University Housing Dining Services:

Logistics System Analysis

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Abstract

The goal of this project was to analyze and make recommendations with regard to the distribution system of the University of Illinois' dining hall. In this report, the joint replenishment problem for five vegan items for the current operating system was examined and new techniques were proposed using the deterministic and stochastic demand models. First, relevant information about the University inventory system was collected, actual demand data was analyzed and necessary assumptions were made. The evaluation of the total cost of the current system per semester was made and compared to the optimal order quantity and schedule based on the two proposed solutions in the deterministic and stochastic models.

Assuming the time-varying deterministic demand, the joint replenishment problem was solved for the items using CPLEX 11.2.0 and the optimal order quantity for each item was found. For the stochastic demand, a cost optimization function that is developed by approximating each individual component of the cost function using a Taylor series expansion was used in order to find optimal order quantity and safety stock for the five items. The solutions between the models were compared and concluded that both models showed dramatic improvement in terms of the total cost per semester. However, the deterministic model is more suitable in a case that the forecasted demand is very accurate, while the stochastic based-solutions are more reasonable in a situation that the demands are high uncertainty or fluctuation.

Acknowledgements

We would like to express our deep gratitude to Professor Yangfeng Ouyang at the University of Illinois at Urbana-Champaign for providing us the fundamental concepts to effectively analyze a logistics system and making valuable suggestions for this work. We would also like to thank Christopher Henning, Administrative IV and Peter Testory, Chef at Food Service, University of Illinois at Urbana-Champaign for their assistance and time in keeping our progress on this project by explaining the operation of University Housing Food Store and providing the relevant data.

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1. INTRODUCTION

1.1 Background

The University Housing Dining Services at the University of Illinois at Urbana-Champaign provides cater service to more than 6,000 events annually and dining service for more than 13,000 students daily, which requires producing approximately 24,000 meals per day. The dining service includes all aspects of the food service operation for residents, retailers, and catering. The high complexity of the food service system at the university is a result of the large and diverse population in the community. Because of the wide spectrum of campus events and diverse venues, the dining hall service anticipates many different culinary trends and contemporary food design and presentation. The goals of University Housing Dining Services are to provide a high quality meal service and satisfy the needs of the diverse college community as a whole. These goals are accomplished by the collective efforts of 225 full-time employees and over 2,000 student employees of the University Housing Dining Services during the academic years.

The University Housing Dining Services operates classic dining halls (buffet style), a la carte location (items priced individually), and specialty restaurants. Classic meal plans range from $2,000 to $6,000 for the academic year, and individual meals rates are $10.35 and $13.60 for lunch and dinner respectively. Most customers utilizing the dining services are undergraduate freshman students, as they are required to live in the university housing and purchase a meal plan. In addition to these students, countless of the university employees and guests also make use of the dining hall services throughout the academic years. There is a total of six dining halls: Busey-Evans, Florida Avenue Residence Halls (FAR), Ikenberry, Illinois Street Residence Halls (ISR), Lincoln Avenue Residence Halls (LAR) & Allen, and Pennsylvania Avenue Residence Halls (PAR). Each of the dining halls is self-managed, meaning that each place orders independently, receive the ordered items daily and directly from suppliers, and have its own food storage area for the inventory.

The role of the Housing Food Stores (HFS) is crucial, as it supports all phases of food production by coordinating orders and overseeing the operation of the dining halls, and managing and distributing certain items to all dining halls. These items include specialty and gourmet food requiring extra preparation (e.g. vegans), and items commonly used by all dining halls requiring longer storage due to vendor contracts and large shipment requirements (e.g. staples). Under the Housing Division, the centralized food warehouse of HFS is responsible for ensuring that the aforementioned items are appropriately received, stored, prepared, and distributed to all dining halls in the residential houses. In addition of the dining halls in the residential houses, HFS provides the catering service to places such as Levis Faculty Center and Illini Union Ballroom. In order to manage and store the items, the centralized food warehouse is equipped with special kitchens and cooking equipment, large refrigerators, freezers, and dry storage areas. Furthermore, all menus, food preparation procedures, nutrition researches, recipes and food quantity calculations are conducted by the senior management team at HFS. Ultimately, the University Housing Dining Services and Housing Food Stores provide a complete logistics food supply management service to the community at the University of Illinois at Urbana-Champaign.

1.2 Supply Chain Overview

A supply chain, or logistics system, can be defined as a system encompassing the people, technology and resources involved in the transportation of goods or services from the supplier to the customer. An overview of the logistics system for the University Housing Dining Services is shown in Figure 1.

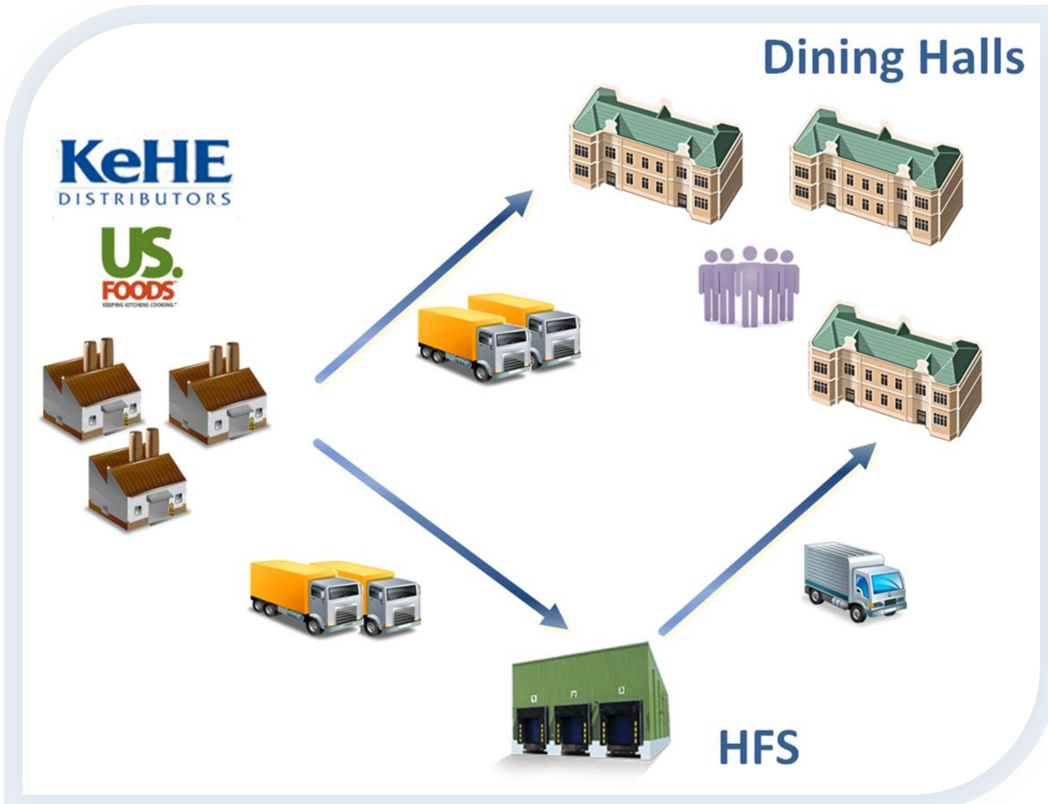


Figure 1. Supply Chain Diagram

As explained previously, suppliers ship directly to either the specific dining halls or central warehouse to meet their corresponding orders. The orders for food ingredients through bidding vendors are placed based on a monthly bidding process. The monthly contract requires private vendors to directly deliver food supplies to each individual dining hall or catering location on a daily basis. Trucks from the warehouse also distribute to and collect food from each dining hall twice a day (8am & 2pm). The local route map of the distribution from the warehouse is shown in Figure 2.

In order to assess the current performance of the logistics system and identify improvement opportunities, the main drivers of supply chain performance were considered. These drivers are: facilities, inventory, transportation, information, sourcing and pricing. The last three drivers are out of the scope of the project. The facilities (e.g. warehouse, dining halls, and suppliers) are fixed, so a little or no improvement can be made. As it can be seen from Figure 2, improving the local routing does not offer much benefit. On the other hand, the inventory management offers the greatest improvement opportunity. Therefore, the analysis of the inventory is the main focus of the project.

Figure 2. Dining Hall Route Map

1.3 Logistics Challenges

In terms of the inventory management, the University Housing Dining Services deals with over 20,000 total items across all dining halls each year. Approximately 1,650 items are delivered directly from their suppliers to each dining hall and stored in each dining hall's refrigerator or food storage for their use. The HFS warehouse receives 387 items directly from their suppliers (mostly from the KEHE Distributors, LLC and U.S. Food Service), stores and re-distributes them to each dining hall. HFS usually receives three food items orders every week from their food suppliers. The foods that HFS usually orders have a large shipment size requirement, and some of the items are ingredients for special events, such as the 'Taste of Asia. The items that the HFS warehouse directly manages are categorized into seven different food types:

- bakery (34 items)
- chemical (26 items)
- frozen (45 items)
- HFS special diet (69 items)
- meats and cheeses(55 items)
- paper and supply (41 items)
- staples (117 items)

Because of the large inventory and independent operation of the different dining hall units, the scope of the project was narrowed down to the operation of HFS. A process diagram of the operation of the HFS warehouse is shown in Figure 3. After conducting a preliminary analysis with HFS, the following challenges were identified:

1) The communication problem between HFS, each dining hall and I-Hotel, etc.: In response to the unusual demand, dining hall units usually cancel or increase orders by phone calls and emails in a short notice before the food is scheduled to be delivered. The HFS warehouse cannot adjust to this sudden change, causing discrepancies in the inventory management.
2) Forecasting demand: HFS currently forecasts the demand for the next meal cycle (about every 4 weeks) based on the consumed and leftover quantities of the previous meal cycle. In addition, the special events (e.g. basketball game) are taking into consideration when forecasting. CBORD, which is a food and nutrition service management system software, is then used to determine the required quantities for the next meal cycle based on the menu recipes, which contain raw materials and serving size information, while the adjustment in the forecasted demand is performed by HFS. A more methodical and accurate forecasting system can be implemented.
3) Inefficient inventory management: CBORD provides a record of the historical data and the current inventory levels. However, order quantities and frequencies are not done systematically. No specific inventory policies are currently in place.
4) The expiration date tracking for perishable goods: HFS is not able to keep track of the expiration dates for perishable goods. This results in the raw ingredients becoming spoiled and wasted before the dining hall warehouse can use them. This seems to be a recurrent issue for vegan items, as they are ordered once or twice a year and kept in inventory for a long time, incurring a high inventory cost.

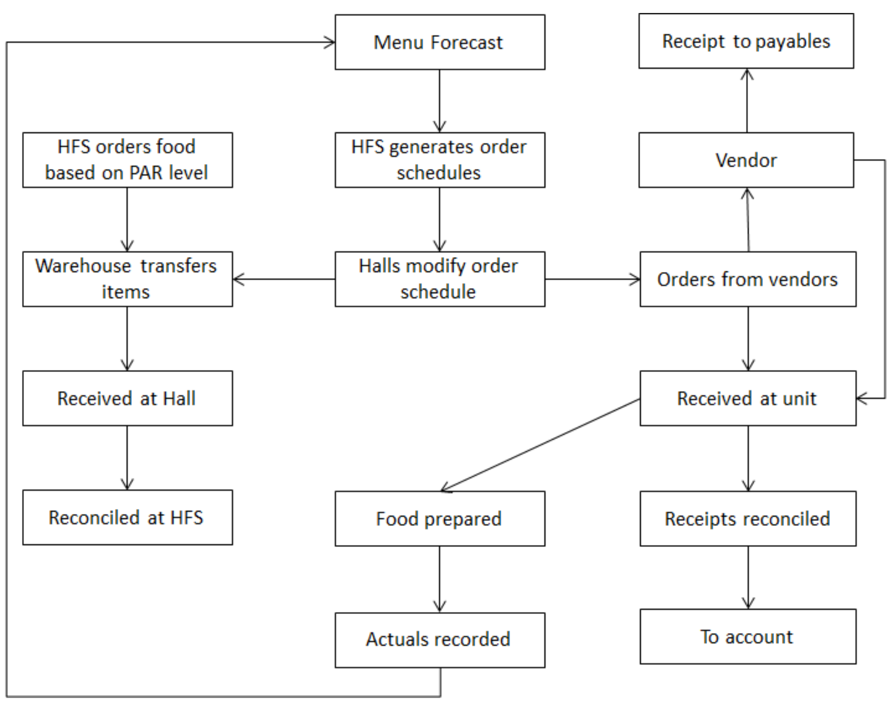


Figure 3. Schematic diagram of HFS's warehouse operation system

2. PROBLEM DESCRIPTION

2.1 Project Scope

Based on the preliminary analysis and challenges discussed in the previous section, the focus of the project was to address the inefficient inventory management (challenge 3) and expiration dates (challenge 4) because of the potential of improvement and applicability to logistics. Since HFS mainly has problems with the perishability and ordering of vegan items from a single supplier, these items are the target improvement of this project. These “special vegan items” can be categorized as:

1) Vegetarian Citrus Chicken (VCC)
2) Vegetarian Fish Filet (VFF)
3) Vegan Chicken Teriyaki (VCT)
4) Vegan Beef Teriyaki (VBT)
5) Vegan Sliced Baked Ham (VSBH)

These special vegan items are all supplied by a single contractor, U.S. Foodservice. Another supplier, KEHE, is also used for some vegan and specialty items. The main problem is that HFS only orders a large quantity of vegan raw ingredients especially at the beginning of each semester. Even though HFS does not have to deal with complex administrative process and several transactions, the large shipment size leads to extremely high inventory cost. The vegan items remain in the refrigerators in the HFS warehouse for an extensive period of time. This takes a large storage area and high electricity cost for maintaining low temperature to prevent food from being perished. Furthermore, the long inventory time reduces the freshness and quality of the food, resulting in an inferior customer experience.

Table 1 gives an overview of the inventory management metrics from various perspectives (Raguraman 2001). Based on the discussion above, the project focus was on the inventory levels, order quantities and critical inventory metrics. The scope of the project was limited to the operation of the HFS and the five listed vegan items. Since the vegan items were tested at the end of 2010, and fully served in the fall semester of 2011, the scope was narrowed down to this period. Because items are ordered from the same supplier, coordinated ordering (or joint replenishment) policies were investigated.

