Financial Instruments

Bus 35100

John Heaton

Homework 1

Due at the beginning of class week 2.

1 Arbitrage and Forward Rates

Consider a one-year forward contract for converting between dollars and Euros. The current exchange rate is \$1.20 for each Euro. The one-year risk-free rate in dollars is 5% in continuously compounded units while the one-year risk-free rate in Euros is 4.5%.

(1) According to the principle of no-arbitrage, what should be the one-year forward rate?

(2) Suppose that the one-year forward contract is currently trading at \$1.15 per Euro. Is there an arbitrage opportunity? If so, explain in detail the trading you would like to do to exploit this arbitrage opportunity.

2 Forward Rates and Covered Interest Rate Parity

The Excel file DataHW1_2024.xls contains data on the \$/Euro exchange rate on the first business day of October of the years 2005 to 2009. In addition, it contains Forward Rate quotes, as well as US and EURO LIBOR rates.

Please, do the following:

(1) For each date, use the Forward Rate formula discussed in Teaching Note 2 to compute the Forward Exchange Rate. Please do so for all of the given maturities (1 month, 3 months, 6 months, 1 year). (Tip#1: Note that LIBOR rates use linear compounding, while the formula in the teaching notes use continuous compounding. You will need the transformation like the one on page 14 of Teaching Note 2 to use the forward pricing formula we developed. Tip#2: The log function in the notes on page 14 is the *natural logarithm*. To implement this in Excel you use the function *LN* NOT the function *LOG*).

(2) Do your forward exchange rates match (approximately) the quoted ones?

(3) If not, pick one particular date in which the parity is violated, and describe the arbitrage strategy you would undertake.

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Homework 2

Due at the beginning of class 3

1 Exploiting an Apparent Arbitrage Opportunity

On Oct 1 2008 you spot a potential arbitrage opportunity in the foreign exchange market (see solution to HW 1). In particular, recall that the 1-year forward exchange rate USD/EUR seems too high to be justified by the current spot exchange rate and the LIBOR differential. You want to take advantage of the trade, and you set up an arbitrage trade. The data for this homework are in the excel file DataHW2_2024.xls

1. Set up the arbitrage trade on Oct 1, 2008 for the *1-year forward*. (*Tip: Yes, this is the same solution as in HW #1*)

2. After six months, on April 1, 2009, you decide to un-ravel the trade.

   (a) Given the new market data (exchange rate and interest rates), is the value of the short forward position positive or negative? Given your answer, do we lose money or gain money on the short forward position? (*Tip: Remember we are only looking at one part of the arbitrage trade, that is, only the short forward position. Note that after six months, the remaining horizon is only 6 months. The data set contains all of the possible data, but you do not need to use all of them. In fact, you need to use a small part of them)*

   (b) Did you make any money in the trade? Explain. (*Tip: You now need to also compute the value of the synthetic long forward. Recall that the synthetic long forward is made up of two pieces, a short leg (borrowing in dollars) and a long leg (investment in euros). To compute the value of the short leg, remember (see solution to HW1) that you borrowed at time 0 the dollar* *amount \$ $M_0 e^{-r_e \times T}$, where $T = 1$. That is, the dollar principal you will have to pay at maturity is \$$N = M_0 e^{(r_\$ - r_e)\times T}$. The time t value of the short leg is therefore the present value of this principal amount, which changes over time as the LIBOR changes. Similarly, to compute the time t value of the long leg, remember that you will receive 1 EUR at T from the investment. The dollar value of such investment at time t is $M_t e^{-r_e \times (T-t)}$, where M_t is the USD/EUR spot exchange rate. The value of the synthetic forward is then the sum of the short and long leg.*)

3. How would your answer to point 2 change if you unraveled the trade after 9 months, that is, on July 1, 2009 (i.e. 3 months before maturity)?

2 Commodity Futures

Consider an oil futures contract at time 0 with delivery T. Let S_t denote the spot oil price, r the continuously compounded interest rate, and u the storage cost in percentage of the oil price (i.e. the storage cost between t and $t + dt$ is $U_t = u \times S_t \times dt$. This is like a negative dividend yield for stocks $u = -q$).

- Question: Does the no arbitrage relation

$$F_{0,T} = S_t e^{(r+u)T}$$

  necessarily hold?

  - Try the alternatives $F_{0,T} < S_t e^{(r+u)T}$ and $F_{0,T} > S_t e^{(r+u)T}$ and see whether it is feasible to carry out the strategy.

3 Hedging with Futures: Southwest jet fuel hedge

When oil skyrocketed between mid 2006 and 2008 airline companies increased their use of commodity derivatives to reduce their exposure to raising jet fuel prices. The following example shows in a simplified fashion the effect of fuel hedging for Southwest.

On Dec 31st 2007 the COO of Southwest comes out with an estimate of expected fuel consumption for year 2008 of 1,511 million of gallons. On the same day the market price of jet fuel per gallon is \$2.71. On Dec 31st 2006 Southwest held positions in derivative contracts sufficient to hedge 100% of its forecasted fuel need in Q1 2007. Given the steep increase in oil (and jet fuel) prices during 2007, the company opted to reduce the hedge to 75% of its expected fuel consumption over Q1 2008.

