Step-Cone Pulley Optimization

by

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ABSTRACT

This study investigates the optimal design of a step-cone pulley. Although the design of the step-cone pulley is simple, it has wide applicability and is commonly used in mechanical systems, such as drilled presses and lathes, to easily change the output speed or power. A step-cone pulley can be defined as series of pulleys in decreasing circumference, stacked on top of one another forming a stepped cone. The step-cone pulley is mainly used to vary the velocity ratios of shafts, allowing shifting the speed of the pulley's movements, with a smaller circumference requiring less work but also producing less work. The increasing costs and intense competition as result of globalization and the recent economic crisis have emphasized the need of manufacturing at the lowest cost possible. However, the minimization of cost should not compromise the structural performance of the step-cone pulley system. Therefore, the motivation for performing an optimization study was to determine optimal dimensions that will minimize manufacturing cost subject to design, performance and structural constraints. The optimization study involved different algorithms and a sensitivity analysis, revealing the different trade-offs as well as providing insights regarding the choice of the design.

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1. INTRODUCTION

The design of the step-cone pulley should be such that the desired power transmitted is achieved without structural failure. The objective of the optimization problem is to minimize the pulley system total weight, which is a surrogate for minimizing manufacturing cost. It is assumed that the total cost (shipping, material handling, production, etc.) is proportional to the weight of the step-cone pulley. The optimization problem is subject to performance requirements, dimension limitations and mechanical behavior of material constraints. The anticipated trade-offs that motivated the optimization study and affected the weight included the dimensions, material property, power transmitted and structural performance.

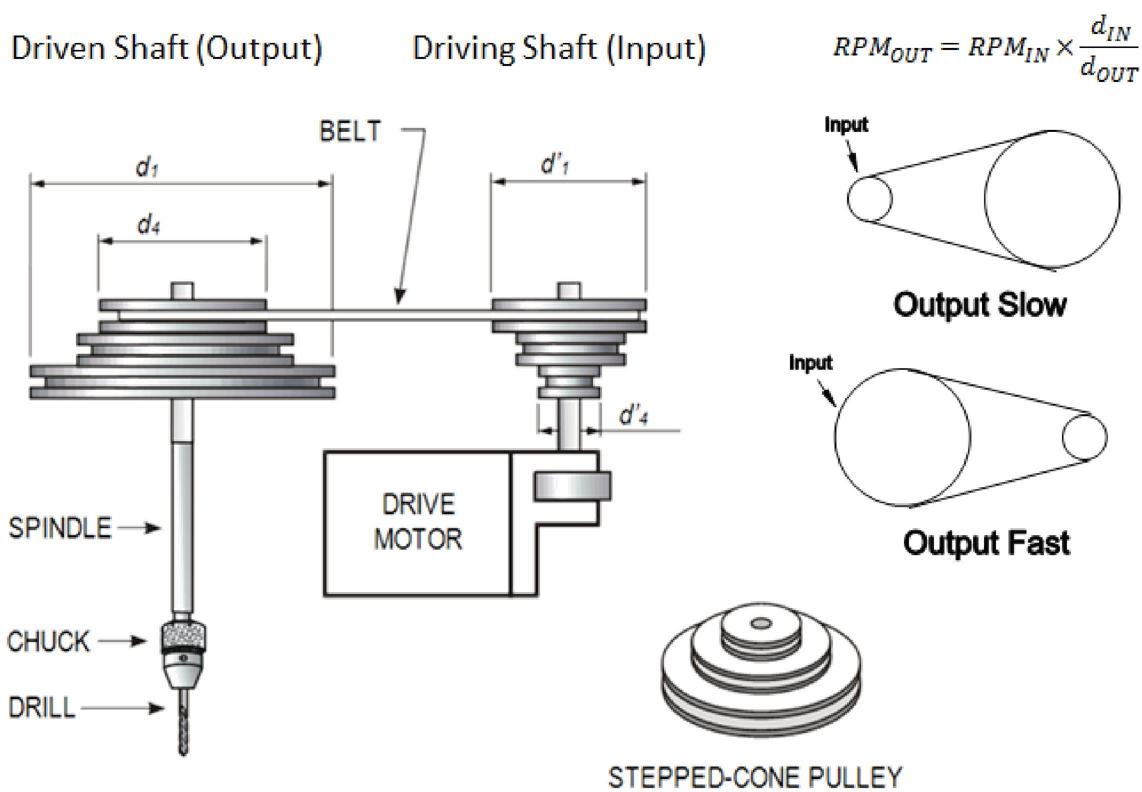


Figure 1. Step-cone pulley system for a drilling press

2. DESIGN PROBLEM STATEMENT

The goal of the project was to optimize the design variables for minimum weight of step-cone pulley system. The different performance requirements will dictate the physical requirements to avoid structural failure and achieved the desired output.

3. NOMECLATURE

Symbol	Description
d_i	Diameter of the ith step on the output pulley ($i = 1,2,3,4$)
d_i'	Diameter of the ith step on the input pulley ($i = 1,2,3,4$)
N	Input speed of the shaft
N_i	Output speed of the ith step ($i = 1,2,3,4$)
a	Center distance between shafts
C_i	Length of the belt needed to obtain output speed N_i ($i = 1,2,3,4$)
w	Width of the belts and steps
ρ	Density of the material of the pulleys
μ	Coefficient of friction between belt and pulley
θ_i	Angle of lap of the belt over the ith pulley step ($i = 1,2,3,4$)
T_1^i	Tensions on the tight sides of the ith step ($i = 1,2,3,4$)
T_2^i	Tensions on the slack sides of the ith step ($i = 1,2,3,4$)
s	Maximum allowable stress in the belt
t	Thickness of the belt
R_i	Tension ratio at the ith step ($i = 1,2,3,4$)
P_i	Power transmitted at the ith step ($i = 1,2,3,4$)
P_0	Minimum required power transmitted by the step pulley
R_0	Minimum required ratio of the tension on the tight side of the belt to that on the slack side

A graphical illustration of the design problem is shown in Figure 2.

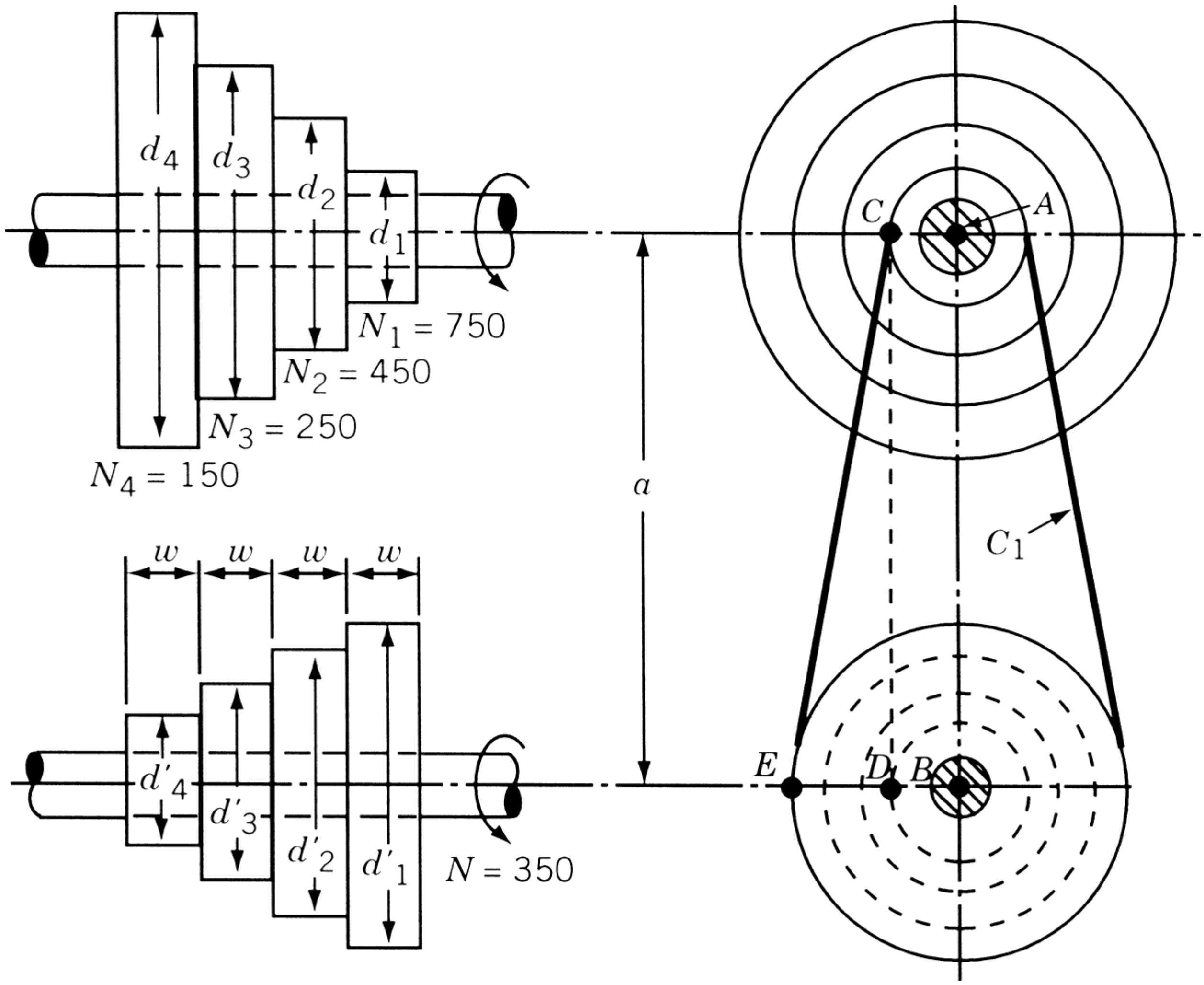


Figure 2. Step-Cone pulley (Rao, 2012)

4. MATHEMATICAL MODEL

4.1 Objective function

The objective is to minimize the total weight of the step-cone pulley system. Thus, the objective function is just the sum of the pulleys steps' weights, which are the product of the densities and cylindrical volume. Assuming the same material and width for the steps of the pulley, the objective function can be written as:

$$f(\boldsymbol{X}) = \sum_{i=1}^{4} \rho w \frac{\pi}{4} \left({d_i}^2 + {d_i'}^2\right) \tag{1}$$

where ρ is the density of the pulleys' material, w is the width of the pulleys, d_i is the diameter of the ith step on the output pulley, and d_i' is the diameter of the ith step on the input pulley. Assuming that the thickness of the belt is small compared to the diameter of the steps, and that there is no slippage between the belt and pulley, then the velocities in the output and input pulleys are equal and the following expression holds:

$$\frac{\pi d_i N_i}{60} = \frac{\pi d_i' N}{60} \tag{2}$$

where N is the input is speed of the shaft and N_i is the output speed of the ith step. Therefore, the relationship between the output and input pulley is given by:

$$\frac{N_i}{N} = \frac{d_i'}{d_i} \tag{3}$$

Substituting d_i' from Eq. 3 into Eq. 1 the objective function can be expressed as:

$$f(\boldsymbol{X}) = \sum_{i=1}^{4} \rho w \frac{\pi}{4} \left\{ {d_i}^2 \left[1 + \left(\frac{N_i}{N} \right)^2 \right] \right\} \tag{4}$$

where $\boldsymbol{X} = (d_1, d_2, d_3, d_4, w)$ is the design vector.

