Bus 35130: Fixed Income Asset Pricing
John Heaton

Final Take-Home Exam
Spring 2024

Instructions (PLEASE, READ CAREFULLY!)

1. The final is about TIPS and Inflation-Dependent securities.
2. You may do this project in groups of **no more than 4 people.** Submit one writeup per group.
3. You must write the final in the form of a report
   - Assume that today is January 15, 2013. You have recently been hired by JCH Asset Management (JCH) and your first assignment is to write a report on inflation-dependent securities. You must write an internal report addressed to the Managing Director of your division. You will have to use data for your exercise and apply what you have learned in class.
   - The MD is very demanding and wants to know all of the details of your "analysis," so do not spare equations and details that are important for understanding **how** how the solution to the problem (or a pricing formula) is obtained and **why** it works (you need to convince the MD).
   - You (as a team) are on your own. You will have to choose methodologies, data, make assumptions as necessary, and defend them in the report.
   - The zip on canvas that accompanies this document contains a spreadsheet with data as of January 15 2013, and guide-files for Matlab, Excel and Python,[1]
   - There are also a few readings that you may find useful to obtain additional background information about TIPS and inflation derivatives. For example the reading, "The TIPS-Treasury Bond Puzzle" (in the file "jofi.12032.pdf"), may be of help when answering the last question
     Some of these files are password protected, and the password is "TIPS"
   - Below you will find a number of questions that you should address in your report. **Shape your report to answer all these questions in the order given**. However, do not limit yourself to answering these questions alone. Use your creativity and imagination to address additional issues.
4. The write up should be no longer than 8 single-sided pages with double line spacing, standard indentation (margins), font 12 Times New Roman (plus any additional material, such as tables, figures etc.). The report must be professionally written. Please be clear and concise. *Reports that do not conform to the rules will be penalized.*

[1]The guides are kept at the bare minimum for the final, as you should also adapt previous codes and spreadsheets to do some of the required computations.

5. Each group must upload the write up on Canvas as described below.

    - **When**: The deadline n 11:59 pm (Chicago Time) on Sunday May 26, 2024.
    - **Where**: You must deliver the written solution by uploading it, as a pdf file, to Canvas by the due date and time.

6. Before 11:59 pm on Sunday, May 26, you can only speak about the material of this exam with other members of your own group. A violation to this rule is considered a violation of the Honor Code.

Honor Code: I pledge my honor that I have not violated the Honor Code during this examination.

Signatures (each group member must sign):

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Questions

Do not limit yourself to answering these questions.
Use your creativity and imagination to address additional issues.
Remember to shape the report to answer these questions in the order provided.

I. TIPS (10 points)

1. Describe the Inflation Indexed securities and their uses.
2. How are nominal and real rate interest rates related to each other?
3. What is the difference between an inflation-linked bond and a real bond?
4. What type of expected return should you expect from an inflation-linked bond as compared to a nominal bond?

II. Real and nominal rates: Empirical Analysis (15 points)

It is January 15, 2013, and JCH has a $ 200 million holdings (par value) of the .125%, Jan 15 2022 TIPS. The Principals of JCH are worried about their exposure to potential changes in interest rates and inflation expectations. Before you compute risk measures (Question III below), you should discuss some properties of real and nominal rates in the data, using both historical data and today's prices. The dataset *DataFinal2021.xls* contains the data required for the analysis. Do the following:

1. Use the Nelson-Siegel model to fit the term structure of real interest rates $Z_{real}(0,T)$ on January 15, 2013.
   - Plot the term structure of real rates $r_{real}(0,T)$ and the "real" forward curve.
   - Comment on your findings
2. Similarly, use the Nelson-Siegel model to fit the term structure of nominal interest rates $Z(0,T)$ as of January 15, 2013.
   - Plot the term structure of nominal rates $r_{real}(0,T)$ and the "nominal" forward curve
   - Comment on your findings
3. The quantity $\pi(0,T) = r(0,T) - r_{real}(0,T)$ is defined as the (continuously compounded, annualized) *breakeven inflation* rate.
   - Compute it for January 15, 2013 and plot it against maturity T. This is the term structure of break-even rates.
   - How can you interpret the break-even inflation rate? What components does it have? Interpret the term structure of break-even rates as of Jan 15, 2013.
4. The dataset *DataFinal2021.xls* also contains monthly observations of continuously compounded nominal and real rates estimated from the (extended) Nelson and Siegel model.

- Plot and comment on the time-series of both real and nominal rates.
- Plot and comment on the time series of the term structure of (annualized) break-even rates. In particular, what explains the big variation in late 2008/early 2009? Make sure to interpret such variation using also economics (e.g. real growth, expected inflation, risk premia etc). *(Note that the maturities of real rates start in year 2).*

III. **Duration and Convexity Hedging (20 points)** Consider again JCH the $200 million position in .125%, 01/15/22 TIPS. As mentioned, principals at JCH are worried about its risks.

