

**Fixed Income Asset Pricing**  
**Bus 35130 Spring 2024**  
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**Homework 5**

Due at the beginning of Class 6

**Note 1:** For each section below, there are questions that require a pencil and paper (PP) answer, and questions that require actual computations using data and computer programs (CP). You are supposed to do both.

**Note 2:** As with past homework assignments there are “guides” for doing the homework in Excel, Matlab and Python. In each code provides partial solutions to the questions. To make the code run you are required to complete some formulas or to produce some of the results yourself. You are not required to use any of the guides, but use of one of them is recommended.

**Part I. Using Black’s formula for Caps, Floors, and Swaptions**

1. (CP) Attached are two data screens from Bloomberg. Figure ?? quotes out to 30 years. In using this data use the mid-point quotes. The second, figure ??, provides data on implied flat Black forward-volatility for Caps and Floors and implied Black volatility for Swaptions. This screen also provides the 1-month LIBOR rate needed for the exercise. (There may be some slight inconsistency in the quotes in the figures because they were not extracted from Bloomberg at exactly the same time.) Compute:

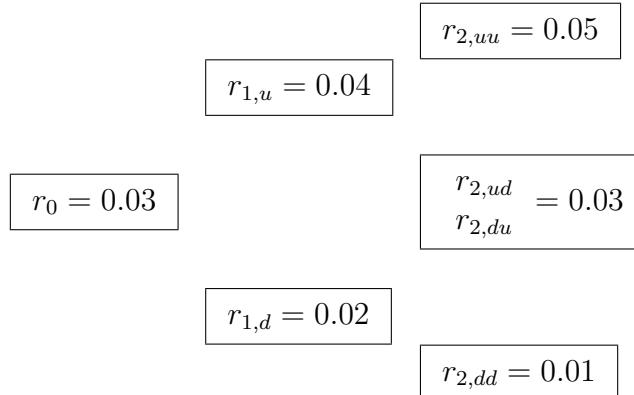
- The dollar value of a two-year Cap (quoted caps are at-the-money, and therefore the strike rate equals the current swap rate). The only think you need to enter in the code is the proper volatility and swap rates (along with 3 month LIBOR) to produce this price and the swaption price below.
- The dollar value of a 1-year swaption to enter into a 5-year swap.

2. (PP) In the exercise above, make sure to illustrate the option-pricing formulas used for (a) the first caplet in the cap (i.e. the one with 6-months to maturity); and (b) the swaption. Show your work. (Look at the code and match it to the notation from the notes class).

## Part II. Interest Rate Trees

1. (PP) Consider the following interest rate tree, where each time interval is 1-year (that is,  $\Delta = 1$ ).

$$i = 0 \quad i = 1 \quad i = 2$$



The probability of an “up” movement can either be

$$p = 40\% \quad \text{or} \quad p = 70\%$$

You also know the current value of a 2-period zero coupon bond is

$$Z_0(2) = 0.94$$

- (a) Find the risk neutral probability  $\pi^*$  (see TN4, page 21) at time 0.
- (b) You have to compute the value of a one-year call option on the interest rate  $r_1$  at the strike rate  $r_K = 3\%$  and notional  $N = 1000$ , that is, with payoff

$$\begin{aligned} \text{payoff}_{1u} &= 1000 * \max(r_{1u} - r_K, 0) \quad \text{in the “up” node; and} \\ \text{payoff}_{1d} &= 1000 * \max(r_{1d} - r_K, 0) \quad \text{in the “down” node} \end{aligned}$$

Use the risk-neutral probability  $\pi^*$  in previous point to compute the price for this option.

