Fixed Income Asset Pricing
Bus 35130 Spring 2019
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Midterm

INSTRUCTIONS:

- There are 3 questions, each worth 35 points.
- The questions cover various aspects of the material covered in class. Read all the question and start from those with which you feel most comfortable.
- Answer all questions to the best of your abilities.
- Do not get hung up in calculations. Sometimes, just setting up an equation or giving an intuitive argument are sufficient for partial credit (if correct!) Move on!
- You may use a calculator and refer to one 8.5" by 11" sheet of notes (two-sided).
- **You must write your answers on this midterm. Use back pages for any calculations. No other piece of paper, besides your formula sheet, is allowed.**

Honor Code: I pledge my honor that I have not violated the Honor Code during this examination.

Name and ID number (Please, type):________________________________

Signature:____

Problem 1. (35 points) True/False Questions. Grade depends on completeness of answer.

(1.A) (7 points) [True/False] The duration of a floating rate note with semi-annual payments is at most equal to 0.5.

(1.B) (7 points) [True/False] If the spot curve is increasing, the forward rate is above the spot curve, while if the spot curve is decreasing, the forward rate is below the spot curve.

(1.C) (7 points) [True/False] The value of a fixed-for-floating swap is always equal to zero, both at inception of the contract and later.

(1.D) (7 points) [True/False] In binomial trees, risk neutral probabilities are crazy because they assume that investors are really risk neutral. Everyone knows that market participants are on average risk averse.

(1.E) (7 points) [True/False] Suppose the current yield-to-maturity on a one-year Treasury Bill is 4% and the market expect inflation of 2% over the next year. The yield-to-maturity on a one-year Treasury inflation protected security would necessarily be 2%.

Problem 2. (35 points) You fit the following model to the current term structure of interest rates, and you came up with the following binomial tree. The probability of moving up the tree is 1/2. (Risk-neutral). For simplicity, the time step is one year.

$i = 0$	$i = 1$	$i = 2$
		$r_{2,uu} = 0.1$
	$r_{1,u} = 0.07$	
$r_0 = 0.04$		$r_{2,ud}$, $r_{2,du}$ $= 0.06$
	$r_{1,d} = 0.03$	
		$r_{2,dd} = 0.02$

Compute the following: (Make sure you show all the computations at all nodes.)

(2.A) (7 points) Compute the price today ($i = 0$) of the one-, two-, three-period zero-coupon bonds.

(2.B) (5 points) Using the zero-coupon bonds prices from point 2.A, compute the one-period forward rates for $i = 1$ and $i = 2$. How do they compare with expected interest rates computed from the tree (using the risk-neutral probabilities)?

(2.C) (7 points) The market expects that because of a tightening of monetary policy, the one-period interest rate at time $i = 1$ will be 4%. What is the implied market price of risk? Comment on its sign and its economic meaning.

(2.D) (8 points) An investor would like to purchase a security with payoff given by

$$\text{Payoff}(i) = \begin{cases} 0 & i = 0 \\ 100 \times max(Average(r_0, r_1) - r_0, 0) & i = 1 \\ 100 \times max(Average(r_0, r_1, r_2) - r_0, 0) & i = 2 \end{cases}$$

Compute the value, at time $i = 0$ of this **American option**. (*Tip 1: At time $i = 0$ and $i = 1$, the investor can either exercise the option or wait to exercise the option. Tip 2: The calculations below may help speed up your own calculations.*)

Quantity	Value
$\frac{1}{2} \times (r_0 + r_{1,u})$	0.055
$\frac{1}{2} \times (r_0 + r_{1,d})$	0.035
$\frac{1}{3} \times (r_0 + r_{1,u} + r_{2,uu})$	0.07
$\frac{1}{3} \times (r_0 + r_{1,d} + r_{2,uu})$	0.0567
$\frac{1}{3} \times (r_0 + r_{1,u} + r_{2,ud})$	0.0567
$\frac{1}{3} \times (r_0 + r_{1,d} + r_{2,ud})$	0.0433
$\frac{1}{3} \times (r_0 + r_{1,u} + r_{2,dd})$	0.0433
$\frac{1}{3} \times (r_0 + r_{1,d} + r_{2,dd})$	0.03