Table 1. Metrics and Benchmarking for Inventory Management (Raguraman 2001)

INVENTORY METRICS	CUSTOMER PERSPECTIVE	FORECAST ACCURACY METRICS
Turnover Inventory Turns	**In-Stock Percent** at Point of Sale	**Sales Forecast Accuracy** Track actual sales vs. forecasted sales variances **Forecasting Utilization of Inventory Assets** The higher the inventory turns, the better the firm uses its inventory assets.
Inventory Levels Dollars and/or units at various points in supply chain	**Purchase Order Fill Rate Percent** Percent Shipped on time Percent Delivered on time	**Order Forecast Accuracy**
Order Quantity Order Processing/ Setup Cost, Inventory Carrying cost, Back Order Cost, Excess and Obsolete Stock Cost	**Reliability** Stock out percentage, delays, loss and paperwork involved	**Order Forecast Accuracy**
Critical Inventory Average Inventory, EOQ	**Stock out percentage, Percentage of orders fulfilled**	**Maintaining proper Inventory levels** Determining method for reordering inventory
Space Utilization & Layout % space utilized (cu.m.)		**Forecasting Storage Space and Lead Times involved**
Stock to Sales Ratios Weeks/days of supply	**Lost Sales Analysis** Evaluate actual/potential lost sales due to lack of inventory	Track actual order qty. vs. forecasted order qty. variances at critical times that could influence production
Industry Ratio Comparison with others	**Part Count Accuracy Percentage** **Quality-Percent Defects** **Price of Non-Conform. (PONC)**	**Percent Variability in Lead Time**
Engineering Changes per Month	**Communication Effectiveness** Measure collaborative cycle time/issue resolution time	Customer Satisfaction
Markdown Dollars/ Liquidation Loss	**Cycle Time** May include series of time segments measured for specific aspects of the process from raw materials to merchandise delivered to POS (e.g., manufacturing time cycles, order time cycles, packing time cycle, shipping time cycle, etc.)	**Frequency of Emergency or Cancelled orders** Track- quantity, cause, actions taken to limit reoccurrence

2.2 Project Objective

The main goal of the project is to evaluate, analyze and improve the current inventory management system of the University of Illinois Housing Food Stores. More specifically, the objective is to determine joint replenishment policies for the vegan items to minimize the total cost consisting of ordering and holding.

2.3 Motivation

Inventory is critical component of a logistic system that is a result of the mismatch between supply and demand. The motivation of holding inventory is to increase the responsiveness to the customer and take advantage of economies of scale. Therefore, there is a fundamental trade-off between the holding cost and the ordering cost. For instance, holding excessive inventory increases the responsiveness and reduces the ordering cost but incurs a high holding cost. On the other hand, holding too little inventory translates into lower holding costs but higher ordering costs. The purpose of this project is to find a balance between the ordering frequency and the ordered quantity such that the total cost is minimized.

Inventory costs an important consideration because inventory is one of the most expensive assets of many companies, representing as much as 50% of total invested capital (Heizer 2005). According to the United States Department of Commerce, the U.S. business logistics system cost in 2009 was equivalent to 7.7% of the GDP in 2009, with the total U.S. business inventories in 2009 totaling over $1.85 trillion (Wilson 2011). Perishability is also a relevant consideration in inventory management of foods. The U.S. food industry annually discards $35 billion worth of spoiled goods (Forbes Magazine, April 24, 2006). According to the FDA, up to 20% of food is discarded due to spoilage (Black 2003).

Furthermore, in a multi-item inventory environment, the coordinated replenishment policies have been shown to result in substantial savings of often 20% compared to the uncoordinated replenishment policies (Atkins 1988). The joint replenishment problem can be defined as follows: “In multi-item inventory replenishment contexts, cost savings can be achieved by coordinating the replenishment of some items. Each order is made up of a subset of item types that are jointly replenished and it generates two types of fixed costs: the common ordering cost associated with the order itself and the individual ordering cost associated with each item type” (Boctor 2004).

As discussed above, the inventory costs can be substantial. The motivation of the project is to aid the University Housing Dining Services in the management of inventory by providing insights on the replenishment policies, and to provide a better dining experience for students, employees and guests at the University of Illinois of at Urbana-Champaign.

3. ANALYSIS

3.1 Data Collection

Relevant data gathered for the project mainly included the demand (actual quantities consumed) of the vegan items and miscellaneous costs related to the value of items, namely the ordering and inventory costs. The demand data was obtained from the inventory transaction report (Appendix 10.1) generated by CBORD software. The report was generated for the five special vegan items, covering the period from December 4th, 2010 to April 14th, 2012. The report shows the following transaction information for the five vegan items:

- the types and quantities of the vegan items that the warehouse received from the supplier or transferred to each dining hall
- the remaining quantities in warehouse
- the time period that each transaction was made
- the unit cost price of each items presented in the transaction

The information from above was used to determine the demand for each vegan item. First, all units of each item was converted to the same unit in packs (1 pack = 6.62 pounds = 1/4 case), and the quantities were rounded to the nearest integer and grouped by week. Although the entire data covered 72 weeks, only the data from the 17-week period (Fall 2011 semester) from August 27, 2011 to December 10th, 2011 was considered in the evaluation of the current system and determination of the replenishment policies. This data is shown in Table 2 below.

Table 2. Demand (in packs) for each item for one semester (Fall 2011)

d_{it}	1	2	3	4	5
1		6		20	
2			4		8
3	17	6			4
4		4		40	8
5	16			12	
6	12	4	4	12	
7		2	8	12	
8	12	2	4	16	
9	12	6		4	
10		4		12	
11				16	
12		2	16	24	
13	12			12	
14					
15	12	2		4	
16	8	2	4	12	
17			4	8	

Other periods were excluded from the analysis because:

- The data for the 38-week period from December 4th, 2010 to August 20th contained only five scattered transaction records. This period was when items were first introduced into the menus.
- Since one semester seemed the natural planning horizon, the data for Spring 2012 semester was excluded since it was incomplete (Appendix 10.2).

3.2 Demand Analysis

The demand of each item by period was analyzed to get an insight on the behavior of the system, capture any trends, determine if the demand follows a specific distribution, and guide the approach to find the optimal inventory policies discussed in the methodology sections. Figure 4 shows the distribution demand per week by item for the fall 2011 semester. From the graph, it can be noted that the demand varies with time for each item differently, without following any evident pattern. Also it can be seen that for week 14, there is zero demand due to thanksgiving break. The distribution of the total demand by item is shown in Figure 5, which suggests that item 4 has the highest demand, accounting for about half of the total demand. In addition, Figure 6 shows that item 4 is the most frequently demanded (i.e. has the highest number of weeks with demand). The opposite observations can be corroborated for item 5. These insights have some implications in the optimal joint ordering policies, which are mentioned later in the report.

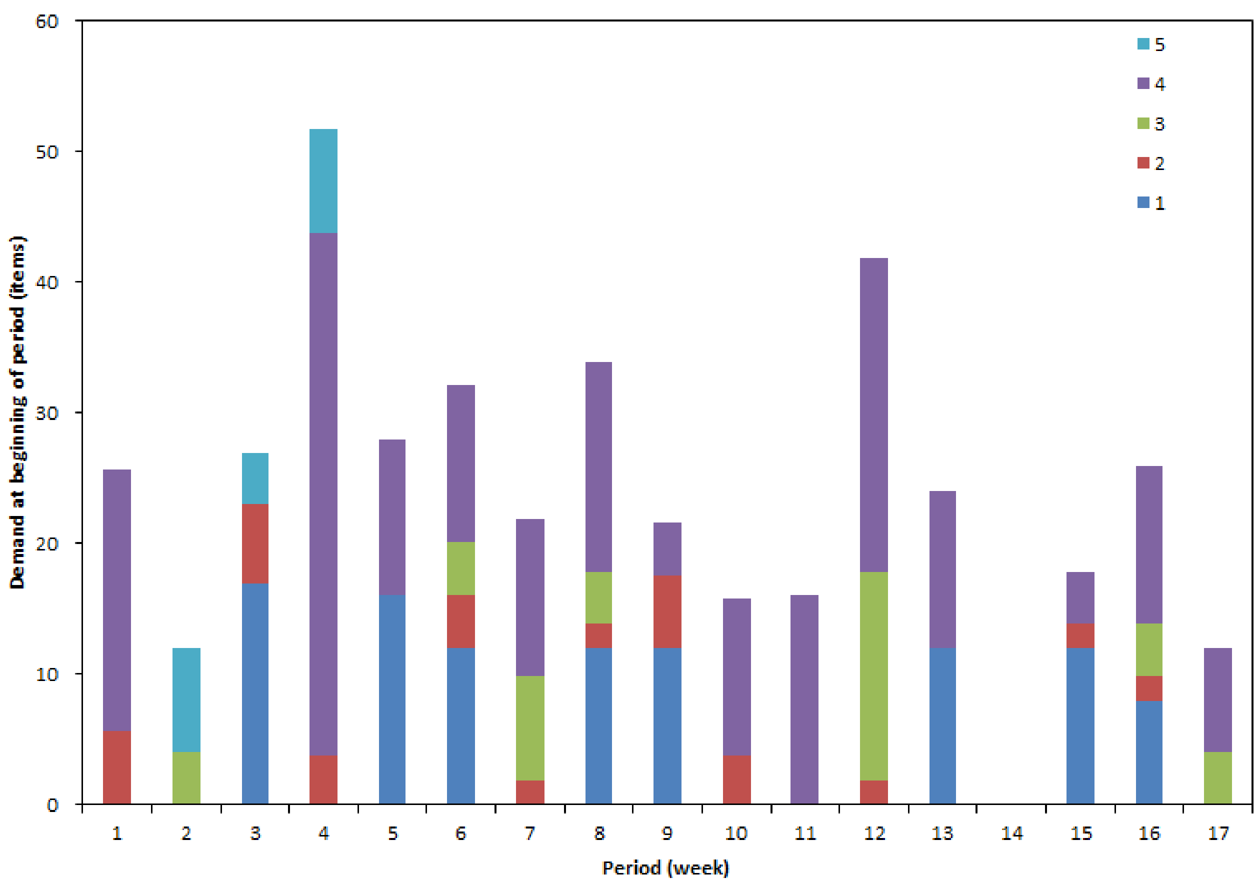


Figure 4. Demand per week by item (Fall 2011)

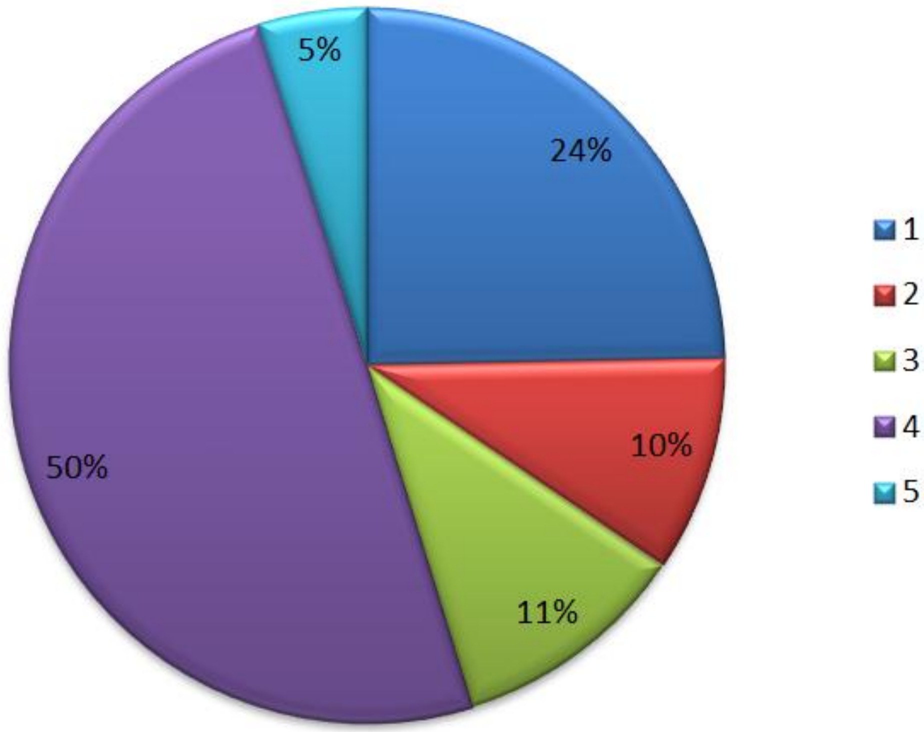


Figure 5. Distribution of total demand by type of item (Fall 2011)

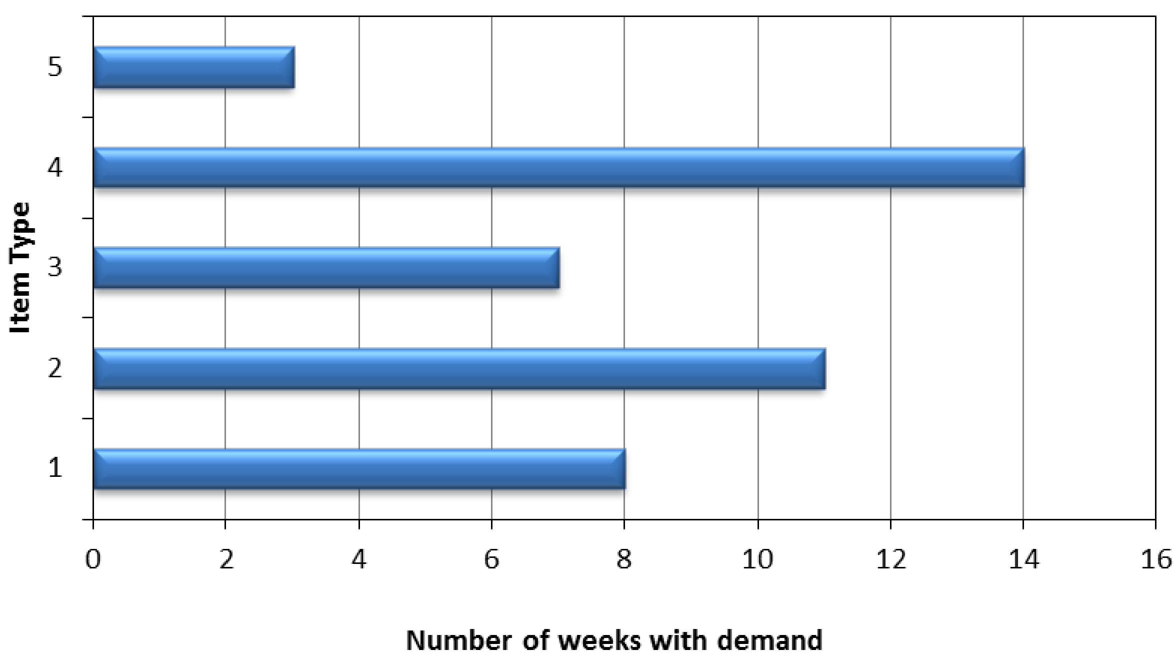


Figure 6. Number of weeks with demand by item type (Fall 2011)

In order to further understand the behavior of the demand and select appropriate mathematical models for the joint replenishment, the probability distribution of the five items was analyzed. Because the Poisson distribution is widely used to model arrivals and demand events, this distribution was chosen for the goodness of fit test. The Normal distribution was also tested due to its robustness and wide applicability. Figure 7 shows the histogram and fitted Poisson distribution (with maximum likelihood parameter estimates) for item 1 to item 4 for the entire data set (Fall 2011 and Spring 2012). Item 5 was excluded due to the small sample.