4.2 Constraints

Constant Belt Length

The first set of constraints is to assure that the total belt length remains the same for all speeds. This will ensure equal tightness of the bell on each pair of opposite steps. Thus, the equality constraints needed to satisfy this are:

$$C_1 - C_2 = 0 \tag{5}$$

$$C_1 - C_3 = 0 \tag{6}$$

$$C_1 - C_4 = 0 \tag{7}$$

where C_i is the length of the belt needed to obtain output speed N_i $(i = 1,2,3,4)$ and can be approximated as (Juvinall, 1991 and Shigley, 1989):

$$C_i = \frac{\pi d_i}{2}\left(1 + \frac{N_i}{N}\right) + \frac{\left(\frac{N_i}{N} - 1\right)^2}{4a} + 2a, \quad i = 1,2,3,4 \tag{8}$$

where a is the center distance between shafts.

Tension Ratios

The second set of constraints is to assure that for each pair of opposite steps, the ratios of the tension on the tight side of the belt to that on the slack side are at least equal to the minimum required tension ratio. This is expressed with the following inequality constraint:

$$R_i \geq R_0, \quad i = 1,2,3,4 \tag{9}$$

where R_i is ratio of the tension on the tight side of the belt to that on the slack side of the ith step, and R_0 is the minimum required tension ratio. The difference in tensions in the belt is caused by the friction between the pulley and belt, which enables the power transmission from one pulley to the other by creating a relative motion between the belt and pulley surface. If the tension ratio is smaller than the allowable tension ratio, then unnecessary higher tensions will be needed to transmit the same power, causing a reduction in the belt life. The relationship between the tight side and slack side tensions in the belt are given by:

$$R_i = \frac{T_1^i}{T_2^i} = e^{\mu\theta_i}, \quad i = 1,2,3,4 \qquad (10)$$

where T_1^i and T_2^i are the tensions in the tight side and slack side of the belt of the ith step respectively, μ is the coefficient of friction between belt and pulley, and θ_i is the angle of lap of the belt over the ith pulley step. The angle of lap of can be expressed as:

$$\theta_i = \pi - 2\sin^{-1}\left[\left(\frac{N_i}{N} - 1\right)\frac{d_i}{2a}\right], \quad i = 1,2,3,4 \qquad (11)$$

Substituting θ_i from Eq. 11 into Eq. 10, the tension ratio can be rewritten as:

$$R_i = \exp\left\{\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_i}{N} - 1\right)\frac{d_i}{2a}\right\}\right]\right\}, \quad i = 1,2,3,4 \qquad (12)$$

Power transmitted

The third set of constraints ensures that the step-cone pulley system is capable of transmitting a power of at least the minimum required power P_0. This requirement translates into the following inequality constraint:

$$P_i \geq P_0, \quad i = 1,2,3,4 \qquad (13)$$

where P_i is the power transmitted at the ith step. The power transmitted between the belt and the pulley can be expressed as the product of the difference of tension and belt velocity:

$$P_i = (T_1^i - T_2^i)\frac{\pi d_i N_i}{60}, \quad i = 1,2,3,4 \tag{14}$$

where the 60 is included assuming the speed of the shaft is in rpm units.

Given that the maximum allowable stress (s) in the belt can be expressed as:

$$s = \frac{T_1^i}{tw}, \quad i = 1,2,3,4 \tag{15}$$

where t is the thickness of the belt, then the power transmitted at the ith step can be rewritten by substituting T_1^i with the expression in Eq. 13 and using the relationship between T_1^i and T_2^i stated in Eq. 10:

$$P_i = stw\left[1 - \exp\left\{-\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_i}{N} - 1\right)\frac{d_i}{2a}\right\}\right]\right\}\right]\frac{\pi d_i N_i}{60}, \quad i = 1,2,3,4 \tag{16}$$

Design Variable Bounds

The next set of constraints deals with limiting the design variables in the form of upper and lower bounds. The constraints on the design variables are

$$d_{iL} \le d_i \le d_{iU}, \quad i = 1,2,3,4 \tag{17}$$

$$w_L \le w \le w_U \tag{18}$$

Here d_{iL} and d_{iU} denote the lower bound (minimum allowed value) and upper bound (maximum allowed value) respectively for the ith diameter, and w_L and w_U denote the lower bound (minimum allowed value) and upper bound (maximum allowed value) respectively for the width.

4.3 Design Variables and Parameters

The above model and constraints contained several parameters and design variables. Table 1 shows the design variables and the parameters for the system.

Table 1. Design variables and parameters

Design Variables	Parameters	Value	Units
d_1	N	350	rpm
d_2	N_1	750	rpm
d_3	N_2	450	rpm
d_4	N_3	250	rpm
w	N_4	150	rpm
	a	3	m
	ρ	7,200	kg/m^3
	μ	0.35	
	s	1.75	MPa
	t	8	mm
	P_0	0.75	hp
	R_0	2	
	d_{1L}, d_{1U}	40,500	mm
	d_{2L}, x_{2U}	40,500	mm
	d_{3L}, x_{3U}	40,500	mm
	d_{4L}, x_{4U}	40,500	mm
	w_L, w_U	16,100	mm

Note that after converting to appropriate units, the values for the following parameters are:

Table 2. Unit conversion

Parameters	Converted Value	Units
a	3×10^3	mm
ρ	$7{,}200 \cdot 1 \times 10^{-9}$	kg/mm^3
s	$1.75 \cdot 1 \times 10^6 \cdot 1 \times 10^{-3}$	$kg/mm \cdot s^2$
P_0	$0.75 \cdot 745.6998 \cdot 1 \times 10^6$	$kg \cdot mm^2/s^3$

A set of values for the design variables that satisfy all of the constraints is shown in Table 3. This indicates that there is at least one feasible solution in the model stated. Details of the solution can be found in Appendix A.

Table 3. Feasible solution

Design Variables	Value	Units
d_1	70	mm
d_2	96.25	mm
d_3	128.333	mm
d_4	154	mm
w	50	mm
Objective	**Value**	**Units**
f	29.6661	kg

The final model formulation consists of minimizing the objective function for the total weight (Eq. 4) by finding the values of 5 continuous design variables (d_1, d_2, d_3, d_4, w), subject to 3 equality constraints for the constant belt length (Eqs. 5, 6 and 7), 4 inequality constraints for the tension ratios (Eq. 9), 4 inequalities constraints for the power transmitted (Eq. 13), and 10 inequality constraints for the lower and upper bounds of the design variables (Eq. 17 and 18).

4.4 Summary Model

The model consists of

$$Min\ f = \rho w \frac{\pi}{4}\left\{{d_1}^2\left[1+\left(\frac{N_1}{N}\right)^2\right]+{d_2}^2\left[1+\left(\frac{N_2}{N}\right)^2\right]+{d_3}^2\left[1+\left(\frac{N_3}{N}\right)^2\right]+{d_4}^2\left[1+\left(\frac{N_4}{N}\right)^2\right]\right\}$$

Subject to:

$$h_1\colon\ C_1 - C_2 = 0$$

$$h_2\colon\ C_1 - C_3 = 0$$

$$h_3\colon\ C_1 - C_4 = 0$$

$$g_1\colon\ R_0 - \exp\left\{\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_1}{N}-1\right)\frac{d_1}{2a}\right\}\right]\right\} \le 0$$

$$g_2\colon\ R_0 - \exp\left\{\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_2}{N}-1\right)\frac{d_2}{2a}\right\}\right]\right\} \le 0$$

$$g_3\colon\ R_0 - \exp\left\{\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_3}{N}-1\right)\frac{d_3}{2a}\right\}\right]\right\} \le 0$$

$$g_4\colon\ R_0 - \exp\left\{\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_4}{N}-1\right)\frac{d_4}{2a}\right\}\right]\right\} \le 0$$

$$g_5\colon\ P_0 - stw\left[1 - \exp\left\{-\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_1}{N}-1\right)\frac{d_1}{2a}\right\}\right]\right\}\right]\frac{\pi d_1 N_1}{60} \le 0$$

$$g_6\colon\ P_0 - stw\left[1 - \exp\left\{-\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_2}{N}-1\right)\frac{d_2}{2a}\right\}\right]\right\}\right]\frac{\pi d_2 N_2}{60} \le 0$$

$$g_7\colon\ P_0 - stw\left[1 - \exp\left\{-\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_3}{N}-1\right)\frac{d_3}{2a}\right\}\right]\right\}\right]\frac{\pi d_3 N_3}{60} \le 0$$

$$g_8\colon\ P_0 - stw\left[1 - \exp\left\{-\mu\left[\pi - 2\sin^{-1}\left\{\left(\frac{N_4}{N}-1\right)\frac{d_4}{2a}\right\}\right]\right\}\right]\frac{\pi d_4 N_4}{60} \le 0$$

$$g_9, g_{10}\colon\ d_{1L} \le d_1 \le d_{1U}$$

$$g_{11}, g_{12}\colon\ d_{2L} \le d_2 \le d_{2U}$$

$$g_{13}, g_{14}\colon\ d_{3L} \le d_3 \le d_{3U}$$

$$g_{15}, g_{16}\colon\ d_{4L}, \le d_4 \le d_{4U}$$

$$g_{17}, g_{18}\colon\ w_L \le w \le w_U$$

where

$$C_i = \frac{\pi d_i}{2}\left(1+\frac{N_i}{N}\right)+\frac{\left(\frac{N_i}{N}-1\right)^2}{4a}+2a, \qquad i = 1,2,3,4$$

5. MODEL ANALYSIS

A monotonicity analysis of the system was used to determine well-boundedness of the problem formulation. The monotonicity table is shown below in Table 4.

	d_1	d_2	d_3	d_4	w
f	+	+	+	+	+
h1	+/-	+/-			
h2	+/-		+/-		
h3	+/-			+/-	
g1	+				
g2		+			
g3			-		
g4				-	
g5	-				-
g6		-			-
g7			-		-
g8				-	-
g9	-				
g10	+				
g11		-			
g12		+			
g13			-		
g14			+		
g15				-	
g16				+	
g17					-
g18					+

Table 4. Monotonicity analysis of the mathematical model

The monotonicity of the function and constraints with respect to the design variables were found by taking the partial derivative with respect to the corresponding decision variable. According to the First Monotonicity Principle 1 (MP1), which states that in a well-constrained minimization problem every increasing variable is bounded below by at least one non-increasing active constraint, the problem seems well-bounded because all objective variables have opposite monotonicity in the constraints. This conclusion holds regardless of the direction of the equalities constraints. No redundant constraints were detected.

