Financial Instruments
Fall 2022
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MOCK FINAL

1. (30 points)

   The term structures in both the U.S. and Euro zone are flat. The continuously-compounded risk-rate rates are $r_{\$} = 4\%$ in the U.S. and $r_e = 6\%$ in Euros. You are interested in entering into a three-year currency swap in which each year you pay 3% per year in dollars and receive Euros. The principal amounts in the two currencies are \$10 million and 15 million Euros. (Today you receive \$10 million today and pay 15 million Euros. At maturity you pay \$10 million and receive 15 million Euros.) The current exchange rate is 0.6600 USD/EUR.

   (a) (15 points) What is the market Euro coupon rate for this swap?

   Solution:

   Let us consider the party who pays dollars and receives Euros. Each year the dollar payment is $3\% \times \$10$ million $= \$0.3$ million and the Euro receipt is x million Euros where x is to be determined. At initiation of the swap contract, the value of the swap to this party (as well as to the counter party) is zero. Therefore we have:

$$\begin{aligned} 0 &= 10 - 0.66 \times 15 \\ &\quad -0.3 \times (e^{-0.04} + e^{-2\times 0.04} + e^{-3\times 0.04}) \\ &\quad +0.66 \times x \times (e^{-0.06} + e^{-2\times 0.06} + e^{-3\times 0.06}) \\ &\quad -10 \times e^{-3\times 0.04} + 0.66 \times 15 \times e^{-3\times 0.06} \end{aligned}$$

   which gives $x = 1.268$ million Euros. The Euro coupon rate is therefore $0.757171/15 = 8.45\%$.

   (b) (15 points) Six months after entering into this swap, the market exchange rate becomes 0.64 USD/EUR. What is the value of the swap contract to you? What is the value of the swap contract to the counter party to the swap?

   Solution:

   Six months later the value of the swap to you is:

$$\begin{aligned} V &= -0.3 \times (e^{-0.5\times 0.04} + e^{-1.5\times 0.04} + e^{-2.5\times 0.04}) \\ &\quad +0.64 \times 0.757171 \times (e^{-0.5\times 0.06} + e^{-1.5\times 0.06} + e^{-2.5\times 0.06}) \\ &\quad -10 \times e^{-2.5\times 0.04} + 0.64 \times 15 \times e^{-2.5\times 0.06} \\ &= -\$303,379 \end{aligned}$$

   The swap contract is worth \$303,379 to the counter party.

2. (50 points)

It is September 1, 2011 and you have received a job offer from Company ABC. The compensation package includes a salary of \$100,000 (in present value) plus 5,000 options on the company stock, whose features are described below. The deadline to accept the offer is in six months, that is March 1, 2012. Denote by $t = 0, 1, 2, 3$ the dates 9/1/2011, 3/1/2012, 9/1/2012 and 3/1/2013 respectively. Assume that the company stock price today ($t = 0$) is $S_0 = \$100$ and that every 6 months the stock moves up or down by 10% with equal probability $p = 1/2$. Finally, the *simple* semi-annual interest rate is 2%. In other words if r is the continuously compounded six-month rate, $\exp(0.5 \times r) = 1.02$.

(a) (15 points) Assume that the options will be issued at-the-money on the day of your acceptance (March 1, 2012). That is, the strike price of the options will be set equal to the stock price S_1 (whatever that will be) on $t = 1$. These options will expire in one year from the data of issuance, that is on March 1, 2013 ($t = 3$). What is the value of your compensation package?

Solution:

The risk-neutral probability is

$$q^* = \frac{\exp(0.5 \times r) - d}{u - d} = \frac{1.02 - .9}{1.1 - .9} = .6$$

If stock goes up to $S_u = 110$, then $K_u = 110$. The value of the option then is

$$C_u = \frac{{q^*}^2 (133.1 - 110) \times 5000}{1.02^2} = 39,965.4$$

If stock goes down to $S_d = 90$, then $K_d = 90$. The value of the option then is

$$C_d = \frac{{q^*}^2 (108.9 - 90) \times 5000}{1.02^2} = 32,695.96$$

The value of the option today is

$$C^a = \frac{q^* C_u + (1 - q^*) C_d}{1.02} = 36,332.1799$$

The value of the compensation package is \$136,332.18.

