IE413 / CS482

Computer Simulation

Group Project: A Bowling Alley Problem

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Appendix

1. Problem Description

The system being modeled is a bowling center that opens at 12:00 PM (seven days per week), and closes when all bowlers have left the center. It currently has 16 available lanes that become occupied as customers arrive the center. Arrivals occurs in groups (1,2,3 or 4 people), with each group utilizing one lane, and stop at 11:00 PM. At this time, groups are not added to the waiting list, but groups that are on the waiting list stay until they have bowled. If no lanes are available, then the group is put on a waiting list until the first available lane becomes available. The probability that a group balks (when told that there is a waiting list), and the distribution of the inter-arrival time, group size, and bowling time are detailed in the model description.

The objectives of the project were to build a discrete-event simulation model of the system to analyze the operation of the bowling center. In particular, simulation was used to determine if additional lanes should be installed, and to provide the owner with an optimal business decision.

2. Model Description

SIGMA was used to model the system and run the simulations. The event graph is shown Figure 1, and the descriptions of the variables and events are discussed in the following sections.

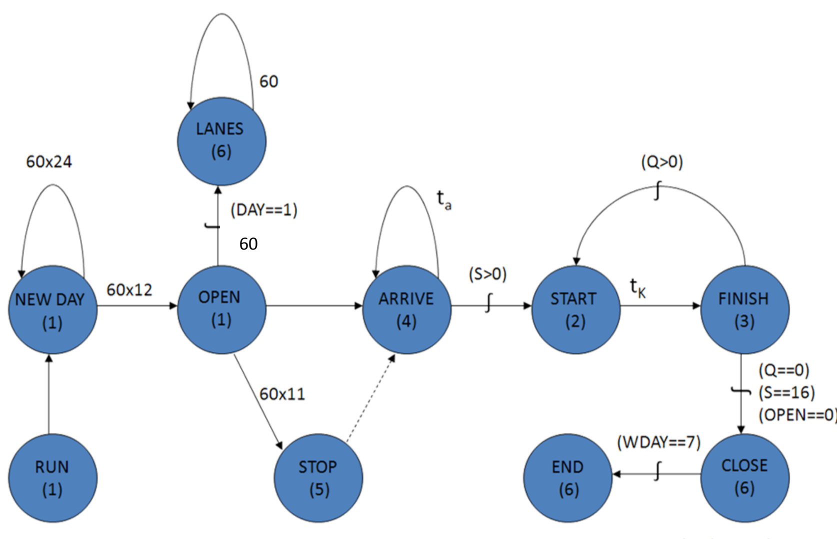


Figure 2.1. Event graph for the simulation model

2.1 Entities & Attributes

One entity is the bowling center, which is permanent and has the number of available lanes (dynamic, quantitative) as an attribute. The other entity is a group of customers, which is transient, and has group size and bowling time (both dynamic, qualitative) as an attribute.

2.2 Variables

- **S** (integer): the number of available lanes
- **Q** (integer): the number of groups in line (includes all groups to be served)
- **B** (integer): the number of groups that balk (leave without being served)
- **AI** (integer): arrival index, ID of group at arrival (not including groups that balk)
- **SI** (integer): service index, ID of group at start of service
- **DAY** (integer): running day of simulation
- **WDAY** (integer): working day of bowling alley (from opened to empty after closed). This represents the actual number of iterations (simulated operating days) of the system.
- **OPEN** (integer): status of bowling alley (0 if closed, 1 if opened); closed refers to stopping the addition of people to the waitlist
- **A:** holder for current index of group that arrived (0 if group balks, AI otherwise) in ARRIVE event
- **K[]** (integer): array containing the group size.
- **ARRIVE[]** (real): array documenting the arrival time of each group. **ARRIVE[0]**: holder for random variable used in START and END event. **ARRIVE[15000]**: holder for simulation clock time in STOP event.
- **WAIT** (real): total waiting time of groups
- **CTIME** (real):r cumulative time in minutes that the alley is opened (all groups finished bowling) after manager stops adding groups to the waiting list (23:00).
- **L []** (integer): array containing the cumulative (for all days) number of lanes in use for each hour of the day (00:30, 1:30, 2:30…, 23:30).
- **N** (integer): total number of people served

2.4 Events

- **RUN** (Priority 1): Starts the simulation. The system is initialized to be empty and idle.

State Changes:

S = 16, Q = 0, B = 0, AI = 0, SI = 0, DAY = 0, WDAY = 0, WAIT = 0, N = 0, CTIME =0

Schedule:

Unconditionally, **NEW DAY** with no time delay

- **NEW DAY** (Priority 1): A new day starts at 0:00 hours. Open the center at 12:00 PM.

State Changes:

DAY = DAY + 1

Schedule:

Unconditionally, **OPEN** in 60x12 minutes (at 12:00 PM)

Unconditionally, **NEW DAY** in 60x24 minutes (every 24hours)

- **OPEN** (Priority 1): The alley is opened at 12:00 PM and a new working day starts. One group is waiting to enter. Schedule STOP to cancel arrivals at 23:00.

State Changes:

OPEN = 1, WDAY = WDAY + 1

Schedule:

Unconditionally, **STOP** in 60x11 minutes (at 11:00 PM/23:00)

If DAY == 1, **LANES** in 60 minutes (at 13:00)

Unconditionally, **ARRIVE** with no time delay (at 12:00 PM)

- **STOP** (Priority 5): The doors of the alley are closed at 23:00 ; No more groups are added to the waiting list/queue. Store the stopping time to use in CLOSE event.

State Changes:

OPEN = 0, ARRIVE [15000]=CLK

Schedule:

Unconditionally, cancel **ARRIVE** with no time delay (at 23:00)

- **LANES** (Priority 6): Check the number of lanes in use for a specific hour of the day

State Changes:

L[CLK/60 – 24*(DAY-1)]=L[CLK/60 – 24*(DAY-1)]+(16-S)

Schedule:

Unconditionally, **LANES** in 60 minutes

- **ARRIVE** (Priority 4): Groups arrive at the bowling alley distributed exponentially with mean 4 minutes. Groups that balk are excluded from the queue and the arrival index. Schedule service if there are idle servers (free lanes available).