6. OPTIMIZATION STUDY

6.1 Local Results

The constrained mathematical model to be optimized was solved using the *fmincon* function in MATLAB's Optimization Toolbox to find a local minimum. Appendix B shows the details of the code with the default parameters (e.g. maximum iterations and tolerances).

The first set of numerical results was obtained using the solution described in the mathematical model section as the initial starting point for three algorithms: sequential quadratic programming (SQP), active set (AS), and interior point (IP). The results of the MATLAB output can be found in Appendix C and are summarized below in Figure 3 and Table 5.

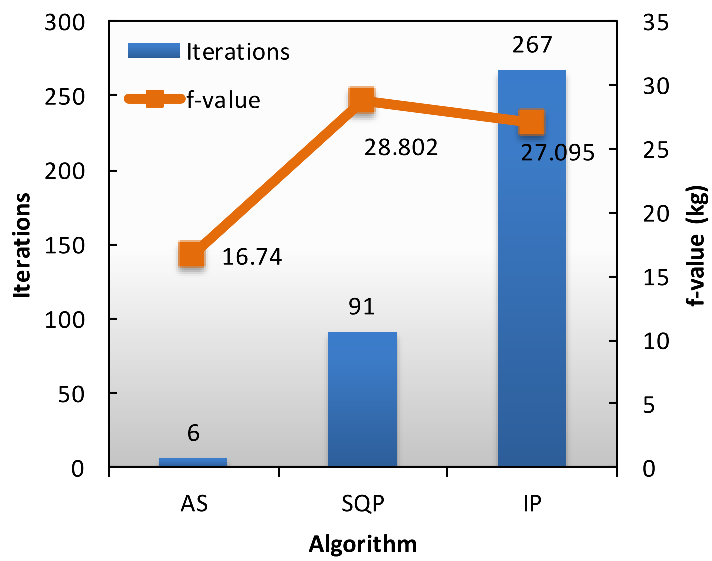


Figure 3. Graphical summary of first round of optimization results

Table 5. Summary of first round of optimization results

Algorithm	Iterations	Exitflag	Comp Time (s)	f	d1	d2	d3	d4	w
SQP	91	0	5.915	28.802	68.965	94.827	126.436	151.724	50.011
AS	6	4	1.727	16.740	40.000	55.000	73.333	88.000	86.406
IP	267	0	17.993	27.095	64.811	89.116	118.821	142.585	53.271

The MATLAB output indicated that SQP and IP terminated prematurely because the maximum number of iterations or function evaluations being was exceeded. On the other hand, AS reached a solution within 6 iterations and stopped because the size of the current search direction was less than twice the default value of the step size tolerance and constraints were satisfied to within the default value of the constraint tolerance. The MATLAB output also stated that the solution was a possible local minimum and that all constraints were satisfied. Only the constraint on the lower bound of d_1 was found to be active, and thus the solution is a boundary solution. Comparing the solutions when the algorithms terminated, it can be seen that the function value is much lower for AS than for SQP or IP.

Because SQP and IP stopped prematurely, a different set of initial starting points was tested. The results for the initial starting point with diameters equal to 50 mm and width equal to 80 mm can be found in Appendix C and are summarized below in Figure 4 and Table 6.

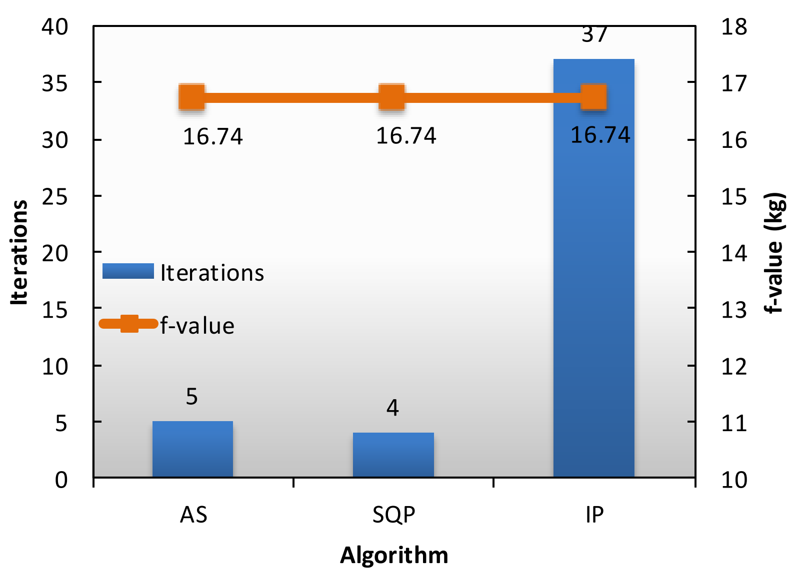


Figure 4. Graphical summary of second round of optimization results summary

Table 6. Summary of second round of optimization results

Algorithm	Iterations	Exitflag	Comp Time (s)	f	d1	d2	d3	d4	w
SQP	5	1	1.594	16.740	40.000	55.000	73.333	88.000	86.406
AS	4	1	1.727	16.740	40.000	55.000	73.333	88.000	86.406
IP	37	1	3.871	16.740	40.000	55.000	73.333	88.000	86.406

The MATLAB output indicated that all algorithms terminated because first-order optimality measure and maximum constraint violation was less than the tolerance. In other words, the objective function was non-decreasing in feasible directions, to within the default value of the function tolerance, and constraints were satisfied to within the default value of the constraint tolerance. From Table 6 it can be seen that SQP and AS had similar number of iterations and computation time, and these were lower than those of IP. The MATLAB output also stated that a local minimum was found that satisfies the constraints, indicating that the solution satisfied the KKT conditions. AS also indicated that the inequalities constraints for power transmitted and tension ratio, and the lower bound of d_1 were found to be active, implying a boundary solution. All algorithms converged to the same solution.

6.2 Global Results

The constrained mathematical model to be optimized was solved using the *GlobalSearch* in MATLAB's Optimization Toolbox to find a global minimum. The algorithm starts the local solver *fmincon* from multiple starting points, analyzes them and rejects those points that are unlikely to improve the best local minimum found so far. The process is repeated until a global solution vector is generated.

Appendix B shows the details of the code with the default parameters (e.g. maximum iterations and tolerances). The results of the MATLAB numerical output using *GlobalSearch* and SQP as the algorithm can be found in Appendix C, and the graphical output is shown below in Figure 5.

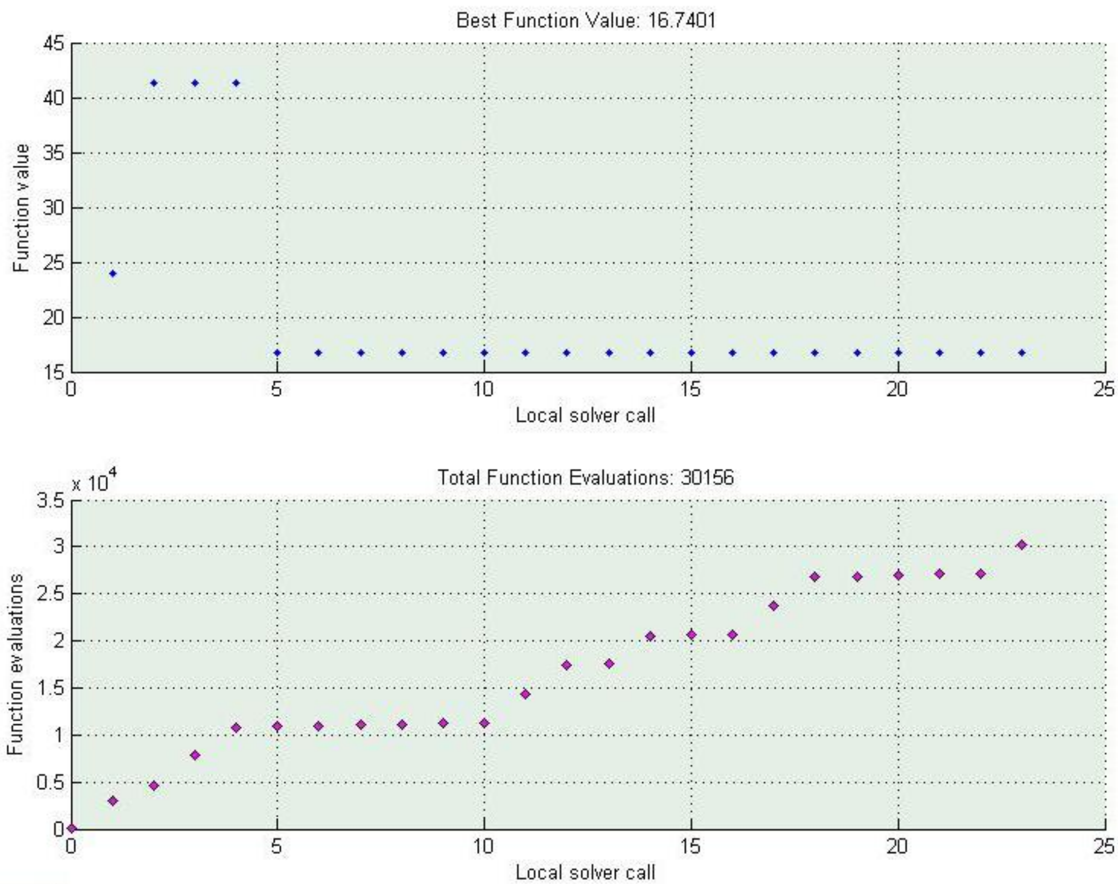


Figure 5. Function value and evaluations vs. Solver call

The plot indicates that after the fifth solver call, the solver reached a solution with the same function value (i.e. f = 16.740) as the one in the previous section. The MATLAB output indicates that *GlobalSearch* stopped because it analyzed all the trial points, of which 14 out of 23 local solver runs converged with a positive local solver exit flag (e.g. feasible solution). The computation time was 38.99 seconds and the final solution matched same local solution found in the previous section.

The numerical results obtained so far used algorithms (e.g. SQP, IP) that are gradient-based methods, meaning that information of derivatives (e.g. gradients and hessians) are used in the computation. On the other hand, direct search methods (e.g. simulated annealing) do not involve derivative information in the calculations of finding a solution. Even though simulated annealing is mainly used for problems involving discrete variables and a large search space, the metaheuristic was tested to find a global minimum of the constrained mathematical model and compared the solution to the one found by the gradient-based methods. The results of the MATHEMATICA output using *NMinimize* (Appendix A) and simulated annealing as the method agree with the results found previously with gradient-based methods.

The results of the optimization study are summarized below in Table 7.

