Useful Formulas for Financial Instruments Final Exam

- Forward contract, currency:
  $F_{0,T} = M_0 \times e^{(r_\$ - r_e)T}$

- Value of a forward contract to sell the underlying at $F_{0,T}$:

$$f_{t,T} = e^{-r_\$ \times (T-t)} \times (F_{0,T} - F_{t,T})$$

- Example of other forward contracts:
  - Stock with a fixed dividend yield q: $F_{0,T} = S_0 \times e^{(r-q)T}$
  - Stock: known dividend: $F_{0,T} = \left(S_0 - D \times e^{-rT_1}\right) \times e^{rT}$
  - Commodity: Storage cost U. $F_{0,T} = (S_0 + PV(U)) \times e^{rT}$
  - Commodity: % storage cost u. $F_{0,T} = S_0 \times e^{(r+u)T}$
  - Commodity: % storage cost u, convenient yield y $F_{0,T} = S_0 \times e^{(r+u-y)T}$

- Swap rate for one euro paid and K dollars received for every six months for the next in each of the next T years:

$$\text{Currency Swap Rate} = K = M_0 \frac{e^{-r_e \times 0.5} + e^{-r_e \times 1} + ... + e^{-r_e \times T}}{e^{-r_\$ \times 0.5} + e^{-r_\$ \times 1} + ... + e^{-r_\$ \times T}}$$

- Value of swap after initiation:

$$V_t^{swap} = K \times \left(\sum_{i=1}^{n} e^{-r_\$ (T_i - t)} \right) - M_t \times \left(\sum_{i=1}^{n} e^{-r_e (T_i - t)} \right)$$

- Plain vanilla swap (exchange of principal and coupons):

$$K = M_0 \times \frac{B^e(0,T)}{B^\$(0,T)}$$

- Put-Call Parity (no dividends, European Options):

$$\text{Put} = \text{Call} + e^{-r \times T} \times K - S_0$$

- Put-Call Parity, European Options with constant dividend yield q

$$\text{Put} = \text{Call} + e^{-r \times T} \times K - e^{-qT} S_0$$

- Binomial Trees (e.g. with one-period):
  - Let $V_{1,u}$ be the value of the derivative when the stock goes up and $V_{1,d}$ be the value of the option when the stock goes down
  - Delta: $\Delta = \frac{V_{1,u}-V_{1,d}}{S_{1,u}-S_{1,d}}$
  - $B_0 = e^{-rT} \times (V_{1,u} - \Delta \times S_{1,u})$
  - Value of option at time 0: $V_0 = \Delta \times S_0 + B_0$
  - Risk-neutral pricing: $q^* = \frac{S_0 \times e^{r \times T} - S_{1,d}}{S_{1,u}-S_{1,d}}$
  - $V_0 = e^{-rT}(q^* V_{1,u} + (1-q^*)V_{1,d})$

- Black-Scholes formula with known dividend yield q.

$$\begin{aligned} c &= Se^{-qT}N(d_1) - Ke^{-rT}N(d_2); \quad p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \\ d_1 &= \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T} \end{aligned}$$

- Options Sensitivities from Black-Scholes: *"The Greeks"*
  - **Delta:**

$$\Delta = \frac{d \text{ Option Price}}{d\ S} = \begin{cases} e^{-qT}N(d_1) & \text{for Calls} \\ -e^{-qT}N(-d_1) & \text{for Puts} \end{cases}$$

  - **Gamma:**

$$\Gamma = \frac{d\ \Delta}{d\ S} = \frac{e^{-qT}N'(d_1)}{S\sigma\sqrt{T}} \quad \text{with} \quad N'(x) = \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}}$$

  - **Theta:**

$$\Theta = \frac{d \text{ Option Price}}{d\ t} = \text{Long ugly formula}$$

  - **Rho**:

$$\text{Rho} = \frac{d \text{ Option Price}}{d\ r} = \begin{cases} KTe^{-rT}N(d_2) > 0 & \text{for Calls} \\ -KTe^{-rT}N(-d_2) < 0 & \text{for Puts} \end{cases}$$

  - **Vega**:

$$\text{Vega} = \frac{d \text{ Option Price}}{d\ \sigma} = e^{-qT}S\sqrt{T}N'(d_1) > 0$$

- Merton Model for Corporate Securities
  - Value of the Firm:

$$V_T = V_0 \times e^{\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}\epsilon}$$

- Value of Equity

$$E_0 = \text{Call}\left(V_0, F, r, T, \sigma\right)$$

$$\text{Call}\left(V_0, F, r, T, \sigma\right) = V_0 \; N(d_1) - F \; e^{-rT} \; N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}; \qquad d_2 = d_1 - \sigma\sqrt{T}$$

- Value of Debt

$$D_0 = V_0 - E_0 = V_0 - Call(V_0, F, r, T, \sigma)$$

- Credit Spread

$$\text{Credit Spread} = y - r = -\frac{1}{T} log\left[1 - e^{r \times T} Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right)\right]$$

- KMV Model:

$$\begin{aligned}
\text{Expected Default Frequency (EDF)} &= p_T = \Pr\left[V_T < F | V_0\right] = N\left(-d_2\right) \\
\text{Distance to Default (DD)} &= d_2 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}}
\end{aligned}$$