To set up the hedge, the CFO decides to utilize NYMEX Crude Oil futures. Crude Oil Futures trade in units of 1,000 U.S. barrels (42,000 gallons), and they are available for maturities of 30 consecutive months[1]. In addition trading of such instruments terminates at the close of business on the third business day prior to the 25th calendar day of the month preceding the delivery month.

Table 1 provides the list of FEB.08, MAR.08 and APR.08 as of Dec 31st 2007.

Table 1. Crude oil Future prices on Dec 31st 2007, \$

Contract	Price/barrel
FEB.08	95.98
MAR.08	95.78
APR.08	95.24

Knowing that the expected continuously compounded interest rate was 0% for year 2008,

1. What strategy would you suggest to the CFO to best hedge the expected fuel consumption for Q1 2008 using the three contracts listed above? (*Tip: You can assume that fuel consumption and purchase occurs on the last futures contract trading date of each month (e.g. fuel needed for January is purchased and consumed on Jan 22 2008) and that fuel consumption is uniform across months.*)

On March 22nd 2008 the CFO wants to assess the hedging effectiveness of the adopted strategy. The excel file DataHW2_2024.xls provides daily data on the future prices of FEB.08, MAR.08 and APR.08 contracts until their last trading date, as well as the jet fuel prices during the same period.

[1]Long-dated futures are also available. They are initially listed 36, 48, 60, 72, and 84 months prior to delivery

2. What is the P&L of the hedging strategy? Was the hedging beneficial? To assess it, compute the jet-fuel price that Southwest would implicitly pay in Jan, Feb, and Mar 2008 and compare it to the actual prices. (*Tips: (i) Don't forget the size of each contract and the fact that each barrel contains 42 gallons. (ii) The implicit jet fuel price is the effective price per gallon that Southwest pays, given the total gains/losses from the future positions.*)

3. How would your result to point 2 change if the CFO had decided to keep the same 100% hedging strategy that was in force up to Dec 2007?

4. Compute the correlation between *changes* in jet fuel price and *changes* in Apr 08 futures prices. Is the correlation one? If not, can you provide an explanation? Think of at least two reasons why the correlation between jet fuel prices and oil futures may different from 1. (*Tip: Think about your result from the exercise 2 "Commodity Futures" above*)

On June 30th 2008, the CFO is asked to set up a similar strategy for the next three months. Table 2 provides the list of relevant future contracts and their prices on June 30th.

Table 2. Crude oil Future prices on June 30th 2008, \$

Contract	Price/barrel
AUG.08	140.00
SEP.08	140.58
OCT.08	140.95

5. Using the data contained in the Excel file DataHW2.xls compute the P&L of the hedging strategy between June 30th and September 22nd. Was the hedging beneficial? Compute again the implicit jet fuel prices and comment on the effectiveness of the hedging strategy. In doing so, please compare this result with the performance of the hedging strategy in point 2 above.

4 BONUS QUESTION (i,e, not required): Speculating with Futures: Amaranth calendar spread trade

Amaranth Advisors L.L.C. was a hedge fund that, as of Sep 2006, massively invested in energy derivatives. The fund held very large positions in natural gas futures. The trades consisted mainly of buying and selling natural gas futures contracts with a variety of maturity dates, setting up what is called a calendar spread.

The fund had positions on most of the futures contracts available, covering almost every month between September 2006 and December 2011. We will consider the positions in two of these many contracts[2] to get a practical understanding of what happened in Sep 2006.

Suppose that, as of Aug 31st 2006 Amaranth book was as follows:

Table 1. Amaranth simplified portfolio on Aug 31st 2006

Contract	**# of contracts**	**Futures price, $**
NOV.06	59,247	$ 8.23
APR.07	(77,527)	$ 8.34

The Excel file DataHW2_2924.xls contains daily futures prices of the two contracts between Aug 31st 2006 and Sep 21st 2006. The prices are per MMBtu (millions of British Thermal Unites). A contract is for 10,000 MMBtu.

Using these prices, compute:

(1) The daily and cumulative P&L from the strategy

(2) The value of the portfolio between Aug 31st and Sep 21st 2006, assuming that the portfolio was initiated on Aug 31st 2006

(3) The cumulative cash required by the strategy, knowing that NYMEX requires $5,400 initial margin and $2,700 maintenance margin per contract and assuming

[2] The positions considered are actual NYMEX Futures Equivalent Contracts: in a nutshell, Amaranth held positions in a variety of instruments (futures, options, swaps) with a given maturity, that, with some degree of approximation, are equivalent to the positions in the single future contracts reported in Table 1

that the cash in excess of the initial margin (if any) is daily withdrawn from the account.

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Homework 3

Due at the beginning of class 4

1 Greece Currency Swaps

On June 1st 2001 Greece issued a 10 year dollar denominated note for a value of USD 50 billion and a semi-annual coupon rate $c = 6\%$ *Hint:* The bond pays semiannual coupons in the amount of USD $(6\%/2) \times 50$ billion = USD 1.5 billion along with the face value of USD 50 billion at maturity.