(2.E) (8 points) Compute the replicating portfolio for the security in 2.D *at each node along the tree*. What other securities do you use to replicate? Why? What weights do you get? Check that the replication in fact replicates the security's payoff at $i = 1$ and that the price of your replicating portfolio at $i - 0$ equals the price from 2.D. (*Tip: If you are unable to solve 2.D, you can still receive partial credit for a correct setup in terms of generic payoffs.*)

Problem 3. (**35 points**) The pension plan of BBB Corp has $1,000,000 in liabilities and is 80% funded. Also there are three bonds available to invest in for the pension plan asset portfolio. The following table contains the main characteristics of the pension plan's assets, liabilities and available bonds:

	Price	Duration	Convexity
Liability	$1,000,000	10	100
Assets	$800,000		
Bond_1	$100	14	150
Bond_2	$100	8	50
Bond_3	$100	2	4

(3.a) (5 points) If we believe that interest rate term structure will only move in a parallel fashion in the future, then what kind of bond portfolio will always outperform liability for all possible interest rate movements and by the greatest possible amount out of all bond portfolios based on only three available bonds? Using the available three bonds, determine what bond or bonds are required and determine the exact position or positions. Assume that only long positions can be entered into (no short positions) and you are free to use any set of bonds you wish along with Cash if needed. (You don't need to use all of the bonds)

(3.b) The plan sponsor would like to create a bond portfolio that exactly matches duration and convexity of the liability to that of the assets.

(3.b.i) (5 points) Discuss the pros and cons of this strategy.

(3.b.ii) (8 points) Select the two most appropriate bonds out of three available and determine the bond positions. Once the bond positions are determined, please illustrate that the total bond portfolio is not greater than the total assets available (it is okay to be smaller). As before, no short positions are allowed (i.e. bond weights can not be negative). Explain intuitively why any other combination of bonds out of three available will not be suitable.

(3.c) (4 points) Suppose now that you conduct a Principal Component Analysis of the yield curve and determine that three principal components are need to fit changes in the yield curve. Would you be comfortable with your hedge from part (3.b.ii)? Why or why not? If not, discuss how you might construct a better hedge. Would you need all three bonds in this case? Why or why not?

(3.d) XYZ insurance company has liability cash flows (aka benefit payouts), as summarized in the following table, for their annuity line of business. The table also contains continuously compounded spot rates, corresponding Z-factors and three PCA β's for Level, Slope and Curvature factors respectively. "Time" column in the table is the number of years from today until the payments are due, i.e. all the payments are made annually, with spot rates, Z-Factors, and PCA β's shown at the same annual frequency (TIP: this is a standalone question and not a continuation of questions 3a-3c)

Time	Spot Rates	Z-Factors	β_{Level}	β_{Slope}	$\beta_{Curvature}$	Liability CF
1	0.0300	0.9704	0.0968	0.3189	0.2003	\$1,000
2	0.0360	0.9305	0.1453	0.0802	(0.1099)	\$1,100
3	0.0432	0.8784	0.1298	(0.0500)	(0.1348)	\$1,200
4	0.0518	0.8129	0.1227	(0.0945)	(0.0257)	\$1,300
5	0.0622	0.7327	0.1216	(0.0988)	0.1314	\$1,400

(3.d.i) (8 points). Using only information from the table above (and not the information for questions 3a-3c), calculate factor sensitivities (i.e. factor duration) for Level, Slope and Curvature factors a) for XYZ insurance company's Liability, using liability cash flows provided in the table above, and b) for three hypothetical zero coupon bonds with 1, 3 and 5 years to maturity respectively and principal \$1000 each

(3.d.ii) (5 points) Using factor sensitivities for XYZ insurance company's liability and three bonds calculated in the previous question (3.d.i), discuss how you can establish individual bond positions in a three bond portfolio such that bond portfolio's factor sensitivities would match liability's factor sensitivities for all three factors. Write down all the necessary equations but there is no need to solve them, i.e. no need to calculate actual bond positions