Table 7. Optimization results

Design Variables	**Value**	**Units**
d_1	40.000	mm
d_2	55.000	mm
d_3	73.333	mm
d_4	88.000	mm
w	86.406	mm
Objective	**Value**	**Units**
f	16.740	kg

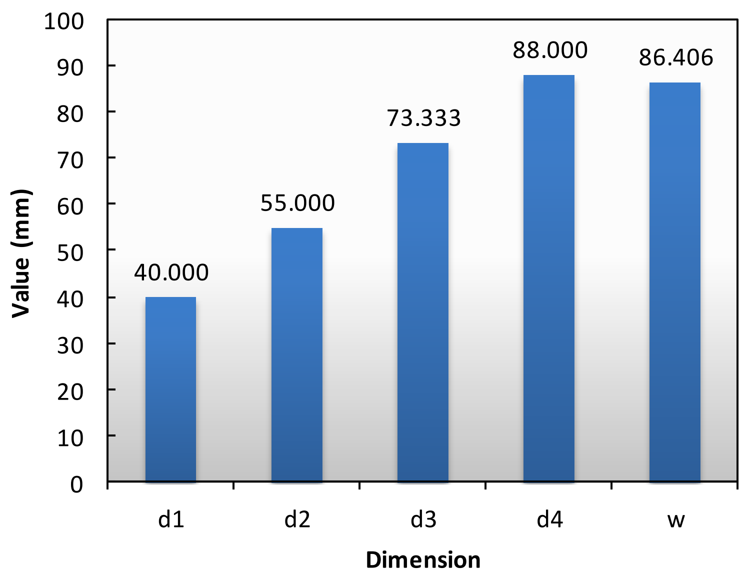


Figure 6. Optimization results graphical summary

7. PARAMETRIC STUDY

A parametric study was conducted for different sets of parameters to gain insights and test the feasibility and optimality of the problem formulation. The chosen parameters for the sensitivity analysis were the input speed of the shaft (N), the center distance between shafts (a), the maximum allowable stress in the belt (s), the minimum required power transmitted by the step pulley (P_0) and the minimum required ratio of the tension on the tight side of the belt to that on the slack side (R_0). The latter two would provide additional insights on the sensitivity of constraints to evaluate the benefits of moving or removing the constraints.

The parametric study was conducted in MATLAB using *GlobalSearch* and SQP as the algorithm for *fmincon*. The numerical results can be found in Appendix D. The sensitivity to shaft center distance (Figure 7) shows that the optimal solution is not very sensitive to this parameter. The optimal weight slightly changes due to a small change in the optimal width.

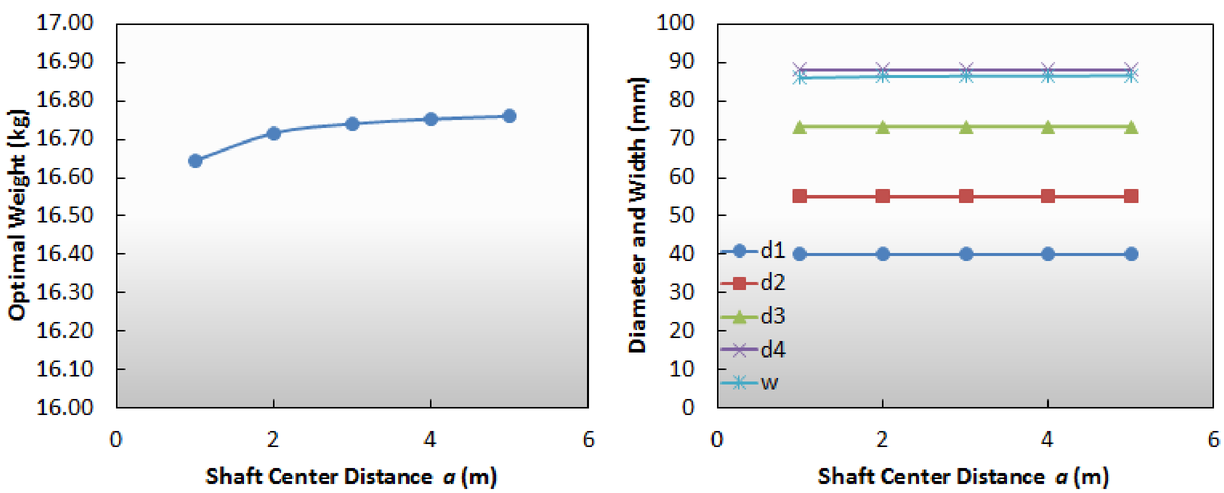


Figure 7. Sensitivity to shaft center distance

The sensitivity to input speed (Figure 8) shows that the optimal solution is very sensitive to this parameter. The optimal weight decreases as the input speed increases mainly due to a decrease in the optimal diameters, which have a greater effect than the increase in optimal width.

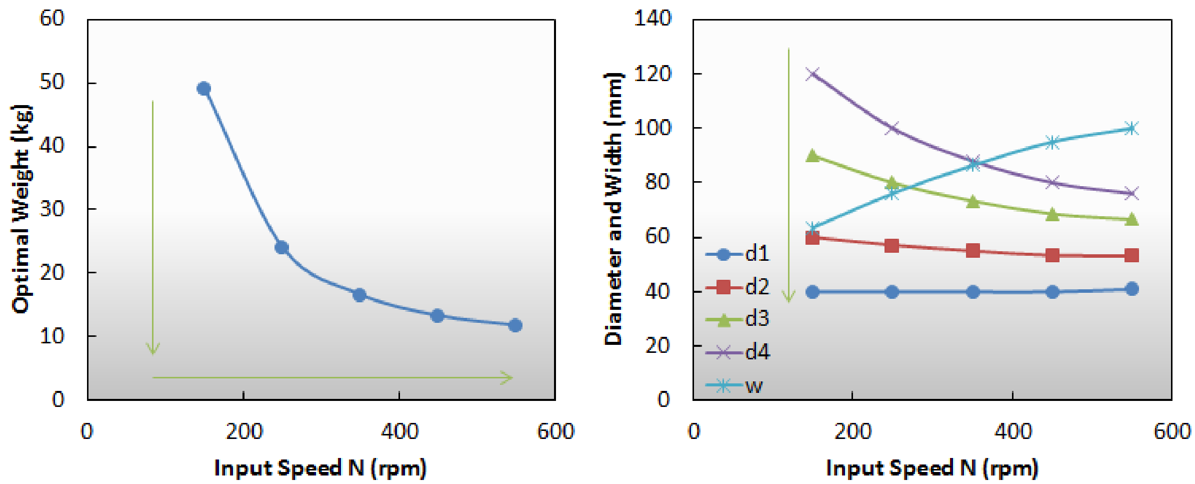


Figure 8. Sensitivity to input speed

The sensitivity to minimum tension ratio (Figure 9) shows that the optimal solution is insensitive to this parameter. The optimal solution does not change with changes in the minimum required tension ratio, suggesting that this constraint should not be an important concern for the designer.

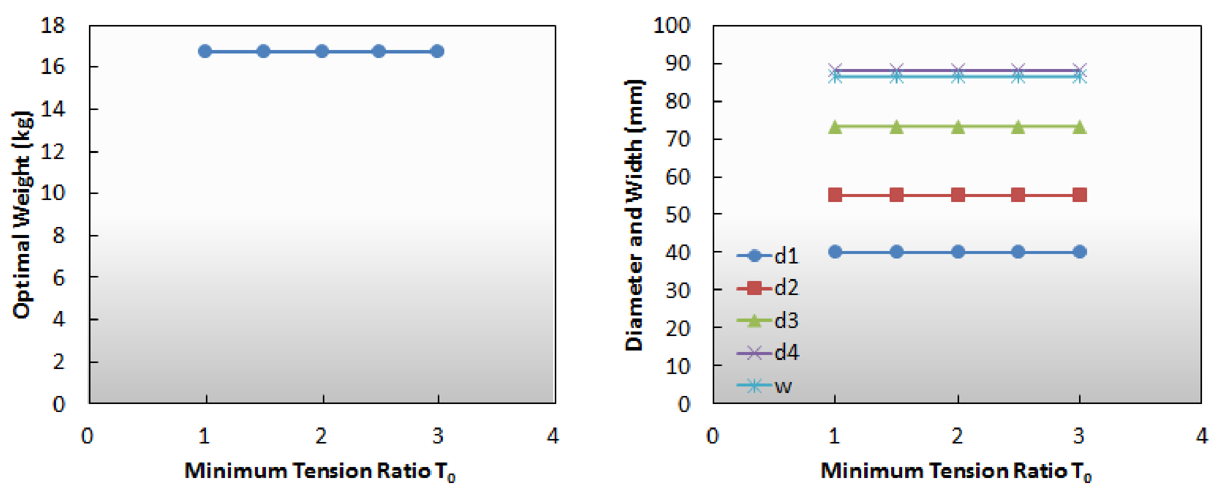


Figure 9. Sensitivity to minimum tension ratio

The sensitivity to maximum allowable stress (Figure 10) shows that the optimal solution is very sensitive to this parameter. As expected, the optimal weight decreases as the constraint is relaxed (i.e. maximum allowable stress is increased). In addition, there is a point at which for higher values of the parameter (i.e. constraint is relaxed more) only the width changes, and for lower values only the diameters change.

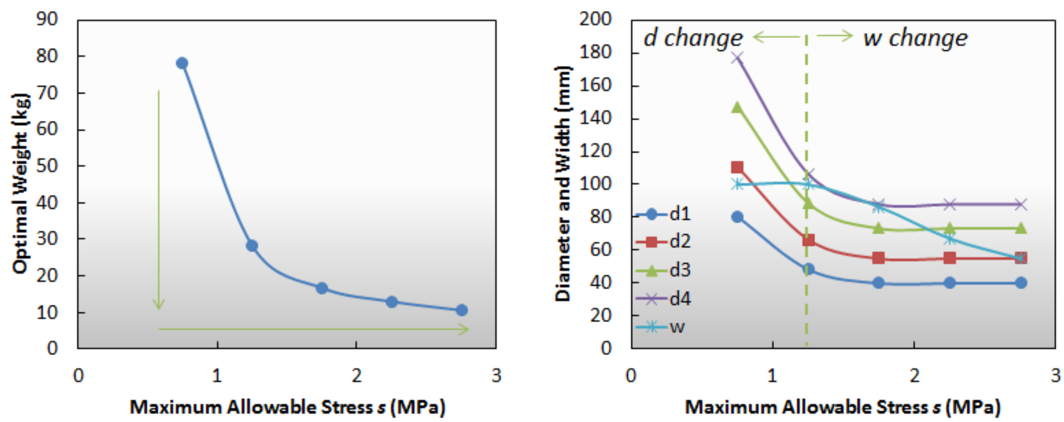


Figure 10. Sensitivity to allowable stress

Interestingly, the sensitivity to the minimum required power (Figure 11) shows the same pattern. As expected, the optimal weight decreases as the constraint is relaxed (i.e. minimum power is decreased). In addition, there is a point at which for lower values of the parameter (i.e. constraint is relaxed more) only the width changes, and for higher values only the diameters change.