Issuing debt denominated in a foreign currency is common. There are few reasons why this happens: a country might need to make payments in a foreign currency or foreign investors' appetites for diversification allows the country to issue at more favorable conditions. In this example we will assume that Greece has obtained more favorable conditions issuing in US dollars and the money will be used for domestic operations. At issuance Greece therefore has to convert into Euros any US dollars obtained from the note. To make the payments (semi-annual coupons and principal) to the investors that purchased the note, Greece must to convert Euros into US dollars at coupon dates and at maturity. This exposes the country to exchange rate risk. In particular if the US dollar appreciates, Greece will have to use more Euros to get an amount equal to the US dollar value of coupons and principal.

The Greek Finance Minister contacts VeroTende Investment Bank to ask about how to hedge the position, and the bank suggests a currency swap. The swap will require an initial transfer of principal, in which Greece will pay USD (US dollar) 50 billion and receive EUR (Euro) 59 billion at initiation of the contract. Every six months after that Greece will receive USD 1.5 billion in exchange for a fixed payment in EUR. *At maturity Greece will pay 59 billion EUR and receive 50 billion USD.*

Table 1 reports the prices of European and American Government zero coupon bonds on June 1st 2001.

Maturity	Greek ZCB	US ZCB
0.0	1.0000	1.0000
0.5	0.9786	0.9822
1.0	0.9588	0.9647
1.5	0.9388	0.9431
2.0	0.9191	0.9192
2.5	0.8989	0.8973
3.0	0.8788	0.8749
3.5	0.8583	0.8520
4.0	0.8379	0.8287
4.5	0.8177	0.8051
5.0	0.7977	0.7812
5.5	0.7780	0.7603
6.0	0.7583	0.7397
6.5	0.7370	0.7194
7.0	0.7155	0.6993
7.5	0.6953	0.6795
8.0	0.6751	0.6600
8.5	0.6559	0.6407
9.0	0.6369	0.6218
9.5	0.6208	0.6032
10.0	0.6050	0.5848

Table 1. Greek and US ZCB prices on June 1st 2001

Hint: to use these number consider the value of the value (in dollars) of a dollar denominated bond with annualized coupon rate of c and face value N. This value is given by:

$$B_0^{\$} = \sum_{t=0.5}^{T} \frac{c}{2} \cdot N^{\$} \cdot Z^{\$}(0,t) + N^{\$} \cdot Z^{\$}(0,T)$$

(1) Using the data above, and knowing that the USD/EUR spot exchange rate on June 1st 2001 is equal to 0.8475 USD/EUR (=50/59 USD/EUR), compute the swap rate (i.e. the annualized coupon rate for the semi-annual payments in EUR) that sets the value of the currency swap to 0 at initiation of the contract.

The VeroTende Bank executives meet Greek officials, describe the currency swap and quote the rate calculated in point (1). To their surprise, the Greek Finance Minister

explains that they have decided to be advised by another bank. They shake hands and the finance Minister leaves.
Some rumors say that Goldman Sachs has set up a currency swap with structured as above, but with two minor differences:[1]

a) For the exchange of principal at time t = 0 the exchange rate utilized is not the market spot rate (i.e. 0.8475 USD / EUR), but rather the historical average of the spot exchange rate between March 12th 2001 and June 1st 2001, that is 0.8148 USD / EUR;

b) The rate quoted for the payments in euro is 7.00%

(2) Compute the value of the Goldman swap. Why do you think Greece has decided to accept Goldman proposal rather than the one suggested by VeroTende Bank? In answering the question give particular thought to the cash flows of the swap and their timing.

2 Hedging with Options: Southwest Jet Fuel Hedging Program

Consider again the hedging strategy of Southwest Airlines (see Homework 2). We are back on December 31st 2007. Instead of using commodity futures, the CFO is considering buying insurance on oil prices using options. Table 2 reports the prices, on December 31st 2007, of a set of European call and put options on crude oil. All options expire on March 31st 2008 and can only be exercised at maturity. Each options' underlying is one lot of 1,000 crude oil barrels, which equals 42,000 gallons of crude oil.

[1]This exercise is motivated by a real deal between Goldman Sachs and Greece. However, **all** of the details in this exercise are fictitious, as details of the deal are not known. For initial reference, please visit https://www.goldmansachs.com/media-relations/in-the-news/archive/greece.html

Call options		Put options	
Strike Price	Option Price	Strike Price	Option Price
60.00	35,674	60.00	3
65.00	30,744	65.00	17
70.00	25,859	70.00	76
75.00	21,098	75.00	259
80.00	16,593	80.00	698
85.00	12,513	85.00	1,562
88.61	9,923	88.61	2,541
90.00	9,016	90.00	3,009
95.00	6,195	95.00	5,132
100.00	4,060	100.00	7,941
105.00	2,541	105.00	11,366
110.00	1,522	110.00	15,292
115.00	875	115.00	19,589
120.00	485	120.00	24,142
125.00	259	125.00	28,861
130.00	135	130.00	33,680

Table 2. Crude oil options prices on December 31, 2007, USD per lot

Given the data in Table 2:

1. Assume that the CFO decides to buy the \$105 strike call options on oil (we can refer to this strategy as a **straight insurance**).

   (a) How many options should be bought? In computing the number of options, make the (somewhat unreasonable) assumption that 1 barrel of oil is sufficient to produce 1 barrel of jet fuel, that a \$1 price change of crude oil per barrel barrel, always causes a \$1 price change of jet fuel per barrel, that is the same as a $\frac{1}{42} = \$0.02381$ price change of jet fuel per gallon and that all fuel is consumed on March 31st 2008.

