Fixed Income Asset Pricing

Bus 35130 Spring 2024

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Homework 6

Due at the beginning of Class 8

Note As with past homeworks there are "guides" for doing the homework in Excel, Matlab and Python. In each code provides partial solutions to the questions. To make the code run you are required to complete some formulas or to produce some of the results yourself. You are not required to use any of the guides, but use of one of them is recommended.

Pricing the Freddie Mac 6% Callable Bond

Attached below you will find the prospectus of Freddie Mac 6%, 20-year callable bond, issued on June 7, 2007. Your task in this homework is to obtain its fair valuation, using both the Ho-Lee model and the Simple BDT model. Proceed as follows:

1. (CP) Use the data in *"HW6_Data_Bonds.xls"* and extract the discount curve $Z(0,T)$ from the Treasuries using Nelson-Siegel model.

2. (CP) Build the Ho-Lee tree, given by

$$r_{i+1,j} = r_{i,j} + \theta(i) \times \Delta + \sigma \times \sqrt{\Delta} \times \epsilon_{i+1}$$

where σ is the volatility of interest rates, $\theta(i)$ are chosen to fit the term structure of interest rates exactly, and

$$\epsilon(i) = \begin{cases} +1 & \text{with probability} \quad 1/2 \\ -1 & \text{with probability} \quad 1/2 \end{cases}$$

Let r_0 match the first zero-coupon bond $Z_0(0.5)$ from Nelson Siegel model. The methodology to fit the model to the term structure of interest rates is explained in TN4. Both the matlab file and the guide spreadsheet that are available contain the routine to build it. You need a value of σ. Use the data on six-months rates

available in the dataset ($HW6\_FRB\_H15.csv$) to estimate σ (this can be done by taking the standard deviation of first differences in interest rates, over six-month periods, which is one time step). Remember to *annualize* the volatility estimates, as σ in Ho-Lee and BDT are annualized.

3. (CP) After fitting the tree to the data, please plot the zero-coupon bond yields from the tree and from the original zero-coupon bonds $Z(T)$ obtained from Nelson and Siegel (and used as inputs for the tree). Are they the same? Show also the first 10 time-steps of the interest rate tree (table 10 x 10)

4. (CP) Use the tree to price the Freddie Mac Callable bond. Recall from the teaching notes that a callable bond can be decomposed in a non-callable bond minus a call option. Therefore

   - Obtain the price of the non-callable bond
   - Obtain the price of the call option written on the non-callable bond
   - Obtain the price of the callable bond as the difference.

   Please, make sure to incorporate the fact that the callable bond becomes callable only after the First-Call-Date

   (Matlab users: please see below for a generic algorithm to compute prices through backward calculation)

5. (CP) Show the first 10 nodes of the non-callable bond, the option to call, and the callable security.

6. (CP) Plot the price of the non-callable and of the callable security against interest rates at call time, as well as 1, 2, 3 semesters before. How does the callable and non-callable bonds compare? Comment and discuss.

7. (CP) Compute the duration and convexity of callable and non-callable bond at time 0. Comment on the difference between the two securities.

8. (CP) **Ho Lee versus Simple BDT.** Redo all of the points above for the Simple BDT model (note: in both the matlab file and the spreadsheet, this amounts to change BDT_Flag from 0 to 1 and re-run the routine to build the tree. Everything else should be automatic, except for the estimate of σ which now should use log differences in rates). Comment on the difference in price, if any, from the two methodologies.

Additional notes for Matlab users:

Note 1: Below you will find a generic backward algorithm for any security P with periodic cash flows c, maturity at time n, and generic payoff $gg = [g_1, ..., g_n]$.

```
PP(:,:)=zeros(n,n);     % initialize matrix for security
PP(1:n,n)=gg;  % set final payoff of bond i equal to 1 for all nodes

for j=n-1:-1:1     % move backward on the tree, from i-1, back to 1
  PP(1:j,j)=exp(-ImTree(1:j,j)*dt).*(0.5*PP(1:j,j+1)+0.5*PP(2:j+1,j+1,i)+c*dt);
end
```

(Note: The recursive formula above computes at time j the whole vector of zeros for all nodes, from $1 : j$. Using vectors as opposed to loops speeds up computations)

Note 2: To plot bond prices against interest rates at a given time j, you just need to use the following command: *plot(ImTree(1:j,j),PP(1:j,j))*

PRICING SUPPLEMENT DATED May 22, 2007
(to Offering Circular Dated July 28, 2006)

$100,000,000

Freddie Mac

6.00% Fixed Rate Medium-Term Notes Due June 7, 2027
Redeemable periodically, beginning June 7, 2011

Issue Date:	June 7, 2007
Maturity Date:	June 7, 2027
Subject to Redemption:	Yes. The Medium-Term Notes are redeemable at our option, in whole only, upon notice of not less than 5 Business Days, at a price of 100% of the principal amount, plus accrued interest to the Redemption Date.
Redemption Date(s):	Semiannually, on June 7 and December 7, commencing June 7, 2011
Interest Rate Per Annum:	6.00%
Frequency of Interest Payments:	Semiannually, in arrears, commencing December 7, 2007
Interest Payment Dates:	June 7 and December 7
Principal Payment:	At maturity, or upon redemption
CUSIP Number:	3128X6AT3

You should read this Pricing Supplement together with Freddie Mac's Global Debt Facility Offering Circular, dated July 28, 2006 (the "Offering Circular"), and all documents that are incorporated by reference in the Offering Circular, which contain important detailed information about the Medium-Term Notes and Freddie Mac. See "Available Information" in the Offering Circular. Capitalized terms used in this Pricing Supplement have the meanings we gave them in the Offering Circular, unless we specify otherwise.

The Medium-Term Notes may not be suitable investments for you. You should not purchase the Medium-Term Notes unless you understand and are able to bear the redemption, yield, market, liquidity and other possible risks associated with the Medium-Term Notes. You should read and evaluate the discussion of risk factors (especially those risk factors that may be particularly relevant to this security) that appears in the Offering Circular under "Risk Factors" before purchasing any of the Medium-Term Notes.

The Medium-Term Notes, including any interest or return of discount on the Medium-Term Notes, are not guaranteed by and are not debts or obligations of the United States or any federal agency or instrumentality other than Freddie Mac.

Any discussion of tax issues set forth in this Pricing Supplement and the related Offering Circular was written to support the promotion and marketing of the transactions described in this Pricing Supplement. Such discussion was not intended or written to be used, and it cannot be used, by any person for the purpose of avoiding any tax penalties that may be imposed on such person. Each investor should seek advice based on its particular circumstances from an independent tax advisor.

	Price to Public [(1)(2)]	Underwriting Discount [(2)]	Proceeds to Freddie Mac [(1)(3)]
Per Medium-Term Note	100%	.415%	99.585%
Total	$100,000,000	$415,000	$99,585,000

(1) Plus accrued interest, if any, from June 7, 2007.
(2) See "Distribution Arrangements" in the Offering Circular.
(3) Before deducting expenses payable by Freddie Mac estimated at $1,000.

Goldman, Sachs & Co.

OFFERING:

1.	Pricing date:	May 22, 2007
2.	Method of Distribution:	x Principal _ Agent
3.	Concession:	N/A
4.	Reallowance:	N/A
5.	Underwriter:	Goldman, Sachs & Co.
6.	Underwriter's Counsel:	Sidley Austin LLP