(b) (10 points) If the options were issued at-the-money today ($t = 0$) and had one year to maturity (i.e. they would expire on $t = 2$), what would be the value of your compensation package today? How does it compare with your answer to (a)? Can you provide a brief intuitive explanation for your results?

Solution:

The value of the option

$$C^b = \frac{{q^*}^2 (121 - 100) \times 5000}{1.02^2} = 36332.1799 = C^a$$

It is the same value as in (a). In the risk-neutral world all securities have an expected growth rate equal to r (continuously compounded). Since the strike price

K to be set in six months is a security, its expected growth rate is r. Hence, when we discount back the expected value of C to today, we are discounting the expected strike prices at their expected growth rate leading to the result.

(c) (10 points) Suppose that company ABC gives you the choice between (A) the options described in part (a) and (B) a sign-on check for \$35,000 to be paid on the date of your acceptance. When you call ABC to accept their offer, you will have to communicate whether you prefer (A) or (B). What is the value of your compensation package now?

Solution:

If S goes up, we keep the options and hence the value is $C_u = 39965.4$. If S goes down, we have $C_d = 32695.96$ and hence we want the sign-in check of \$35,000. The value of the compensation package is

$$10000 + \frac{q^* \times 39965.4 + (1 - q^*) \times 35000}{r} = 137,234.5478$$

(d) (15 points) It is now March 1, 2012 ($t = 1$) and you decided to accept the offer in (a). You need money upfront to pay for moving expenses but the company restricts you from selling the options in the compensation package before maturity. However, you are allowed to sell (short) the stock. Assume that the stock price moved up between $t = 0$and $t = 1$. Devise a dynamic strategy equivalent to the sale of the options, which would allow you to raise around \$8,000 today and yet bear no risk at all.

Solution

The value of the options along the tree (starting from $S_1 = 110$) are: $C_1 = 7.993$, $C_{2u} = 13.588$, $C_{2d} = 0$, $C_{3uu} = 23.1$, $C_{3du} = 0$, $C_{3dd} = 0$. Hence, the deltas of the option are:

$$\begin{aligned}
\Delta_1 &= \frac{13.588 - 0}{121 - 99} = .6176 \\
\Delta_{2u} &= \frac{23.1 - 0}{133.1 - 108.9} = .9545 \\
\Delta_{2d} &= \frac{0 - 0}{108.9 - 89.1} = 0
\end{aligned}$$

Hence, to replicate one short call we sell $\Delta_1 = .6176$ stocks for $\Delta_1 S_1 = 67.9412$, keep $C_1 = 7.993$ and invest in T-bills $L_1 = \Delta_1 S_1 - C_1 = 59.9481$. If the stock goes up we sell $(\Delta_{2u} - \Delta_1) S_{2u} = 40.7647$ and invest in T-bills. Overall, we are short $\Delta_{2u} = .9545$. If the stock goes up again to $S_{2uu} = 133.1$, our calls (in the compensation package) are in the money and we buy it back using $59.9481 \times 1.02^2 + 40.7647 \times 1.02 + (133.1 - 110) = 127.05 = \Delta_{2u} S_{2uu}$. If the stock goes down to $S_{2ud} = 108.9$, the calls in the compensation package are out of the money and we buy the stock back with $59.9481 \times 1.02^2 + 40.7647 \times 1.02 = 103.95 = \Delta_{2u} S_{2ud}$. Similarly if the stock went down at time $t = 2$.

Finally, to obtain around \$8000 we just need to multiply all the above by 1000.

3. (55 points) Suppose that stock JCH is trading at $S_0 = \$100$, it pays a continuous dividend yield equal to 5% and it has a volatility equal to 20%. Assume that the (annualized, continuously compounded) interest rates is 3% and that all the following options are European.

(a) (10 points) What is the delta of a short position in a 1 year, at-the-money call option on JCH stock? What about the delta of a short position in a 1-year, at-the-money put option?