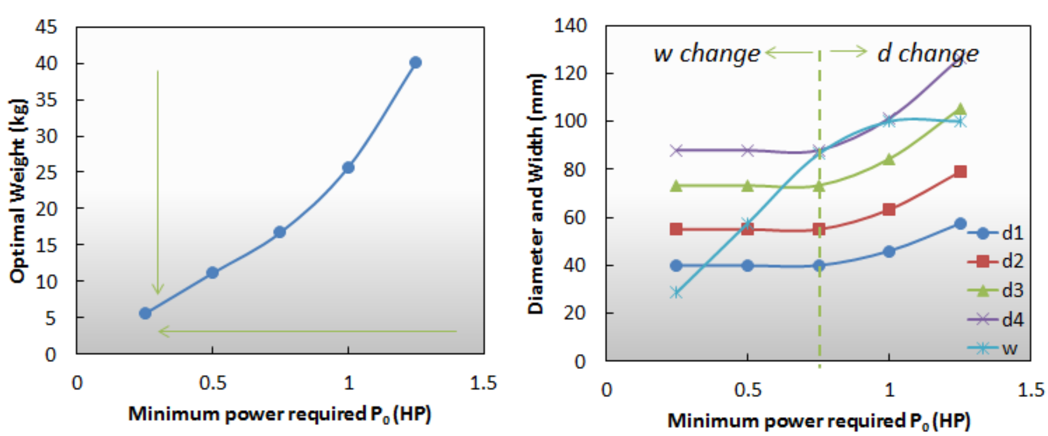


Figure 11. Sensitivity to minimum power required

8. DISCUSSION OF RESULTS

The optimization results revealed that an optimal set of design parameters can be found that minimize the weight. The results also showed constraint activity, satisfaction of KKT points, the type of solution (e.g. boundary optima), the agreement between local and global results, and the impact of the different starting points with the algorithms.

In addition, since SQP guarantees global convergence from any arbitrary point, then the solution found in this optimization study is guaranteed to be a local minimum since the algorithm converged. However, SQP was found to be highly sensitive to the initial starting points, which might be an indication of ill-conditioning. Thus, future work might involve normalizing the variables so a proper scale is used and faster convergence can be achieved. Trust regions might also help with poor starting points. Other issues that should be verified include checking the regularity (i.e. gradients of active inequality constraints are linearly independent).

The optimal solution made practical sense. For example, the size of the diameters increased from d_1 to d_4, with the increase from each step being not very different. One of the most interesting results is that d_1 seems insensitive to most of the parameters As expected, the difference between diameters remained almost constant across all parametric studies. Regarding the sensitivity analysis, the results revealed interesting insights as described in the previous section. For instance, diameters should be the main concern of the design, while the width should consider last to satisfy the constraints. Future work might involve conducting sensitivity analysis on the material of the pulley since they type of material has great impact on the cost. However, the coefficient of friction should be taken into account because it changes based on the material. Further sensitivity analysis might also help identify design rules for the optimal solution, and reveal trade-offs between parameters that would aid in the best design of the step-cone pulley.

REFERENCES

R. C. Juvinall and K. M. Marshek, *Fundamentals of Machine Component Design*, 2nd ed., Wiley, New York, 1991.

P.Y. Papalambros and D.J. Wilde, *Principles of optimal design: modeling and computation* , 2nd ed., Cambridge University Press, 2000.

Rao, R V, and Vimal J. Savsani. *Mechanical Design Optimization Using Advanced Optimization Techniques*. London: Springer, 2012. Internet resource

J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989.

APPENDIX

A. MATHEMATICA Code & Output

Define Function

```
f[d1_,d2_,d3_,d4_,w_]:=π/4*ρ*w*(d1^2*(1+(N1/N)^2)+d2^2*(1+(N2/N)
^2)+d3^2*(1+(N3/N)^2)+d4^2*(1+(N4/N)^2));
```

Define constraints

```
h={C1-C2==0, C1-C3==0,C1-C4==0};
ga=ToExpression[Table["R"<>ToString[i]<>">=2",{i,1,4}]]
{R1≥2,R2≥2,R3≥2,R4≥2}
```

```
gb=ToExpression[Table["P"<>ToString[i]<>">=0.75*745.6998*1000^2"
,{i,1,4}]]
{P1≥5.59275×10^8,P2≥5.59275×10^8,P3≥5.59275×10^8,P4≥5.59275×10^8}
```

Length of belt

```
Ci[di_,Ni_]:=π*di/2*(1+Ni/N)+(Ni/N-1)^2/(4*a)+2a
Ca={Ci[d1,N1],Ci[d2,N2],Ci[d3,N3],Ci[d4,N4]};
```

Tension Ratios

```
Ri[di_,Ni_]:=Exp[μ*(π-2*ArcSin[(Ni/N-1)*di/(2*a)])];
Ra={Ri[d1,N1],Ri[d2,N2],Ri[d3,N3],Ri[d4,N4]};
```

Power Transmitted

```
pi[di_,Ni_]:=s*t*w*(1-Exp[-μ*(π-2*ArcSin[(Ni/N-
1)*di/(2*a)])])*π*di*Ni/60;
Pa={pi[d1,N1],pi[d2,N2],pi[d3,N3],pi[d4,N4]};
```

Parameters

```
parameters={ρ→7200/1000^3,a→ 3*1000,μ→ 0.35,s→
1.75*10^6/1000,t→ 8,N→ 350,N1→ 750,N2→ 450,N3→ 250,N4→ 150};
```

Objective

```
objective=f[d1,d2,d3,d4,w]/.parameters;
```

Equality constraints: Length of belt

```
eqconstraints=h/.{C1→Ca[[1]],C2→Ca[[2]],C3→Ca[[3]],C4→Ca[[4]]}/.
parameters;
```

Inequality constraints: Tension Ratios

```
ineqconstraints1=ga/.{R1→Ra[[1]],R2→Ra[[2]],R3→Ra[[3]],R4→Ra[[4]
]}/.parameters;
```

Inequality constraints: Power Transmitted

```
ineqconstraints2=gb/.{P1→Pa[[1]],P2→Pa[[2]],P3→Pa[[3]],P4→Pa[[4]
]}/.parameters;
```

Inequality constraints: Lowerbounds

```
lowerbounds={d1≥ 40,d2≥ 40,d3≥ 40,d4≥ 40,w≥ 16};
```

Inequality constraints: Upperbounds

```
 upperbounds={d1≤  500,d2≤  500,d3≤  500,d4≤  500,w≤  100};
```

Find a feasible solution

```
 sol=Solve[eqconstraints/.{d1→70},{d2,d3,d4}];
x0={d1→70, sol[[1,1]],sol[[1,2]],sol[[1,3]],w→50};
eqconstraints/.x0
ineqconstraints1/.x0
ineqconstraints2/.x0
lowerbounds/.x0
upperbounds/.x0
x0//N
objective/.x0//N
 {True,True,True}
 {True,True,True,True}
 {True,True,True,True}
 {True,True,True,True,True}
 {True,True,True,True,True}
 {d1→70.,d2→96.25,d3→128.333,d4→154.,w→50.}
 29.6661
```

Optimization Study

```
NMinimize[{objective,eqconstraints[[1]],eqconstraints[[2]],eqcon
straints[[3]],ineqconstraints1[[1]],ineqconstraints1[[2]],ineqco
nstraints1[[3]],ineqconstraints1[[4]],ineqconstraints2[[1]],ineq
constraints2[[2]],ineqconstraints2[[3]],ineqconstraints2[[4]],lo
werbounds[[1]],lowerbounds[[2]],lowerbounds[[3]],lowerbounds[[4]
],lowerbounds[[5]],upperbounds[[1]],upperbounds[[2]],upperbounds
[[3]],upperbounds[[4]],upperbounds[[5]]},{d1,d2,d3,d4,w},Method→
"SimulatedAnnealing" ]
 {16.7401,{d1→40.,d2→55.,d3→73.3334,d4→88.,w→86.406}}
```

B. MATLAB Code

A.1 FUN.m

```
%% IE 513 - Final Project

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% File Name: FUN.m
% Date: 12/01/12
% Author: Luis S Lin
% Description: Objective for step-cone pulley weight minimization
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%
function [f]=FUN(x,P)
x1 = x(1);
x2 = x(2);
x3 = x(3);
x4 = x(4);
x5 = x(5);

% Objective = sum of weights (weight = density * volume of cylinder)

f = P.p*x5*pi/4*(x1^2*(1+(P.N1/P.N)^2)+...
                 x2^2*(1+(P.N2/P.N)^2)+...
                 x3^2*(1+(P.N3/P.N)^2)+...
                 x4^2*(1+(P.N4/P.N)^2));
```

A.2 NONLCON.m

```
%% IE 513 - Final Project

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% File Name: NONLCON.m
% Date: 12/01/12
% Author: Luis S Lin
% Description: Nonlinear constraints for step-cone pulley weight minimization
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


function [g,h]=NONLCON(x,P)
x1 = x(1);
x2 = x(2);
x3 = x(3);
x4 = x(4);
x5 = x(5);

% Output speed of the step i
N = [P.N1 P.N2 P.N3 P.N4];
```

```
% Diameter of step i
d = [x1 x2 x3 x4];

% C(i): length of the belt needed to obtain output speed Ni
C = zeros(1,4);
for i = 1:4
    C(i) = pi*d(i)/2*(1+N(i)/P.N)+(N(i)/P.N-1)^2/(4*P.a)+2*P.a;
end

% RT(i): ratio of the tension on the tight side of the belt to that
% on the slack side of the ith step
RT = zeros(1,4);
for i = 1:4
    RT(i) = exp(P.mu*(pi-2*asin((N(i)/P.N-1)*d(i)/(2*P.a))));
end

% PW(i): power transmitted at the ith step
PW = zeros(1,4);
for i = 1:4
    PW(i) = P.s*P.t*x5*(1-exp(-P.mu*(pi-2*asin((N(i)/P.N-
1)*d(i)/(2*P.a)))))*pi*d(i)*N(i)/60;
end

% inequality constraints
g1 = P.R0 - RT(1);
g2 = P.R0 - RT(2);
g3 = P.R0 - RT(3);
g4 = P.R0 - RT(4);
g5 = P.P0 - PW(1);
g6 = P.P0 - PW(2);
g7 = P.P0 - PW(3);
g8 = P.P0 - PW(4);

g=[g1;g2;g3;g4;g5;g6;g7;g8];

% equality constraints
h1= C(1) - C(2);
h2= C(1) - C(3);
h3= C(1) - C(4);

h=[h1;h2;h3];
```