State Changes:

ARRIVE[0] = RND

AI = AI + I{S>0} + I{S==0}*I{Q<=5} +

I{S==0}*I{Q>=6)*I{Q<=15}*I{ARRIVE[0]>=(Q/10-0.5)}

A= AI*I{S>0} + AI*I{S==0}*I{Q<=5} +

AI*I{S==0}*I{Q>=6)*I{Q<=15}*I{ARRIVE[0]>=(Q/10-0.5)}

B = B + I{S==0}*I{Q>=6)*I{Q<=15}*I{ARRIVE[0]<(Q/10-0.5)} + I{S==0}*I{Q>15}

Q = Q+ I{S>0} + I{S==0}*I{Q<=5} + I{S==0}*I{Q>=6)*I{Q<=15}*I{ARRIVE[0]>=(Q/10-0.5)}

ARRIVE[A]=CLK

Schedule:

Unconditionally, **ARRIVE** in t_a minutes, where t_a = 4*ERL{1}

If S > 0, **START** with no time delay

- **START** (Priority 2): A group starts service (enters the bowling lane). Finish service is normally distributed with mean 30K and standard deviation 4K. compute the aggregated waiting time and number of people served.

State Changes:

ARRIVE[0]=RND , S=S-1, Q=Q-1, SI=SI+1

WAIT = WAIT+ (CLK-ARRIVE[SI])

K[SI] = 1*I{ARRIVE[0]<0.3} + 2*I{ARRIVE[0]>=0.3}*I{ARRIVE[0]<0.5} +

3*I{ARRIVE[0]>=0.5}*I{ARRIVE[0]<0.6} + 4*I{ARRIVE[0]>=0.6}

N = N + K[SI]

Schedule:

Unconditionally, **FINISH** in t_K minutes, where t_K = NOR{30*K[SI];4*K[SI]}

- **FINISH** (Priority 3): A group finishes service (leaves the bowling alley). Schedule CLOSE if there are no more groups in the waiting list after 23:00.

State Changes:

S = S+1

Schedule:

If Q>0, **START** with no time delay

If Q==0 & S==16 & OPEN==O, **CLOSE** with no time delay

- **CLOSE** (Priority 6): System is empty; close alley and determine working time after 23:00

State Changes:

CTIME = CTIME + (CLK-ARRIVE[15000])

Schedule:

If WDAY==7, **END** in 7*24*60+11*60-CLK minutes

- **END** (Priority 6): Terminate simulation when 7 WDAY is done (12:00 PM of the 8^{th} DAY)

2.3 Assumptions

One assumption (besides the ones described in the problem statement) is that a customer that arrives at 23:00 is added to the waiting list (or served if idle lanes). Thus, event ARRIVE has higher priority than event STOP. In addition, event START has higher priority than FINISH and ARRIVE to avoid “phantom” groups or lanes (i.e. two START’s for the same group or idle lane).

2.4 Validation & Verification

The model was first validated and verified by studying the code and then tracing the simulation output model (using the single step run option in SIGMA) to ensure that event and state changes were in accordance with the model. In addition, different parameters were tested to ensure agreement with expected behavior. For instance, decreasing the inter-arrival time increased the number of services and queue as expected.

3. Statistical Output Analysis

3.1 Data Collection Procedure

The model described in the previous sections simulates 7 working days (i.e. one working day goes from 12:00 until the alley closes). One run, or sample, represents 7 simulated working days with a unique seed. Each simulation run was started with the same initial conditions (empty and idle). To use the assumption that the sampling distribution is approximately normal (central limit theorem), 30 runs (samples) independently seeded were made. This procedure (using the same 30 different seeds) was done for 16, 18, 20, 22, 24 and 26 lanes. Thus, for each lane capacity, 30 independent replications were obtained, giving a total number of 180 simulations runs.

Once the model was validated, simulations were made using the high speed option in SIGMA and only event END was traced to collect the necessary data. Thus, for each capacity, 30 SIGMA output files were generated with a different seed each (refer to the CD). Each file contained one row with the aggregated data for 7 days. Then, the 30 output files were combined into a single file that consisted of 30 rows that represented the 30 independent replications with the same capacity. This procedure was repeated for the other lane capacities.

The six files (one for each capacity) containing the 30 rows of data (replications) were imported into EXCEL. Since the data represented aggregated statistics over the 7 days, the appropriate statistics were averaged to obtain a mean value per day for each replication and each statistic. Then, the mean, standard deviation and confidence intervals were determined for each statistic by considering their corresponding 30 replications. A sample output for the raw data collected, the calculations and equations used to compute the statistics, and a sample with the averaged data per day for each replication can be found in appendices A-1, A-2 and A-3 respectively.

For the sensitivity analysis, only one seed (12345) was used and two changes for each parameter (mean inter-arrival time, bowling time mean and standard deviation) were made for each capacity. Thus, a total of 36 simulations runs were made in a slightly modified SIGMA model in which only the necessary data (people served and groups that balked) were collected.

3.2 Results & Discussion

After running the simulations and calculating the appropriate statistics, the data analysis was done in EXCEL. The variables listed below are used in the following discussions (averages refer to averages per day):

- **F =** fraction of groups that balk (leave the center due to their are unwillingness to wait)
- **AL[i] =** average number of lanes at hour i
- **AWAIT,AGWAIT =** average waiting time for all groups and per group respectively
- **AVEOT, AVECT =** average over time spent after 23:00 and average closing time
- **AVER, AVEC, AVEP =** average revenue ,average cost and average profit

3.2.1 Lane expansion decision

Table 3.1 shows only the mean values for all statistics and each capacity. The complete summary of the statistics with their confidence intervals can be found in appendix A-4.