Solution:

From the data it easy to see that

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} = 0$$

hence $\mathcal{N}(0) = 1/2$. However, since the call is paying a continuous dividend yield we have

$$\Delta_c = e^{-qT}\mathcal{N}(d_1) = .4756$$

Similarly, the delta of the at the money put is

$$\Delta_p = -e^{-qT}\mathcal{N}(-d_1) = -.4756$$

The short positions have opposite signs.

(b) (10 points) You have a short straddle position (short one call and put with the same strike price and maturity). What is the delta of the short straddle? What can you conclude about the "riskiness" of your position? Be brief but exhaustive (you may want to plot the value of the straddle with respect to the stock price).

Solution:

Since a straddle is $SS = -P - C$ we have

$$\Delta_{SS} = -\Delta_c - \Delta_p = 0$$

Small changes in the stock are not going to affect the value of the straddle. Big changes may (see part (c)).

(c) (10 points) What is the Gamma of the short straddle in part (b)? What does it imply for your answer in part (b)?

Solution:

Since the call and put have same maturity and strike price, we have

$$\Gamma_c = \Gamma_p = \frac{e^{-qT}\mathcal{N}'(0)}{S\sigma\sqrt{T}} = \frac{e^{-qT}\frac{1}{\sqrt{2\pi}}}{S\sigma\sqrt{T}} = 0.018974$$

Hence the Gamma of the short straddle is

$$\Gamma_{SS} = -\Gamma_c - \Gamma_p = -0.03795$$

A negative Gamma implies that substantial changes in the stock will affect the value of the position substantially. In this case, any movement (positive or negative) in stock will decrease the value of the straddle by a substantial amount.

(d) (25 points) Suppose that six months ago you sold 100 at-the-money straddles for \$1,850 with maturity 18 months, that you invested at the risk-free rate. So it happens that after the stock underwent some ups and downs, today the straddles are again at-the-money. The value of a 1-year, at-the-money call option on GSB stock is $C = 6.7309$.

i. What is the value of your overall position, including the proceeds of the sale of the straddles? What happens to the value of your position if there is a sudden increase in volatility?

Solution:

First, we need to find the value today of a put. We use Put-Call parity

$$P = C - Se^{-qT} + Ke^{-rT} = \$8.6525$$

Hence, the value today of the 100 short straddle position is $-(C + P) \times 100 = -1538.34$. The value of our investment in T-Bills is now$\$1850 \times \exp(.02 \times .5) = \$1877.85$. Hence, the overall position is \$339.511. The effect of a sudden increase in volatility can obtained from the Vega

$$v_c = v_p = S\sqrt{T}\mathcal{N}'(0)e^{-qT} = 37.9485$$

The Vega of the position is

$$v_{SS} = 100 \times (-v_c - v_p) = -7589$$

which is highly negative. An increase of volatility by .01% will decrease the value of the position by $v_{SS} \times .01 = \$75.89$

ii. Suppose that you want to make up a portfolio of JCH stock, calls and puts that has the same value of the current position, but it is delta and gamma neutral. What is the composition of this portfolio?

Solution:

We want the following system of equations to be satisfied

$$
\begin{aligned}
n_S S + n_c C + n_p P &= 339.511 && (1)\\
n_S + n_c \Delta_c + n_p \Delta_p &= 0 \\
n_c \Gamma_c + n_p \Gamma_p &= 0 && (2)
\end{aligned}
$$

yielding $n_S = 3.3279$, $n_c = -n_p = -3.4986$.

iii. What happens to the value of the portfolio in part (ii) if there is a sudden increase in volatility? Can you express the portfolio using some other derivative security?

Solution:

An increase in volatility does not affect this portfolio: the stock has zero Vega and we have

$$v_c = v_p = S^2 T \Gamma$$

Hence, equation (2) implies a zero Vega for the portfolio. Since we are short calls and long puts with the same strike price and maturity, it is as if we are short a forward contract with delivery price K and maturity one year.

4. (30 points)

Suppose that the LIBOR rates (annualized, annually compounded) for maturities up to three years are given by:

Maturity	1 year	2 years	3 years
LIBOR	5.5%	5.7%	6%

(a) (10 points) What is the market fixed rate (annualized, annual compounding) for a three year, fixed-for-floating swap with annual payments?