A.3 RUN_1.m (executable file, same as RUN_2.m)

```
%% IE 513 - Final Project
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% File Name: RUN1.m
% Date: 12/01/12
% Author: Luis S Lin
% Description: Step-cone pulley weight minimization
%
% Files used: FUN.m
%             NONLCON.m
%
% Inputs: 1) Algorithm: 'sqp', 'active-set','interior-point'
%
% Outputs: 1) Optimal Design Variables
%          2) Optimal Function Value
%          3) Iterations Table
%          4) Functiona Value and Evaluations vs Iterations Plot
%          5) Computation Time
%          6) Exit Flag
%          7) Multipliers
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Information
clear all %clear out old variables
clc % clear command window

fprintf('\n-------------------------------------\n');
fprintf('\nAuthor: Luis S Lin\n');
fprintf('\nCourse: IE 513 - Optimal System Design\n');
fprintf('\nFinal Project: Step-cone pulley optimization \n');
fprintf('\n-------------------------------------\n');


%%
clear all %clear out old variables

% Options and Common Parameters for fmincon
% algorithm & output
% large scale optimization (usually should be set to OFF)
options = optimset('algorithm','active-set','Display','iter',...
    'LargeScale','off','MaxFunEvals',3000,'MaxIter',1000,...
    'Diagnostics','on','PlotFcns',{@optimplotx,@optimplotfval,...
    @optimplotfirstorderopt,@optimplotfunccount,...
    @optimplotconstrviolation,@optimplotstepsize});

% matrix/vectors for defining linear constraints (not used)
A=[]; b=[]; Aeq=[]; beq=[];

% lower bounds on the problem
lb = [ 40 40 40 40 16];
```

```
% upper bounds on the problem (not used)
ub = [500 500 500 500 100];

% initial starting point
% x1 = d1 (diameter of step 1 in mm)
% x2 = d2 (diameter of step 2 in mm)
% x3 = d3 (diameter of step 3 in mm)
% x4 = d4 (diameter of step 4 in mm)
% x5 = w  (width of the belts and steps in mm)

x0 = [70 96.25 128.333 154 50];
%x0 = [50 50 50 50 80]; For Run_2.m

% parameters
P.N  = 350;    % Input speed of the shaft (RPM)
P.N1 = 750;    % Output speed of the step 1 (RPM)
P.N2 = 450;    % Output speed of the step 2 (RPM)
P.N3 = 250;    % Output speed of the step 3 (RPM)
P.N4 = 150;    % Output speed of the step 4 (RPM)
P.a  = 3;      % Center distance between shafts (m)
P.p  = 7200;   % Density of the material of the pulleys (kg/m^3)
P.mu = 0.35;   % Coefficient of friction between belt and pulley
P.s  = 1.75;   % Maximum allowable stress in the belt (MPa)
P.t  = 8;      % Thickness of the belt (mm)
P.P0 = 0.75;   % Minimum required power transmitted by the step pulley (hp)
P.R0 = 2;      % Minimum required ratio of the tension on the tight side of
               % the belt to that on the slack side

% convert parameters to appropriate units
P.a  = P.a*(10^3);   % (mm)
P.p  = P.p*(10^-9); % (kg/mm^3)
P.s  = P.s*(10^6)*(10^-3); % (kg/mm^2)
P.P0 = P.P0*745.6998*(10^6); % (kg*mm^2/s^3)

% solve
tic % Start clock

[xopt,fval,exitflag,ouput,lambda] = fmincon(@(x)FUN(x,P),...
    x0,A,b,Aeq,beq,lb,ub,@(x)NONLCON(x,P),options)

toc  % End Clock
```

A.3 RUN_GS.m (executable file)

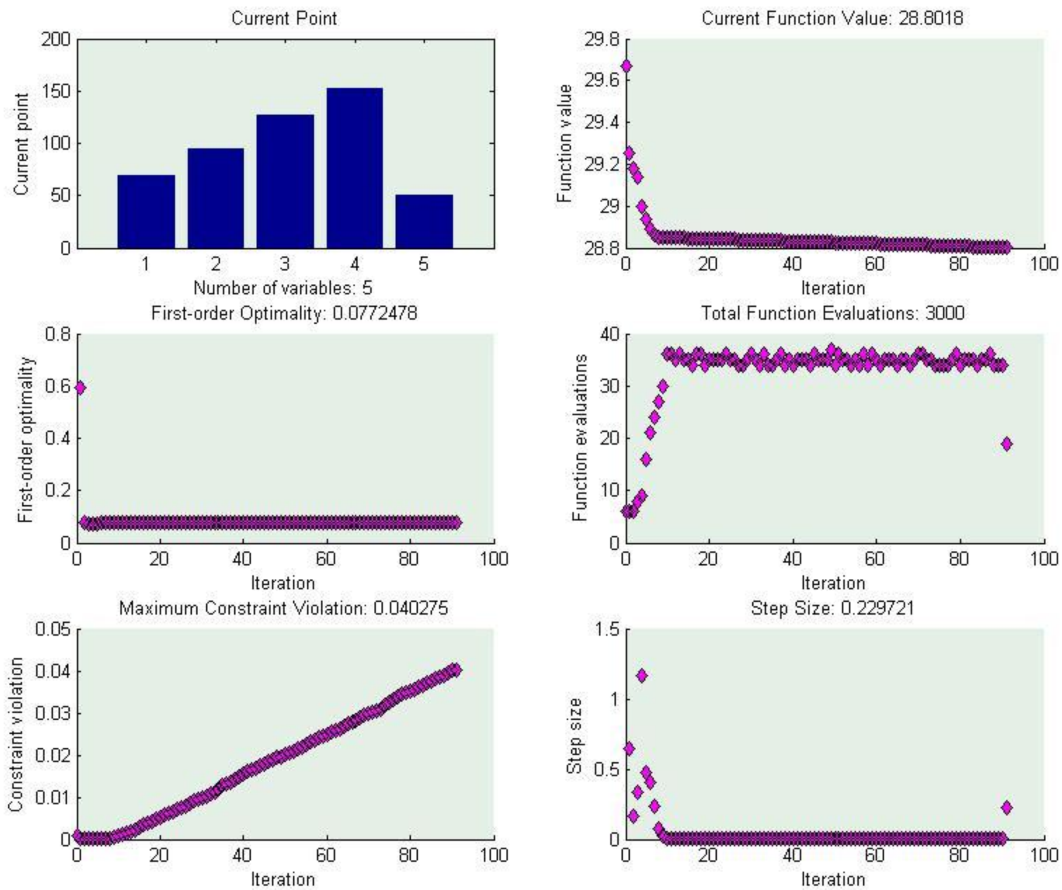
```
 %% IE 513 - Final Project

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% File Name: RUN_GS.m
% Date: 12/01/12
% Author: Luis S Lin
% Description: Step-cone pulley weight minimization
%
% Files used: FUN.m
%             NONLCON.m
%
% Inputs: 1) Algorithm: 'sqp', 'active-set','interior-point'
%
% Outputs: 1) Optimal Design Variables
%          2) Optimal Function Value
%          3) Iterations Table
%          4) Functiona Value and Evaluations vs Iterations Plot
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Information
clear all %clear out old variables
clc % clear command window

fprintf('\n-------------------------------------\n');
fprintf('\nAuthor: Luis S Lin\n');
fprintf('\nCourse: IE 513 - Optimal System Design\n');
fprintf('\nFinal Project: Step-cone pulley optimization \n');
fprintf('\n-------------------------------------\n');


%%
clear all %clear out old variables

% Options and Common Parameters for fmincon
    % algorithm & output
    % large scale optimization (usually should be set to OFF)
options = optimset('algorithm','sqp' ,'Display','OFF',...
'LargeScale','off','MaxFunEvals',3000,'MaxIter',1000,'Diagnostics','off');

% matrix/vectors for defining linear constraints (not used)
A=[]; b=[]; Aeq=[]; beq=[];

% lower bounds on the problem
lb = [ 40 40 40 40 16];

% upper bounds on the problem (not used)
ub = [500 500 500 500 100];

% initial starting point
```

```matlab
% x1 = d1 (diameter of step 1 in mm)
% x2 = d2 (diameter of step 2 in mm)
% x3 = d3 (diameter of step 3 in mm)
% x4 = d4 (diameter of step 4 in mm)
% x5 = w  (width of the belts and steps in mm)

x0 = [100 100 100 100 20];

% parameters
P.N  = 350;    % Input speed of the shaft (RPM)
P.N1 = 750;    % Output speed of the step 1 (RPM)
P.N2 = 450;    % Output speed of the step 2 (RPM)
P.N3 = 250;    % Output speed of the step 3 (RPM)
P.N4 = 150;    % Output speed of the step 4 (RPM)
P.a  = 3;      % Center distance between shafts (m)
P.p  = 7200;   % Density of the material of the pulleys (kg/m^3)
P.mu = 0.35;   % Coefficient of friction between belt and pulley
P.s  = 1.75;   % Maximum allowable stress in the belt (MPa)
P.t  = 8;      % Thickness of the belt (mm)
P.P0 = 0.75;   % Minimum required power transmitted by the step pulley (hp)
P.R0 = 2;      % Minimum required ratio of the tension on the tight side of
               % the belt to that on the slack side

% convert parameters to appropriate units
P.a  = P.a*(10^3);  % (mm)
P.p  = P.p*(10^-9); % (kg/mm^3)
P.s  = P.s*(10^6)*(10^-3); % (kg/mm^2)
P.P0 = P.P0*745.6998*(10^6); % (kg*mm^2/s^3)

% solve

tic % Start Clock

problem = createOptimProblem('fmincon','objective',@(x)FUN(x,P),'x0',x0,...
    'lb',lb,'ub',ub,'nonlcon',@(x)NONLCON(x,P),'options',options);

gs =
GlobalSearch('Display','iter','PlotFcns',{@gsplotbestf,@gsplotfunccount});

[x,f] = run(gs,problem)


toc % Stop Clock
```

C. MATLAB Output

C.1 SQP (RUN_1.m)

```
Solver stopped prematurely.

fmincon stopped because it exceeded the function evaluation limit,
options.MaxFunEvals = 3000 (the selected value).


xopt =

   68.9653   94.8273  126.4364  151.7237   50.0107


fval =

   28.8018


exitflag =

     0


ouput =

         iterations: 91
          funcCount: 3000
          algorithm: 'sequential quadratic programming'
            message: [1x144 char]
    constrviolation: 0.0403
           stepsize: 0.0023
       firstorderopt: 0.0772


lambda =

         eqlin: [0x1 double]
      eqnonlin: [3x1 double]
       ineqlin: [0x1 double]
         lower: [5x1 double]
         upper: [5x1 double]
    ineqnonlin: [8x1 double]

Elapsed time is 5.915466 seconds.
```