   (b) How does your answer in (1.a) change if we assume that a \$1 price change of crude oil per barrel, always causes a \$1.1964 price change of jet fuel per barrel? Maintain the (still unrealistic) assumption that 1 barrel of oil is sufficient to produce 1 barrel of jet fuel.

(c) Suppose that you have bought the number of call options determined in (1.a). If the assumptions mentioned in point (1.b) are true, how would your (1.a) strategy perform? Would you be over-hedged or under-hedged?

(d) How does your answer in point (1.b) change if we assume that to produce 1 barrel of jet fuel one needs to employ 1.1964 barrels of oil? Why are the answers in point (1.b) and (1.d) different or the same? (Tip: don't spend more than 2 minutes on this. Just think about the implication for the relationship between jet fuel price and oil price if you assume that you need 1.1964 units of oil to produce 1 unit of fuel.)

(e) Under no hedging, Southwest position is *de facto* a short position on jet fuel between December 31st 2007 and March 31st 2008. Indeed if fuel price goes up Southwest is loosing money as it has to spend more to buy the same amount of fuel; conversely if jet fuel price falls, Southwest experiences a gain as it can buy the same quantity of fuel at a lower price. With this in mind, draw the payoff diagram, at maturity, of the implicit short position that Southwest has on jet fuel.

(f) Now draw the payoff diagram, at maturity, of the **straight insurance** strategy determined in (1.a) and, in another chart, the diagram of the overall position of Southwest, that is the implicit short position in jet fuel plus the **straight insurance**; please label the axes of the diagram, specify the units of measure and show some values on each axis. Compare the costs and benefits of the **straight insurance** with the costs and benefits of hedging with futures.

2. Assume that the CFO is a Booth alumni, and reasons that one drawback of the straight insurance is that it costs money upfront. Instead he decides to set up a **collar**, that is, sell some out-of-the-money put options to finance the purchase of out-of-the-money calls. The goal of this collar is that it must **cost nothing** (or near to nothing) to the firm at initiation. In other words, the total amount from the puts must compensate for the calls.

(a) Keeping the same strike for the insurance (call) as in point (1.a) above, find the strike of the put option that the CFO must sell to achieve the **zero-cost collar strategy**. To determine the total number of options (calls and puts), you make the same assumptions as in point (1.a) above.

(b) Draw the payoff diagram, at maturity, of the **collar** strategy determined in (2.a) and, in another chart, the diagram of the overall position of Southwest: the implicit short position in jet fuel plus the **collar**; please label the axes of the diagram, specify the units of measure and show some values on each axis. Compare the costs and benefits of the **collar** with the costs and benefits of both the straight insurance and the hedging with futures.

3. Assume now that the CFO is a Booth student who is taking the Financial Instruments course and is very excited about the variety of different options strategies. Instead of selling just one put option to finance the purchase of the call option (used for insurance) as in point 2, the CFO wants to experiment different quantities of put options, still maintaining the constraint that the cost of the strategy must be zero.

   (a) If the CFO chooses the \$80 strike put options, how many options does he have to sell to set up the collar?

   (b) How would your answer in point (3.a) change if the CFO chooses the \$90 strike put options?

   Draw the payoff diagrams, at maturity, of these two new strategies as well as of the overall Southwest position in each case. Please label the axes of the diagrams, specify the units of measure and show some values on each axis. Compare the costs and benefits of the two strategies with the costs and benefits of both the straight insurance and the first collar determined in point (2.a).

   (c) If you would have set up a bear spread, how would its payoff have compared with the payoff of the portfolio discussed in point (2.b) (i.e. the collar determined in point (2.a) plus the implicit short position on fuel)? Which strategy is more expensive? Why? (No need to carry out calculations; just provide an intuition. Tip: drawing payoff diagrams helps intuition with intuition.)

(d) While the CFO is on a plane to Italy, he thinks that he would like to know what is the continuously compounded interest rate implied by the options markets market. As "all electronic devices have been turned off", he does not have access to any additional data. He just remembers that the spot price of oil on December 31, 2007 was \$ 95 per barrel and that the maturity of the quoted options was $T = 0.25$ years. How can he compute the implied continuously compounded interest rate using option prices? (Tip: is there any relationship that holds between put and call prices?)

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Homework 4

Due at the beginning of class 5

1 Barings / Leeson position on Nikkei options

On February 26, 1995, Barings, Britain's oldest merchant bank, went bankrupt due to USD 1.3 billion (bn) losses in derivative securities from a single trader, Nick Leeson. The collapse occurred because Barings could not meet the enormous trading obligations established by Leeson in the name of the bank. When it went into receivership on February 27, 1995, Barings, via Leeson, had outstanding notional futures positions on Japanese equities and interest rates of USD 27 bn: USD 7 bn on the Nikkei 225 equity contract and USD 20 bn on Japanese government bond (JGB) and Euroyen contracts. But that's just part of the story. In addition to these positions, on December 24th 1994, Leeson also sold 35,500 Nikkei calls and 35,500 put options with the same strike price of $K = 19,750$ index points and the same maturity of $T = 2$ months. On the day Leeson sold the options, the Nikkei was trading at 19,634 points[1].