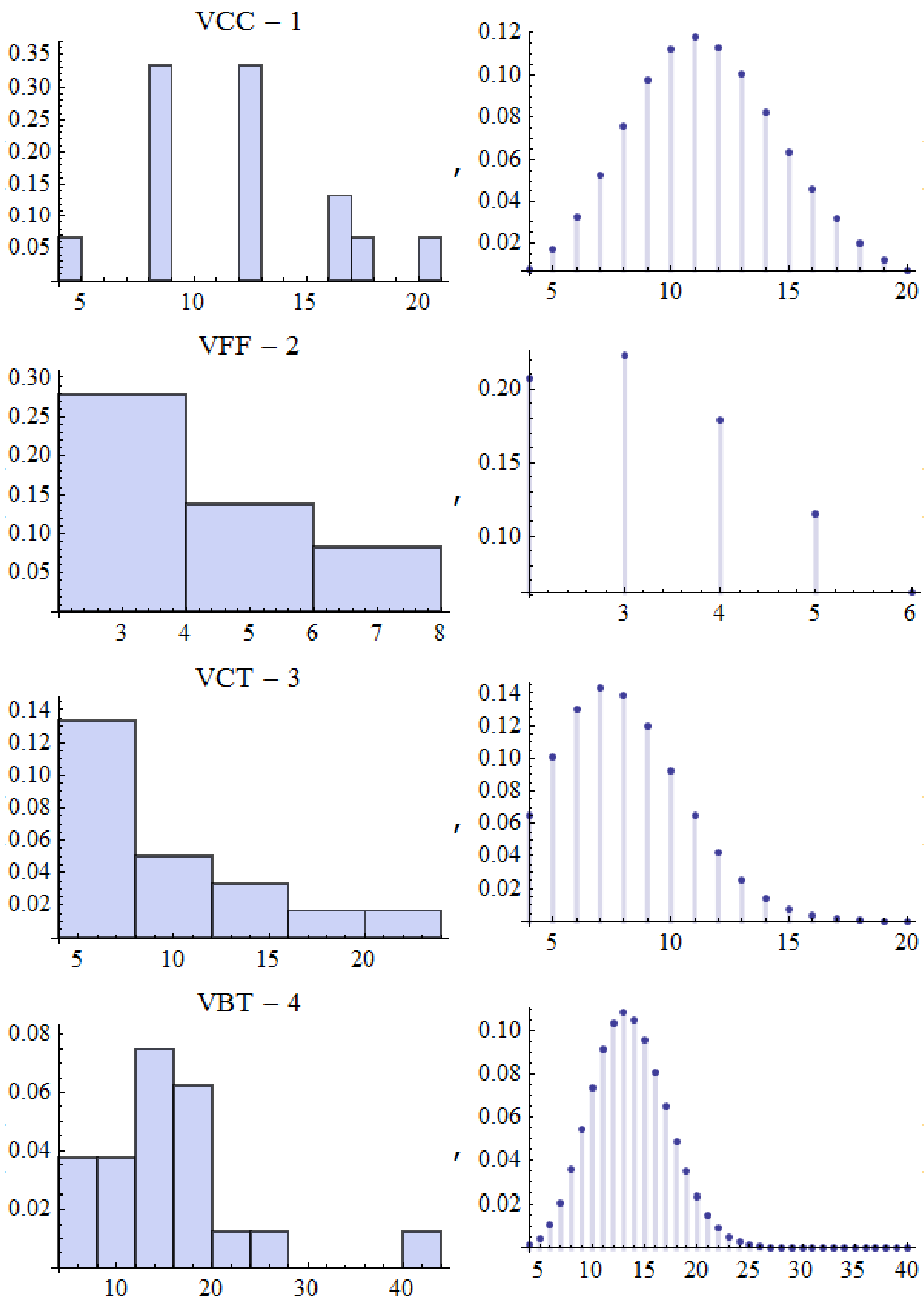


Figure 7. Histogram and Estimated (Poisson) distribution for items 1-4

The plots above suggest that the demand for all items might be reasonably estimated by a Poisson distribution. Figure 8 shows the empirical and theoretical (Poisson) distribution, and suggests that only item 1 and item 4 might follow a Poisson distribution.

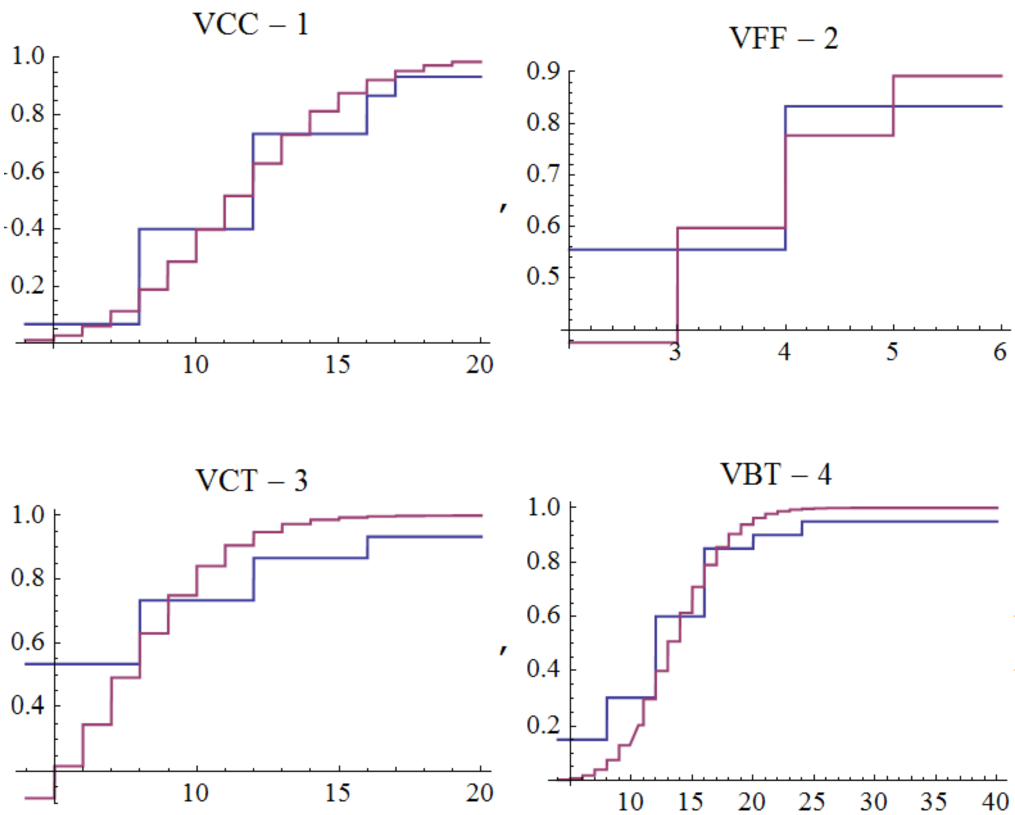


Figure 8. Empirical and Theoretical (Poisson) Cumulative Distributions for items 1-4

However, the outcomes for the goodness of fit tests lead to different conclusions. The results (Table 3) show that the null hypothesis (the data are from some null distribution) is not rejected (p-value > 0.05) for item 4 in the Poisson test, and item 1 in the Normal test at the 0.05 significance level. Thus, there is no reason to doubt the assumption of the Poisson distribution for item 4 and the Normal distribution for item 1. In other words, these distributions seem to be a good fit for the corresponding items. All other combinations of tests and items are rejected by the null hypothesis, suggesting that there is strong indication that the data from the corresponding item did not come from the tested distribution. Note that Item 5 was not tested due to the small sample of points.

Table 3. Goodness of fit tests

Item	Poisson Test (p-value χ^2 test)	Poisson Distribution Reasonable?	Normality Test (p-value Watson U^2)	Normal Distribution Reasonable?
1	0.0068	No	0.0623	Yes
2	0.0003	No	0.0004	No
3	0.0090	No	0.0009	No
4	0.1068	Yes	0.0227	No

3.3 Cost Estimates

The focus of the project is to determine the optimal inventory policies involving coordinated ordering in a multi-item inventory system, which is commonly known in the literature as the joint replenishment problem (JRP). In general, the classical joint replenishment problem generally seeks to minimize the ordering (setup) costs and inventory holding (carrying) by determining the optimal order quantities and frequencies for multiple items ordered from a single supplier. Goyal points out, “because of the major ordering cost, using group replenishment may lead to substantial cost savings. The savings from group replenishment are more significant the higher the major ordering cost.”

The following costs were considered and estimated for the JRP:

S	common ordering cost (units: $/order)
s_i	individual ordering cost for item type i (units: $/order)
c_i	unit cost for item type i (units: $/item)
p	percentage of unit cost during period t for holding an item (units: %)
h_i	unit inventory holding cost for item type i during period t (units: $/item)
L	Lead time (units: week)

The ordering cost represents the cost of preparing and receiving the order and the transportation cost. The cost of placing an order to the supplier for a number of different items can be divided into two components (Goyal 2008):

- a major ordering cost, which is independent of the number of different products in the order
- a minor ordering cost, which depends on the number of different products in the order

A major order cost is incurred anytime a review takes place and is associated with order placement. The major order cost also includes the cost to assess and update the inventory status (Viswanathan 1997). A minor order cost, or line-item cost is associated with each item included in the order to cover the cost of picking, packing, or other special handling required to process the item for shipment (Atkins 1988).

The inventory holding cost, also known as carrying cost, is the cost of keeping inventory until a item is used or sold. The major components of the holding cost include cost of capital ("Lost opportunity cost" is the return that could have been obtained if the capital had been invested in anything other than inventory), storage and handling, insurance, pilferage and spoilage, and obsolescence and deterioration (Richardson 1995). The last two are the most relevant in the food industry. Figure 9 shows a complete and detailed breakdown of the holding cost.

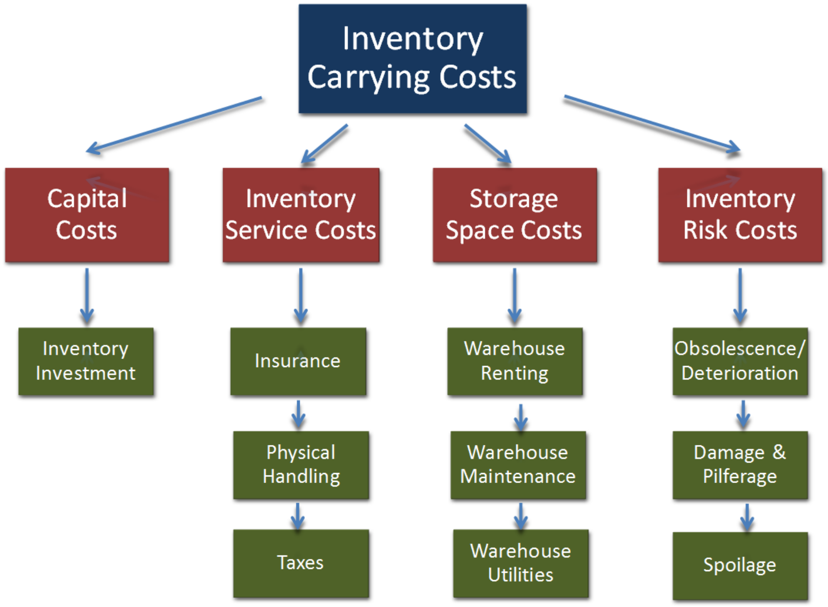
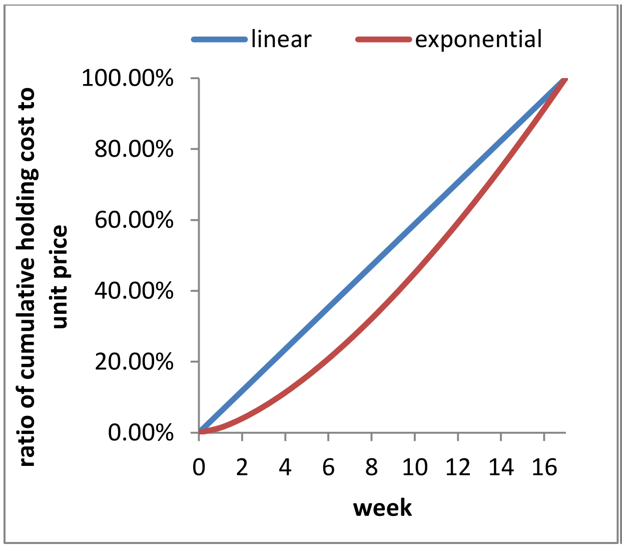


Figure 9. Holding Cost Breakdown

In the current system, the major setup cost including transportation is $500. However, the minor setup cost is not directly given. According to the contract with U.S. Foodservice, it uses the markup cost (4.7%) of the total cost in a transaction and not ordering costs based on item categories. The markup cost depends on the ordering quantity, and this is not allowed in our model because one of the decision variables in the proposed models depends on this. Therefore, for our project an assumption about the minor setup cost needs to be made. The minor ordering cost was estimated as the markup cost (4.7%) of the unit cost times 4 packs/case multiplied by 10 cases/pack, which represents 40 packs in an order. This number was chosen because HFS currently orders 60 cases from the supplier for six items, which means that each order includes approximately 10 cases of each item assuming equal distribution of the items in the order. This assumption seems reasonable for the project because, based on the definition, the minor ordering cost should be about the same (i.e. all items are similar) for all items and related in some way to the value of the item.