1. Obtain a formula for duration and convexity for TIPS, where they are both defined against shifts in the nominal term structure of interest rates, assuming nothing else changes.
   (Hints. (1) It is useful to use continuous compounding as in TN 2. (2) Use the notion of break-even rate $\pi(0,T)$ obtained earlier, to transform real rates into nominal rates minus break-even inflation. (3) Assume $\pi(0,T)$ is not affected by a parallel shift in the nominal term structure of interest rates. (4) Exploit this formula and thus the relation between $Z_{real}(0,T)$ and $Z(0,T)$ to obtain the formulas for duration and convexity. The steps are the same as in TN 2.)
2. Use the data in *DataFinal2021.xls* to compute duration and convexity for the .125%, 01/15/22 TIPS. Comment on your findings.
3. Similarly, use the data in *DataFinal2021.xls* to compute duration and convexity of the 2%, 02/15/2022 (nominal) U.S. Treasury. Compare the duration to the one of TIPS and comment on your findings.
4. Use your results in these two points to compute a duration hedge for the 0.125%, 01/15/22 TIPS using the 2%, 02/15/2022 Treasury notes.
   - Comment on your hedge ratio.
   - How would you implement the hedging strategy? Describe the methodology.
5. How would you hedge against large changes in interest rates? Compute hedge ratios against large variation in interest rates and discuss the results.

IV. **Inflation Duration and Inflation Swaps (20 points)**

1. TIPS are affected by variation in break-even inflation. What is the "inflation duration" of TIPS? That is, the sensitivity of TIPS to changes in break-even inflation *holding constant the nominal rates?*
   - Using the methodology used for deriving TIPS's nominal duration, obtain a formula for inflation duration and comment on your results.
   - Is the hedging strategy discussed in previous point III.3 addressing the exposure of TIPS to changes in breakeven $\pi(0,T)$?
2. *Inflation swaps* are popular derivatives to trade inflation claims and hedge inflation risk. The dataset *DataFinal2021.xls* contains data on zero-coupon inflation swaps on January 15, 2013. The reading from Lehman Brothers contains a discussion of zero-coupon inflation swaps.

- Use the methodologies discussed in the teaching notes to obtain a valuation formula for zero-coupon inflation swaps. *(Hints: The reading from Lehman Brothers also contains a formula which does not use continuous compounding rates. I would encourage you to derive the valuation formula again using continuously compounding and then use the Lehman Brothers formula for comparison purposes.)*
- Compute the term structure of *real* interest rates on January 15, 2013 using inflation swaps and compare it with the one obtained from TIPS. Discuss similarities and differences.
- Compute the term structure of break-even inflation rates and compare it with the one obtained from TIPS and Nominal Bonds. Comment.

3. Using the valuation formula for inflation swaps, can a position in inflation swap help hedge independent variation in break-even rates? (i.e. independently of the variation in nominal yields). Compute the following:
   - The sensitivity of an inflation swap *value* to changes in nominal rates right after inception of the contract (i.e. from 0 to dt). (*Hint: Can you compute the traditional (percentage) duration for a swap? Make sure you use the proper formulas for the case of swaps.*)
   - The sensitivity of an inflation swap *value* to changes in break-even rates right after inception of the contract (i.e. from 0 to dt).
4. Use your results on inflation swaps to compute an optimal duration and inflation hedge for the position in TIPS discussed in point II.
5. Is the hedge working? Use the data after January 15, 2013 in *DataFinal2021.xls* (tab "TimeSeries_TIPS_Treasury") to check on the performance with and without the inflation-hedge.

V. **Factor Analysis of Nominal and Break-even Rates (20 points)**

1. The hedging strategy illustrated earlier uses duration and convexity. As we know, the nominal rates are affected by "level," "slope", and "curvature". Is the same true for break-even rates?
   - Perform a Principal Component Analysis on *break-even inflation rates* and comment on the results. (*Hint: Use the methodology illustrated in class for yields, but for break-even inflation rates.*)
2. TIPS are thus affected by "nominal factors" and by "break-even" factors. Because hedging would uses nominal bonds (or derivatives) and inflation-based derivatives (such as inflation swaps), how would you set up a hedging strategy that hedges against both nominal and break-even rate factors? Discuss. *(Hints: This is more of an open ended question, but we do expect some formal analysis to take into account both nominal and break-even factors.)*

VI. **TIPS, Nominal Bonds, and Inflation Swaps (15 points)** As you examine your hedging strategy (previous points), you notice that Inflation Swaps, TIPS, and Nominal

bonds do not seem to be perfectly aligned with each other. Is there a potential arbitrage opportunity?

1. Examine the data and set up a trade to exploit the potential misspricing of inflation swaps versus TIPS and nominal bonds.
2. Discuss the trade and its implementation.
3. What could be the source of such arbitrage opportunity in the data? Discuss potential risks from the arbitrage. *(Hints: Examine the time series data about inflation swaps and real and nominal rates to support your argument. Make calculations as necessary to provide any potential evidences for the risks).*
4. Should you advise JCH to exploit the arbitrage?