(1) What is the name of the option strategy (i.e. short 1 call and short 1 put with same strike price and same maturity) that Leeson set up?

(2) Under what condition is such a strategy profitable at maturity? What are the risks for a trader that sets up this strategy?

(3) Consider just 1 call and 1 put. Draw, on the same chart, the profit diagram at maturity, in Japanese Yen (JPY), of each option, as well as the profit diagram, still at maturity and in JPY, of the entire strategy (i.e. the combination of the

[1]The actual numbers have been slightly modified to simplify the exercise. For further reference please visit https://www.sciencedirect.com/science/article/pii/S0927538X0100004X

two options). Label the points where the profit of the strategy intersects the x axis, and show their values.

To carry out the computations you need to know that, on December 24th 1994, 1 Nikkei call option with strike equal to $K = 19,750$ was priced JPY 9.90 m and 1 Nikkei put option with strike equal to $K = 19,750$ was priced JPY 9.80 m. Assume that the 2-months interest rate is 0 (in this way you can show the full value of the option premium in the profit diagram, without worrying about time value of money) and that each Nikkei option trades on JPY 10,000 times the index value. (*Tip: You can think about everything using the contract size where 1 option is on 10,000 units of the index*)

(4) Consider now the scale of the entire strategy (i.e. 35,500 positions). Draw the profit diagram at maturity, in JPY, of the entire strategy. Label the points where the profits of the entire strategy intersects the x axis, and show their values.

On January 17 1995 the well known Kobe earthquake hit Japan's industrial heartland. That day the Nikkei 225 was at 19,350. It ended that week (on Friday January 20 1995) slightly lower at 18,950. Three days later, on Monday January 23, the Nikkei dropped 1,000 points to 17,950. The large declines in Japanese equities, post-earthquake, made the market more volatile.

(5) What is the effect of an increase in stock market volatility on Leeson position? No calculations needed, just a directional answer (i.e. position value increases / decreases) and provide intuition for why this happens. (*Tip: you may find it helpful to draw two profit diagrams for the strategy and to assume two different distributions of the Nikkei values. An very extreme, but helpful, simplification in the first chart you could assume that Nikkei prices at maturity are normally distributed, with mean equal to option strike price and with a given unknown variance - just draw a bell curve centered at K -, while in the second chart you could assume that Nikkei prices also have a normal distribution with the same mean as in the first chart, but with a much larger variance, so much bigger tails*)

By February 24th 1995 (the option maturity date) the Nikkei reached 17,473 points.

(6) What was the final profit / loss of Leeson Nikkei 225 options strategy on February 24th 1995?

As a final remark, the Baring / Leeson example is an interesting case study for many reasons. First, besides the option portfolio, Leeson's positions included massive bets on index futures. Second, most of the transactions were not authorized by Barings headquarters; Leeson was indeed able to create an account (named "Error Account 88888") through which he executed his unauthorized trades and, at the same time, hid the true size of his positions and P&L to the bank. The Baring / Leeson episode is often cited as an example of bad operational risk management. Even though one may tempted to view the Barings debacle as being caused by just one individual - the "rogue trader" (there is a movie about it!) - the fiasco should also be attributed to the underlying structure of the firm, in particular, to the lack of internal checks and balances.

2 Binomial Trees - The case of FDA drug approval

Vanda Pharmaceutical is a biopharmaceutical company with a focus on the development and commercialization of clinical-stage product candidates for central nervous system disorders. On July 28 2008 the company was waiting for the FDA's response to a Drug Approval Application submitted for "iloperidone", an investigational atypical antipsychotic that was reviewed for the treatment of schizophrenia. We will analyze FDA approval implications on Vanda stock price using binomial trees.
Today, $t = 0$ is July 28 2007; in 1 year's time, that is $t = 1$, depending on whether FDA approves the drug application, analysts believe that Vanda stock price can either be $S_{1,u} = 21$ with probability $q = 0.7$, or $S_{1,d} = 10$ with probability $(1 - q) = .3$. Assume that the firm has a CAPM beta equal to $\beta = 2$, that the annualized continuously compounded risk free rate is $r = 5.00\%$, and that the annually compounded Expected Excess Return on the market (or market risk premium) is $E[RP] = E[r_m] - r_f = 6.44\%$

(1) What is the expected return on the stock according to CAPM? (*Tip: note that the risk free rate provided is continuously compounded, but in the CAPM equation you need to plug the annually compounded equivalent*)

(2) What is the value S_0 of the stock on July 28 2007, that is 1 year before FDA's approval decision?

(3) Compute value of an at-the-money European call option with maturity $t = 1$ year using:

(a) **The dynamic replication methodology.** Please follow the steps listed below:

(i) Compute the time $t = 0$ position in stocks and bonds.

(ii) Compute the value of the portfolio at time $t = 0$.

(iii) Show that the replicating portfolio replicates the option payoffs on the tree.

(b) **The risk neutral methodology.** Please follow the steps listed below:

(i) Compute the risk neutral probabilities.

(ii) Compute the expected stock price at $t = 1$ (July 28 2008) under the risk neutral probabilities.