Table 3.1 Summary of statistics

	Means					
LANES	16	18	20	22	24	26
F	0.14	0.08	0.04	0.01	0.00	0.00
AWAIT	3351.60	2319.94	1329.67	689.32	276.49	113.66
AGWAIT	3.36	2.16	1.18	0.60	0.24	0.10
AVEOT	148.82	137.86	126.81	123.58	122.18	118.80
AVECT	1.48	1.30	1.11	1.06	1.04	0.98

As it can be seen from data, increasing the number of lanes reduces the fraction of costumers that balk, the average waiting time and average closing time. Appendix A-5 shows the average number of busy lanes, and its distribution is illustrated in Figure 3.1. From this plot, it can be concluded that the current operation of 16 lanes is not optimal since all lanes are used during most of the normal operating hours. As the plot shows, adding more lanes will accommodate for more groups that otherwise would be waiting when the capacity is 16 lanes. Based on the data collected, it is recommended to expand the number of available lanes.

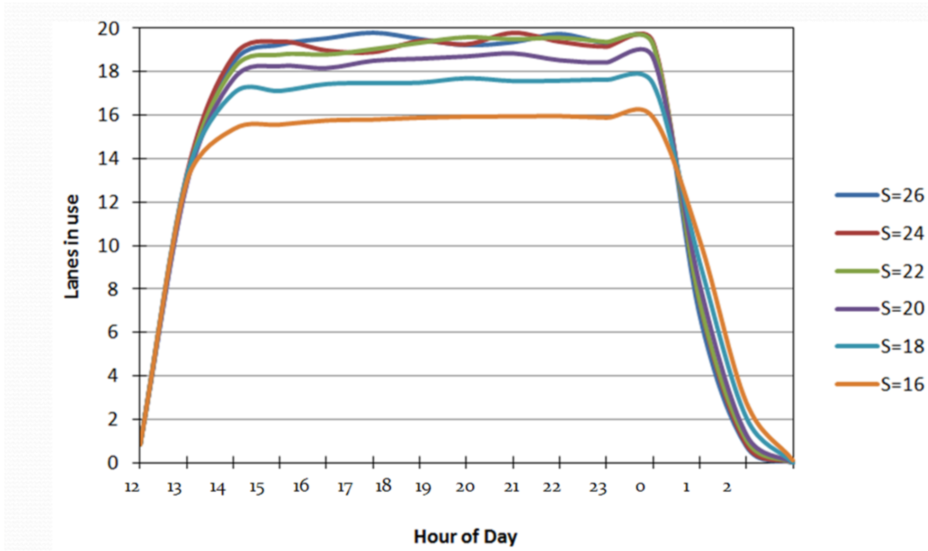


Figure 3.1 Distribution of number of lanes in use

3.2.2 Maximum pair of lanes cost and Optimal decision

The optimal business decision of how many pair of lanes to add depends on the actual cost of the pair of lanes. Given that revenue of $5 per bowler is earned and a cost of $10 per group that balks is incurred, the average revenue and average costs per day can be calculated using the equations in appendix A-2. The average profit is just the difference between the two. Table 3.2 shows the average revenue per day and average cost per days calculated from the data collection. The projected 5-year profit if operating with the corresponding lane capacity is also illustrated. From the table, it can be seen that increasing the number of lanes increases the revenue since more people bowl, and decreases the costs since fewer groups balk. Thus, the profit always increases with increasing number of available lanes.

Table 3.2 Average revenue, costs and profits

LANES	16	18	20	22	24	26
AVER/day	$1,862.98	$2,010.33	$2,080.86	$2,134.10	$2,145.79	$2,146.95
AVEC/day	$243.48	$133.10	$59.19	$22.00	$6.67	$2.14
5-Year Revenue	$3,399,932	$3,668,858	$3,797,564	$3,894,724	$3,916,059	$3,918,188
5-Year Cost	$444,344	$242,899	$108,023	$40,150	$12,167	$3,911
5-Year Profit	$2,955,588	$3,425,960	$3,689,542	$3,854,574	$3,903,892	$3,914,277

The optimal decision depends on the actual cost of a pair of lane. Because the owner is willing to spread the cost over 5 years, and money neither appreciates nor devalues over time, the maximum cost that the owner is willing to pay for the total number of pair of lanes added (i.e. total investment) is equal to the additional profit made in 5 years as a result of increasing the capacity from 16 lanes to the desired number of lanes. This is the maximum cost since this is the amount that can be recovered by the end of 5-year for the corresponding lanes added to the available 16 lanes. The maximum cost that the owner is willing to pay for each pair of lane can be found by just dividing the additional profit (total investment) by the number of pair of lanes. The results are illustrated in Table 3.3 and Figure 3.2. The plot shows that the optimal business decision would be to add pair(s) of lanes corresponding to their ranges of cost of pair per lane that the actual cost per lane falls in (e.g. if the actual cost of a pair lane is $200,000, then the optimal number of lanes to add is 8 since it falls within $191,738 and $237,076). The optimal decision of upgrading from pair to pair of lanes (not from 16) is shown in appendix A-6.

Table 3.3 Maximum cost per lane for upgrading from 16 lanes

Added Lanes	2	4	6	8	10
Increase in 5-year profit	$470,372	$733,954	$898,986	$948,305	$958,690
Cost per pair of Lane	$470,372	$366,977	$299,662	$237,076	$191,738

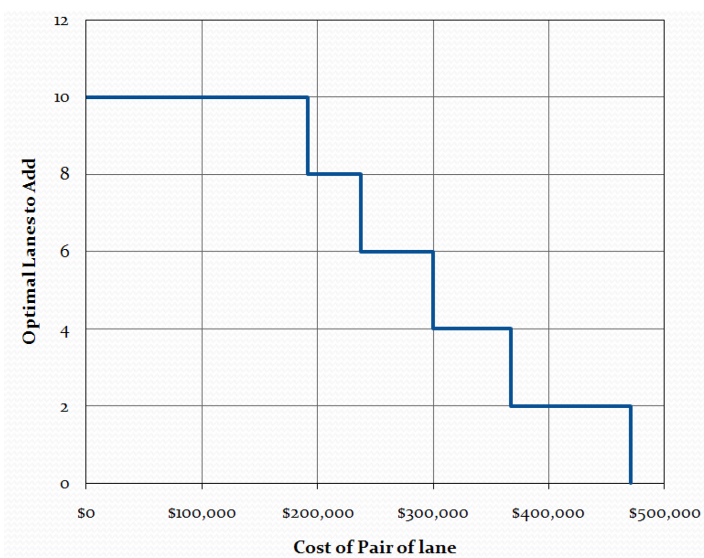


Figure 3.2 Optimal lanes upgrade from 16 lanes

3.2.3 Sensitivity analysis

Appendix A-7 shows the resulting changes in the maximum cost per lane (optimal decision) with respect to the original parameters. The summary is illustrated in Figure 3.3, which shows the changes in cost per pair of lane as a result of a decrease (a) and increase (b) one at a time in the listed parameters. As expected, decreasing (a) the first three parameters increased the maximum cost per pair of lane (e.g. more arrivals means more profit, and hence willing to pay more or invest), while decreasing the last parameter decreased the maximum cost per pair of lane. In general, the opposite trend was shown when parameters were increased (b).