Since HFS does not operate for profit, information on the holding cost for each item is not available. The unit holding cost for carrying each item in inventory for 1 week can be computed as a function of the percentage of the unit cost ($h_i = p * c_i$). The percentage (p or p-value) of unit cost during period t for holding an item was estimated based on the different components that make up the holding cost shown in Figure 9. Specifically in our case, the vegan items can last for the whole semester with 17 weeks (the average shelf life time is approximately 6 months, but we will only consider it as 17 weeks, as there will be no transaction during the summer or winter break). If perishability is considered, then at the end of the shelf life, the item will be completely spoiled, meaning that the item cannot be used and the product value equal to zero. This implies that at the end of the shelf life, the holding cost is the same as the value of the item ($p = 100\%$). So assuming a constant deterioration rate (linearly increase of holding cost with the percentage of spoilage), then p can be estimated as 100% / (17 weeks), which turns out to be about 5.88% per week. Nevertheless, based on Ferguson (2007), the cumulative holding cost specifically for perishables will follow an exponential curve, representing a lower deterioration rate at the beginning and a much higher rate toward the end of the product shelf life time. The exponential curve show a result of a lower average holding cost per week than the spoilage linear assumption. Also with the consideration of the food quality for the university students, the warehouse would like to use more fresh food and thereby shorten the inventory holding period less than 17 weeks; we assume that 2 months will be a reasonable time period for us to estimate the p value. With the estimation from both models presented in Figure 10 , p is assumed to be 5%.



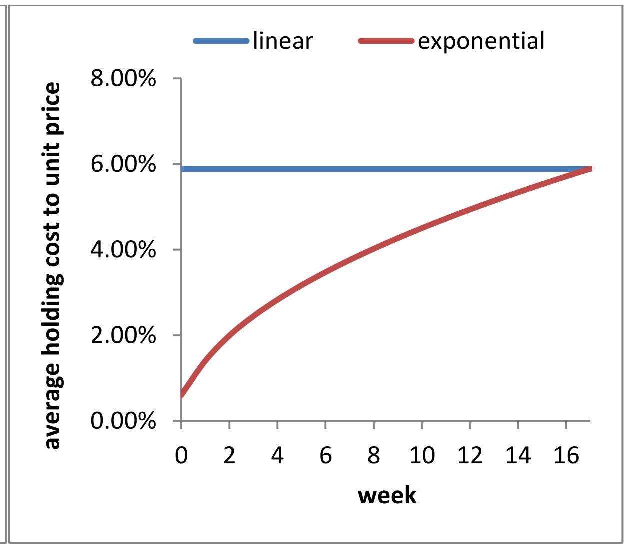


Figure 10. Cumulative and average holding cost in classical model and perishable proposed model

According to the information provided by HFS, the lead time is in a range from 10 to 14 days. So two week lead time is used as the estimated value for the rest of the computations. The estimated parameters based on the above discussion are shown in Table 4. Other particular costs and estimated parameters used in the project are discussed in the corresponding methodology sections.

Table 4. Parameters

Item Name	VCC	VFF	VCT	VBT	VSBH
Item Number i	1	2	3	4	5
S (\$/order)	\$500				
c_i (\$/pack)	\$34.59	\$37.97	\$39.24	\$38.30	\$41.07
s_i (\$/order)	\$65.03	\$71.38	\$73.77	\$72.00	\$77.21
p (%)	5%	5%	5%	5%	5%
h_i (\$/pack)	\$1.73	\$1.90	\$1.96	\$1.92	\$2.05
L (week)	2				

3.4 Current System Evaluation

The current system refers to how HFS currently places an order and holds inventory during the semester. The current system can be evaluated by calculating the ordering cost and the holding cost of the current replenishment policy—ordering once at the beginning of the semester to cover the demand for the entire semester. Table 5 shows the inventory and the holding cost if the aforementioned inventory management procedure is followed. The total holding cost is the sum of the holding cost computed in the table across all periods, which results in a total of $6,226. Assuming the major ordering and minor costs listed in the previous section, the total ordering cost is equal to $860 ($500 + ($65.03 + $71.38 + $73.77 + $72.00 + $77.21)). The former procedure for calculating the total ordering cost (i.e. using estimated minor costs) is used for the rest of the paper for comparison purposes with the proposed policies. Thus, the total cost of the current system is equal to $7,086 ($6,226 + $860).

Table 5. Inventory at end of period and corresponding holding cost

	Inventory at end of period (packs)					Holding cost ($)				
Week/Item	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
0	101	38	44	204	20	175	72	86	391	41
1	101	33	44	184	20	175	62	86	352	41
2	101	33	40	184	12	175	62	78	352	25
3	84	27	40	184	8	145	50	78	352	16
4	84	23	40	144	0	145	43	78	276	0
5	68	23	40	132	0	118	43	78	253	0
6	56	19	36	120	0	97	36	71	230	0
7	56	17	28	108	0	97	32	55	207	0
8	44	15	24	92	0	76	28	47	176	0
9	32	9	24	88	0	55	18	47	169	0
10	32	6	24	76	0	55	11	47	146	0
11	32	6	24	60	0	55	11	47	115	0
12	32	4	8	36	0	55	7	16	69	0
13	20	4	8	24	0	35	7	16	46	0
14	20	4	8	24	0	35	7	16	46	0
15	8	2	8	20	0	14	4	16	38	0
16	0	0	4	8	0	0	0	8	15	0
17	0	0	0	0	0	0	0	0	0	0

4. METHODOLOGY I: DETERMINISTIC MODEL

4.1 Literature Review

The first approach to determine a joint replenishment policy is based on the assumption of a time-varying deterministic demand. In other words, the demand for each item changes by period but the quantity demanded and period that the demand occurs is known exactly. This problem is commonly known in the literature as the Dynamic Joint Replenishment Problem (DJRP).

The objective of the DJRP is to determine the ordering quantities and frequency of each item, such that the total replenishment cost (major and minor ordering costs) and inventory holding costs are minimized over the whole planning horizon. Arkin (1989) showed that the DJRP is NP-hard.

For the DJRP, exact algorithms can be classified into four main categories: “(1) dynamic programming (Zangwill 1966, Veinott 1969, Kao 1979, Silver 1979); (2) branch-and-bound (Erenguc 1988, Federgruen and Tzur 1994, Kirca 1995,Robinson and Gao 1996); (3) branch-and-cut (Raghavan and Rao 1991, Raghavan1993); and (4) Dantzig–Wolfe decomposition (Raghavan and Rao 1992)” (Boctor 2004). Some well-known heuristics proposed in the literature for solving the DJRP problem include: Fogarty and Barringer (1987), Silver and Kelle (1988), Atkins and Iyogun (1988), Silver–Meal (1973), and Iyogun (1991). Boctor (2004) also developed improvement heuristics that involves making a small random change to a feasible solution, maintaining feasibility, and optimizing the perturbed solution. For a detail review of the literature in DJRP see Boctor (2004) and Goyal (2008).

4.2 Assumptions

The most important assumption is that the demand being time-varying and deterministic. This assumption is reasonable because it is known exactly when the demand is going to occur (i.e. university dining housing prepares the menu one semester in advance), and because the number of students served from a semester to another semester is almost constant (i.e. housing knows the number of students with meal plans). Furthermore, the demand pattern is predictable due to the seasonality and campus-wide events observed from past forecasts. The following assumptions were also made (Golany 1992 and Boctor 2004):

- replenishments are made at the beginning of each period
- items are consumed during period t generate no holding cost
- initial inventory is zero
- no quantity discounts from supplier are available
- no backlogging is allowed
- no limit is imposed on order sizes
- no limit is imposed on inventory levels
- no exogenous constraints (e.g. storage volume limitations, capital investment in inventory, etc)
- joint order cost of two or more items is smaller than the sum of their order costs when they are ordered separately
- assume deterministic lead time (2 weeks), so an order is made 2 weeks before desired replenishment period

4.3 Formulation

The following notation is defined for the integer linear program formulation (Boctor 2004):

N	maximum number of items
T	maximum number of periods
S	common ordering cost (units: \$/order)
s_i	individual ordering cost for item type i (units: \$/order)
M	sufficiently large number (max of total demand of an item)
c_i	unit cost for item type i (units: \$/item)
p	percentage of unit cost during period t for holding an item (units: %)
h_i	unit inventory holding cost for item type i during period t (units: \$/item)
D_{it}	demand for item type i for period t (units: item)
$I_{0,i}$	initial inventory level of item type i (units: item)
I_{it}	Inventory level of item type i at the end of period t (for t>0) (units: item)
x_{it}	Replenishment quantity of item type i at the beginning of period t (units: item)
y_{it}	Binary variable = 1 if and only if item type i is replenished at the beginning of period t, i.e. y_{it} = 1 if x_{it} >0
z_t	Binary variables taking the value 1 if an order is placed for period t

The formulation is then:

$$min \sum_{t=1}^{T} \left[Sz_t + \sum_{i=1}^{N} \{s_i y_{it} + h_i I_{it}\} \right] \tag{4.1}$$

subject to

$$I_{i,t-1} + x_{it} - I_{it} = d_{it} \qquad \forall\, i, t \tag{4.2}$$

$$x_{it} \leq M y_{it} \qquad \forall\, i, t \tag{4.3}$$

$$\textstyle\sum_{i=1}^{N} y_{it} \leq N z_t \qquad \forall\, t \tag{4.4}$$

$$I_{it} \geq 0 \qquad \forall\, i, t \tag{4.5}$$

$$x_{it} \geq 0 \qquad \forall\, i, t \tag{4.6}$$

$$y_{it} = 0 \ or\ 1 \qquad \forall\, i, t \tag{4.7}$$

$$z_t = 0 \ or\ 1 \qquad \forall\, t \tag{4.8}$$

The objective function minimizes the major ordering cost (S), minor ordering cost (s_i), and holding cost (h_i) across all periods (T) items (N). The first constraint (4.2) balances the inventory, demand and order quantity for each item and period. The constraint ensures that the demand at period t is satisfied by the inventory from the end of the previous period ($I_{i,t-1}$) carried to the current one, and the quantity order at the present period (x_{it}), with the leftover being held during the current period (I_{it}) to the next one. The second constraint (4.3) ensures that for every period, the replenishment quantity of an item type (x_{it}) can only be greater than zero if that item type is included in the order (y_{it}), or in other words, if the minor ordering cost for that item is incurred. The third constraint (4.4) states that individual item types (y_{it}) can be included in the joint replenishment order (z_t) only if an order is placed. The rest of the constraints enforce that the appropriate decisions variables should be positive (4.5 and 4.6) and binary (4.7 and 4.8).

Note that the formulation could be simplified and computation time improved if optimal properties from Wagner and Within (1958) and Silver (1979) are considered. For instance, the optimal solution should satisfy ${x_{it}}^* \cdot {I_{i,t-1}}^* = 0$ for all periods and items, since it will not be optimal to both replenish an item type i in period t and pay to hold this item i during period $t-1$. However, since only few items and periods are analyzed, the presented mathematical formulation was deemed suitable for the purpose of the project, and a commercial software package (e.g. CPLEX) was used to solve for the optimal solution. An analysis of different mathematical formulations and their performance in terms of computation times is conducted by Boctor (2004).

4.4 Solution

The optimal integer solution is shown in Table 6. The mathematical program was solved using CPLEX 11.2.0, resulting in an objective value of **$ 4,060.99**. Because few variables were considered, CPLEX was able to find the solution in less than 1 second. For details on the solution, see the AMPL output in the Appendix 10.1.

Table 6. Optimal Order Quantities

Week / x_{it}	1	2	3	4	5
1	**33**	**16**	**4**	**72**	**20**
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	**36**	**24**	**16**	**72**	**0**
7	0	0	0	0	0
8	0	0	0	0	0
9	0	0	0	0	0
10	0	0	0	0	0
11	0	0	0	0	0
12	**32**	**0**	**24**	**60**	**0**
13	0	0	0	0	0
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	0	0	0
17	0	0	0	0	0

As it can be seen, the optimal solution is to place 3 orders that replenish the quantities at week 1, week 6 and week 12. As expected, when a joint order is placed, all items are included in the joint replenishment if there is demand for that item, since the major ordering cost is incurred. Item 2 is the only exception, because for this item, it is less expensive to carry a small inventory to satisfy the small future demand than to include the item (and pay the minor ordering cost) in the joint order in period 12. Item 5 is only replenished with the first order because there is only demand for this item in the first 4 weeks.

A graphical illustration of the inventory levels of all items for the optimal solution is shown in Figure 11. The different curves correspond to the inventory on hand at the end of the period for the different items, and the circled periods correspond to the period when the item is replenished (arrival of order). As it can be seen from the plot, the optimal solution coordinates the replenishments to minimize the ordering cost and the inventory cost, with the item with the most demand (item 4) apparently dictating the timing of the joint replenishment order. This makes sense since this item will incur the most holding cost, so ordering by matching the ordering frequency of this item seems a reasonable approach. The exception of excluding item 2 in the last order (discussed previously), can be also corroborated from the plot. As it can be seen, the solution indicates that is optimal to carry the inventory for this item from period 6 (second order) all the way until all demand is consumed (period 16), instead of ordering less in period 6 and include this item in the order of period 12.

