**Financial Instruments**

**Bus 35100**

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**Homework 2**

Due at the beginning of class 3

# 1 Exploiting an Apparent Arbitrage Opportunity

On Oct 1 2008 you spot a potential arbitrage opportunity in the foreign exchange market (see solution to HW 1). In particular, recall that the 1-year forward exchange rate USD/EUR seems too high to be justified by the current spot exchange rate and the LIBOR differential. You want to take advantage of the trade, and you set up an arbitrage trade. The data for this homework are in the excel file DataHW2_2024.xls

1. Set up the arbitrage trade on Oct 1, 2008 for the *1-year forward*. (*Tip: Yes, this is the same solution as in HW #1*)

2. After six months, on April 1, 2009, you decide to un-ravel the trade.

   (a) Given the new market data (exchange rate and interest rates), is the value of the short forward position positive or negative? Given your answer, do we lose money or gain money on the short forward position? (*Tip: Remember we are only looking at one part of the arbitrage trade, that is, only the short forward position. Note that after six months, the remaining horizon is only 6 months. The data set contains all of the possible data, but you do not need to use all of them. In fact, you need to use a small part of them)*

   (b) Did you make any money in the trade? Explain. (*Tip: You now need to also compute the value of the synthetic long forward. Recall that the synthetic long forward is made up of two pieces, a short leg (borrowing in dollars) and a long leg (investment in euros). To compute the value of the short leg, remember (see solution to HW1) that you borrowed at time 0 the dollar*

*amount \$ $M_0 e^{-r_e \times T}$, where $T = 1$. That is, the dollar principal you will have to pay at maturity is $\$N = M_0 e^{(r_\$ - r_e)\times T}$. The time t value of the short leg is therefore the present value of this principal amount, which changes over time as the LIBOR changes. Similarly, to compute the time t value of the long leg, remember that you will receive 1 EUR at T from the investment. The dollar value of such investment at time t is $M_t e^{-r_e \times (T-t)}$, where $M_t$ is the USD/EUR spot exchange rate. The value of the synthetic forward is then the sum of the short and long leg.*)

3. How would your answer to point 2 change if you unraveled the trade after 9 months, that is, on July 1, 2009 (i.e. 3 months before maturity)?

# 2 Commodity Futures

Consider an oil futures contract at time 0 with delivery $T$. Let $S_t$ denote the spot oil price, $r$ the continuously compounded interest rate, and $u$ the storage cost in percentage of the oil price (i.e. the storage cost between $t$ and $t + dt$ is $U_t = u \times S_t \times dt$. This is like a negative dividend yield for stocks $u = -q$).

- Question: Does the no arbitrage relation

$$F_{0,T} = S_t e^{(r+u)T}$$

  necessarily hold?

  - Try the alternatives $F_{0,T} < S_t e^{(r+u)T}$ and $F_{0,T} > S_t e^{(r+u)T}$ and see whether it is feasible to carry out the strategy.

# 3 Hedging with Futures: Southwest jet fuel hedge

When oil skyrocketed between mid 2006 and 2008 airline companies increased their use of commodity derivatives to reduce their exposure to raising jet fuel prices. The following example shows in a simplified fashion the effect of fuel hedging for Southwest.

On Dec 31st 2007 the COO of Southwest comes out with an estimate of expected fuel consumption for year 2008 of 1,511 million of gallons. On the same day the market price of jet fuel per gallon is $2.71. On Dec 31st 2006 Southwest held positions in derivative contracts sufficient to hedge 100% of its forecasted fuel need in Q1 2007. Given the steep increase in oil (and jet fuel) prices during 2007, the company opted to reduce the hedge to 75% of its expected fuel consumption over Q1 2008.

To set up the hedge, the CFO decides to utilize NYMEX Crude Oil futures. Crude Oil Futures trade in units of 1,000 U.S. barrels (42,000 gallons), and they are available for maturities of 30 consecutive months[1]. In addition trading of such instruments terminates at the close of business on the third business day prior to the 25th calendar day of the month preceding the delivery month.

Table 1 provides the list of FEB.08, MAR.08 and APR.08 as of Dec 31st 2007.

