

**Financial Instruments**  
**Winter 2024**  
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**Final Exam**

**INSTRUCTIONS:**

- There are **5** questions. **NOTE: Each question is on a separate page of the document. Make sure you try to do them all!**
- Points for each question: Question 1: 20 points, Question 2: 30 points, Question 3: 30 points, Question 4: 45 points, Question 5: 55 points.
- The allotted time on Canvas is 200 minutes (3 hours and 20 minutes). It should take you approximately 180 minutes to complete the exam. The number of points for each question are exactly equal to the number of minutes you should spend on the question. This should give you plenty of time to upload your solution.
- You are to submit a “pdf” file. **NO EXCEL** files or **CHATGPT output**. Please write with care and label all solutions.
- **NOTE: solutions without justification** (i.e. formulas showing what you calculated) will receive **no credit**. In other words: **show your work**.
- The questions cover various aspects of the class material. Read all the questions and start from those that you feel more comfortable with.
- Answer all questions as well as you can.
- Do not get hung up on calculations. Sometimes, just setting up an equation or providing a good intuitive argument will be sufficient for partial credit Remember to keep moving!
- Approximate all your calculations to **2 decimal points**.

1. (20 points) Short Answer questions. *Your grade depends on completeness of your answer.*
  - (a) (10 points) Discuss the advantages of Monte Carlo Simulation over the binomial model for the pricing and hedging of options. Are there any disadvantages to the Monte Carlo method relative to the binomial approach?
  - (b) (5 points) Consider an at-the-money European call option and an at-the-money European put option on the same underlying security with the same maturity. Which one has a higher theoretical market value? Why?
  - (c) (5 points) A colleague of yours is interested in forecasting the US dollar/Euro exchange rate to occur in the future. They are currently using traded futures prices for this purpose. Explain why futures prices are unlikely to be very good predictors of the future exchange rate.

2. (30 points)

Suppose that the current exchange rate between the US dollar and the Euro is 1.1 dollars per Euro. Consider the following information about risk-free yields in US dollars and in Euros (all in continuously-compounded annualized units):

Maturity	Dollar Rate	Euro Rate
6 months	4%	5%
1 year	4%	5%
18 months	4%	5%

- (a) (6 points) What should be the forward exchange rates for maturities of 6 months, 1 year, and 18 months?
- (b) (10 points) Suppose that a US firm currently has a bond outstanding with 18 months to maturity, face value of 100 million Euros, and a semi-annual coupon rate of 5% (stated at an annual rate and paid in Euros). The firm would like to enter into a swap contract to swap the payments in Euros (coupon payments and principal) for dollars at a fixed rate (pay dollars and receive Euros). What should the swap rate be?
- (c) (7 points) Suppose that the firm is offered a contract to swap coupon payments at the rate you calculated in part 2b but the principal would be swapped using the current exchange rate? Should the firm enter this contract? Why or why not? If it does, what would be the profit (or loss) from this transaction?
- (d) (7 points) Suppose that immediately after entering the swap contract in part 2b the yields on dollar and Euro instruments move to the following:

Maturity	Dollar Rate	Euro Rate
6 months	4%	4%
1 year	4%	4%
18 months	4%	4%

What would be the value of the swap contract that the firm entered into in part 2b?

## 3. (30 points)

JCH Corporation has assets with market value \$500 million. The expected return on assets is 10% per year (continuously compounded) with standard deviation of 25%. JCH has debt and equity outstanding. The debt is in the form of 400 zero coupon bonds each with a maturity of seven years and face value of \$1 million. Suppose that the risk-free rate is constant at 2% (annualized, continuously-compounded). Because of its bond covenants, JCH will not pay dividends over the next seven years. There are currently 500,000 shares outstanding.

- (a) (7 points) Apply the Black-Scholes model equation to price one share of JCH common stock.
- (b) (5 points) What would be the standard deviation of the return to equity?
- (c) (8 points) What would be the default risk premium for the debt of JCH and the probability of default?
- (d) (10 points) Suppose now half of the debt outstanding has a senior claim over the rest of the debt. What would be the risk-premia on the two types of debt in this case? What the probability of default by JCH be different? Why or why not?

