Fixed Income Asset Pricing
Bus 35130
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Midterm, Spring 2022

INSTRUCTIONS:

- The questions cover various aspects of the material covered in class. Read all the question and start from those with which you feel more comfortable.
- Answer all questions as well as you can. Provide details of your calculations.
- Although we award most of the points for your work and not the final number, we want numerical answers to three significant decimal places.
- You have 200 minutes to upload your solutions. There are 4 questions. Points are equal to the number of minutes you should spend on a question. The total comes to 180 points or 180 minutes. This gives you twenty (20) minutes to organize your submission.
- Do not get hung up in calculations. Sometimes, just setting up an equation or giving an intuitive argument are sufficient for partial credit (if correct!) Keep moving.
- For your submission:
  - You are to submit a pdf of your scanned answers. You are to use your own paper for your answers.
  - **Submit a pdf file. NO EXCEL FILES.**
  - With your pdf file you must include this completed first page
  - Start each question (1, 2, 3, 4) on a new page.
  - Write your name on EVERY PAGE in the upper left-hand corner.
  - Write the page number, sequentially, on EVERY PAGE in the upper right-hand corner.
  - Write the total number of pages on the LAST PAGE of your answers and circle i

Honor Code: I pledge my honor that I have not violated the Honor Code during this examination.

Name and ID number (Please, type):___________________________________

Signature:____

1. (60 points, 10 points each) Short-Answer/True-False Questions. Grade depends on completeness of answer, however you should try to be short and to the point in your response. We're only looking for the obvious big point(s). We don't give credit for long winded answers and will take off points if you add things that are wrong or irrelevant.

   (a) The duration of zero coupon bonds is given by its time to maturity. Therefore, it is independent of current interest rates. It follows that the convexity of a zero coupon bond has to be zero.

   (b) Changes in the slope of the term structure of interest rates can be effectively hedged through the so called "Duration and Convexity" hedging.

   (c) The following quotes are from Bloomberg and are from a single day:

Maturity (T)	LIBOR	3 month Forward Rate $F(T, T+3\ \text{months})$	
3 months	1.18438%	$F(3\ month, 6\ month)$:	1.231 %
6 months	1.66 %	$F(6\ month, 9\ month)$:	1.281 %
9 months	1.82375%	$F(9\ month, 1\ year)$:	1.45 %
1 year	1.975 %		

   There is an arbitrage in the quoted forward rate $F(9\ months, 1\ year) = 1.45\%$ compared to LIBOR rates. Discuss.

   (d) The expectation hypothesis – the fact that the long-term yield equals the market forecast of future short-term rates – is strongly supported by the historical data.

   (e) The payoff of a receiver swaption is equivalent to the payoff of a call option on a coupon bond with coupon rate equal to the swap rate. Such call option has a strike price of par (= 100).

   (f) The duration of an inverse floater may be higher than the time to maturity of the floater.

2. (50 points) You asked your assistant to fit some bond prices, and he came back with the following binomial trees for *zero-coupon bonds*. The true probability to go up the tree is $p = 0.5$ and the time step is one year. Unfortunately, as you and your assistant were communicating, one entry ($Z_{1,u}(3)$) did not go through, hence the question marks ?? in the tree. Your assistant left for vacation and cannot be reached.

$i = 0$	$i = 1$	$i = 2$
		$Z_{2,uu}(2) = 1.00$ $Z_{2,uu}(3) = 0.91$
	$Z_{1,u}(1) = 1.00$ $Z_{1,u}(2) = 0.92$ $Z_{1,u}(3) =??$	
$Z_0(1) = 0.96$ $Z_0(2) = 0.90$ $Z_0(3) = 0.85$		$Z_{2,ud}(2) = Z_{2,du}(2) = 1.00$ $Z_{2,ud}(3) = Z_{2,du}(3) = 0.96$
	$Z_{1,d}(1) = 1.00$ $Z_{1,d}(2) = 0.97$ $Z_{1,d}(3) = 0.94$	
		$Z_{2,dd}(2) = 1.00$ $Z_{2,dd}(3) = 0.99$

(a) (5 points) Compute the risk neutral probability at time $i = 0$.

(b) (5 points) What is the value of $Z_{1,u}(3)$?

(c) *Expected Bond Returns in Excess of Risk Free Rate* (not risk neutral expectations here! Use the true ones for this part of the quesiton.)

i. (5 points) What is the expected *excess* return between $i = 0$ and $i = 1$ from an investment at time 0 in the 2-period bond $Z_0(2)$?

ii. (10 points) Is this expected excess return (over the same period) the same as for an investment at time 0 in the 3-period bond $Z_0(3)$? If not, why not? Be as precise as possible.