**Table 1. Crude oil Future prices on Dec 31st 2007, $**

| Contract | Price/barrel |
|---|---|
| FEB.08 | 95.98 |
| MAR.08 | 95.78 |
| APR.08 | 95.24 |

Knowing that the expected continuously compounded interest rate was 0% for year 2008,

1. What strategy would you suggest to the CFO to best hedge the expected fuel consumption for Q1 2008 using the three contracts listed above? (*Tip: You can assume that fuel consumption and purchase occurs on the last futures contract trading date of each month (e.g. fuel needed for January is purchased and consumed on Jan 22 2008) and that fuel consumption is uniform across months.* )

On March 22nd 2008 the CFO wants to assess the hedging effectiveness of the adopted strategy. The excel file DataHW2_2024.xls provides daily data on the future prices of FEB.08, MAR.08 and APR.08 contracts until their last trading date, as well as the jet fuel prices during the same period.

[1]Long-dated futures are also available. They are initially listed 36, 48, 60, 72, and 84 months prior to delivery

2. What is the P&L of the hedging strategy? Was the hedging beneficial? To assess it, compute the jet-fuel price that Southwest would implicitly pay in Jan, Feb, and Mar 2008 and compare it to the actual prices. (*Tips: (i) Don't forget the size of each contract and the fact that each barrel contains 42 gallons. (ii) The implicit jet fuel price is the effective price per gallon that Southwest pays, given the total gains/losses from the future positions.*)

3. How would your result to point 2 change if the CFO had decided to keep the same 100% hedging strategy that was in force up to Dec 2007?

4. Compute the correlation between *changes* in jet fuel price and *changes* in Apr 08 futures prices. Is the correlation one? If not, can you provide an explanation? Think of at least two reasons why the correlation between jet fuel prices and oil futures may different from 1. (*Tip: Think about your result from the exercise 2 "Commodity Futures" above*)

On June 30th 2008, the CFO is asked to set up a similar strategy for the next three months. Table 2 provides the list of relevant future contracts and their prices on June 30th.

**Table 2. Crude oil Future prices on June 30th 2008, $**

| Contract | Price/barrel |
|---|---|
| AUG.08 | 140.00 |
| SEP.08 | 140.58 |
| OCT.08 | 140.95 |

5. Using the data contained in the Excel file DataHW2.xls compute the P&L of the hedging strategy between June 30th and September 22nd. Was the hedging beneficial? Compute again the implicit jet fuel prices and comment on the effectiveness of the hedging strategy. In doing so, please compare this result with the performance of the hedging strategy in point 2 above.

# 4 BONUS QUESTION (i,e, not required): Speculating with Futures: Amaranth calendar spread trade

Amaranth Advisors L.L.C. was a hedge fund that, as of Sep 2006, massively invested in energy derivatives. The fund held very large positions in natural gas futures. The trades consisted mainly of buying and selling natural gas futures contracts with a variety of maturity dates, setting up what is called a calendar spread.

The fund had positions on most of the futures contracts available, covering almost every month between September 2006 and December 2011. We will consider the positions in two of these many contracts[2] to get a practical understanding of what happened in Sep 2006.

Suppose that, as of Aug 31st 2006 Amaranth book was as follows:

Table 1. Amaranth simplified portfolio on Aug 31st 2006

| **Contract** | **# of contracts** | **Futures price, $** |
|---|---|---|
| NOV.06 | 59,247 | $ 8.23 |
| APR.07 | (77,527) | $ 8.34 |

The Excel file DataHW2_2924.xls contains daily futures prices of the two contracts between Aug 31st 2006 and Sep 21st 2006. The prices are per MMBtu (millions of British Thermal Unites). A contract is for 10,000 MMBtu.

Using these prices, compute:

(1) The daily and cumulative P&L from the strategy

(2) The value of the portfolio between Aug 31st and Sep 21st 2006, assuming that the portfolio was initiated on Aug 31st 2006

(3) The cumulative cash required by the strategy, knowing that NYMEX requires $5,400 initial margin and $2,700 maintenance margin per contract and assuming

[2]The positions considered are actual NYMEX Futures Equivalent Contracts: in a nutshell, Amaranth held positions in a variety of instruments (futures, options, swaps) with a given maturity, that, with some degree of approximation, are equivalent to the positions in the single future contracts reported in Table 1

that the cash in excess of the initial margin (if any) is daily withdrawn from the account.