Figure 3.3 Sensitivity to a) increase in parameters b) decrease in parameters

4. Conclusion

As the confidence intervals for the statistics showed, there is some overlap in the statistics among the different lanes capacities. Therefore, a larger sample size or the use reduction of variance technique might eliminate uncertainty and this possible source of error that might have lead to incorrect conclusions. For the sensitivity analysis, larger ranges in smaller increments as well as simultaneous changes should be explored to make the analysis more robust. The results of the simulation model in general agree with the expected behavior of the system. However, due to the high sensitivity to the parameters and variation in the sampling, the model should be mostly used as a qualitative tool to make decisions and draw conclusions.

A – 1 Sample Output of raw data (capacity = 16 lanes)

n	Seed	SI	B	WAIT	CTIME	N	L[12]	L[13]	L[14]	L[15]	L[16]	L[17]	L[18]	L[19]	L[20]	L[21]	L[22]	L[23]	L[0]	L[1]	L[2]
1	12345	1001	180	24792.9	1022.3	2631	6	87	110	108	112	112	112	111	112	112	112	111	77	20	1
2	16807	1012	155	21650.4	1031.5	2575	6	94	109	107	112	111	112	112	112	112	112	112	68	14	0
3	34981	990	195	25442.8	1065.9	2623	6	89	110	112	112	112	112	112	112	112	112	112	73	17	1
4	60193	995	202	26167.1	1102.5	2639	6	77	105	112	112	112	112	112	112	111	112	112	80	27	0
5	45819	1007	179	25236.0	1061.1	2658	6	103	111	112	112	110	110	112	112	112	112	112	78	23	1
6	59231	1008	140	20242.9	1011.9	2593	6	87	97	105	106	112	111	112	110	112	112	112	77	15	2
7	29227	990	154	23799.5	1035.4	2612	6	90	107	112	112	110	106	112	112	112	112	112	75	21	1
8	39839	989	141	20287.0	1126.5	2590	6	88	100	100	112	108	112	109	112	111	112	112	74	24	2
9	12393	977	181	21809.6	1022.3	2585	6	85	104	107	111	112	111	112	109	112	110	112	72	21	0
10	63913	1002	180	23025.1	1157.7	2595	6	85	106	108	107	105	112	110	111	112	112	112	73	26	2
11	51213	1012	210	26374.6	1022.0	2615	6	84	111	112	111	112	112	112	112	112	112	112	66	18	2
12	24231	1005	129	18537.6	951.6	2569	6	102	102	105	108	112	112	109	112	107	108	107	73	13	0
13	31623	1016	179	24310.3	1055.3	2598	6	97	102	109	109	109	112	111	110	112	112	112	71	17	2
14	10209	997	165	24315.4	973.0	2599	6	108	111	112	112	112	112	112	112	112	109	110	63	16	1
15	62065	987	67	17313.9	899.3	2550	6	102	111	109	112	111	106	112	107	112	107	103	53	10	0
16	27491	997	185	26138.5	1157.4	2636	6	76	109	106	110	112	112	112	112	112	112	112	89	26	1
17	46177	985	251	28488.7	1074.5	2627	6	84	111	112	111	112	112	112	112	112	112	112	70	22	1
18	36939	1003	226	26756.7	1060.3	2636	6	93	111	112	112	112	112	112	112	112	112	112	66	18	1
19	50257	1004	129	20323.5	992.0	2607	6	99	112	110	111	108	109	112	111	112	108	112	68	17	0
20	61551	997	118	18094.7	1029.7	2557	6	80	99	109	108	109	103	108	111	111	109	112	73	20	0
21	2549	1025	176	23484.5	1118.1	2636	6	86	109	107	112	112	112	112	112	112	111	112	70	25	3
22	5325	1016	192	25739.2	1051.3	2627	6	102	111	112	112	112	112	112	112	112	112	108	68	22	1
23	41499	1008	154	22100.1	964.6	2579	6	92	108	112	112	109	112	112	112	109	107	112	68	18	0
24	13091	1003	173	24859.3	1037.3	2642	6	99	108	112	112	112	112	112	112	112	112	112	78	19	0
25	8863	1009	143	21872.5	1067.6	2609	6	93	107	108	106	109	112	110	112	112	112	112	75	29	1
26	7575	1003	126	20366.1	1056.4	2576	6	93	112	106	99	100	112	112	112	112	112	112	73	17	1
27	9201	1009	159	23244.2	993.3	2624	6	89	99	109	111	112	112	111	112	111	112	112	78	15	0
28	19301	973	208	26354.5	1037.1	2619	6	86	110	107	110	112	112	112	112	112	112	111	68	18	1
29	51197	973	208	26354.5	1037.1	2619	6	86	110	107	110	112	112	112	112	112	112	111	68	18	1
30	44597	973	208	26354.5	1037.1	2619	6	86	110	107	110	112	112	112	112	112	112	111	68	18	1

A – 2 Equations

Calculations of statistics:

- F = B/(SI+B)
- AWAIT = WAIT/WDAY
- AGWAIT = AWAIT/SI
- AL[hour] = L[hour]/WDAY
- AVEOT= CTIME/WDAY
- AVECT= 23 + AVEOT/60 if AVEOT < 60
- AVECT = AVEOT/60-1 if AVEOT>=60
- AVER = 5*N/WDAY
- AVEC = 10*B/WDAY
- AVEP = AVER - AVEC