(iii) Compute the expected stock price at $t = 1$ under the probabilities assumed by the analysts. Are the two time $t = 1$ expected prices computed above equal or different? Discuss.

(iv) Compute the option value under the risk neutral probabilities. Is it the same as in (3.a.ii)?

(4) Fix the strike price of the call option to the value you calculated in part (3). How would the value of the option change if:

(a) the probability (assumed by the analysts) that FDA approves the drug becomes $q^{new} = 80\%$? Provide some intuition. (Hint: remember that this changes the implications for the current stock price given the other assumptions ...)

(b) the probability (assumed by the analysts) that FDA approves the drug becomes $q^{new} = 80\%$ and CAPM beta of the firm becomes $\beta^{new} = 3.14$? Provide some intuition.

(5) What is the value of a European at the money put option? To compute it you can use any methodology you like. Please show calculations.

(6) What is the value of an at the money forward contract (forward price equal to the current stock price)? As above, to compute it you can use any methodology you like. Please show calculations.

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Homework 5

Due at the beginning of class 6

1 Multiperiod binomial tree

In each of the next two periods ($i = 1$ and $i = 2$), a stock whose value in period $i = 0$ is $S_0 = 100$, can either rise or fall by 10% (using the notation of the teaching notes, $u = 1.1$ and $d = \frac{1}{u}$). Suppose the per-period risk-free rate is $r = 5\%$. A financial intermediary sold a European call option on the stock, with exercise price equal to $K = 100$.

1. What actions would the firm need to take in order to hedge its risk from writing the call? (*Tip: describe how you would set up a portfolio that the intermediary can use to mirror the payoffs of the option.* Note: answering that the firm can buy the same call will not get full credit!)

   (a) Using a two-period binomial tree, construct the replicating portfolio at each node. How much would you charge for the call option at period $i = 0$? What is the "delta" (the number of shares bought or sold in the replicating portfolio) and how much borrowing or lending is required at each node?

   (b) Let's see what happens if we change the initial stock price S_0 by a little. In particular I would like you to figure out how the value of the replicating portfolio changes when you change S_0. To do this compute the value of the replicated portfolio, V^{RP} when the initial stock price is equal to $S_0 + \triangle S$ with $\Delta S = 1, 2, ..., 10$. When doing this remember that future stock prices are also changed under the assumption that the stock price rises or falls by 10% going forward.

i. Is the change in value of the portfolio linear in ΔS? (*Tip: the change in value is linear in* ΔS *if* $V^{RP}(k \cdot \triangle S) = k \cdot V^{RP}(\triangle S)$ *).* Show your calculations.

ii. If your answer in the previous point was "No", would you describe the relationship between V^{RP} and S as concave or convex? What would be the sign of the second derivative of a function $V^{RP}(S)$?

(c) Use the initial assumption for S_0 for this and the remaining questions.

Is the portfolio "self-financing"? (*Tip: The portfolio is self-financing if you do NOT need any additional capital infusion between period* $i = 0$ *and the period of the final payoff* $T = 0$)

(d) What is the profit/loss on the replicating portfolio?

(e) What is the price of the call option if the stock pays a 5% dividend yield in period $i = 1$? Show the resulting trees for the stock price and for the option as well as your computations. (Hint: when the dividend is paid the stock price must decline in response. In the "up" state in period 1 before the dividend is paid, the stock prices is $S_0 \times u$. If y is the dividend yield, the stock price next period in the "up" state *after the payment of the dividend* is $S_0 \times u \times (1 - y)$. A similar adjustment must be made to the stock price in the "down" state after the dividend is paid.)

(f) What is the price of the call option if the stock pays a \$ 5 dividend in period $i = 1$? In this case the dividend is a fixed dollar amount that is independent of the stock price. Show the resulting trees for the stock price and for the option as well as your computations. Comment on the difference with part (e).

2 Black and Scholes (and Merton) Formula

Today is January 4, 2022. The stock price of Verotende Inc. (a well known publicly traded investment bank in early 2022) is trading at at USD 42. In addition the annualized stock return volatitiy is 20% and the annualized continuously compounded yield on 6 months T-bills is 10%.

1. Using the Black-Scholes formula, compute the price of a European call and a European put option, with identical exercise price $K^c = K^p = 40$ dollars, and six months to maturity. Show your calculations.

2. Compute the price of the call and put options using several initial stock prices values, from 10% in the money to 10% out of the money. Plot the price of the the two options as a function of the stock price in a chart. Would you describe the relationship between the two option prices and S as concave or convex?

3. Consider changing the inputs: compute the price of the call and of the put using different values for the parameters listed in the first column of Table (1). Report your findings on the relationship between options prices and inputs, by filling the white spaces in Table (1) (simply write: "increase", "decrease" or "uncertain" in each white space). Provide some intuition for your findings

	If input increases...	
Input	Change in call price:	Change in put price:
Stock Price		
Strike Price		
Volatility		
Maturity		
Risk free rate		

Table 1: Sensitivity of Black and Scholes option prices to inputs

4. If you sell an at-the-money put option on Verotende Inc. to a counterparty, and you want to hedge the short put using a position in stocks and bonds, what is going to be your position in stocks and bonds?

