1 = 0 \Rightarrow \omega_3(2) = \omega_3(1) - x_1 = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$$

继续循环迭代直到满足$d_1(k) \leq d_2(k), d_1(k) \leq d_3(k)$ (k 是迭代次数,分类结果正确。得到权值向量为：

$$\omega_1 = \begin{pmatrix} -1 & -1 & 1 \end{pmatrix}^T$$
$$\omega_2 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$$
$$\omega_3 = \begin{pmatrix} 2 & 2 & -2 \end{pmatrix}^T$$

对应判别函数为:

$$d_1(x) = -x_1 - 2x_2 - 1$$
$$d_2(x) = 0$$
$$d_3(x) = 2x_1 + 2x_2 - 2$$

6

$$\frac{\partial J}{\partial \omega} = \frac{1}{4||x||^2}[(\omega^T x - b)|\omega^T x - b|^T][x - xsign(\omega^T x - b)]$$

其中，

$$sign(\omega^T x - b) = \begin{cases} 1, & \omega^T x - b > 0 \\ -1, & \omega^T x - b \leq 0 \end{cases}$$

由梯度法则可得

$$\omega(k+1) = \omega(k) + \frac{C}{4||x||^2}[(\omega(k)^T x - b)|\omega(k)^T x - b|^T][x - xsign(\omega^T x - b)]$$

其中x_k 是第k 次迭代的训练模式样本：

$$\omega(k+1) = \omega(k) + C\begin{cases} 0, & \omega^T x - b > 0 \\ \frac{x_k}{||x_k||^2}(b - \omega_k^T x_k), & \omega^T x - b \leq 0 \end{cases}$$

上式$\omega^T x - b > 0$ 时不对权向量进行修正，而$\omega^T x - b \leq 0$ 时进行修正。

7 由第一类势函数的定义可得势函数：

$$K(x, x_k) = \sum_{i=1}^{9} \varphi_i(x)\varphi_i(x_k)$$

给定训练样本：

$$\begin{aligned} x_{(1)} &= \begin{pmatrix} 0 & 1 \end{pmatrix}^T \\ x_{(2)} &= \begin{pmatrix} 0 & -1 \end{pmatrix}^T \\ x_{(3)} &= \begin{pmatrix} 1 & 0 \end{pmatrix}^T \\ x_{(4)} &= \begin{pmatrix} -1 & 0 \end{pmatrix}^T \end{aligned}$$

累计势位$K(x)$ 的迭代计算如下：

$$\begin{aligned} &1 : x_{(1)} = \begin{pmatrix} 0 & 1 \end{pmatrix}^T \in \omega_1, K_1(x) = -15 + 20x_2 + 40x_2^2 + 24x_1^2 - 32x_1^2 x_2 - 64x_1^2 x_2^2 \\ &2 : x_{(2)} = \begin{pmatrix} 0 & -1 \end{pmatrix}^T \in \omega_1, K_1(x_{(2)}) = 5 > 0, K_2(x) = K_1(x) \end{aligned}$$

将全部的样本进行循环迭代，直到所有样本都分类正确为止。此时的判别函数为：

$$d(x) = K_{10}(x) = -32x_1^2 + 32x_2^2$$

8 取$\alpha = 1$ 此时的势函数为：

$$K(x, x_k) = e^{-||x-x_k||^2} = e^{-[(x_1-x_{k1})^2+(x_2-x_{k2})^2]}$$

给定训练样本：

$$\begin{aligned} x_{(1)} &= \begin{pmatrix} 0 & 1 \end{pmatrix}^T \\ x_{(2)} &= \begin{pmatrix} 0 & -1 \end{pmatrix}^T \\ x_{(3)} &= \begin{pmatrix} 1 & 0 \end{pmatrix}^T \\ x_{(4)} &= \begin{pmatrix} -1 & 0 \end{pmatrix}^T \end{aligned}$$

累计势位$K(x)$ 的迭代计算如下：

$$\begin{aligned} &1 : x_{(1)} = \begin{pmatrix} 0 & 1 \end{pmatrix}^T \in \omega_1, K_1(x) = -e^{-x_1^2-(x_2-1)^2} \\ &2 : x_{(2)} = \begin{pmatrix} 0 & -1 \end{pmatrix}^T \in \omega_1, K_1(x_{(2)}) = e^{-4-0)} > 0, K_2(x) = K_1(x) \end{aligned}$$

将全部的样本进行循环迭代，直到所有样本都分类正确为止。此时的判别函数为：

$$d(x) = K_{10}(x) = e^{-x_1^2-(x_2+1)^2} + e^{-x_1^2-(x_2-1)^2} - e^{-(x_1+1)^2-x_2^2} - e^{-(x_1-1)^2-x_2^2}$$