Fixed Income Asset Pricing

Bus 35130 Spring 2024

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Homework 7

Relative Value Trades and Mortgage Back Securities

Due at the beginning of class 9

Note 1: The first part of this assignment is about a simple relative value trade on Trees. The others parts of assignment are on Mortgage Backed Securities, and valuation using both trees and Monte Carlo simulations. It appears long as we describe all of the steps, but the guide matlab and excel codes performs automatically large parts of the exercise. Available on Canvas are guideline files in Excel, Matlab and Python that implement many of the steps.

Note 2: Parts I and II are "pencil and paper" (PP) questions. The other parts use data and computer programs (CP). Your are supposed to do both.

(PP) Part I: Relative Value Trades on Binomial Trees (PP) Consider the following binomial tree. Each time interval is 1-year (that is, $\Delta = 1$).

$i = 0$	$i = 1$	$i = 2$
		$r_{2,uu} = 0.05$
	$r_{1,u} = 0.04$	
$r_0 = 0.03$		$\begin{matrix} r_{2,ud} \\ r_{2,du} \end{matrix} = 0.03$
	$r_{1,d} = 0.02$	
		$r_{2,dd} = 0.01$

The 2-period bond is $Z_0(2) = 0.94$, which implied a risk neutral probability $\pi^* = 0.59618073$. The true probability to move up the tree is $p = 0.5$.

The three period, fixed-coupon bond with coupon rate $c = 3\%$ is trading at par 100. Your model, instead, prescribes it should trade at $P_0(3) = 99.34716$. The three-period zero coupon bonds $Z_0(3) = 90.88915$ seems to be trading at the proper price.

1. Design a relative value trading strategy to take advantage of the miss-pricing (according to your model). Please, be clear on what you sell and what you buy. Recall that the only traded securities are the coupon bond (which is miss-priced) and the 3-period zero-coupon bond (which is correctly price). In addition, you can buy and sell 1-period bonds (i.e. borrow and lend at the rates on the tree).

2. Make a binomial tree with the trading strategy up to maturity: Indicate the positions in traded securities at any point. (You can use excel for this. But make sure to report the trees in the assignment).

3. What is the total profit and loss?

Part II. Interest Rate Trees and MBS

1. (PP) Consider the interest rate tree examined in HW5, reported below. Recall that each time interval is 1-year (that is, $\Delta = 1$).

$i = 0$	$i = 1$	$i = 2$
		$r_{2,uu} = 0.05$
	$r_{1,u} = 0.04$	
$r_0 = 0.03$		$\begin{matrix} r_{2,ud} \\ r_{2,du} \end{matrix} = 0.03$
	$r_{1,d} = 0.02$	
		$r_{2,dd} = 0.01$

The probability of an "up" movement is $p = 40\%$, and the risk neutral probability π^* is in the solution to HW5 (or recompute it for this homework). You also know the current value of a 2-period zero coupon bond is $Z_0(2) = 0.94$.

(a) Compute the value of a 3% Pass-Through MBS with maturity $i = 3$. The underlying pool has a coupon rate of $r^m = 3.469\%$, principal value $N = 100$, and maturity $i = 3$. Follow the steps in TN 5 (but recall the risk neutral probability is not 0.5).

Part III. Pricing MBS on Trees

On November 1, 2014, the Ginnie Mae Pass-Through security GNSF 4 was trading at \$106.5781. [1] This pass-through security was collateralized by a pool of mortgages with a weighted average coupon (WAC) = 4.492%, and a weighted average maturity (WAM) = 311 (months). As a junior analyst at the JCH Investment Group, you are examining the value of the GNSF 4 and trying to assess whether this price is in line with current interest rates and option prices.