Confidence interval:

- After using the previous equations to find the averages for each replication, the average per day for each statistics and each capacity was computed as follows (6 sets of 30 n data points each)
  1. Compute the mean for each statistics: $\bar{X} = \frac{1}{30}\sum_{i=1}^{30} X_i$

     Each X_i represents a data point with a different seed, same capacity
  2. Compute the standard deviation for each statistic: $S^2 = \frac{1}{29}\sum_{i=1}^{30}(X_i - \bar{X})^2$
  3. Compute a confidence interval of 95% for the each mean of the statistic:

     $\left[\bar{X} - t_{29,0.025}\frac{S}{\sqrt{n}}, \bar{X} + t_{29,0.025}\frac{S}{\sqrt{n}}\right]$

A -3 Averaged data per day for each replication (capacity = 16 lanes) ** For presentation purposes, values were rounded*

n	F	AWAIT	AGWAIT	AVEOT	AVECT	AL[12]	AL[13]	AL[14]	AL[15]	AL[16]	AL[17]	AL[18]	AL[19]	AL[20]	AL[21]	AL[22]	AL[23]	AL[0]	AL[1]	AL[2]	AVEP	AVEC
1	0.15	3542	3.54	146.0	1.4	0.9	12.4	15.7	15.4	16.0	16.0	16.0	15.9	16.0	16.0	16.0	15.9	11.0	2.9	0.1	1879	257
2	0.13	3093	3.06	147.4	1.5	0.9	13.4	15.6	15.3	16.0	15.9	16.0	16.0	16.0	16.0	16.0	16.0	9.7	2.0	0.0	1839	221
3	0.16	3635	3.67	152.3	1.5	0.9	12.7	15.7	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	10.4	2.4	0.1	1874	279
4	0.17	3738	3.76	157.5	1.6	0.9	11.0	15.0	16.0	16.0	16.0	16.0	16.0	16.0	15.9	16.0	16.0	11.4	3.9	0.0	1885	289
5	0.15	3605	3.58	151.6	1.5	0.9	14.7	15.9	16.0	16.0	15.7	15.7	16.0	16.0	16.0	16.0	16.0	11.1	3.3	0.1	1899	256
6	0.12	2892	2.87	144.6	1.4	0.9	12.4	13.9	15.0	15.1	16.0	15.9	16.0	15.7	16.0	16.0	16.0	11.0	2.1	0.3	1852	200
7	0.13	3400	3.43	147.9	1.5	0.9	12.9	15.3	16.0	16.0	15.7	15.1	16.0	16.0	16.0	16.0	16.0	10.7	3.0	0.1	1866	220
8	0.12	2898	2.93	160.9	1.7	0.9	12.6	14.3	14.3	16.0	15.4	16.0	15.6	16.0	15.9	16.0	16.0	10.6	3.4	0.3	1850	201
9	0.16	3116	3.19	146.0	1.4	0.9	12.1	14.9	15.3	15.9	16.0	15.9	16.0	15.6	16.0	15.7	16.0	10.3	3.0	0.0	1846	259
10	0.15	3289	3.28	165.4	1.8	0.9	12.1	15.1	15.4	15.3	15.0	16.0	15.7	15.9	16.0	16.0	16.0	10.4	3.7	0.3	1854	257
11	0.17	3768	3.72	146.0	1.4	0.9	12.0	15.9	16.0	15.9	16.0	16.0	16.0	16.0	16.0	16.0	16.0	9.4	2.6	0.3	1868	300
12	0.11	2648	2.64	135.9	1.3	0.9	14.6	14.6	15.0	15.4	16.0	16.0	15.6	16.0	15.3	15.4	15.3	10.4	1.9	0.0	1835	184
13	0.15	3473	3.42	150.8	1.5	0.9	13.9	14.6	15.6	15.6	15.6	16.0	15.9	15.7	16.0	16.0	16.0	10.1	2.4	0.3	1856	256
14	0.14	3474	3.48	139.0	1.3	0.9	15.4	15.9	16.0	16.0	16.0	16.0	16.0	16.0	16.0	15.6	15.7	9.0	2.3	0.1	1856	236
15	0.06	2473	2.51	128.5	1.1	0.9	14.6	15.9	15.6	16.0	15.9	15.1	16.0	15.3	16.0	15.3	14.7	7.6	1.4	0.0	1821	96
16	0.16	3734	3.75	165.3	1.8	0.9	10.9	15.6	15.1	15.7	16.0	16.0	16.0	16.0	16.0	16.0	16.0	12.7	3.7	0.1	1883	264
17	0.20	4070	4.13	153.5	1.6	0.9	12.0	15.9	16.0	15.9	16.0	16.0	16.0	16.0	16.0	16.0	16.0	10.0	3.1	0.1	1876	359
18	0.18	3822	3.81	151.5	1.5	0.9	13.3	15.9	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	9.4	2.6	0.1	1883	323
19	0.11	2903	2.89	141.7	1.4	0.9	14.1	16.0	15.7	15.9	15.4	15.6	16.0	15.9	16.0	15.4	16.0	9.7	2.4	0.0	1862	184
20	0.11	2585	2.59	147.1	1.5	0.9	11.4	14.1	15.6	15.4	15.6	14.7	15.4	15.9	15.9	15.6	16.0	10.4	2.9	0.0	1826	169
21	0.15	3355	3.27	159.7	1.7	0.9	12.3	15.6	15.3	16.0	16.0	16.0	16.0	16.0	16.0	15.9	16.0	10.0	3.6	0.4	1883	251
22	0.16	3677	3.62	150.2	1.5	0.9	14.6	15.9	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	15.4	9.7	3.1	0.1	1876	274
23	0.13	3157	3.13	137.8	1.3	0.9	13.1	15.4	16.0	16.0	15.6	16.0	16.0	16.0	15.6	15.3	16.0	9.7	2.6	0.0	1842	220
24	0.15	3551	3.54	148.2	1.5	0.9	14.1	15.4	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	16.0	11.1	2.7	0.0	1887	247
25	0.12	3125	3.10	152.5	1.5	0.9	13.3	15.3	15.4	15.1	15.6	16.0	15.7	16.0	16.0	16.0	16.0	10.7	4.1	0.1	1864	204
26	0.11	2909	2.90	150.9	1.5	0.9	13.3	16.0	15.1	14.1	14.3	16.0	16.0	16.0	16.0	16.0	16.0	10.4	2.4	0.1	1840	180
27	0.14	3321	3.29	141.9	1.4	0.9	12.7	14.1	15.6	15.9	16.0	16.0	15.9	16.0	15.9	16.0	16.0	11.1	2.1	0.0	1874	227
28	0.18	3765	3.87	148.2	1.5	0.9	12.3	15.7	15.3	15.7	16.0	16.0	16.0	16.0	16.0	16.0	15.9	9.7	2.6	0.1	1871	297
29	0.18	3765	3.87	148.2	1.5	0.9	12.3	15.7	15.3	15.7	16.0	16.0	16.0	16.0	16.0	16.0	15.9	9.7	2.6	0.1	1871	297
30	0.18	3765	3.87	148.2	1.5	0.9	12.3	15.7	15.3	15.7	16.0	16.0	16.0	16.0	16.0	16.0	15.9	9.7	2.6	0.1	1871	297