3 Big binomial tree

Use the file BinomialTree.xls available on chalk to confirm that as you increase the number of steps, the option price and delta from the Tree converges to the Black and Scholes ones. Report the prices and deltas in a table for $n = 2, 5, 10, 25, 50, 125, 250$ tree steps.

Financial Instruments

Winter 2024

John Heaton

Homework 6

Due at the beginning of class 8

1 Implied Volatility

Replicate the Implied Volatility of Teaching Note 6, using current options data on S&P500 maturing in June (3 months or one quarter from now). Please state the assumptions you make, if any, to compute the time to maturity of the options, that is the value of T that you use in your formulas. (No need to use many options. Choose just a few ITM, OTM and ATM. Note that the current SP500 index value is on the top right corner of the table.) The spreadsheet "Options.xls" provides an example of the implied volatility calculation. See the worksheet "Implied Volatility."

- Current Options Data can be found at CBOE. The ticker is SPX.
  `http://www.cboe.com/DelayedQuote/QuoteTable.aspx`

  NOTE: I have downloaded all of this data for you! Please see the file "Quote-Data_2024.xls."

- You can use federal reserve website (link below) to retrieve the value of the risk free rate. Use the Treasury Constant Maturity rate that most closely matches the maturity of the options. Note that TCM are compounded annually, so be sure to make the relevant adjustments. Please report the value of the rate that you use.
  `http://www.federalreserve.gov/releases/h15/Current/`

- The dividend yield can be estimated using the data collected by Robert Shiller which are available at the link below. You can estimate the dividend yield as the average dividend yield over the last available 12 months. Be sure to make the relevant compounding adjustments and report the dividend yield that you use.

`http://www.econ.yale.edu/~shiller/data.htm`

You should use the data in the "Excel file" used in his book. This gives monthly prices, dividends, earnings and other information.

2 Valuing and analyzing a structured security

(1) A wealthy investor hires you to help her evaluate a recent security issued by Morgan Stanley, called PLUS (see prospectus: The security was issued by Morgan Stanely in April 2008. Assume the same security is issued today and has one-year to maturity, but it is otherwise identical). To assess its fair value, you decide to use the appropriate implied volatility from the options markets as in Exercise (1).

- (a) How can you decompose the PLUS into more basic securities? (*Tip: The solution to the Mock Midterm might be helpful...*)
- (b) Use the decomposition obtained in point (2.1.a) and the information obtained in exercise (1) to value the PLUS.
- (c) If the value you obtain is not at par, what might you modify to make sure the value of the security is par as of today? (Words only, but for a bonus, see if you can actually set the value to par by changing one of the terms.)

(2) What is the sensitivity of this security to changes in the underlying stock price? How can you compute its market "beta", namely, the **percentage** sensitivity of the security to **percentage** changes in the underlying? Compute the "beta" of the PLUS for several stock prices "S" as of (a) today, (b) 6 month from now, (c) 1 year from now. Discuss your findings.

Morgan Stanley

March 2008

Pricing Supplement No. 528
Registration Statement No. 333-131266
Dated March 20, 2008
Filed pursuant to Rule 424(b)(2)

STRUCTURED INVESTMENTS

Opportunities in Equities

PLUS based on the Value of the S&P 500® Index due April 20, 2009

Performance Leveraged Upside Securities℠

The PLUS are senior unsecured obligations of Morgan Stanley, will pay no interest, do not guarantee any return of principal at maturity and have the terms described in the prospectus supplement for PLUS and the prospectus, as supplemented or modified by this pricing supplement. At maturity, you will receive for each stated principal amount of PLUS that you hold an amount in cash that may be more or less than the stated principal amount based upon the closing value of the underlying index on the valuation date.

FINAL TERMS			
Issuer:	Morgan Stanley		
Maturity date:	April 20, 2009		
Underlying index:	S&P 500® Index		
Aggregate principal amount:	$47,500,000		
Payment at maturity:	If final index value is *greater than* initial index value, $10 + leveraged upside payment *In no event will the payment at maturity exceed the maximum payment at maturity.* If final index value is *less than or equal to* initial index value, $10 x (final index value / initial index value) *This amount will be less than or equal to the stated principal amount of $10.*		
Leveraged upside payment:	$10 x leverage factor x index percent increase		
Index percent increase:	(final index value – initial index value) / initial index value		
Initial index value:	1,329.51, the index closing value of the underlying index on the pricing date		
Final index value:	The index closing value of the underlying index on the valuation date		
Valuation date:	April 16, 2009, subject to adjustment for certain market disruption events		
Leverage factor:	300%		
Maximum payment at maturity:	$11.90 (119% of the stated principal amount) per PLUS		
Stated principal amount:	$10 per PLUS		
Issue price:	$10 per PLUS (see "Commissions and Issue Price" below)		
Pricing date:	March 20, 2008		
Original issue date:	March 31, 2008		
CUSIP:	61747W166		
Listing:	The PLUS have been approved for listing on the American Stock Exchange LLC under the ticker symbol "SKE," subject to official notice of issuance. It is not possible to predict whether any secondary market for the PLUS will develop.		
Agent:	Morgan Stanley & Co. Incorporated		
Commissions and Issue Price:	**Price to Public(1)**	**Agent's Commissions(1)(2)**	**Proceeds to Company**
Per PLUS	$10	$0.15	$9.85
Total	$47,500,000	$712,500	$46,787,500

(1) *The actual price to public and agent's commissions for a particular investor may be reduced for volume purchase discounts depending on the aggregate amount of PLUS purchased by that investor. The lowest price payable by an investor is $9.95 per PLUS. Please see "Syndicate Information" on page 4 for further details.*

(2) *For additional information, see "Plan of Distribution" in the accompanying prospectus supplement for PLUS.*

The PLUS involve risks not associated with an investment in ordinary debt securities. See "Risk Factors" beginning on page 7.