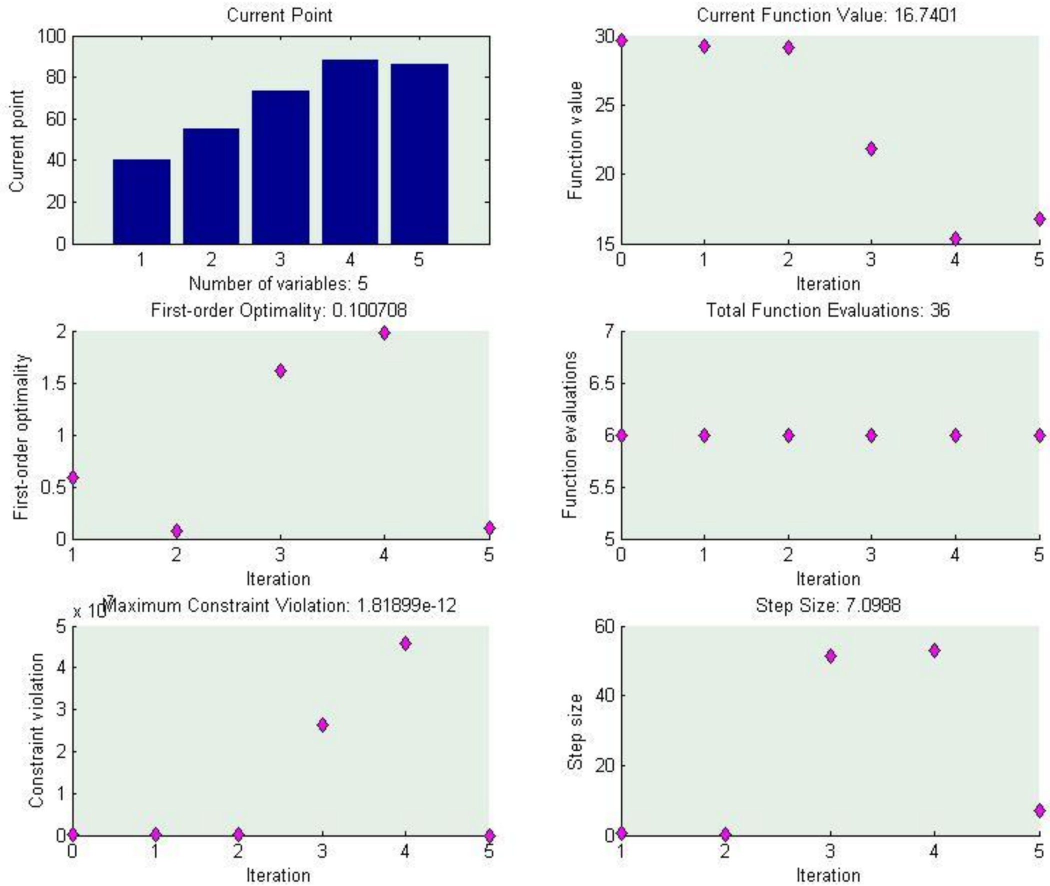


Current Point
Current point
Number of variables: 5
Current Function Value: 28.8018
Function value
Iteration
First-order Optimality: 0.0772478
First-order optimality
Iteration
Total Function Evaluations: 3000
Function evaluations
Iteration
Maximum Constraint Violation: 0.040275
Constraint violation
Iteration
Step Size: 0.229721
Step size
Iteration

C.2 AS (RUN_1.m)

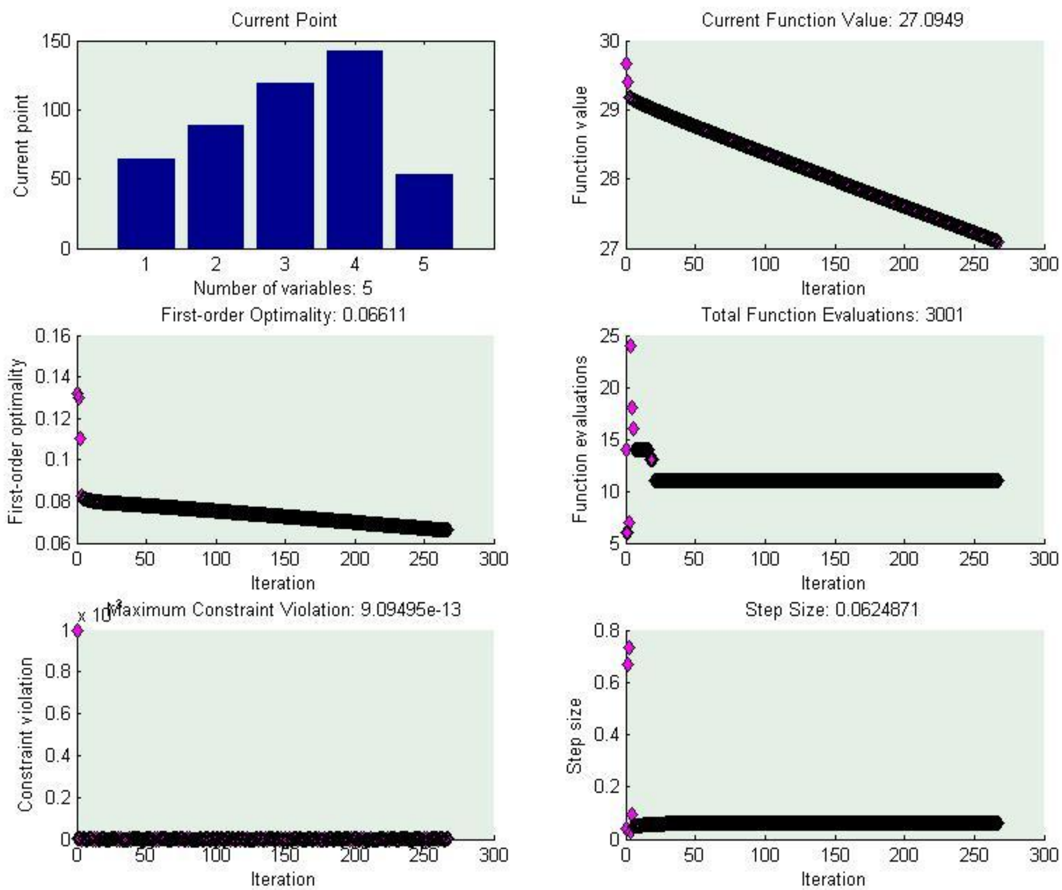
```
____________________________________________________________
   Diagnostic Information

Number of variables: 5

Functions
Objective:                            @(x)FUN(x,P)
Gradient:                             finite-differencing
Hessian:                              finite-differencing (or Quasi-
Newton)
Nonlinear constraints:                @(x)NONLCON(x,P)
Nonlinear constraints gradient:       finite-differencing

Constraints
Number of nonlinear inequality constraints: 8
Number of nonlinear equality constraints:   3

Number of linear inequality constraints:    0
Number of linear equality constraints:      0
Number of lower bound constraints:          5
Number of upper bound constraints:          5

Algorithm selected
   medium-scale: SQP, Quasi-Newton, line-search


____________________________________________________________
   End diagnostic information

                                Max     Line search  Directional  First-
order
 Iter F-count        f(x)   constraint   steplength   derivative
optimality Procedure
    0      6      29.6661    0.0009996
Infeasible start point
    1     12        29.25    1.955e-08            1       -0.646
0.592
    2     18      29.1804    2.754e-08            1       -0.423
0.0755
    3     24      21.8183    2.627e+07            1       -0.122
1.62  Hessian modified
    4     30      15.3648    4.595e+07            1      -0.0906
1.98
    5     36      16.7401    1.819e-12            1        0.194
0.101

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current search direction is less
than
twice the default value of the step size tolerance and constraints are
```

```
satisfied to within the default value of the constraint tolerance.

<stopping criteria details>

Active inequalities (to within options.TolCon = 1e-06):
  lower      upper     ineqlin   ineqnonlin
    1

xopt =

   40.0000   55.0000   73.3334   88.0000   86.4060


fval =

   16.7401


exitflag =

     4


ouput =

         iterations: 6
          funcCount: 36
       lssteplength: 1
           stepsize: 2.1253e-07
          algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'
      firstorderopt: 0.1937
    constrviolation: 1.8190e-12
            message: [1x762 char]


lambda =

         lower: [5x1 double]
         upper: [5x1 double]
         eqlin: [0x1 double]
      eqnonlin: [3x1 double]
       ineqlin: [0x1 double]
    ineqnonlin: [8x1 double]

Elapsed time is 1.727288 seconds.
```


Current Point
Current point
Number of variables: 5
Current Function Value: 16.7401
Function value
Iteration
First-order Optimality: 0.100708
First-order optimality
Iteration
Total Function Evaluations: 36
Function evaluations
Iteration
Maximum Constraint Violation: 1.81899e-12
Constraint violation
Iteration
Step Size: 7.0988
Step size
Iteration

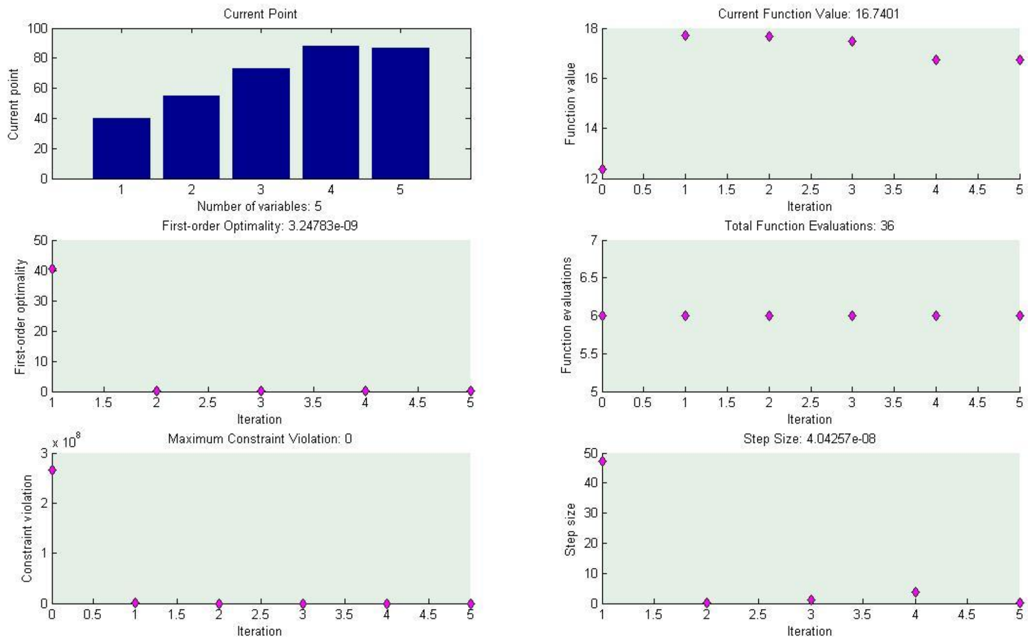
C.3 IP (RUN_1.m)

```
Solver stopped prematurely.

fmincon stopped because it exceeded the function evaluation limit,
options.MaxFunEvals = 3000 (the selected value).


xopt =

   64.8114   89.1157  118.8209  142.5850   53.2710


fval =

   27.0949


exitflag =

     0


ouput =

         iterations: 267
          funcCount: 3001
    constrviolation: 9.0949e-13
           stepsize: 0.0625
          algorithm: 'interior-point'
      firstorderopt: 0.0661
       cgiterations: 293
            message: [1x144 char]


lambda =

         eqlin: [0x1 double]
      eqnonlin: [3x1 double]
       ineqlin: [0x1 double]
         lower: [5x1 double]
         upper: [5x1 double]
    ineqnonlin: [8x1 double]

Elapsed time is 17.993146 seconds.
```


Current Point
Current point
Number of variables: 5
Current Function Value: 27.0949
Function value
Iteration
First-order Optimality: 0.06611
First-order optimality
Iteration
Total Function Evaluations: 3001
Function evaluations
Iteration
Maximum Constraint Violation: 9.09495e-13
Constraint violation
Iteration
Step Size: 0.0624871
Step size
Iteration

