Useful Formulas for 35100

- Forward contract, currency:

  $F_{0,T} = M_0 \times e^{(r_\$ - r_e)T}$

- Value of a forward contract to sell the underlying at $F_{0,T}$:

$$f_{t,T} = e^{-r_\$ \times (T-t)} \times (F_{0,T} - F_{t,T})$$

- Example of other forward contracts:

  - Stock with a fixed dividend yield q: $F_{0,T} = S_0 \times e^{(r-q)T}$
  - Stock: known dividend: $F_{0,T} = \left(S_0 - D \times e^{-rT_1}\right) \times e^{rT}$
  - Commodity: Storage cost U. $F_{0,T} = (S_0 + PV(U)) \times e^{rT}$
  - Commodity: % storage cost u. $F_{0,T} = S_0 \times e^{(r+u)T}$
  - Commodity: % storage cost u, convenient yield y $F_{0,T} = S_0 \times e^{(r+u-y)T}$

- Swap rate for one euro paid and K dollars received for every six months for the next in each of the next T years:

$$\text{Currency Swap Rate} = K = M_0 \frac{e^{-r_e \times 0.5} + e^{-r_e \times 1} + ... + e^{-r_e \times T}}{e^{-r_\$ \times 0.5} + e^{-r_\$ \times 1} + ... + e^{-r_\$ \times T}}$$

- Value of swap after initiation:

$$V_t^{swap} = K \times \left(\sum_{i=1}^{n} e^{-r_\$ (T_i - t)} \right) - M_t \times \left(\sum_{i=1}^{n} e^{-r_e (T_i - t)} \right)$$

- Plain vanilla swap (exchange of principal and coupons):

$$K = M_0 \times \frac{B^e(0,T)}{B^\$(0,T)}$$

- Put-Call Parity (no dividends, European Options):

$$\text{Put} = \text{Call} + e^{-r \times T} \times K - S_0$$

- Binomial Trees (e.g. with one-period):

  - Let $V_{1,u}$ be the value of the derivative when the stock goes up and $V_{1,d}$ be the value of the option when the stock goes down
  - Delta: $\Delta = \frac{V_{1,u}-V_{1,d}}{S_{1,u}-S_{1,d}}$
  - $B_0 = e^{-rT} \times (V_{1,u} - \Delta \times S_{1,u})$
  - Value of option at time 0: $V_0 = \Delta \times S_0 + B_0$
  - Risk-neutral pricing: $q^* = \frac{S_0 \times e^{r \times T} - S_{1,d}}{S_{1,u}-S_{1,d}}$
  - $V_0 = e^{-rT}(q^* V_{1,u} + (1-q^*)V_{1,d})$

- Black-Scholes formula with known dividend yield q.

$$c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2); \quad p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

- Options Sensitivities from Black-Scholes: *"The Greeks"*

  - **Delta:**

$$\Delta = \frac{d \text{ Option Price}}{d\,S} = \begin{cases} N(d_1) & \text{for Calls} \\ -N(-d_1) & \text{for Puts} \end{cases}$$

  - **Gamma:**

$$\Gamma = \frac{d\,\Delta}{d\,S} = \frac{e^{-qT}N'(d_1)}{S\sigma\sqrt{T}} \quad \text{with} \quad N'(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

  - **Theta:**

$$\Theta = \frac{d \text{ Option Price}}{d\,t} = \text{Long ugly formula}$$

  - **Rho:**

$$\text{Rho} = \frac{d \text{ Option Price}}{d\,r} = \begin{cases} KTe^{-rT}N(d_2) > 0 & \text{for Calls} \\ -KTe^{-rT}N(-d_2) < 0 & \text{for Puts} \end{cases}$$

  - **Vega:**

$$\text{Vega} = \frac{d \text{ Option Price}}{d\,\sigma} = S\sqrt{T}N'(d_1) > 0$$