## 4. (45 points)

Consider a stock with current price of 100. In each of the next 3 years the price will either increase by 20% (i.e.  $u = 1.2$ ) or decrease by 20% ( $d = 1/1.2$ ). The risk-free rate is 2% in annually continuously-compounded units. (Please use continuous compounding for discounting when completing your answers to the questions below.)

- (a) (10 points) If the stock does not pay a dividend, what should the price of an at-the-money European call option with **2 years** to maturity.
- (b) (5 points) Suppose that the European call option is trading at the price you calculated in part 4a and that an at-the money American call option also with **2 years** to maturity is trading but at a price of \$0.50 more than the European option. Is there an arbitrage possibility? Why or why not? If there is, indicate how you would take advantage of this opportunity.
- (c) (15 points) Suppose now that the stock will pay a special dividend in period 2 (end of the second year) in the amount of \$10 if the stock is trading at or above \$100 at that time. The proportional changes in the stock prices after dividend payments are unchanged by this announcement. What should be the price of an at-the-money American call option with **3 years to maturity**? In what period(s) and what state(s) for the stock price would you expect to exercise the option?
- (d) (15 points). Suppose that you sold the option in part 4c and you set up a dynamic hedge using the stock and bond. When you enter period 2 with your hedged position and before the dividend is paid you find out that the person to whom you sold the option will not be exercising the option regardless of the stock price. How would you design your hedge for each state so that you cover your obligation in period 3? Evaluate the combined value of the hedged position in stocks and bond in period 2 for each node. Do you make a profit on the hedged position?

5. (55 points)

Consider the following “Structured Product” with maturity of one year which is benchmarked against an index called “JCH.” The index is currently trading at \$1,000. The payoff at maturity to the structured product is a function of the return to the index over the next year:

**Payoff of the Security in One Year**

Index Return	Payoff
Less than 0	\$1,000
Between 0 and 20%	$\$1,000 + \$1,000 \times 200\% \times \text{Index Return}$
Greater than 20%	\$1,400

- (a) (5 points) Sketch the payoff diagram for this security as a function of the value of the index at maturity.
- (b) (5 points) Suppose that risk-free yields are 2% (continuous-compound annual units) at all maturities. Suppose further that you observe two European put options trading on JCH. These put options are written on one unit of the index and have maturity of 1 year. The index does not pay dividends. The first put option has a strike price of \$1,000 and is trading at a price of \$50. The second put option has a strike price of \$1,200 is trading at a price of \$188. Given these prices, what should be the price of the structured product?
- (c) (10 points) Again assume that the index is currently trading at \$1,000, pays no dividends and risk-free rates are 2%. Suppose further that you are convinced that the assumptions of the Black-Scholes model are appropriate and your forecast for the standard deviation of the index return is 30%. According to the model would you want to sell the structured product in the market at the price from part 5b? Why or why not?
- (d) (15 points) Assume that you plan to sell the structured product and would like to hedge the position by holding a position in the underlying and bonds.
  - i. If the index is currently trading at \$1,000 what positions in the index and bonds would you take to hedge your position? What would be the  $\beta$  of your position (with respect to the index)?
  - ii. If the index is currently trading at \$1,300, what positions in the index and bonds would you take to hedge your position? What would be the  $\beta$  of your position (with respect to the index)?
- (e) (10 points) Are the answers to part 5(d)i and 5(d)ii different? If they are different, what explains the differences? Provide intuition for why the  $\beta$ 's are different (if they are).
- (f) (5 points) In constructing your hedging strategy and theoretical value you made the Black-Scholes assumptions. The model value was compared to the “market” value of part 5b to determine if there was an opportunity. By selling the product, with what aspect of the market's views are you disagreeing? How might you change your assumptions as inputs to the Black-Scholes model to better reflect the market's views and why? (Answer this qualitatively, i.e. in words only.)