C.4 SQP (RUN_2.m)

```
                                                          Norm of First-
order
 Iter F-count            f(x)  Feasibility  Steplength        step
optimality
    0       6    1.237146e+01   2.654e+08
2.530e-01
    1      12    1.770088e+01   2.451e-05   1.000e+00   4.736e+01
4.062e+01
    2      18    1.766212e+01   4.547e-12   1.000e+00   2.001e-01
1.937e-01
    3      24    1.746834e+01   0.000e+00   1.000e+00   1.000e+00
1.937e-01
    4      30    1.674013e+01   0.000e+00   1.000e+00   3.759e+00
1.456e-01
    5      36    1.674013e+01   0.000e+00   1.000e+00   4.043e-08
3.248e-09

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function
tolerance,
and constraints are satisfied to within the default value of the
constraint tolerance.

<stopping criteria details>

xopt =

   40.0000   55.0000   73.3334   88.0000   86.4060

fval =

   16.7401


exitflag =

     1

ouput =

         iterations: 5
          funcCount: 36
          algorithm: 'sequential quadratic programming'
            message: [1x777 char]
    constrviolation: 0
           stepsize: 1
      firstorderopt: 3.2478e-09

Elapsed time is 1.593999 seconds.
```

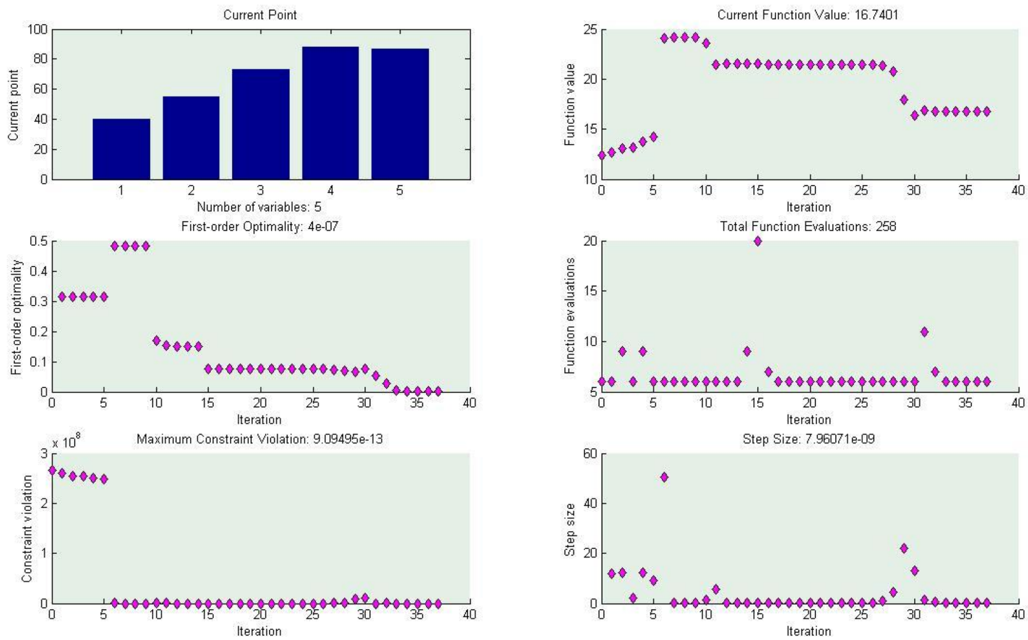

Current Point
Current point
Number of variables: 5
Current Function Value: 16.7401
Function value
Iteration
First-order Optimality: 3.24783e-09
First-order optimality
Iteration
Total Function Evaluations: 36
Function evaluations
Iteration
Maximum Constraint Violation: 0
Constraint violation
x 10^8
Iteration
Step Size: 4.04257e-08
Step size
Iteration

C.5 AS (RUN_2.m)

```
                                Max     Line search  Directional  First-
order
 Iter F-count        f(x)   constraint   steplength   derivative
optimality Procedure
    0      6      12.3715    2.654e+08
Infeasible start point
    1     12      17.7009    2.451e-05            1        0.073
101
    2     18      17.6617    4.547e-12            1       -0.194
0.194  Hessian modified
    3     24      16.7401       0.8354            1       -0.194
0.00773  Hessian modified
    4     30      16.7401    1.192e-07            1        0.194
5.63e-09  Hessian modified

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function
tolerance,and constraints are satisfied to within the default value of the
constraint tolerance.

<stopping criteria details>

Active inequalities (to within options.TolCon = 1e-06):
  lower      upper     ineqlin   ineqnonlin
    1                                 8
xopt =

   40.0000   55.0000   73.3334   88.0000   86.4060

fval =

   16.7401

exitflag =

     1

ouput =

         iterations: 4
          funcCount: 30
       lssteplength: 1
           stepsize: 1.2906e-07
          algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'
      firstorderopt: 5.6340e-09
     constrviolation: 1.1921e-07
            message: [1x783 char]

Elapsed time is 2.160478 seconds.
```

C.5 IP (RUN_2.m)

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function
tolerance,
and constraints are satisfied to within the default value of the
constraint tolerance.

<stopping criteria details>


xopt =

   40.0000   55.0000   73.3334   88.0000   86.4060


fval =

   16.7401


exitflag =

     1


ouput =

         iterations: 37
          funcCount: 258
    constrviolation: 9.0949e-13
           stepsize: 7.9607e-09
          algorithm: 'interior-point'
      firstorderopt: 4.0000e-07
       cgiterations: 9
            message: [1x777 char]


lambda =

         eqlin: [0x1 double]
      eqnonlin: [3x1 double]
       ineqlin: [0x1 double]
         lower: [5x1 double]
         upper: [5x1 double]
    ineqnonlin: [8x1 double]

Elapsed time is 3.870883 seconds.
```


Current Point
Current point
Number of variables: 5
Current Function Value: 16.7401
Function value
Iteration
First-order Optimality: 4e-07
First-order optimality
Iteration
Total Function Evaluations: 258
Function evaluations
Iteration
Maximum Constraint Violation: 9.09495e-13
x 10^8
Constraint violation
Iteration
Step Size: 7.96071e-09
Step size
Iteration

C.6 SQP (RUN_GS.m)

```
 Num Pts                  Best         Current    Threshold         Local
Local
Analyzed  F-count          f(x)         Penalty      Penalty          f(x)
exitflag        Procedure
       0     3000         23.97                                      23.97
0     Initial Point
     200     4565         41.25                                      41.25
2     Stage 1 Local
     300     4667         41.25           615.4        86.61
Stage 2 Search
     400     4767         41.25            1386          217
Stage 2 Searc
     437     7804         41.25           105.2        312.9         26.05
0     Stage 2 Local
     438    10805         41.25           81.96        105.2          33.8
0     Stage 2 Local
     441    10886         16.74           63.06        81.96         16.74
1     Stage 2 Local
     478    11001         16.74           73.63        75.87         16.74
1     Stage 2 Local
     480    11081         16.74           73.29        73.63         16.74
1     Stage 2 Local
     482    11161         16.74           72.86        73.29         16.74
1     Stage 2 Local
     493    11244         16.74           72.34        72.81         16.74
1     Stage 2 Local
     495    11318         16.74           72.17        72.34         16.74
1     Stage 2 Local
     500    11323         16.74           79.61        72.17
Stage 2 Search
     545    14369         16.74           90.12        95.48         39.71
0     Stage 2 Local
     600    14424         16.74           174.8        108.9
Stage 2 Search
     617    17441         16.74           107.1        130.9         57.05
0     Stage 2 Local
     619    17524         16.74           83.95        107.1         16.74
1     Stage 2 Local
     644    20549         16.74           88.89        100.9         51.17
0     Stage 2 Local
     645    20631         16.74           86.64        88.89         16.74
1     Stage 2 Local
     657    20722         16.74           84.77        86.64         16.74
1     Stage 2 Local
     668    23733         16.74           81.96        84.77         52.49
0     Stage 2 Local
     694    26759         16.74           98.09        98.55         59.31
0     Stage 2 Local
     700    26765         16.74           228.5        98.09
Stage 2 Search
```

```
     708    26858       16.74         84.49        98.09        16.74
1    Stage 2 Local
     718    26954       16.74          84.3        84.49        16.74
1    Stage 2 Local
     789    27110       16.74         107.1        146.4        16.74
1    Stage 2 Local
     793    27199       16.74         84.23        107.1        16.74
1    Stage 2 Local
     800    27206       16.74           773        84.23
Stage 2 Search
     900    27306       16.74         96.88        58.23
Stage 2 Search
     978    30156       16.74         115.4        121.8        71.86
-2    Stage 2 Local
    1000    30178       16.74         151.2        138.7
Stage 2 Search

GlobalSearch stopped because it analyzed all the trial points.

14 out of 23 local solver runs converged with a positive local solver exit
flag.

x =

   40.0000   55.0000   73.3334   88.0000   86.4060


f =

   16.7401

Elapsed time is 38.986607 seconds.
```

D. Sensitivity Analysis

N	Fval	d1	d2	d3	d4	w
150	49.233	40.000	60.000	90.000	120.000	63.549
250	24.199	40.000	57.143	80.000	100.000	76.0825
350	16.740	40.000	55.000	73.333	88.000	86.406
450	13.433	40.000	53.333	68.571	80.000	95.0300
550	11.911	40.931	53.211	66.514	76.015	100.000

a	Fval	d1	d2	d3	d4	w
1	16.644	40.000	55.000	73.333	88.000	85.910
2	16.716	40.000	55.000	73.333	88.000	86.281
3	16.740	40.000	55.000	73.333	88.000	86.406
4	16.752	40.000	55.000	73.333	88.000	86.469
5	16.760	40.000	55.000	73.333	88.000	86.507

s	Fval	d1	d2	d3	d4	w
0.75	78.292	80.411	110.564	147.419	176.903	100.000
1.25	28.316	48.358	66.492	88.657	106.388	100.000
1.75	16.740	40.000	55.000	73.333	88.000	86.406
2.25	13.020	40.000	55.000	73.333	88.000	67.205
2.75	10.653	40.000	55.000	73.333	88.000	54.986

P0	Fval	d1	d2	d3	d4	w
0.25	5.580	40.000	55.000	73.333	88.000	28.802
0.5	11.160	40.000	55.000	73.333	88.000	57.604
0.75	16.740	40.000	55.000	73.333	88.000	86.406
1	25.692	46.063	63.337	84.449	101.339	100.000
1.25	40.077	57.531	79.105	105.473	126.568	100.000

R0	Fval	d1	d2	d3	d4	w
1	16.740	40.000	55.000	73.333	88.000	86.406
1.5	16.740	40.000	55.000	73.333	88.000	86.406
2	16.740	40.000	55.000	73.333	88.000	86.406
2.5	16.740	40.000	55.000	73.333	88.000	86.406
3	16.740	40.000	55.000	73.333	88.000	86.406