A – 4 Summary of statistics with their confidence intervals for all capacities

	LANE = 16				LANE = 18			
	Mean	Stdev	Up 95% CI	Lo 95% CI	Mean	Stdev	Up 95% CI	Lo 95% CI
F	0.14	0.03	0.16	0.13	0.08	0.02	0.09	0.07
AWAIT	3351.60	413.61	3506.05	3197.16	2319.94	309.66	2435.57	2204.31
AGWAIT	3.36	0.43	3.52	3.20	2.16	0.28	2.26	2.05
AVEOT	148.82	8.11	151.85	145.79	137.86	7.49	140.66	135.07
AVECT	1.48	0.14	1.53	1.43	1.30	0.12	1.34	1.25
AL[12]	0.86	0.00	0.86	0.86	0.86	0.00	0.86	0.86
AL[13]	12.96	1.14	13.39	12.54	13.20	1.08	13.61	12.80
AL[14]	15.34	0.63	15.58	15.11	17.00	0.84	17.31	16.68
AL[15]	15.55	0.43	15.71	15.39	17.10	0.71	17.36	16.84
AL[16]	15.74	0.40	15.89	15.59	17.42	0.49	17.60	17.24
AL[17]	15.79	0.38	15.93	15.64	17.46	0.53	17.66	17.26
AL[18]	15.87	0.32	15.98	15.75	17.48	0.51	17.67	17.29
AL[19]	15.92	0.16	15.98	15.86	17.69	0.36	17.82	17.55
AL[20]	15.93	0.16	15.99	15.87	17.56	0.40	17.71	17.41
AL[21]	15.94	0.15	16.00	15.89	17.57	0.40	17.72	17.42
AL[22]	15.87	0.24	15.96	15.78	17.61	0.38	17.75	17.47
AL[23]	15.89	0.28	15.99	15.78	17.43	0.58	17.64	17.21
AL[0]	10.25	0.91	10.59	9.91	9.22	1.33	9.72	8.73
AL[1]	2.78	0.63	3.02	2.55	2.09	0.66	2.33	1.84
AL[2]	0.13	0.11	0.17	0.09	0.02	0.05	0.04	0.00
AVER	1862.98	19.22	1870.15	1855.80	2010.33	34.44	2023.19	1997.47
AVEC	243.48	53.37	263.40	223.55	133.10	33.13	145.47	120.72

	LANE = 20				LANE = 22			
	Mean	Stdev	Up 95% CI	Lo 95% CI	Mean	Stdev	Up 95% CI	Lo 95% CI
F	0.04	0.01	0.04	0.03	0.01	0.01	0.02	0.01
AWAIT	1329.67	280.78	1434.52	1224.83	689.32	214.34	769.36	609.29
AGWAIT	1.18	0.24	1.27	1.09	0.60	0.18	0.66	0.53
AVEOT	126.81	7.30	129.54	124.08	123.58	6.91	126.17	121.00
AVECT	1.11	0.12	1.16	1.07	1.06	0.12	1.10	1.02
AL[12]	0.86	0.00	0.86	0.86	0.86	0.00	0.86	0.86
AL[13]	12.85	1.03	13.24	12.47	13.09	1.32	13.58	12.59
AL[14]	17.68	0.86	18.00	17.36	18.13	1.11	18.54	17.71
AL[15]	18.26	0.92	18.61	17.92	18.77	1.20	19.22	18.32
AL[16]	18.17	1.10	18.58	17.76	18.78	1.25	19.24	18.31
AL[17]	18.51	0.91	18.85	18.17	19.02	1.06	19.42	18.63
AL[18]	18.60	1.04	18.99	18.22	19.32	1.04	19.71	18.93
AL[19]	18.71	0.92	19.05	18.37	19.57	1.06	19.97	19.18
AL[20]	18.85	0.85	19.16	18.53	19.48	1.07	19.88	19.08
AL[21]	18.53	0.82	18.84	18.23	19.55	1.08	19.95	19.14
AL[22]	18.43	0.70	18.69	18.17	19.36	0.73	19.63	19.09
AL[23]	18.65	1.08	19.06	18.25	19.23	1.42	19.76	18.70
AL[0]	8.19	1.21	8.64	7.74	7.43	0.95	7.79	7.08
AL[1]	1.28	0.44	1.44	1.12	1.04	0.41	1.20	0.89
AL[2]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AVER	2080.86	47.02	2098.41	2063.30	2134.10	56.01	2155.01	2113.18
AVEC	59.19	21.06	67.05	51.33	22.00	13.41	27.01	16.99