- (c) Does it matter which probability  $p$  is the true one (i.e. whether  $p = 40\%$  or  $p = 70\%$ )? Why?
- (d) Find the portfolio of securities that replicates the payoff of the option in point (b). What is the value of this portfolio at time 0? How does it compare with the value of the option computed in point (b)? Show your work and comment.
- (e) Assume the risk neutral probability  $\pi^*$  computed in point (a) is constant throughout the binomial tree. Use the risk neutral methodology to compute
- the value of a 3-year zero-coupon bond
  - the value of a 3-year coupon bond with annual coupon equal to 3%.
  - Compute a replicating portfolio at time 0 that replicates the payoff of the coupon bond at time  $i = 1$ . That is:
    - Define a portfolio with  $N_0$  units of the 3-period zero-coupon bond  $Z_0(3)$  and  $N_1$  units of the 1-period zero  $Z_0(1)$

$$P_0 = N_0 Z_0(3) + N_1 Z_0(2)$$

so that  $P_{1,u}$  and  $P_{1,d}$  corresponds to the total value of an investor in the coupon bond would receive (remember that such an investor is entitled to the coupon at time  $i = 1$  in both nodes).

- Check that the replicating portfolio “replicates”, that is, that indeed  $P_{1,u}$  and  $P_{1,d}$  corresponds to the payoffs of the coupon bond.

# Figure 1: Swap Rates

United States		# Export		# Settings		Interest Rate Swap Rates												
						Date Range:	24/04/2020	25/04/2020	Health									
							1M Swap	1Y Swap	GDP Growth		1M SOFR/FF Basis		1Y SOFR/10YFR Basis		4M SOFR		PCB	
USD SemiAnnual 30/360 Swap Rates	Tenor	Bid	Ask	Mkt Change	Today	#SOFR/d.	Low	Range	High	Avg	+/-%	-/+%	PCB	BGN	PCB	BGN	PCB	BGN
1 YR	0.358 / 0.366	0.362	0.001		0.0	0.352	0.308	0.308 - 0.352	0.716	0.535	-16.9	-1.7						
2 YR	0.317 / 0.328	0.322	0.007		0.1	0.308	0.337	0.308 - 0.337	0.550	0.417	-8.9	-1.5						
3 YR	0.314 / 0.328	0.321	0.000		0.0	0.308	0.381	0.308 - 0.381	0.541	0.413	-8.4	-1.4						
4 YR	0.349 / 0.365	0.357	0.009		0.0	0.337	0.433	0.337 - 0.433	0.582	0.446	-8.1	-1.3						
5 YR	0.401 / 0.415	0.408	0.001		0.0	0.381	0.683	0.381 - 0.683	0.639	0.492	-7.8	-1.2						
6 YR	0.455 / 0.468	0.461	0.000		0.0	0.433	0.722	0.433 - 0.722	0.691	0.550	-8.2	-1.2						
7 YR	0.509 / 0.523	0.516	0.005		0.1	0.402	0.525	0.402 - 0.525	0.735	0.595	-7.2	-1.0						
8 YR	0.554 / 0.568	0.561	0.005		0.1	0.525	0.561	0.525 - 0.561	0.774	0.637	-7.0	-1.0						
9 YR	0.592 / 0.604	0.598	0.002		0.0	0.561	0.591	0.561 - 0.591	0.809	0.676	-7.3	-1.0						
10 YR	0.629 / 0.643	0.636	0.005		0.1	0.591	0.683	0.591 - 0.683	0.845	0.711	-6.8	-0.9						
15 YR	0.737 / 0.748	0.741	0.000		0.0	0.683	1.043	0.683 - 1.043	0.960	0.811	-6.6	-0.8						
20 YR	0.792 / 0.806	0.799	0.007		0.1	0.722	1.020	0.722 - 1.020	1.020	0.860	-5.4	-0.6						
25 YR	0.800 / 0.815	0.808	0.002		0.0	0.734	1.043	0.734 - 1.043	1.043	0.876	-6.1	-0.7						
30 YR	0.805 / 0.821	0.813	0.001		0.0	0.737	1.058	0.737 - 1.058	1.058	0.885	-6.4	-0.7						

