Fixed Income Asset Pricing
Bus 35130
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Midterm, Spring 2024

INSTRUCTIONS:

- The questions cover various aspects of the material covered in class. Read all the question and start from those with which you feel more comfortable.
- Answer all questions as well as you can. Provide details of your calculations.
- Although I award most of the points for your work and not the final number, I would like numerical answers to three significant decimal places.
- You have 200 minutes to upload your solutions. There are 4 questions. Points are equal to the number of minutes you should spend on a question. The total comes to 180 points or 180 minutes. This gives you twenty (20) minutes to organize your submission.
- Do not get hung up in calculations. Sometimes, just setting up an equation or giving an intuitive argument are sufficient for partial credit (if correct!) Keep moving.
- For your submission:
  - You are to submit a pdf of your scanned answers. You are to use your own paper for your answers.
  - **Submit a pdf file. NO EXCEL FILES.**
  - With your pdf file you must include this completed first page
  - Start each question (1, 2, 3, 4) on a new page.
  - Write your name on EVERY PAGE in the upper left-hand corner.
  - Write the page number, sequentially, on EVERY PAGE in the upper right-hand corner.
  - Write the total number of pages on the LAST PAGE of your answers and circle the number.

1. **Short Answer/ True, False 25 points, 5 points each, points awarded for a complete explanation!)**. You should answer each question in 2 or 3 sentences.

    (a) True or False. We can use the relative prices of TIPS and nominal treasury bonds to back out the market's expectation of future inflation.

    (b) Short answer. Explain how and why an "at the money" (strike prices equal to swap rates) interest floor can be priced based on the prices of an interest rate cap with the same strike price and maturity.

    (c) True or False. Risk neutral models should be designed to help us forecast future interest rates.

    (d) True or False. The goal of hedging of with factors models is not to eliminate all risks, but to eliminate exposure to important risks.

    (e) Short answer. Consider the Ho-Lee model binomial model with dynamics for short rates given by:

    $$\begin{aligned} r_{i+1,j} &= r_{i,j} + \theta_i \times \Delta t + \sigma \times \sqrt{\Delta t} \quad \text{for a ``up'' movement} \\ r_{i+1,j+1} &= r_{i,j} + \theta_i \times \Delta t - \sigma \times \sqrt{\Delta t} \quad \text{for a ``down'' movement} \end{aligned}$$

    How are the values of θ_i calibrated? Are their features of the world that are at odds with this type of model (provide 2 and explain)?

2. **(65 points)**

Consider the following binomial model for 6-month continuously compounded risk-free rates reported in annualized units. Each period is six months. Notice that the tree is NOT RECOMBINING. The true or objective probability of an "up" move at each node is 50%.

$i = 0$	$i = 1$	$i = 2$
		$r_{2,uu} = 0.06$
	$r_{1,u} = 0.05$	
		$r_{2,ud} = .02$
$r_0 = 0.03$		
		$r_{2,du} = .04$
	$r_{1,d} = 0.01$	
		$r_{2,dd} = 0$

(a) (10 points) At each node in period 1, what are the expected *changes* in short-term interest rates (difference between expected rates at time 2 and current rates)? Do the expected changes differ across nodes? What underlying model of interest rates would reflect your findings and why? (HINT: remember assignment #1 ...).

(b) (3 points) What is the time 0 price of a one-period (6-month) zero coupon bond with face value of \$1?

(c) (10 points) A 1-year zero coupon bond with face value \$1 is trading at time zero at $Z_0(2) = 0.972$ Create and report the evolution of the price of this bond at each node of the above interest rate tree.

(d) (5 points) Use the calculations in points (2b) and (2c) to compute the risk neutral probability π^* of moving from time 0 to node $\{1, u\}$.