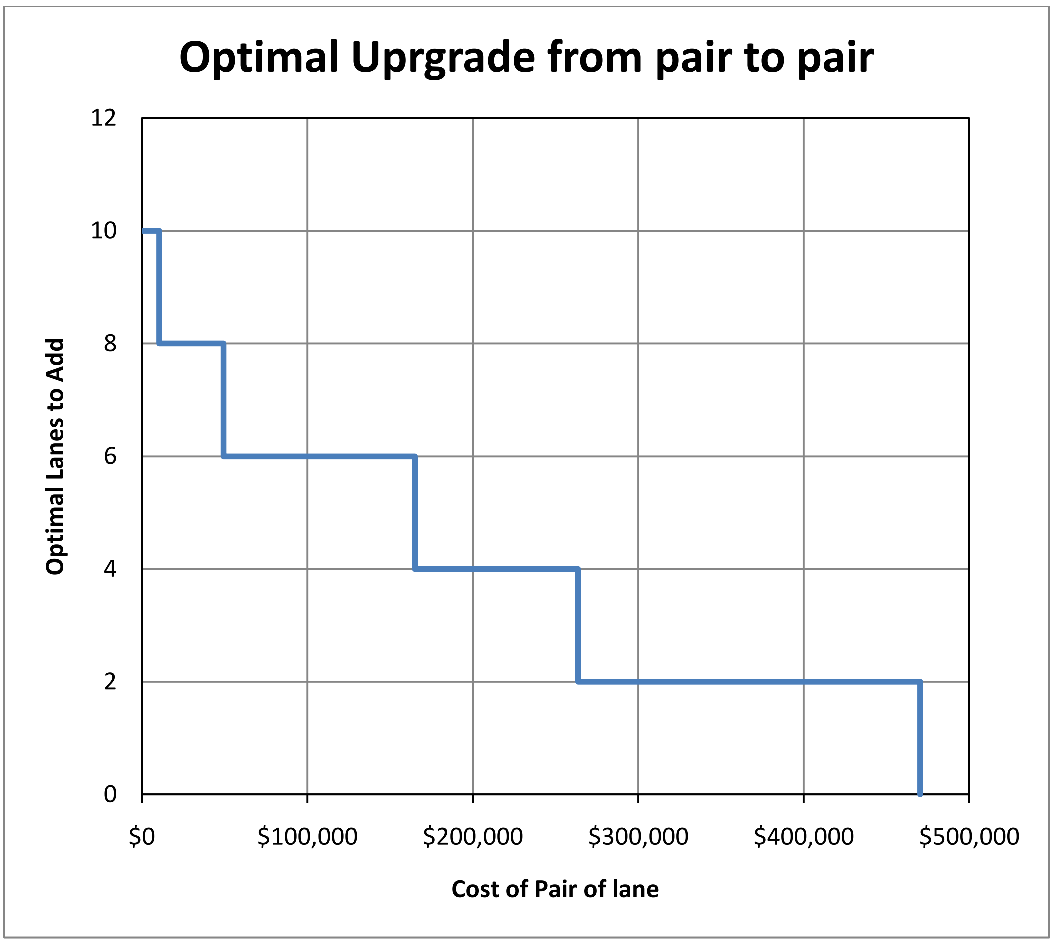
	LANE = 24				LANE = 26			
	Mean	Stdev	Up 95% CI	Lo 95% CI	Mean	Stdev	Up 95% CI	Lo 95% CI
F	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
AWAIT	276.49	135.02	326.90	226.07	113.66	65.86	138.25	89.07
AGWAIT	0.24	0.11	0.28	0.20	0.10	0.06	0.12	0.08
AVEOT	122.18	6.21	124.50	119.87	118.80	6.38	121.18	116.42
AVECT	1.04	0.10	1.08	1.00	0.98	0.11	1.02	0.94
AL[12]	0.86	0.00	0.86	0.86	0.86	0.00	0.86	0.86
AL[13]	13.30	1.30	13.79	12.82	12.94	1.03	13.32	12.56
AL[14]	18.74	1.19	19.18	18.29	18.42	1.33	18.92	17.93
AL[15]	19.41	1.53	19.98	18.84	19.24	1.45	19.78	18.70
AL[16]	19.00	1.37	19.51	18.48	19.53	1.56	20.11	18.95
AL[17]	18.92	1.58	19.51	18.33	19.79	1.48	20.34	19.24
AL[18]	19.43	1.38	19.95	18.92	19.50	1.48	20.05	18.94
AL[19]	19.28	1.53	19.85	18.71	19.22	1.80	19.89	18.55
AL[20]	19.81	1.05	20.20	19.42	19.36	1.32	19.85	18.87
AL[21]	19.40	1.79	20.07	18.73	19.73	1.53	20.30	19.16
AL[22]	19.17	1.73	19.82	18.53	19.30	1.59	19.90	18.71
AL[23]	19.37	1.68	20.00	18.74	19.29	1.63	19.90	18.68
AL[0]	7.38	1.30	7.86	6.89	6.80	1.03	7.19	6.41
AL[1]	0.82	0.35	0.95	0.69	0.76	0.39	0.90	0.61
AL[2]	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.00
AVER	2145.79	64.33	2169.81	2121.76	2146.95	56.31	2167.98	2125.93
AVEC	6.67	5.66	8.78	4.55	2.14	4.76	3.92	0.37

A – 5 Average number of busy lanes. * *For all other hours, the average number of busy lanes was zero.*

	Average number of busy lanes					
HOUR	16	18	20	22	24	26
12	0.86	0.86	0.86	0.86	0.86	0.86
13	12.96	13.20	12.85	13.09	13.30	12.94
14	15.34	17.00	17.68	18.13	18.74	18.42
15	15.55	17.10	18.26	18.77	19.41	19.24
16	15.74	17.42	18.17	18.78	19.00	19.53
17	15.79	17.46	18.51	19.02	18.92	19.79
18	15.87	17.48	18.60	19.32	19.43	19.50
19	15.92	17.69	18.71	19.57	19.28	19.22
20	15.93	17.56	18.85	19.48	19.81	19.36
21	15.94	17.57	18.53	19.55	19.40	19.73
22	15.87	17.61	18.43	19.36	19.17	19.30
23	15.89	17.43	18.65	19.23	19.37	19.29
24	10.25	9.22	8.19	7.43	7.38	6.80
25	2.78	2.09	1.28	1.04	0.82	0.76
26	0.13	0.02	0.00	0.00	0.00	0.00