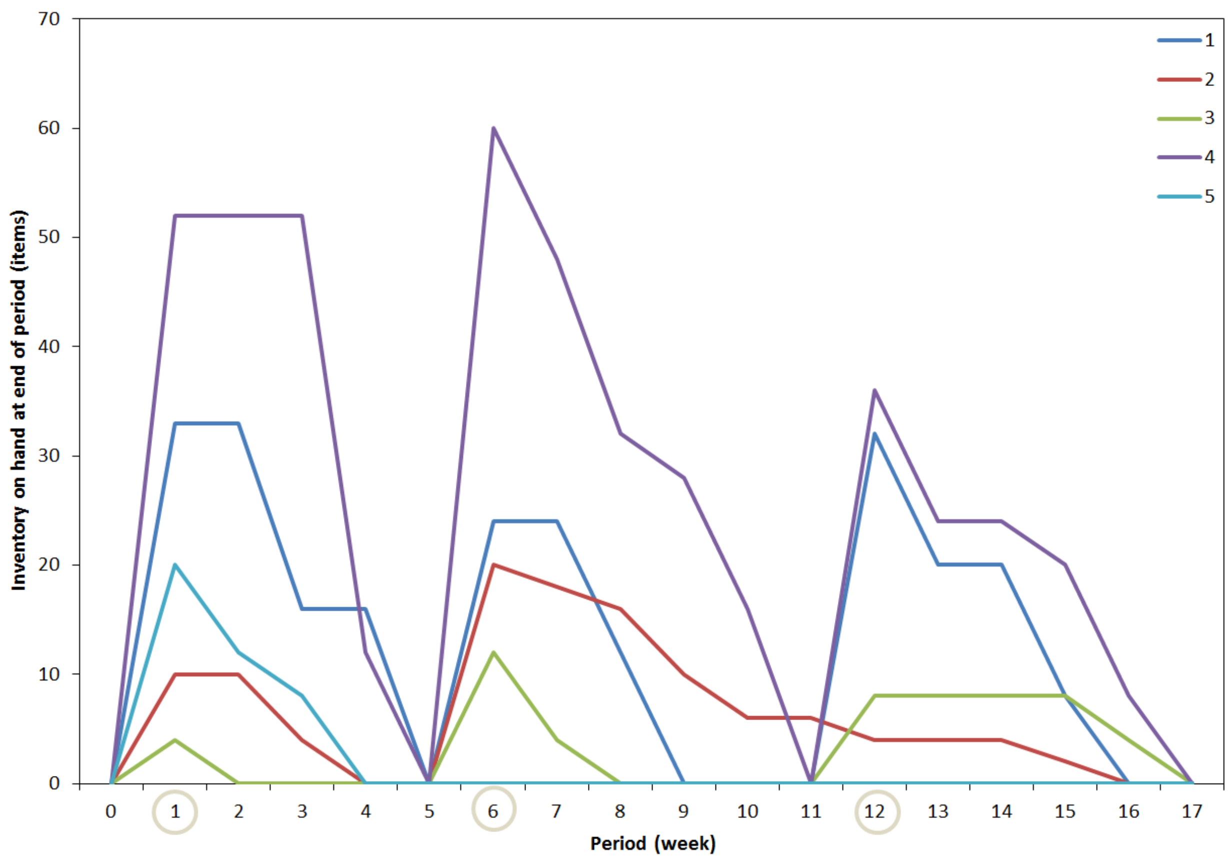


Figure 11. Inventory level with optimal order quantities

A sensitivity analysis for the holding cost (percentage of unit cost) was done to determine how the optimal solution is affected. As Table 7 suggests, increasing the holding cost increases the optimal number of orders because carrying inventory becomes more expensive than placing orders. When the holding cost is negligible, it is optimal to place only one order at the beginning of the period and satisfy all demand from the inventory. Therefore, an accurate estimate of the holding cost is very important to find the optimal solution.

Table 7. Sensitivity Analysis of holding cost

p	# orders	Objective
1%	1	$1,956
5%	3	$4,061
10%	4	$5,395
15%	5	$6,290
20%	6	$6,939
25%	7	$7,506

5. METHODOLOGY II: STOCHASIC MODEL

5.1 Literature Review

When dealing with stochastic demands, there are two approaches available: continuous and periodic reviews. The paper by Amit Eynan and Dean Kropp (2007) focuses on the period review since it gives advantages of coordinating replenishments of multi-items—providing a decision of when and how much to order for the whole system. The period review solution proposes an order of varying quantity at fixed intervals. The target quantity takes an account of the quantity needed for a stochastic demand until the next order arrives, and the quantity varies in order to accommodate the uncertainty nature of the demand rate. When the supply cannot meet the demand, backlogging cost occurs.

When a service level and variable stockout costs are given, the cycle time that minimizes the total cost—the holding cost, the major and minor costs and the backlogging cost—can be determined. Eynan and Kropp(2007) states that this solution to the stochastic joint replenishment problem resembles the deterministic Economic Order Quantity solution, yet it provides a near optimal value.

5.2 Assumptions

The most important assumption is that the demand is stationary and normally distributed. Based on the probability distribution analysis, there was only strong evidence that item 1 followed Normal distribution and item 4 Poisson distribution. Thus, for the stochastic model presented in this section, a normal distribution was considered. Eynan (1997) states that empirically, in many cases the normal distribution provides a better fit to data than most other distributions, and such distribution leads to analytically tractable results. Another important assumption is that the backlogging is allowed. Even though in the food industry backlogging should never occur, but this is a different case in the university dining hall service. The university dining halls may use certain products to substitute out-stocked products. The backlogging penalty cost is assumed to be equal to the unit price of a particular product. The additional assumptions were also made below:

- replenishments are made at the beginning of each period
- items are consumed during period t generate no holding cost
- initial inventory is zero
- no quantity discounts from supplier are available
- no limit is imposed on order sizes
- no limit is imposed on inventory levels
- no exogenous constraints (e.g. storage volume limitations, capital investment in inventory, etc.)
- joint order cost of two or more items is smaller than the sum of their order costs when they are ordered separately
- lead time is assumed to be two weeks

5.3 Formulation

The following notations are defined:

A	major setup cost (units: \$/order)
a	minor setup cost (units: \$/order)
h	holding cost per unit per unit of time (units: \$/pack-week)
D	average demand during one unit of time (units: pack/week)
σ	standard deviation of demand during one unit of time (units: pack/week)
z	multiplier of σ, safety stock factor (units: (-/-))
T	cycle time or periodic review time (units: week)
L	lead time (units: week)
B	penalty cost per unit short (units: \$/pack)
T_{det}	Cycle time based on the deterministic EOQ model (units: week)

The cost function for an *n* product family is:

$$\widehat{TC} = \frac{A}{T} + \frac{\sum_{i=1}^{n}\frac{a_i}{k_i}}{T} + \sum_{n=1}^{n}\left[\frac{D_i k_i T h_i}{2} + \frac{B_i}{k_i T}\sigma_i\sqrt{k_i T + L} f[z_i(k_i T)]\right] \tag{5.1}$$

where

$$T = \sqrt{\frac{2\left(A + \sum_{i=1}^{n}\frac{u_i}{k_i}\right)}{\sum_{i=1}^{n} k_i v_i}} \tag{5.2}$$

$$F[z_i(k_i T)] = 1 - \frac{h_i}{B_i} k_i T \tag{5.3}$$

The following heuristic procedure was taken to get the value of T.

Step (i) Calculate T_i for $\forall i$

$$T_i = \sqrt{\frac{2u_i}{v_i}} \tag{5.4}$$

where:

$$u_i = a_i + B_i\sigma_i\left(b_i - b'_i T_{det,i} + \frac{b_i'' T_{det,i}^2}{2}\right) \tag{5.5}$$

$$v_i = D_i h_i + B_i \sigma_i b_i'' \tag{5.6}$$

$$T_{det,i} = \sqrt{\frac{2a_i}{D_i h_i}} \tag{5.7}$$

$$b_i = \sqrt{T_{det,i} + L_i} f[z(T_{det,i})] \quad (5.8)$$

$$b_i' = \frac{f[z(T_{det,i})]}{2\sqrt{T_{det,i} + L_i}} + \sqrt{T_{det,i} + L_i} z(T_{det,i}) \frac{h_i}{B_i} \quad (5.9)$$

$$b_i'' = \frac{z(T_{det,i})}{\sqrt{T_{det,i} + L_i}} \frac{h_i}{B_i} - \frac{f[z(T_{det,i})]}{4(T_{det,i} + L_i)^{3/2}} - \frac{\sqrt{T_{det,i} + L_i}}{f[z(T_{det,i})]} \frac{h_i^2}{B_i^{\,2}} \quad (5.10)$$

Note: $f(z)$ and $F(z)$ are defined as the corresponding standard normal p.d.f. and c.d.f., respectively.

Step (ii) Sort items in non-decreasing order of the value T_i that was calculated previously.

Step (iii) Let *s* denote the smallest *j*

Subject to

$$T_{j,i}' \equiv \sqrt{\frac{2(A + \sum_{i=1}^{j} u_i)}{\sum_{i=1}^{j} v_i}} < T_{j+1,i} \quad (5.11)$$

Step (iv) Calculate

$$\frac{T_i}{T_j'} = \frac{T_i^*}{\min[T_i^*, T_2^*, \dots T_n^*]} \quad (5.12)$$

Step (v) find the value for *q* where s demote the smallest *j*

$$k_i = 1 \quad i = 1, \dots, s$$

$$k_i = q \quad (positive\ integer)$$

Subject to

$$\sqrt{(q-1)q} \le \frac{T_i}{T_s'} \le \sqrt{q(q+1)} \qquad i = s+1 \quad (5.13)$$

As with Silver (1976), Kaspi and Rosenblatt (1983), and Eynan and Kropp (2007), there is an order for at least one product in every *T* periods. Further, k_i is the multiplier of T for product *i*; thus product *i* is reviewed and ordered every k_iT periods. From the heuristic procedure, Step (i) to (v), all necessary variables (u_i, v_i, k_i, etc) for the equation (5.2) are obtained. Using the equation (5.2) *T* period is calculated, and the total cost per unit time can be calculated through equation (5.1).

5.4 Solution

The inputs and results for the stochastic model are shown in Table 8. The safety stocks for item 1 to item 5 are 19, 6, 12, 28, and 7 respectively. The optimal review period or cycle time is 6.50 weeks (45.5 days). The average total cost per week is $290.10, and the total cost in a semester (17 weeks) is $4945.32.

Table 8. Inputs and Results for the Stochastic Model

item	Unit Price ($/pack)	A ($/order)	a ($/order)	h ($/pack-week)	D (pack/week)	σ (pack/week)	L (week)	B ($/pack)	Safety Stock (pack)
1	34.59	500	65.03	1.7295	5.94	6.75	2	34.59	19
2	37.97	500	71.38	1.8985	2.35	2.26	2	37.97	6
3	39.24	500	73.77	1.962	2.59	4.23	2	39.24	12
4	38.3	500	72	1.915	12	10	2	38.3	28
5	41	500	77.21	2.05	1.18	2.74	2	41	7

Review Period or Cycle Time = 6.50 weeks
Average Cost Per Week = $290.10/week
Total Cost In a Semester = $4945.32

The safety stock values are directly proportional to the standard deviation of the demand. The higher standard deviation means that the item has a highly fluctuating demand rate. For example, item 4 has a standard deviation of 10 and safety stock of 28 items, while item 2 has a standard deviation of 2.26 and a safety stock value of only 6 items. The safety stock takes an account for the uncertainty in the stochastic demand; however, this does not guarantee that shortage never occurs. In the case of shortage, the university dining hall has to pay the backlogging penalty cost.

In order to optimize the total cost, the periodic review is determined. The periodic review or cycle time cannot be too long because the inventory cost will be too high, and it cannot be too short as well because the university dining hall has to pay the major setup cost and the minor setup cost per order transaction. There must be a balance between the two, which leads to the optimal cycle time of 6.50 weeks.

Table 9 shows a numerical value for the restocking quantity. The restocking quantity can be calculated by summing the demand quantity for each cycle time (including demand during the lead time period) and the safety stock quantity.

Table 9. Restocking Quantity Calculation

Item	Restocking Quantity (pack)	Demand Quantity for Each Cycle Time with Lead Time (pack)	Safety Stock Quantity (pack)
1	130	102	28
2	69	51	19
3	26	20	6
4	34	22	12
5	17	10	7

The inventory level for a semester is generated based on the collected demand data and the stochastic model solution shown in Table 10 and Figure 12. These table and figure present the inventory values for week 1 to week 17 in a semester.

Table 10. Inventory Level and Ordering Quantity Based on the Stochastic Model

	Inventory at the end of period (pack)					Ordering Quantity (pack)				
Week /Item	1	2	3	4	5	1	2	3	4	5
1	69	20	34	110	17	69	26	34	130	17
2	69	20	30	110	9	0	0	0	0	0
3	52	14	30	110	5	0	0	0	0	0
4	52	10	30	70	-3	0	0	0	0	0
5	36	10	30	58	0	0	0	0	0	0
6	24	6	26	46	0	0	0	0	0	0
7	24	4	18	34	0	0	0	0	0	0
8	57	22	22	102	0	45	20	8	84	0
9	45	16	22	98	0	0	0	0	0	0
10	45	12	22	86	0	0	0	0	0	0
11	45	12	22	70	0	0	0	0	0	0
12	45	10	6	46	0	0	0	0	0	0
13	33	10	6	34	0	0	0	0	0	0
14	33	10	6	34	0	0	0	0	0	0
15	21	8	6	30	0	0	0	0	0	0
16	13	6	2	18	0	0	0	0	0	0
17	13	6	-2	10	0	0	0	0	0	0

As Table 10 and Figure 12 demonstrate, the inventory level at week 1 represents the value of restocking quantity for each item. The university dining halls must make an order and obtain the items prior to the beginning of the semester. The next order point occurs at the review period, 6.5 weeks. A reorder for these items is made to achieve their corresponding restocking quantities, which is shown in the left section of Table 10. Ordered items are delayed by the lead time period, which is 2 weeks after the reorder point. The result illustrates that backlogging occurs two times: at week 4 for item 5 and at week 17 for item 3.