The Securities and Exchange Commission and state securities regulators have not approved or disapproved these securities, or determined if this pricing supplement or the accompanying prospectus supplement and prospectus is truthful or complete. Any representation to the contrary is a criminal offense.

YOU SHOULD READ THIS DOCUMENT TOGETHER WITH THE RELATED PROSPECTUS SUPPLEMENT AND PROSPECTUS, EACH OF WHICH CAN BE ACCESSED VIA THE HYPERLINKS BELOW.

Amendment No. 2 to Prospectus Supplement for PLUS dated October 24, 2007

Prospectus dated January 25, 2006

Financial Instruments

Bus 35100 Winter 2024

John Heaton

Homework 7

Due at the beginning of class 9

1 Part 1: American Options

Let $S_0 = 100$, and consider a 3-period binomial tree model in which in every period, 1-year long, there is 60% chance of $u = 1.1$ increase, or a 40% chance of $d = 1/u$ decrease. Assume that the stock pays no dividends, and the continuously compounded interest rate is 2%.

(a) Compute the value of an American call and put option with strike price $K = 100$ (at-the-money).

(b) Are the American options computed in point (a) exercised before maturity? Discuss the intuition.

(c) Let the continuously compounded interest rate be 5%. What are the values of the two ATM American options? Compute the times of early exercise and discuss.

(d) In case (a)-(b), with interest rate still at $r = 2\%$, suppose that the firm now pays a 5% proportional dividend per year. How does your answer to point(c) change? Discuss.

2 Part 2: Citigroup's Default Probability during the Credit Crisis

It is Feb 2009, and you have been hired by the US Government to evaluate the effectiveness of the Paulson's plan to save the banks, a plan that was announced on October 13 2010. In particular, you have been assigned to evaluate the impact of the Paulson's plan on Citigroup's probability of default. In this homework you must compute the (true) probability of default of Citigroup by using the KMV model. The file *HW7_data.xls* contains important information about Citigroup (as well as JPMorgan and Goldman Sachs, but computing their probability of default is optional). In addition, it contains a simple solver to compute the Implied Volatility *and* Implied Stock value to match both a call option value and its volatility (see Teaching Notes 9). As you will see, you will need a lot of assumptions to use the KMV model, and so make sure to write the assumptions you make in your report (see below for some tips). For instance, how do you treat deposits and short-term debt?

Please, compute the probabilities of default at the following two dates:

10/10/2008 The day before the government announcement of the bailout program.

10/14/2008 The day after the announcement.

Proceed as follows:

- Compute the number of shares (you will need them to compute the market capitalization in the next point.)
- Compute the value of assets and volatility of assets to match the market cap and equity volatility at the two dates above. The assets and volatility of assets will be different across dates. Please, note the tips at the end.
- Compute the probability of default with one year to maturity. Note that KMV identifies the "Default Point" as short term debt + 1/2 long term debt.

After you do the calculations above, answer the following:

(a) What was the effect of the bailout announcement on the probability of default? Discuss.

(b) How does your answer to point (a) changes if you keep the volatility of assets σ constant to the value estimated on 10/10/2008 when you recompute the asset value at time 10/14/2008? (it is a simple change in the solver: only change assets but not volatility) Provide an intuition for the difference with (a).

(c) What was the credit spread before and after the announcement (choose what you think is most reasonable of your answers in (a) or (b), if you find any difference).

(d) The Paulson's Plan promised Citigroup a cash infusion of 25 billion from the US Treasury. Consider your calculated asset and equity values on 10/10/2008, and assume Citi suddenly gets the 25 bil cash infusion. What is the transfer to bond holders, if any? Discuss.

TIPS

This exercise is complicated by the existence of multiple type of securities. Clearly, we need assumptions, and no set of assumptions is particularly good (we just need a better model). Here is a set of assumptions that may be reasonable.

- To compute the value of equity, the residual claim after all of the debt, other liabilities, and deposits have been paid, you may assume:

  1. All deposits and short-term debt is super short-term, and in fact it will be paid instantly. This implies that for equity calculation purposes, $V - D - S$ is the relevant variable, where V is the total value of the firm, D is the total amount of deposits, and S is the total amount of short-term debt.

  2. Even after all short-term debt S and deposits D are paid out, you can still use Black and Scholes on the residual $V - D - S$ to compute the value of equity. Note that you still have long term debt L and other liabilities O to repay at T.

  3. Assume that the value of assets V has percentage volatility σ both before and after you pay short-term debt and deposits. This implies that the volatility of equity today (before paying out everything) is

  $$\text{Volatility of Equity} = N(d_1) \times \frac{V}{E} \times \sigma$$

  where both d_1 and E use $V - D - S$ instead of V.