A-6 Optimal upgrade from pair to pair of lanes

Uprgrade from pair to pair					
Pair to Pair	16 to 18	18 to 20	20 to 22	22 to 24	24 to 26
Marginal Profit	$470,372	$263,582	$165,032	$49,318	$10,385
Cost per pair of lane	$470,372	$263,582	$165,032	$49,318	$10,385



A-7 Sensitivity to parameters & Change with respect to original parameters

	Changes in Parameters			Sigma Ouput			$ Per day			
	Inter-arrival Mean	Bowling Time Mean	Bowling Time Stdev	C	N	B	Revenue	Cost	Profit	5-year Profit
Sensitivity in Inter-arrival Mean	3.5	30k	4k	16	2666	369	$1,904	$527	$1,377	$2,513,286
				18	2934	181	$2,096	$259	$1,837	$3,352,786
				20	3167	121	$2,262	$173	$2,089	$3,812,946
				22	3193	15	$2,281	$21	$2,259	$4,123,196
				24	3459	7	$2,471	$10	$2,461	$4,490,804
				26	3352	2	$2,394	$3	$2,391	$4,364,357
	4.5	30k	4k	16	2455	89	$1,754	$127	$1,626	$2,968,232
				18	2658	43	$1,899	$61	$1,837	$3,352,786
				20	2591	1	$1,851	$1	$1,849	$3,374,946
				22	2902	9	$2,073	$13	$2,060	$3,759,500
				24	2572	0	$1,837	$0	$1,837	$3,352,786
				26	2556	0	$1,826	$0	$1,826	$3,331,929
Sensitivity in Normal Mean	4	25k	4k	16	2921	56	$2,086	$80	$2,006	$3,661,732
				18	3061	28	$2,186	$40	$2,146	$3,917,232
				20	3001	9	$2,144	$13	$2,131	$3,888,554
				22	2920	0	$2,086	$0	$2,086	$3,806,429
				24	2903	0	$2,074	$0	$2,074	$3,784,268
				26	2912	0	$2,080	$0	$2,080	$3,796,000
	4	35k	4k	16	2357	256	$1,684	$366	$1,318	$2,405,089
				18	2539	172	$1,814	$246	$1,568	$2,861,339
				20	2776	161	$1,983	$230	$1,753	$3,198,964
				22	2947	85	$2,105	$121	$1,984	$3,620,018
				24	3025	62	$2,161	$89	$2,072	$3,781,661
				26	3023	10	$2,159	$14	$2,145	$3,914,625
Sensitivity in Normal Stdev	4	30k	3k	16	2595	197	$1,854	$281	$1,572	$2,869,161
				18	2727	83	$1,948	$119	$1,829	$3,338,446
				20	3022	40	$2,159	$57	$2,101	$3,835,107
				22	2945	4	$2,104	$6	$2,098	$3,828,589
				24	2926	0	$2,090	$0	$2,090	$3,814,250
				26	2917	0	$2,084	$0	$2,084	$3,802,518
	4	30k	5k	16	2631	185	$1,879	$264	$1,615	$2,947,375
				18	2884	136	$2,060	$194	$1,866	$3,404,929
				20	2944	50	$2,103	$71	$2,031	$3,707,357
				22	3048	18	$2,177	$26	$2,151	$3,926,357
				24	3022	7	$2,159	$10	$2,149	$3,921,143
				26	2873	0	$2,052	$0	$2,052	$3,745,161

	Changes in Parameters			Uprgrade from 16 lanes		
	Inter-arrival Mean	Bowling Time Mean	Bowling Time Stdev	Increase in 5-year Profit	Cost Per Lane	Change wrt original
Sensitivity in Inter-arrival Mean	3.5	30k	4k	-	-	-
				$839,500	$839,500	$369,128
				$1,299,661	$649,830	$282,853
				$1,609,911	$536,637	$236,975
				$1,977,518	$494,379	$257,303
				$1,851,071	$370,214	$178,476
	4.5	30k	4k	-	-	-
				$384,554	$384,554	-$85,818
				$861,661	$430,830	$63,853
				$1,246,214	$415,405	$115,743
				$839,500	$209,875	-$27,201
				$818,643	$163,729	-$28,009
Sensitivity in Normal Mean	4	25k	4k	-	-	
				$1,403,946	$1,403,946	$933,574
				$1,375,268	$687,634	$320,657
				$1,293,143	$431,048	$131,386
				$1,270,982	$317,746	$80,669
				$1,282,714	$256,543	$64,805
	4	35k	4k	-	-	
				$456,250	$456,250	-$14,122
				$685,679	$342,839	-$24,138
				$1,106,732	$368,911	$69,249
				$1,268,375	$317,094	$80,018
				$1,401,339	$280,268	$88,530
Sensitivity in Normal Stdev	4	30k	3k	-	-	
				$469,286	$469,286	-$1,086
				$1,321,821	$660,911	$293,934
				$1,315,304	$438,435	$138,772
				$1,300,964	$325,241	$88,165
				$1,289,232	$257,846	$66,108
	4	30k	5k	-	-	
				$457,554	$457,554	-$12,818
				$1,194,071	$597,036	$230,059
				$1,413,071	$471,024	$171,362
				$1,407,857	$351,964	$114,888
				$1,231,875	$246,375	$54,637

			Uprgrade from 16 lanes		
Revenue per bowler	C	5-year Profit	Increase in 5-year Profit	Cost Per Lane	Change wrt original
3	16	$1,595,615	-	-	-
	18	$1,958,416	$362,801	$362,801	-$107,571
	20	$2,170,516	$574,901	$287,451	-$79,527
	22	$2,296,684	$701,069	$233,690	-$65,972
	24	$2,337,469	$741,854	$185,463	-$51,613
	26	$2,347,002	$751,387	$150,277	-$41,461
7	16	$4,315,560	-	-	-
	18	$4,893,503	$577,943	$577,943	$107,571
	20	$5,208,567	$893,007	$446,504	$79,527
	22	$5,412,463	$1,096,903	$365,634	$65,972
	24	$5,470,316	$1,154,756	$288,689	$51,613
	26	$5,481,553	$1,165,993	$233,199	$41,461