(e) (17 points) Consider a coupon bond with *inverse floating and "snowballing" coupons*:

$$\begin{aligned}
\text{Coupon at time 1} &= max\{(\bar{c} - r_0)/2, 0\} && (1)\\
\text{Coupon at time 2} &= max\{(\text{Coupon at 1}) + (\bar{c} - r_1)/2, 0\} && (2)\\
\text{Coupon at time 3} &= max\{(\text{Coupon at 2}) + (\bar{c} - r_2)/2, 0\} && (3)
\end{aligned}$$

where $\bar{c} = 0.03$

Note that the cash flow at time i depends on the path of interest rates strictly before i. At maturity (time $i = 3$) the security also pays back the principal of 1. Assume the risk neutral probability of moving "up", π^*, computed in part 2d is constant throughout the tree. What is the value the snowball inverse floater? *(Tips: You should first calculate and report the cash flows to be paid in various interest rate* *scenarios. As you do the calculation, outline the binomial tree of prices for the bond, as it may be useful in the next question.)*

(f) (10 points) Assume that the bond from part 2e is in fact *callable*: it can be called back at par by the issuer any time before maturity. What is the value of the callable inverse floater? When would the bond be called by the issuer? How are the times of early exercise related to the level of rates and why?

(g) (10 points) Suppose the bond from part 2f is selling at part (\$1). Is there an arbitrage opportunity? If so, how could you use the bonds of parts 2b and 2c to take advantage of this opportunity?

3. **(45 points)**

The current continuously-compounded yield on a 6-month T-Bill is 4.5%. Suppose that you have the following information about swap rates with maturities of 1 year, 1.5 year and 2 years with semi-annual payments where the floating rate is given by the 6-month T-Bill rate determined 0.5 years before each payment.

Maturity.	Swap rate (annualized)
1 year	5%
1.5 years	5.2%
2 years	5.5%

(a) (15 points). What are the implied prices of zero-coupon bonds with 1, 1.5 and 2 years to maturity?

(b) (10 points) What should be the price of a coupon bond with face value \$100, 6.5% coupon and maturity of 2 years with semi-annual coupon payments?

(c) (15 points) Suppose the bond in part 3b is selling at par? Is there a potential arbitrage trade assuming that 6-month repo rates are current equal to the 6-month T-bill yield? If so, how would you execute this trade?

(d) (5 points). Suppose that you execute the trade in part 3c with as much leverage as you can, what risk would you face in holding the trade to maturity?

4. **(45 points)** Consider a hedge fund with equity of \$0.5 billion that would like to invest in a bond position with a market value of \$5 billion.

(a) (10 points) Suppose that the bond position they would like to invest in has duration of 8 and convexity of 60. To finance this position they will borrow using a combination of cash borrowing (over-night borrowing), 5 year and 15 year zero coupon bonds. What combination of these instruments should the hedge-fund use?

(b) (5 points) Although the hedge fund is hedged using duration and convexity, discuss other potential sources of risk they might face? (Limit the discussion to yield curve variation.)

(c) (10 points) Discuss how principle component analysis could be used to potentially help with hedging.

(d) (15 points) Suppose that you run a PCA analysis as in the class with the following results:

	Maturity						
Factor	1 month	6 months	1 year	2 year	3 year	4 year	5 year
			Panel A: Level				
β_{i1}^{PCA}	0.4416	0.4167	0.4315	0.3762	0.3421	0.3169	0.2935
R^2	61.15%	79.52%	91.66%	88.61%	83.89%	75.06%	73.03%
			Panel B: Slope				
β_{i2}^{PCA}	-0.7307	-0.2774	0.0848	0.2425	0.2909	0.3559	0.3344
R^2	95.91%	86.83%	92.39%	96.25%	96.48%	94.72%	92.71%
			Panel C: Curvature				
β_{i3}^{PCA}	0.5186	-0.7099	-0.2869	-0.0231	0.1411	0.2267	0.2700
R^2	99.99%	97.99%	94.35%	96.27%	97.17%	96.58%	95.70%

Suppose further that you know the exposure of the bond strategy to the first two PCA components are 5 and 12 respectively. You would like to use the 3-year and 5-year zero coupon bonds in your borrowing. What combination should you use?

(e) (5 points) Would this hedging strategy work well if the factors where to make "large" movements? Why or why not? If not, what might you consider in addition?