Bloomberg Terminal Data as of 25/04/2020 12:10:00 AM (ET) Last Update: 25/04/2020 12:10:00 AM (ET) Data Source: Bloomberg

# Figure 2: Cap, Floor and Swap Volatilities

# Regions		# Settings		20:22:59				Swaps Markets: United States											
Region	Market	Swap/Govt	Swap/Hld	Swap/1Y	Swap/3Y	Swap/5Y	Swap/10Y	FNMA	FN/GV	FN/SW	FHLMC	FH/GV	FH/SW	FH/LM	FH/PCB	FH/BGN	FH/PCB	FH/BGN	
2Y	0.182	-0.001	13.00	-0.05	0.323	+0.007	0.237			-8.8	-0.1	0.221	5.1	-1.3	-9.9	+0.0			
3Y	0.242	+0.000	8.01	+0.13	0.321		0.247	2.0	-1.6	-5.8	-0.1	0.289	9.1	-1.7	1.3	-0.2			
4Y	0.314	+0.001	5.33	+0.05	0.357	+0.001													
5Y	0.364	+0.001	4.25	+0.13	0.408	+0.001	0.529	17.0	-0.0	14.2	+0.0	0.545	19.8	-0.0	16.7	+0.0			
7Y	0.525	+0.002	-1.74	-0.05	0.516	+0.005	0.799	28.8	-0.8	33.5	-0.3								
10Y	0.634	+0.003	-0.50		0.631		1.065	41.9	-0.8	45.6	-0.1	1.100	44.6	-1.8	47.2	-0.1			
30Y	1.277	-0.001	-46.75		0.813	+0.001	1.408			72.7	-0.1	1.176	11.2	-0.6	40.6	+1.4			
Dow Jones			S&P 500 Index					NASDAQ Composite Index					Bloomberg European 500						
DJIA	23749.76	+26.07	S&P 500	2842.74	+12.03			CQBP	8210.72	+105.77			BE500	217.35	+6.09				
Cash Market	Active Futures	Swap/1Y	Swap/3Y	Swap/5Y	Swap/7Y	Swap/10Y	Swap/30Y												
3M LIBOR	0.30338	5 Year	125.32	-0.01%	1Y	102.000	103.820	99.120	92.360	92.870	106.970								
3M LIBOR	0.540888	10 Year	138.29	+0.02%	2Y	130.000	108.140	98.620	90.620	85.860	129.170								
6M LIBOR	0.71300	LONG BOND	180.23	+0.00%	3Y	107.030	92.970	87.480	84.400	81.080	136.640								
1Y LIBOR	0.83350	ULTRA BOND	224.13	+0.15%	4Y	91.420	84.740	80.690	78.750	77.320	133.890								
Fed Funds	0.04000	5Y Swap	104.07%	+0.01%	5Y	87.000	79.980	78.750	74.920	74.680	129.390								
O/N Repo	0.05000	10Y Swap	106.00%	+0.03%	7Y	76.020	72.850	71.070	71.120	72.190	122.110								
SOFR	0.03000	30Y Swap	108.27	+0.00%	10Y	78.000	59.390	75.550	59.880	78.400	107.340								
30 Economic Releases   ECD =		Date	Time	C	A	H	R	Event	Period	Surv/MO	Avg/Std	Prior	Review	Revised	Revised	Revised	Revised	Revised	
10	05/04	10:00	US	-	-	-	-	Factory Orders	Mar	-9.7%	-10.3%	0.0%	-0.3%						
10	05/04	10:00	US	-	-	-	-	Factory Orders Ex Trans	Mar	-	-3.7%	-0.9%	-1.3%						
10	05/04	10:00	US	-	-	-	-	Durable Goods Orders	Mar F	-14.4%	-14.7%	-14.4%	--						
10	05/04	19:00	US	-	-	-	-	Durables Ex Transportation	Mar F	-0.2%	-0.4%	-0.2%	--						

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