(d) Consider a forward contract to purchase a 1-period zero coupon bond at time $i = 1$ for a price F. The payoff of the forward contract at $i = 1$ is

$$\text{Payoff at } 1 = Z_1(2) - F$$

The value of a forward contract is always zero at time 0.

i. (15 points) What is the value of F? How does it compare with the risk neutral expected value of $E_0^*[Z_1(2)]$? How about with the forward discount $F(0, 1, 2)$ which can be computed from current bond prices? In each case explain any differences.

ii. (10 points) What is a replicating strategy for the payoff to the forward contract? Do you actually need the binomial tree to create this strategy? Why or why not?

3. (35 points) Using the history of changes in interest rates, you computed the following "betas" from principal component analysis.

	Maturity							
Factor	3 months	6 months	1 year	2 year	4 year	6 year	8 year	10 year
				Panel A: Level				
β_{i1}^{PCA}	0.26	0.28	0.28	0.26	0.22	0.18	0.16	0.15
R^2	75 %	88%	96%	96%	89%	81%	71%	61%
				Panel B: Slope				
β_{i2}^{PCA}	-0.40	-0.24	-0.08	0.08	0.12	0.23	0.24	0.24
R^2	98%	97%	97%	97%	99%	97%	90%	81%
				Panel C: Curvature				
β_{i3}^{PCA}	-0.17	0.07	0.20	0.19	0.01	-0.17	-0.30	-0.39
R^2	99%	97%	99%	99%	99%	100%	99%	98%

(a) (15 points) What are the factor durations of 3-months, 4-year, and 10-year zero-coupon bonds? Compute them for all three factors and provide an intuitive reason for their values.

(b) Your fund is long \$100 million in a particular fixed income portfolio. This portfolio has factor durations $D_{level} = 0$ and $D_{slope} = -15$. The current term structure is flat at 5% (continuously compounded).

i. (5 points) Suppose the level factor increases by 100bps and the slope factor also by 100bps. What is the impact on this portfolio?

ii. (10 points) Design a hedging strategy using the zero coupon bonds in 3a to hedge these positions. (You do not necessarily need to use all of them!) Discuss the choice of bonds in your answer. *(Note: Partial credit given to set up the problem right, even without full numerical solution.)*

iii. (5 points) Would the hedging strategy of part 3(b)ii leave the fund with factor and/or residual risk exposure? Why or why not?

4. (35 points) Freddie Mac is one of the key players in mortgage backed securities market.

   The following statements are from a Freddie Mac's Annual Report at a time when Freddie Mac was a privately owned company:

   > "Our primary interest-rate risk measures are PMVS [Portfolio Market Value Sensitivity] and duration gap. PMVS is an estimate of the change in the market value of our net assets and liabilities from an instantaneous 50 basis point shock to interest rates. [...] PMVS is measured in two ways, one measuring the estimated sensitivity of our portfolio market value to parallel movements in interest rates (PMVS-Level or PMVS-L) and the other to nonparallel movements (PMVS-YC).
   >
   > [...]
   >
   > The duration and convexity measures are used to estimate PMVS under the following formula:
   >
   > PMVS = -*[Duration]* multiplied by [rate shock] plus [0.5 multiplied by *Convexity*] multiplied by [rate shock]^2
   >
   > [...]
   >
   > To estimate PMVS-L, an instantaneous parallel 50 basis point shock is applied to the yield curve, as represented by the US swap curve [...]. This shock is applied to the duration and convexity of all interest-rate sensitive financial instruments.
   >
   > [...]
   >
   > To estimate sensitivity related to the shape of the yield curve, a yield curve steepening and flattening of 25 basis points is applied to the duration of all interest-rate sensitive instruments. [...] The more adverse yield curve scenario is then used to determine the PMVS-yield curve."

   Suppose for simplicity that the portfolio of Freddie Mac's *assets* is given by 1 unit each of the 1-year and the 10-year zero-coupon bonds in the table below. In addition, its liabilities are given by 2 units of the 7-year zero coupon bond (I know all these bonds have positive convexity, and not negative convexity as mortgage backed securities, but for the sake of this exercise, this will do).

Table 1: yields and discounts

	1 year	7 year	10 year
$y(t)$	0.7125	1.7307	2.1456
$Z(0,t)$	99.29	88.59	80.69

(a) (15 points) Compute the duration and the convexity of the net assets and liabilities portfolio. *(That is, I am asking the duration and convexity of Equity = Assets minus Liabilities)*

(b) (10 points) Given the result in part 4a compute PMVS_L from a 50 basis point shock (positive or negative).

(c) (10 points) Compute PMVS_YC from a 25 basis point flattening and steepening of the term structure. *(Hint: This appears to be applied only to duration and not convexity of all interest rate sensitive securities. Hint 2: It is OK to assume plus/minus 25 bps change at the extremes, and 0 at the 7-year level).* What is larger in absolute value? PMVS_L or PMVS_YC?