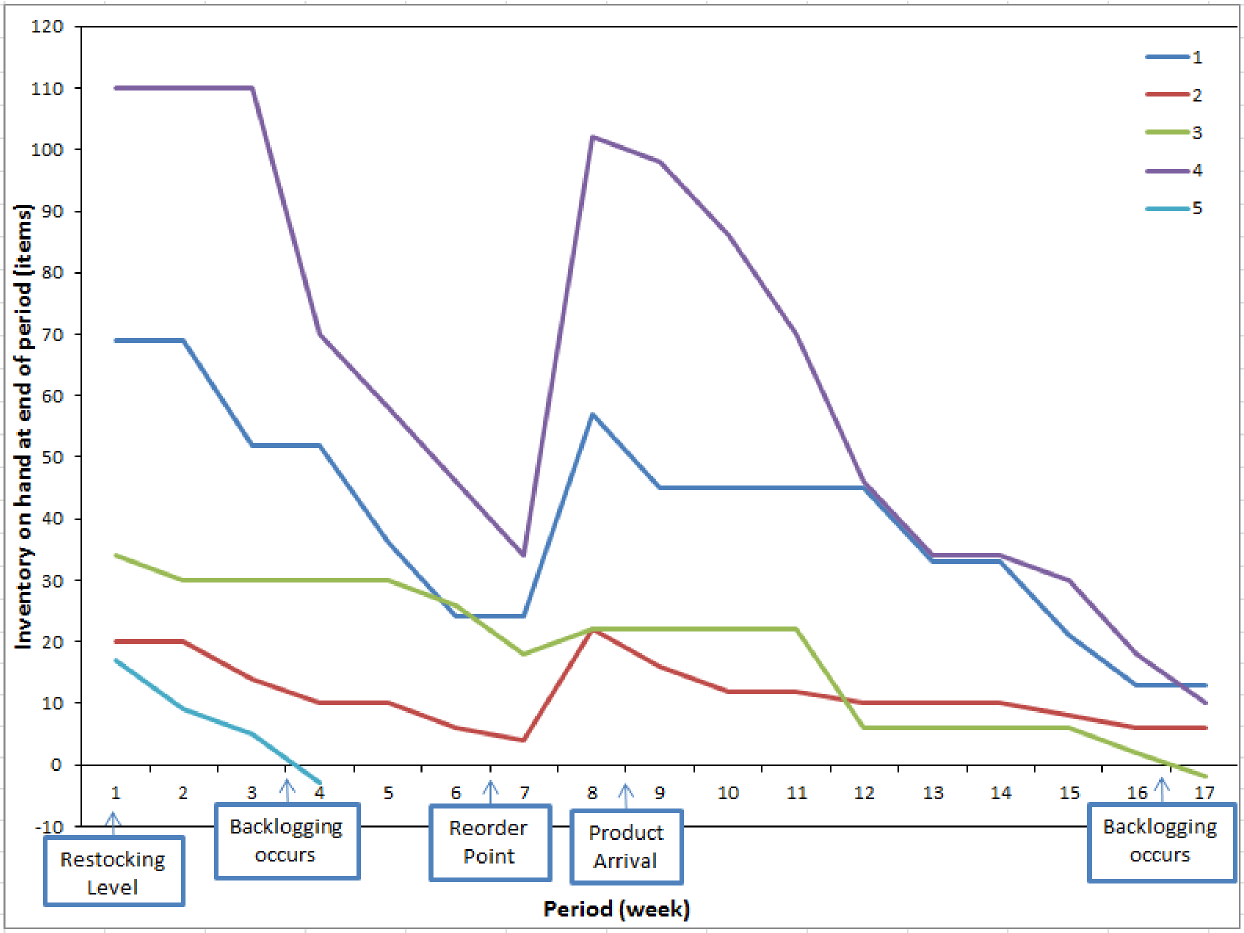


Figure 12. Inventory Level Based on the Stochastic Model

Sensitivity analyses of the holding cost and demand standard deviation for the stochastic model are presented in Table 11 and Table 12 respectively. The analyses are conducted to study the effect of increasing and decreasing the values of holding cost and the demand standard deviation to the stochastic model results, which include the safety stock, the average cost per week and the periodic review time values.

According to Table 11, the increase in the holding costs cause the safety stock and periodic review time values to decrease and the average cost per week to increase. The safety stock values are less sensitive to the holding costs compared to the average cost per week and the period review time, which change more dramatically. This is reasonable because when the holding cost is higher, the review period should be lower to minimize the total holding cost. And, when the periodic review is lowered, the safety stock should be decreased as well. The average cost per week is higher due to the higher holding cost.

According to Table 12, the increase in the demand standard deviation augments the values of the safety stock, the average cost per week and the periodic review time. The safety stock is more sensitive to the change in the demand standard deviation. The average cost per week and periodic review time remains about the "same." The demand standard deviation mostly affects the safety stock. As aforementioned, the safety stock value is directly proportional to the demand standard deviation. When the demand standard deviation changes but the average demand stays the same, the values of the average cost per week and the period review time are insensitive to these changes. Note that item 5 is not included in the sensitivity analysis due to the fact that the item has no demand after week 4.

Table 11. Sensitivity Analysis of the Holding Cost for the Stochastic Model

P	Safety Stock (Item)				Average Cost Per Week ($/Week)	Periodic Review Time (Week)
	1	2	3	4		
1%	22	7	14	33	116.63	12.44
4%	19	6	12	28	234.14	6.98
5%	19	6	12	28	262.71	6.49
6%	18	6	11	27	289.19	6.16
9%	17	6	10	26	379.38	5.52
10%	17	6	9	25	431	5

Table 12. Sensitivity Analysis of the Demand Standard Deviation for the Stochastic Model

σ	Safety Stock (Item)				Average Cost Per Week ($/Week)	Periodic Review Time (Week)
	1	2	3	4		
0.1	2	1	1	3	259.09	6.09
0.5	9	3	6	14	260.58	6.25
0.7	13	4	8	19	261.39	6.34
0.9	17	6	11	25	262.25	6.44
1.0	19	6	12	28	262.71	6.49
1.1	21	7	13	31	263.17	6.54

6. RESULTS AND DISUCSSION

6.1 Results

The current system was evaluated using the current joint replenishment policy and given data, then the deterministic and stochastic joint replenishment solutions were obtained. Table 13 below provides the summary results of the three different inventory systems.

Table 13. Summary Results of the Current system, the Deterministic Model and Stochastic Model

Criterion	Inventory Systems		
	Current System	Deterministic Model	Stochastic Model
Total Cost per semester	$7,086	$4,061	$4,945
Ordering Frequency	1	3	2
Ordering at Week	1	1, 6, 12	1,7

By evaluating the current system, the total cost of the current system was found to be $7,086 per semester, consisting of the holding cost, $6,226, and the ordering cost, $800. There is only one order at the beginning of the semester.

For the deterministic model, based on the time-varying deterministic demand assumption and using the integer program formulation, the expected total cost was found to be $4,061 per semester, and the desired replenishment points were found to be at week 1, 6, and 12 with 2 weeks of the lead time (i.e. order 2 weeks before desired replenishment)

From the stochastic model, the review period (cycle time), the average cost per week, and the total cost per semester were 6.5 weeks, $290.10 per week, and $4,945.32, respectively. The safety stocks in packs for each item were calculated to be 28, 19, 6, 12 and 7 packs for item 1 to item 5, respectively. The desired replenishment points were found to be at week 1 and 7 with 2 weeks of lead time for the items.

6.2 Discussion

The main focus of this project and the inventory problem is to minimize the total cost. The total cost is primarily contributed by the ordering cost and the holding cost and sometimes, the administrative cost and the backlogging cost can play crucial role in some situations. The results of the deterministic and stochastic models show a much lower cost than the current system. The current system, as expected, has an extremely high holding cost because of its ordering policy—only one single shipment at the beginning of a semester. Even though the administrative staff does not have to make an order frequently, the holding cost is impacted by that. One more disadvantage of the current system is that shortage is not considered in the system, and that issue cannot be ignored especially in the service industry. Based on the analysis, the holding cost for the current system, $6,226, actually is higher than the total cost of the deterministic ($4,061) and stochastic ($4,945) models. Table 14 on the next page presents the cost reduction percentage compared to the current system.

Table 14. Cost Reduction Percentage Compared to the Current System

Criterion	Inventory System	
	Deterministic Model	Stochastic Model
Percent Reduction	42.70%	30.20%

According to Table 14, savings of 43% and 30% can be achieved by the deterministic model and stochastic model respectively. This is the results of the improvement in balancing between the cost imposed by the ordering frequency and the inventory holding.

The deterministic solution provides the optimal outcome, but there are number of disadvantages associated with its methodology. The deterministic model use linear integer programming and the program computational time is directly affected by the number of items. The deterministic model is not designed to handle the situation when the demand becomes uncertain. This means that the model requires an extremely accurate forecasted demand in order to provide the true optimal solution. Otherwise, the result of this model is not guaranteed. The lead time is another limiting factor in this model as well. The food service provides must make an order in advance based on the forecast demand; this leads to a high chance of not meeting the actual demand. Consequently, the out stock problem can potentially become a serious issue. Backlogging cost is not taken into an account in this model as well, and it is something that cannot be ignored in the food service industry.

There are some advantages and disadvantages of implementing the stochastic joint replenishment policy over the deterministic model. One of the advantages is that the stochastic model takes into account the uncertainty in the demand by using safety stock values for each item as a buffer zone for fluctuations in the demand. In addition, unlike the deterministic model, the lead time and the demand during that period are taken into an account in the stochastic model. The stochastic model can be implemented easier as it uses close-form solution and the large number of item does not affect the computation time or increase the complexity of obtaining the solution. The stochastic model provides several advantages, yet it also contains some drawbacks. Because the stochastic model considers some uncertainty, the solution obtained is not optimal but only a near optimal solution. Another potential disadvantage is related to its methodology and assumption. The stochastic model uses the Normal Distribution for the model equation derivation and assumption. If the demand of for the items does not follow the Normal Distribution, the result of this solution can be misleading.

The deterministic and stochastic models are suitable for different situations and based on the nature of the joint inventory problems in the real world settings. The deterministic model is favored in the case that the demand is known or can be determine accurately. The result can reach the optimal total cost. On the other hand, the stochastic model can only provide the near optimal solution but can be very useful and potentially outperform the deterministic model in the case that the demand is not known exactly and/or has some uncertainty. Regardless, both of the proposed models provide the improvement in the total cost reduction compared to the current inventory system.

6.3 Limitations

There are many heuristic model for the stochastic demand have been reviewed, such as Goyal & Satir (1989), Viswanathan (1997),Wildeman (1997), and Eynan & Kropp (1998). One of the most appropriate models is the one introduced by Goyal & Satir, whose paper is based on the milestone contribution of Silver (1974). Although some of the equations could be easily solved via commercial softwares, the time constraint is a main problem due to the high complexity of the algorithm. Also, some complex models such as the "can-order" policy by Atkin (1988) have more parameters taken into the consideration, which is the void data in our case. To apply those models, more conditional assumptions and data estimations will be made, which will certainly drive the solution farther away from the actual case we have. Furthermore, Dance and Lee (2012) claimed that some heuristic models for the JRP are not practical for more than 4 or 5 items and will perform badly, since as some items grow more expensive than others, leading to cost rate grows arbitrarily larger. Therefore, some alternative models are selected, which are easier to solve and more suitable to our case, comparatively.

The stochastic model analyzed in the paper is based on Eynan & Kropp (2006), which is the simple EOQ-like solution procedure based on their work in 1998. They assumed the stochastic demand followed a normal distribution, which is moderately acceptable in our case. While more researchers prefer to make a Poisson distribution as the assumption for the stochastic demand, there is no clear evidence showing the distribution of the data conflicted with the normal distribution, and therefore, it is recommended to run this model and hold the solution as a comparable candidate to the deterministic results.

There is another heuristic method for the stochastic demand contributed by Nilsson (2007) with dramatic innovation, by reducing the quotient of the fixed ordering cost to the inventory holding cost less than 1.4 through the iterations to obtain the solution of the reorder policy (see Appendix 10.4). This model has a larger error to the classical JRP models, and the resulting optimal replenishment cycle may quit different from the classical one (Wee, 2010).

7. CONCLUSIONS

7.1 Summary

The current system review gave a result that ordering with large shipment size at the beginning of semester would lead high inventory cost. Due to logistics challenges of collecting and reviewing all different data sets within a limited time, the operation of the HFS was the focus of the project. The proposed goal of this research project was to evaluate the current inventory management system of Housing Food Store (HFS) for the special vegan items and make a recommendation of suggesting how they would be able to improve their current inventory management system.

The quantities consumed of vegan items, their costs, and the current ordering policies were determined by obtaining the special item transaction report and having three meetings with the administrator and staff at HFS warehouse. Demand analysis was conducted based on the demand and the relevant data for the fall semester in 2011, resulting in the determination of the demand per weeks and demand distribution by each item. The demand distribution results suggested that poison and normal distribution are reasonable for item type 4 and 1, respectively.

The total cost for the current system including ordering and holding cost for vegan items was evaluated by calculating ordering and holding costs with the assumptions of the minor ordering cost, constant deterioration rate, and perishability, resulting from review of applicable literature. The total cost, the optimal replenishment periods and order quantities for the DJRP were obtained by assuming time-varying and deterministic demand and applying the classical linear integer formulation. The total cost, the optimal replenishment periods and safety stock, as well as ordering quantity for the SJRP, were obtained by applying a periodic review policy and assuming distribution of vegan item types follow a normal distribution.

The current system, the deterministic model and stochastic models were compared. The results indicate that savings of 43% and 30% can be achieved by the deterministic model and stochastic model respectively. In addition, the advantages and disadvantages for the three systems were examined in detail. Finally, the limitations, future work and recommendations were discussed.

