CS 228 Problem Set 1

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Problem 1

Problem 2

2.1

$$P(C) = \sum_c P(A*, B*, c, D*)$$
$$P(C = 1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$
$$P(C = 1 \mid D = 0) = \frac{\frac{1}{4}}{\frac{1}{8} + \frac{1}{8} + \frac{1}{4}} = \frac{1}{2}$$
$$P(C = 1 \mid D = 1) = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{4}} = \frac{1}{2}$$

Thus, C and D are independent.

2.2

$$P(C = 1) = \frac{1}{2}$$
$$P(C = 1 \mid B = 0) = \frac{\frac{1}{4}}{\frac{1}{8} + \frac{1}{4}} = \frac{1}{3}$$

Thus, C and B are not independent.

Problem 3

3.1

Notice that the observed units form a markov blanket on any given hidden unit. Thus, we can compute $P(h_i \mid v)$ without caring about the rest of the hidden units.

$$P(h_i \mid v) = \frac{P(h_i, v)}{\sum_{h_k} P(h_k, v)} = \frac{P(h_i, v)}{\sum_{h_k} P(h_k, v)}$$

TODO CAN YOU BREAK THIS UP

3.2

Since the hidden units are all independent conditioned on the observed units, we can just factor the probabilities that way.

$$P(h \mid v) = \frac{P(v,h)}{\sum_{i \in \{0,1\}^n} P(v,i)} = \prod_{h_i} \frac{P(v,h_i)}{\sum_{h_k \in \{0,1\}} P(v,h_k)} = \prod_{h_i} P(h_i \mid v)$$