You decide to use caps and floors to assess whether all of the interest rate optionality implicit in mortgage backed securities is taken into account in the current price of MBS. You decide to proceed as follows:

1. **Compute Forward Volatilities from Cap Volatility Quotes**

   (a) Compute the zero-coupon LIBOR curve implicit in the swap curve, as usual (*Note: This part is automatically done by the guide codes available on Canvas*);

   (b) Compute the forward volatilities, also up to maturity of 30 years. Cap volatility quotes are available up to 30-years as well (*Note: This part is automatically done by the guide code*).

   (c) Plot the forward and the flat volatilities.

   - Comment on your findings. Discuss.
   - Provide an intuition about the relation between forward and flat volatilities.

[1] Source: Bloomberg.

2. **Fit the BDT model to the discount curve and forward volatilities.**

   - Use the algorithm in TN 4 to fit the Black-Derman-Toy model (*Note: automatically done by the guides, once fitted LIBOR curves and forward volatilities are carried over. Note that we fit the model at quarterly intervals instead of monthly intervals to keep the computational exercises simpler.*).

3. **Use the BDT model to value GNSF4**. To obtain the price of the the pass-through security GNSF 4, recall you must first do the following steps:

   (a) Compute the dollar coupon and the scheduled interest and principal payments. These calculations allow you to compute the scheduled outstanding principal balance. You can do these computations assuming outstanding principal of 100 (so that, the price will be also in percentage of 100, like the quoted value of the security).

   Note that the coupon must satisfy the equation

$$P_0 = C \sum_{j=1}^{n} \left(\frac{1}{1 + r_m * dt} \right)^j$$

   where n is the number of coupon payments and dt is the time step. A useful formula is the following: define $a = \left(\frac{1}{1+r_m*dt} \right)$, then we have the result that[2]

$$\sum_{j=1}^{n} a^j = \frac{a - a^{n+1}}{1 - a}$$

   so that

$$C = P_0 \frac{(1 - a)}{a - a^{n+1}}$$

   Given C, you can compute the scheduled principal and interest payment and remaining principal over time

[2]To see this, one just need to remember the geometric progression $\sum_{j=0}^{\infty} a^j = 1/(1-a)$. Then:

$$\sum_{j=1}^{n} a^j = \sum_{j=1}^{\infty} a^j - \sum_{j=n+1}^{\infty} a^j = a \sum_{j=0}^{\infty} a^j - a^{n+1} \sum_{j=0}^{\infty} a^j = \frac{a - a^{n+1}}{1 - a}.$$

(b) Compute the value of the non-callable mortgage on the tree (i.e. present value of future coupons. This part is identical to previous cases, except that there is no bullet capital payment at maturity).

(c) Compute the value of the American call option from homeowner perspective, and thus the optimal time of prepayment. (This part again is identical to the previous homework, except that you must compare the present value of future payment at every node (i, j) to the outstanding principal at time i.)

What is the value of the pool of mortgages as of time zero? (I.e. what is the present value of future payments that takes into account the American option?)

(d) Use the GNSF coupon rate (4%) and the optimal prepayment time computed in the previous point to compute the cash flows of the pass-through security and thus its value at time 0.

4. Is the price that you obtained close to the quoted one? If not, explain why not intuitively. What may be going on to explain the discrepancy, if any?

Part IV. Use Monte Carlo Simulations on Trees

From the tree estimated in Part III, do the following:

1. Use Monte Carlo simulations *on the tree* to price an 30-year cap and a 30-year Asian cap with strike rate $r_k = 2\%$. (Note: This is identical to the example in Chapter 13 in the book.)

2. Use Monte Carlo simulations and the prepayment function

$$p_t = c_1 \times max\left(r_m - sp - r_t, c_2\right)$$

to do the following:

- Compute the value of the pass-through, interest-only and principal-only mortgage-backed securities discussed above. Note that both the guide files are already set up to do these calculation for given parameters of the prepayment function. Choose the parameters of the prepayment function to obtain a value of the PT security that is similar to the quoted one. (This can be done by trial and error).

- Vary the parameters of the prepayment functions and compute the values of IO and PO. Discuss your findings intuitively.