7.2 Future Work

As the number of items and periods considered grows, the linear programming formulation is not suitable because the problems grows exponentially in complexity and cannot be solved by solvers or exact algorithms (computational intractable/NP-hard). Federgruen and Tzur (1994) have solved instances with size n=30 and T=30 using exact algorithms. Boctor (2004) compares three different linear programming formulations and six heuristics in terms of computation time and percent of optimal solutions. For the classical DJRP formulation and 20x26 instance size, the computation time was 43.45 seconds. Therefore, heuristics (see section 4.1 for details) should be considered for problems with large size instances.

The Poisson process for slow movers is one of the important assumptions for the stochastic demand (Goyal and Satir, 1989), which can give the time independent parameters. However, another heuristic for the stochastic demand proposed by Atkin & Iyogun (1988) for the determination of the parameter of an (R,k,S) periodic review policy have better adaption for the real case application in general. Compare to the can-order (s,c,S) policy, the (R,k,S) policy has a better weighted leverage of the major ordering cost allocated to each product. Especially, when the ratio of the major ordering cost to the minor cost is large, it will outperform than the (s,c,S) policy, as it does better systematic optimization on the reallocation of each item based on their contribution to the major cost. Also, the heuristic (R,k,S) policy has no dependence on the Poisson processes and constant lead times, so it can perform much better when the time-depended demand varying fast. On average, the (R,k,S) policy is 12% and the can-order policy 28% above the lower bound. Therefore, a comprehensive study on the (R,k,S) policy and the practical application might a potential topic following this project.

8. RECOMMENDATIONS

The significance of this project is to demonstrate some alternative methodologies to evaluate the joint replenishment and inventory cost system of the campus food warehouse. The models considered are based on the different assumptions of the nature of the demand data, leading to the different computational results and ordering policies for the warehouse. The exact ordering policy for the current system can be found in section 6.1 of the results section. Additionally, some criteria of model preference are recommended for the future convenience.

The deterministic model is the first preference to optimize and evaluate the total cost of the inventory system if the warehouse can provide the completed transaction data, and if the demand is almost exactly the same as the previous time period in a single semester, or the potential possible changes in demand could be forecasted accurately. Otherwise, a shortage will be occurred leading to the extra penalty charge to the whole inventory system. However, the campus dining hall system can do the substitution for the food shortage to dramatically reduce the shortage penalty by increasing the flexibility of the menu of daily meals. Also, this model will work specifically well if the number of the type of the item is comparatively not too large (less than 30) because this can affect the time computation of the linear program. As one of the limitation of the deterministic model, the complexity of the calculation will be increased tremendously as the type of the product sharing the joint replenishment increases.

The stochastic model demonstrated in the paper is also an alternative approach. Unlike the deterministic model that can derive the optimal value, the stochastic heuristic can only provide a better solution close the optimum. However, it appears to be a more suitable method to be implemented when the demand for the certain time period cannot be forecasted precisely, and it is varying slowly. This heuristic method will perform remarkably if the data is tested to be reasonable to follow a normal distribution. It will also be a better choice than the deterministic model when the data set is very large for the solver, since the number of the product type has little impact to the complexity of the model; therefore, the computational time will almost remain the same.

However, in the case cannot meet either condition, which means the type of the products is comparatively too large to solve through the deterministic model, and the demand varying fast, some other heuristic model for deterministic and stochastic demand should be considered, as discussed in the future work in section 7.2.

9. REFERENCES

Arkin, E., Joneja, D. and Roundy, R. "Computational Complexity of Uncapacitated Multi-echelon Production Planning Problems," *Operations Research Letters.* Vol 8 (2). Apr 1989. p61–68.

Atkins, D.R., Iyogun, O., 1988. "Periodic versus 'can-order' policies for coordinated multi-item inventory systems. " Management Science 34, 791–796.

Bernard J. LaLonde and Douglas M. Lambert, "A Methodology for Calculating the Cost of Holding Inventory: A food Industry Example. Journal of Food Distribution Research", 1976, vol. 7, issue 3.

Black, A. "E-Logistics in Cold Chain Management," *European Pharmaceutical Contractor (EPC)*. Vol Summer, Jun 2003. P102-106.

Boctor, F. F., Laportes, G., and Renaud, J. "Models and Algorithms for the Dynamic-Demand Joint Replenishment Problem," *International Journal of Production Research*. Vol. 42 (13). p2667-2678.

Dance, C. R., Zoeter, O. R., and Lee, H. Lee. "Simple Policies for Joint Replenishment Can Perform Badly."*Proceedings of the International MultiConference of Engineers and Computer Scientists 2012.* Vol. II, IMECS 2012, March 14-16, 2012, Hong Kong.

Duft, K. D. "Inventory Management and Control," Agribusiness Performance, Agribusiness Extension Newsletters. School of Economic Sciences, Washington State University. Web. 27 Apr 2012. <http://www.agribusiness-mgmt.wsu.edu/ExtensionNewsletters/performance/IventoryMgmtControl.pdf>

Engineering Computing Center (ECC). "Inventory Management Metrics Table." University of Huston ECC. Web. 27 Apr 2012. <http://www2.egr.uh.edu/~araguram/INVENTORY+METRICS-sachin1.htm>

Eynan, A. and Kropp, D. H, "Effective and Simple EOQ-like Solutions for Stochastic Demand Periodic Review Systems", *European Journal of Operational Research*. Vol. 180 (3). Aug 2007. p1135-1143.

Eynan, A. and Kropp, D. H, "Periodic Review and Joint Replenishment in Stochastic Demand Environment", IIE Transaction (1998) 30, 1025-1033

Ferguson, M., Vaidy Jayaraman, and Gilvan C. Souza, "Note: An application of the EOQ model with nonlinear holding cost to inventory management of perishables". European Journal of Operational Research, 2007, vol. 180, issue 1.

Fitzsimmons, A and Fitzsimmons, M. J.," Service Management: Operations, Strategy, and Information Technology", 5th Ed., Irwin/McGraw-Hill, Homewood, IL, 2006.

Golanyi, B. and Lev-er, A. "Comparative Analysis of Multi-item Joint Replenishment Inventory Models," International Journal of Production Research. Vol. 30 (8). Aug 1992. p1791–1801.

Heizer, J. and Render, B. *Principles of Operations Management, 6e.* Upper Saddle River, New Jersey: Prentice Hall, 2005. Print.

Heizer, J. and Render, R. *Operations Management, 8e*. Upper Saddle River, New Jersey: Prentice Hall, 2006. Print.

Kaspi, M. Rosenblatt, M.J. "An Improvement of Silver's Algorithm for the Joint Replenishment Problem," *Institute of Industrial Engineers (IIE) Transactions*. Vol. 15 (3). Sep 1983. p264–267.

Khouja, M. and Goyal, S. "A Review of the Joint Replenishment Problem Literature: 1989-2005," *European Journal of Operational Research*. Vol. 186 (1), Apr 2008. p1-16.

Richardson, H. "Control Your Costs then Cut Them," *Transportation & Distribution*. Vol. 36 (12). Dec 1995. p94-95.

Silver, E. A. "A Simple Method of Determining Order Quantities in Joint Replenishments Under Deterministic Demand," *Management Science*. Vol. 22 (12). Aug 1976. p1351–1361.

Silver, E. A. "Coordinated Replenishments of Items Under Time Varying Demand: Dynamic Programming Formulation," *Naval Research Logistics Quarterly*. Vol. 26 (1). Mar 1979. p141–151.

Tanrikulu, M. M., Şen, A., and Alp, O. "A Joint Replenishment Policy with Individual Control and Constant Size Orders," *International Journal of Production Research*. Vol. 48 (14). Jul 2010. p4253-4271.

Viswanathan, S., 1997. "Periodic review (s,S) policies for joint replenishment inventory systems". Management Science 43, 1447–1454.

Wagner, H. M. and Whitin, T. M. "A Dynamic Version of the Economic Lot Size Model," *Management Science.* Vol. 50 (12). Dec 2004. p86–96.

Wilson, R. "The Great Freight Recession" *CSCMP 21st Annual State of Logistics Report®*. Council of Supply Chain Management Professionals (CSCMP) Balimore Roundtable. 12 Apr 2011. Balimore, MD. Web. 27 Apr 2012. *http://baltimorecscmp.org/wp-content/uploads/2011/04/21st-Annual-State-of-Logistics-Report-Presentation-CSCMP-Baltimore.pdf*

Nilsson, A., Segerstedt, A. and Sluis, E. v. d., 2007, "*A New Iterative Heuristic to Solve the Joint Replenishment Problem using a Spreadsheet Technique*," International Journal of Production Economics, Vol. 108, pp. 399-405.

Wee, H., Praharsi, Y., and Purnomo, H., 2010. "*An Innovative Heuristic for Joint Replenishment Problem with Deterministic and Stochastic Demand*", International Journal of Electronic Business Management, Vol. 8, No. 3, pp. 223-230

10. APPENDICES

10.1 Sample Inventory Transaction Report

Housing Food Stores- Warehouse ***Inventory Transactions Report***

Report Period: 3/15/2010 - 3/22/2012

Item Name: **Vegetarian Fish Filet** Stock Unit: 12.42#/cs

Primary Storage Area: Meats and Cheeses Bin:

Date: 10/27/2011 **Last Physical Count On:** 9/28/2011 **Starting On Hand:** 37

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	1403	Production Request	Lincoln-allen	36

Date: 10/28/2011 **Last Physical Count On:** 9/28/2011 **Starting On Hand:** 36

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	1418	Production Request	Ikenberry	35

Date: 11/11/2011 **Last Physical Count On:** 10/28/2011 **Starting On Hand:** 35

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	1632	Production Request	Ikenberry	34

Date: 11/29/2011 **Last Physical Count On:** 10/28/2011 **Starting On Hand:** 34

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	1806	Production Request	Ikenberry	33

Date: 12/6/2011 **Last Physical Count On:** 11/30/2011 **Starting On Hand:** 33

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	1902	Production Request	Ikenberry	32

Date: 1/23/2012 **Last Physical Count On:** 12/19/2011 **Starting On Hand:** 32

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	2163	Production Request	Ikenberry	31

Date: 2/2/2012 **Last Physical Count On:** 1/31/2012 **Starting On Hand:** 31

	Qty	Cost	Unit Of Measure	PO #/Requisition #	Invoice #/Reason	Vendor/Unit	Qty On Hand
Transfer Out							
	1	154.45	12.42#/cs	2305	Production Request	Lincoln-allen	30

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10.2 Demand Data (Fall 2011 and Spring 2012)

Week	Date	1	2	3	4	5
1	08/27/11		6		20	
2	09/03/11			4		8
3	09/10/11	17	6			4
4	09/17/11		4		40	8
5	09/24/11	16			12	
6	10/01/11	12	4	4	12	
7	10/08/11		2	8	12	
8	10/15/11	12	2	4	16	
9	10/22/11	12	6		4	
10	10/29/11		4		12	
11	11/05/11				16	
12	11/12/11		2	16	24	
13	11/19/11	12			12	
14	11/26/11					
15	12/03/11	12	2		4	
16	12/10/11	8	2	4	12	
17	12/17/11			4	8	
18	12/24/11					
19	12/31/11					
20	01/07/12					
21	01/14/12					
22	01/21/12				16	
23	01/28/12	4	2	4	16	
24	02/04/12		4	4	4	4
25	02/11/12		2		8	
26	02/18/12	8			16	
27	02/25/12	16	2	4	8	
28	03/03/12	8		8		
29	03/10/12	8	2	12		
30	03/17/12	20		20		
31	03/24/12					
32	03/31/12	8	2	8		
33	04/07/12			12		
34	04/14/12		4			
35	04/21/12					
36	04/28/12					