Solution:

The discounts are:

$$\begin{aligned} Z(0,1) &= \frac{1}{1+.055} = .9478 \\ Z(0,2) &= \frac{1}{(1+.057)^2} = 0.895 \\ Z(0,3) &= \frac{1}{(1+.06)^3} = 0.8396 \end{aligned}$$

Hence, the "fixed bond" is

$$B_{Fixed} = C\left(\sum_{i=1}^{3} Z(0,i)\right) + Z(0,3)$$

The rate C is the one that equates B_{Fixed} to the "floating bond" $B_{Floating} = 1$. Hence

$$C = \frac{1 - Z(0,3)}{\left(\sum_{i=1}^{3} Z(0,i)\right)} = 5.98\%$$

(b) (10 points) XYZ entered into such a swap, as the fixed-rate payer with a notional amount of \$10 million. Suppose that one day later due to an announcement of the FED, the market LIBOR rate term structure becomes flat and is equal to 6% (annualized) for all maturities. What is the value of the swap to XYZ?

Solution:

The fixed part becomes

$$B_{Fixed} = \frac{.0598}{1+.06} + \frac{.0598}{(1+.06)^2} + \frac{1.0598}{(1+.06)^3} = .9195$$

The floating part is

$$B_{Floating} = \frac{1}{1+.06} + \frac{.055}{1+.06} = 0.9953$$

In fact, the first coupon (determined one day ago) is .055 times the principal and after the coupon payment is made, the value of the floating bond goes back to 1. Hence, today the value of floating is 0.9953. Hence,

$$V_{swap} = 9.953 - 9.195 = 0.75758 \text{ millions}$$

(c) (10 points) Consider a new swap which is the same as the swap in part (a), except that the floating rate is 2×LIBOR-5.5% (annualized). Using the results from part (a), calculate the market swap rate for this new swap. Do you obtain a higher or a lower value? Briefly interpret your results. (Hint: You only have to use the results in part (a). You should not even need to know how to do part (a)).

Solution:

Consider each payment: fixed pays C^* in exchange for LIBOR+LIBOR-5.5%. This is equivalent to exchange $C^* + 5.5\%$ for LIBOR+LIBOR, or as if the fixed payers is into two identical swaps on LIBOR, paying $\frac{C^*+5.5\%}{2}$ for each of them. Each of these swaps is identical to the one in part (a) and hence

$$\frac{C^* + 5.5\%}{2} = C = .0598$$

This yields $C^* = 6.45\%$. This is much higher than C even though the initial floating payment is the same (because $2 \times LIBOR - 5.5\%$ equals the one year LIBOR rate). The sensitivity of floating payments to changes in LIBOR is much higher though.

5. (15 points)

Consider a six-month, at-the-money European call option on stock XYZ, whose price today is $C = 9.1494$. Suppose the current stock price is $S_0 = \$100$, the (annualized, continuously compounded) interest rate is $r = 3\%$. Its implied volatility is $\sigma_S = 30\%$. Using Black and Scholes formula with $\sigma_S = 30\%$, we obtain the following prices for other maturities and strike prices:

	K=95	K=100	K=105
T=.5	11.778	9.1494	6.9821
T=1	15.777	13.283	11.112

If the stock volatility is negatively correlated with the stock price (that is, it increases when the stock price declines), what bias would the above prices have with respect to market prices? Briefly provide an intuitive answer and also plot:

(a) A curve showing what the implied volatility across strike prices would roughly look like.

(b) A curve showing what the implied volatility across maturities would roughly look like.

Solution:

A negative correlation between volatility and stock implies that in the money calls (K=95) have higher prices and out-of-the-money calls (K=105) have lower prices. It also implies overall higher prices at longer horizons than predicted by the simple Black-Scholes formula.

Cumulative Normal Distribution

$$\Phi\left(x\right)=\int_{-\infty}^{x}\frac{e^{-y^{2}/2}}{\sqrt{2\pi}}dy$$

x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998