10.3 AMPL code and output

```
# CEE 512 - Logistics Systems Analysis
# ---------------------------------------------------------------------------------
# Project: Joint Replenishment Policy for Special Vegan Items
# Client: University of Illinios at Urbana-Champaign Housing Dining Services
# Authors: Luis Steven Lin, Akkaravuth Kopsombut, Maomao Yang, Namdong Yoon
# Date: 4/9/12
# Description: Classical Dynamic Joint Replenishment Problem (DJRP) formulation
# In multi-item inventory replenishment contexts, cost savings can be achieved
# by coordinating the replenishment of some item. The DJRP consists of determining
# the order quantities xit for each item type i and for each period t to minimize
# the sum of replenishment and inventory holding costs over the whole planning
# horizon.(Boctor 2004)
#
# Software:
# Computer Platform:
# ---------------------------------------------------------------------------------

#### Assumptions

# Time-varying deterministic demand
# Replenishments are made at the beginning of each period
# Items consumed during period t generate no holding cost during this period
# No quantity discounts,
# No backlogging is allowed
# No limit is imposed on order sizes and on inventory levels

# ---------------------------------------------------------------------------------

#### Inputs
   param numItems;                    #maximum number of items
   param numPeriods;                  #maximum number of periods
   param M;                           #sufficiently large number
                                      #(max of total demand of an item)

   set ITEM   := 1..numItems;         # types of item i
   set PERIOD := 1..numPeriods;       # demand periods t

# Costs
   param S;          #common ordering cost (units: $/order)
   param s {ITEM};   #individual ordering cost for item type i (units: $/order)
   param c {ITEM};   #unit cost for item type i (units: $/item)
   param p;          #percentage of unit cost for item i for holding cost (units: %)
   param h {i in ITEM} := p*c[i];   #unit inventory holding cost
                                    #for item type i (units: $/item)

# Demand
   param d {PERIOD,ITEM};   # demand for item type i for period t (units: item)

# Initial Inventory
   param Io {ITEM};         # initial inventory level of item type i (units: item)

# ---------------------------------------------------------------------------------

#### Variables

# Inventory level of item type i at the end of period t (for t>0)
   var I {ITEM,PERIOD} >=0; # (units: item)

# Replenishment quantity of item type i at the beginning of period t
   var x {ITEM,PERIOD} >=0; # (units: item)

# Binary variables = 1 if and only if item type i is replenished at the beginning
# of period t, i.e. yit = 1 if xit>0
   var y {ITEM,PERIOD} binary; # (units: item)

# Binary variables taking the value 1 if an order is placed for period t,
   var z {PERIOD} binary; # (units: item)
# ---------------------------------------------------------------------------------
```

```
#### Objective:

# sum of orders and holding costs.
minimize Cost:
   sum{t in PERIOD} (S*z[t] + sum{i in ITEM} (s[i]*y[i,t] + h[i]*I[i,t]) );

# --------------------------------------------------------------------------------

#### Constraints

# Initial demand satisfaction
   subject to DemandInitial {i in ITEM}: x[i,1] + Io[i] - I[i,1] = d[1,i];

# Demand satisfaction
   subject to Demand{i in ITEM, t in PERIOD: t>1}:
                  x[i,t] + I[i,t-1] - I[i,t] = d[t,i];
# Replenishment quantity of an item type can be positive only if that item type is
# replenished
   subject to ReplenishQuantity {i in ITEM, t in PERIOD}: x[i,t] <= M*y[i,t];

# Individual item types can only be included in a joint replenishment if
# that replenishment is made
   subject to ReplenishItem{t in PERIOD}: sum{i in ITEM} y[i,t] <= numItems*z[t];
```

```
#### Inputs

param numItems := 5;
param numPeriods := 17 ;
param M          := 300 ;

#common ordering cost (units: $/order)

param S := 500;

#percentage of unit cost for item i for holding cost (units: %)

param p := 0.05;

#individual ordering cost for item type i (units: $/order)
#unit cost for item type i (units: $/item)

param:        c        s        :=
        1   34.59    65.03
        2   37.97    71.38
        3   39.24    73.77
        4   38.30    72.00
        5   41.07    77.21       ;

# demand for item type i for period t (units: item)

param d:
        1       2       3       4       5       :=
1       0       6       0       20      0
2       0       0       4       0       8
3       17      6       0       0       4
4       0       4       0       40      8
5       16      0       0       12      0
6       12      4       4       12      0
7       0       2       8       12      0
8       12      2       4       16      0
9       12      6       0       4       0
10      0       4       0       12      0
11      0       0       0       16      0
12      0       2       16      24      0
13      12      0       0       12      0
14      0       0       0       0       0
15      12      2       0       4       0
16      8       2       4       12      0
17      0       0       4       8       0 ;


# initial inventory level of item type i (units: item)

param Io := 1 0
            2 0
            3 0
            4 0
            5 0;
```

```
sw: ampl
ampl: model projectFinal.mod;
ampl: data projectFinal.dat;
ampl: options solver cplex;
ampl: solve;
CPLEX 11.2.0: optimal integer solution; objective 4060.992
1053 MIP simplex iterations
30 branch-and-bound nodes
30 cover cuts
28 flow-cover cuts
1 clique cut
1 Gomory cut
47 flow-path cuts
6 implied-bound cuts
1 zero-half cut
ampl: display z,y,x;
z [*] :=
 1  1
 2  0
 3  0
 4  0
 5  0
 6  1
 7  0
 8  0
 9  0
10  0
11  0
12  1
13  0
14  0
15  0
16  0
17  0
;

ampl: display y;
y [*,*] (tr)
:    1   2   3   4   5    :=
1    1   1   1   1   1
2    0   0   0   0   0
3    0   0   0   0   0
4    0   0   0   0   0
5    0   0   0   0   0
6    1   1   1   1   0
7    0   0   0   0   0
8    0   0   0   0   0
9    0   0   0   0   0
10   0   0   0   0   0
11   0   0   0   0   0
12   1   0   1   1   0
13   0   0   0   0   0
14   0   0   0   0   0
15   0   0   0   0   0
16   0   0   0   0   0
17   0   0   0   0   0
;
```

10.4 A New Heuristic by Nilsson

A new iterative heuristic for deterministic and stochastic demand is approached by Nilsson (2007) and Wee (2010), as an alternative method to calculate the optimal joint replenishment cycle. The optimal time interval deriving a minimum total cost is obtained when the quotients of fixed ordering cost to the inventory holding cost less than 1.4 for all products through the iterations. This methodology is much easier to achieve, but comparatively has greater error than most of the other popular models. The notations and equations will be presented below.

T	Time between successive replenishments (years)
S	Major ordering cost associated with each replenishment ($/order)
TC	Total annual holding and ordering costs for all the products ($/year)
i	1, 2, …, *n*, a product index
n	Number of products
D_i	Annual demand for product i (units/year)
h_i	Annual holding cost of product i ($/unit/year)
s_i	The minor ordering cost incurred if product i is ordered in a replenishment ($/order)
Q_i	the ratio between the two costs for item- i (i.e., replenishment cost/ holding cost)
T_i	Time interval between successive replenishment of product *i* (years)
k_i	The integer number of T intervals that the replenishment quantity of item i will last

The joint replenishment decision for each item is based on the EOQ. The total cost consists of the major cost of the fixed transportation cost and the minor setup costs for specific items.

$$TC = \sum_{i=1}^{n} \frac{S + s_i}{k_i T} + \frac{1}{2} \sum_{i=1}^{n} k_i T (h_i D_i)$$

This equation can reach the minimum value iff the time interval:

$$T^*(k_i) = \sqrt{\frac{2 \sum_{i=1}^{n} (\frac{S+s_i}{k_i})}{\sum_{i=1}^{n} (k_i h_i D_i)}}$$

Substituting T^*, the optimal total cost depends only on the set of k_i values:

$$TC^*(k_i) = \sqrt{2(\sum_{i=1}^{n} \frac{S+s_i}{k_i})(\sum_{i=1}^{n} k_i h_i D_i)}$$

The replenishment ordering cost is:

$$C_i^s = \frac{S + s_i}{k_i T}$$

And the inventory holding cost is:

$$C_i^s = \frac{T}{2} k_i h_i D_i$$

Dividing the two equations above to get the ratio between the ordering cost and the inventory cost, and plugging in T^*:

$$Q_i = \frac{\frac{S+s_i}{k_i T}}{\frac{T k_i h_i D_i}{2}} = \frac{2(S+s_i)}{T^2 k_i^2 h_i D_i} = \frac{(S+s_i)(\sum_{i=1}^{n} k_i h_i D_i)}{(\sum_{i=1}^{n} \frac{S+s_i}{k_i})(k_i^2 h_i D_i)}$$

The heuristic procedure is given by

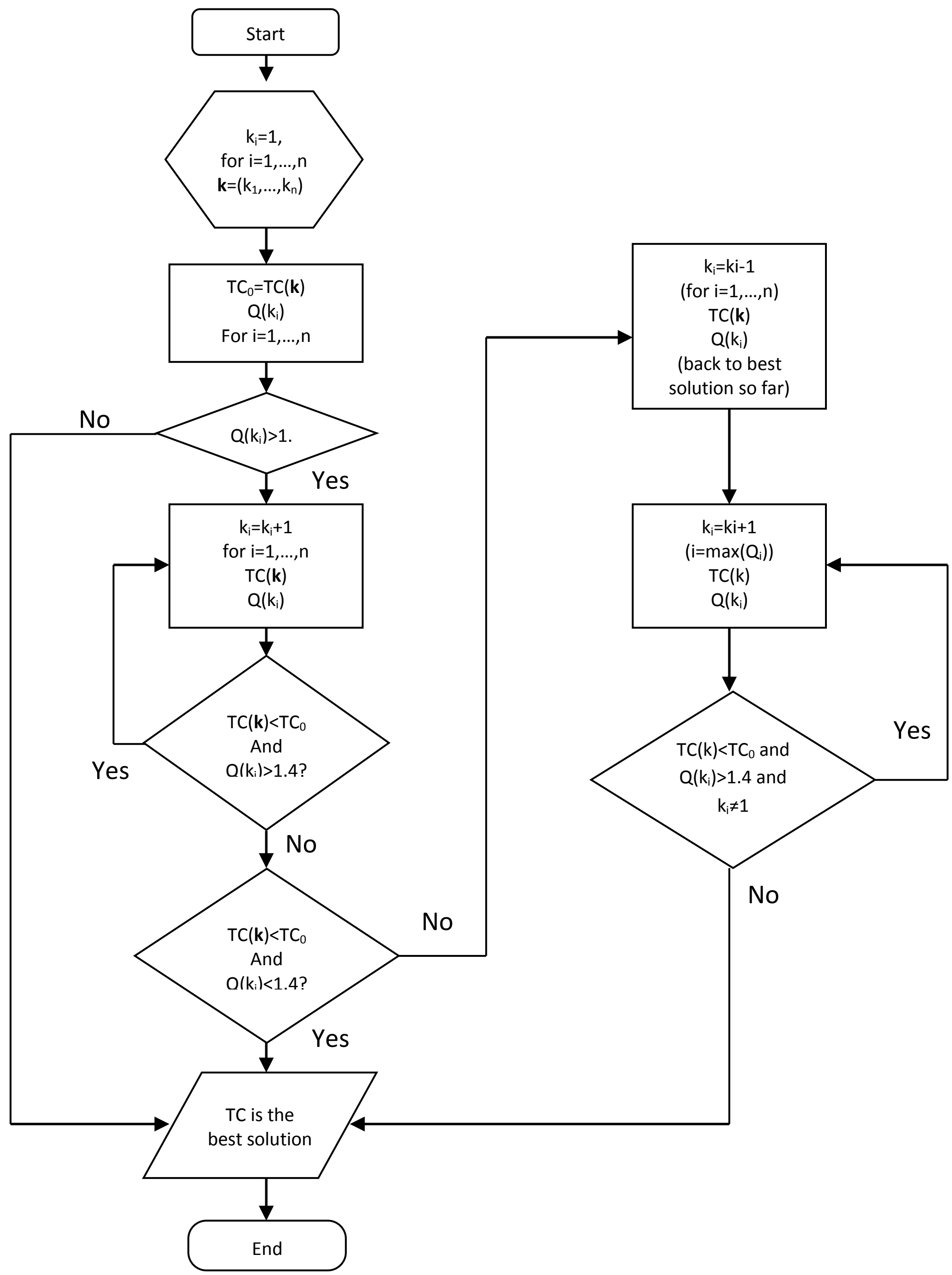

Figure A 1. Flow chart of heuristic procedure

The initial data and the iterative solutions are presented below:

Table A 1. Initial data

Item Name	VCC	VFF	VCT	VBT	VSBH
Item Number i	1	2	3	4	5
S (\$/order)	\$500				
s_i (\$/order)	\$65.03	\$71.38	\$73.77	\$72.00	\$77.21
D_i (pack/week)	5.94	2.35	2.59	12.00	1.18
h_i (\$/pack)	\$1.73	\$1.90	\$1.96	\$1.92	\$2.05

Table A 2 a. Start solution

Item i	1	2	3	4	5	TC/week
k	1	1	1	1	1	
cs	565.03	571.38	573.77	572	577.21	
ch	5.1381	2.2325	2.5382	11.52	1.2095	
K*h*D	10.2762	4.465	5.0764	23.04	2.419	
(S+s)/k	565.03	571.38	573.77	572	577.21	
Q	0.870642	2.026301	1.789709	0.39311	3.778317	\$508.85

Table A 2 b. Second iteration

Item i	1	2	3	4	5	TC/week
k	1	2	2	1	2	
cs	565.03	285.69	286.885	572	288.605	
ch	5.1381	4.465	5.0764	11.52	2.419	
K*h*D	10.2762	8.93	10.1528	23.04	4.838	
(S+s)/k	565.03	571.38	573.77	572	577.21	
Q	1.100632741	0.640393697	0.56562094	0.496954952	1.194102082	\$572.12

Table A 2 c. Back to the best solution so far

Item i	1	2	3	4	5	TC/week
k	1	1	1	1	1	
cs	565.03	571.38	573.77	572	577.21	
ch	5.1381	2.2325	2.5382	11.52	1.2095	
K*h*D	10.2762	4.465	5.0764	23.04	2.419	
(S+s)/k	565.03	571.38	573.77	572	577.21	
Q	0.870642	2.026301	1.789709	0.39311	3.778317	\$508.85

Table A 2 d. Third iteration

Item i	1	2	3	4	5	TC/week
k	1	1	1	1	2	
cs	565.03	571.38	573.77	572	288.605	
ch	5.1381	2.2325	2.5382	11.52	2.419	
K*h*D	10.2762	4.465	5.0764	23.04	4.838	
(S+s)/k	565.03	571.38	573.77	572	577.21	
Q	0.917157415	2.134560625	1.885328029	0.41411263	0.99504543	$522.26

Table A 2 e. Back to the best solution so far

Item i	1	2	3	4	5	TC/week
k	1	1	1	1	1	
cs	565.03	571.38	573.77	572	577.21	
ch	5.1381	2.2325	2.5382	11.52	1.2095	
K*h*D	10.2762	4.465	5.0764	23.04	2.419	
(S+s)/k	565.03	571.38	573.77	572	577.21	
Q	0.870642	2.026301	1.789709	0.39311	3.778317	$508.85

With this iterative heuristic, the results recommend that the warehouse should put the replenishment every week for all of the five products together. Thus, the total cost for an academic semester consisting of 17 weeks will be $508.85*17 = **$8650.45**, which is more than twice of the cost derived from the deterministic model.

10.5 HFS Information

Housing Food Store (HFS) belongs to Division of Housing Food Service under Division of Housing at the University of Illinois at Urbana Champaign. The current staff directory for Housing Division can be found from the current staff directory listing website:

<http://www.dmi.illinois.edu/ddd/getstaff.asp?deptcode=1270>

The relevant contacts are as follows:

Christopher R. Henning, Administrative IV, Food Service, (217) 244-1377, chenning@illinois.edu

Peter M. Testory, CHEF, Food Service, (217) 244-4776, ptestory@illinois.edu

Housing Food Stores, 1321 S. Oak Street, Champaign, IL

There are six classic dining halls at the University of Illinois at Urbana Champaign. Their locations are:

1. Busey-Evans : 1111 W. Nevada, Urbana
2. Florida Avenue Residence Halls (FAR) : 1011 W. College Court, Urbana
3. Ikenberry : 301 E. Gregory Drive, Champaign
4. Illinois Street (ISR) : 1010 W. Illinois Street, Urbana
5. Lincoln Avenue (LAR) & Allen : 1005 S. Lincoln, Urbana
6. Pennsylvania Avenue (PAR) : 906 W. College Ct., Urbana