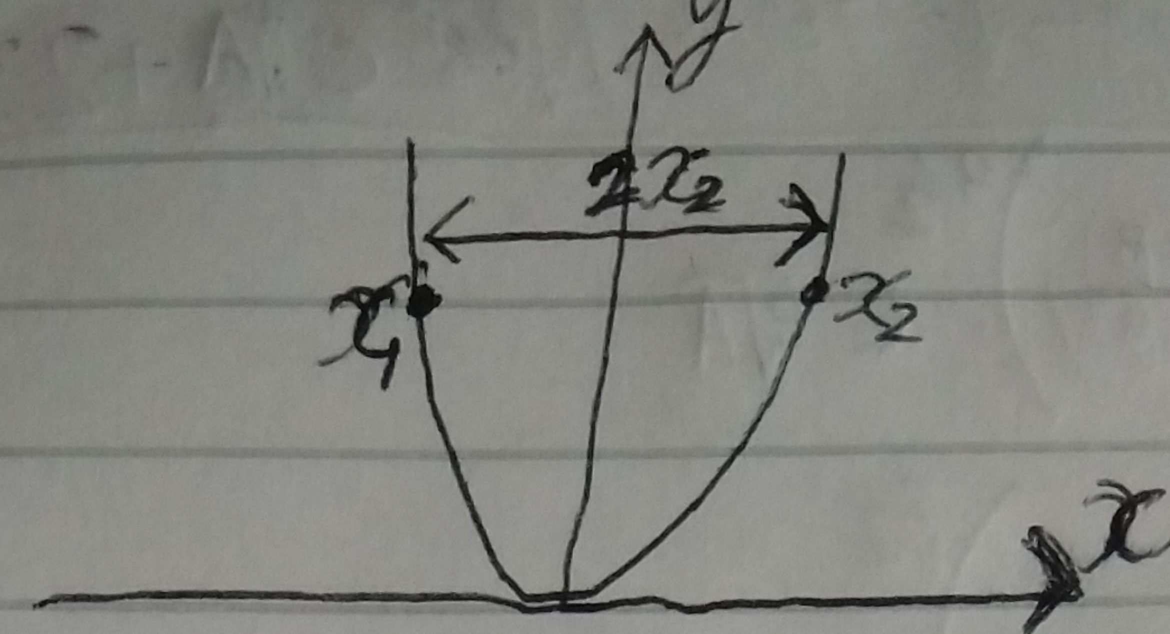
Parabolic

$y = ax^2 \quad a > 0$



$x_1 = -x_2$

$$A = 2\left[x_2 y(x_2) - \int_0^{x_2} ax^2\,dx\right]$$

$$= 2ax_2^3 - 2\left[\frac{a}{3}x^3\right]_0^{x_2}$$

$$= 2ax_2^3 - 2\left(\frac{a}{3}x_2^3\right)$$

$$= 2a\left(\frac{2}{3}x_2^3\right)$$

$$= \frac{4a}{3}x_2^3 \quad \Rightarrow x_2 = \sqrt[3]{\frac{3A}{4a}}$$

$$l = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx \qquad y' = 2ax$$

$$= \int_{x_1}^{x_2} \sqrt{1 + 4a^2x^2}\,dx$$

$$= \left[\frac{1}{4a}\ln\left(\left|\sqrt{1+4a^2x^2} + 2ax\right|\right) + \frac{x\sqrt{1+4a^2x^2}}{2}\right]_{-x_2}^{x_2}$$

$$= \frac{1}{4a}\ln\left(\left|\frac{\sqrt{1+4a^2x_2^2} + 2ax_2}{\sqrt{1+4a^2x_2^2} - 2ax_2}\right|\right) + \frac{(x_2+x_2)\sqrt{1+4a^2x_2^2}}{2}$$

$$= \frac{1}{4a}\ln\left(\left|\frac{\sqrt{1+4a^2x_2^2} + 2ax_2}{\sqrt{1+4a^2x_2^2} - 2ax_2}\right|\right) + x_2\sqrt{1+4a^2x_2^2}$$

$$4a^2x_2^2 = \left(\frac{a^2A}{1/6}\right)^{2/3}$$

$$2ax_2 = \sqrt[3]{6a^2A}$$

$$l(A) = \frac{1}{4a}\ln\left(\left|\frac{\sqrt{1+(6a^2A)^{2/3}} + \sqrt[3]{6a^2A}}{\sqrt{1+(6a^2A)^{2/3}} - \sqrt[3]{6a^2A}}\right|\right) + \sqrt[3]{\frac{3A}{4a}}\sqrt{1+(6a^2A)^{2/3}}$$

$$0 = \frac{\partial A}{\partial t} + \sqrt{\frac{g}{f}}\frac{\partial}{\partial s}\left(A^{3/2}\sqrt{\sin(\alpha)}\sqrt{\frac{1}{\frac{1}{4a}\ln\left(\left|\frac{\sqrt{1+(6a^2A)^{2/3}} + \sqrt[3]{6a^2A}}{\sqrt{1+(6a^2A)^{2/3}} - \sqrt[3]{6a^2A}}\right|\right) + \sqrt[3]{\frac{3A}{4a}}\sqrt{1+(6a^2A)^{2/3}}}}\right)$$

$$= \frac{\partial A}{\partial t} + \sqrt{\frac{g}{f}}\left(\frac{\sqrt{\sin(\alpha)}\left(28\sqrt[3]{6^7a^{11}A} + 1008a^3\sqrt{1+(6a^2A)^{2/3}}\right)A^2 + \left(\sqrt[3]{3^{11}4^4a^7}\ln\left(\left|\frac{\sqrt{1+(6a^2A)^{2/3}}+\sqrt[3]{6a^2A}}{\sqrt{1+(6a^2A)^{2/3}}-\sqrt[3]{6a^2A}}\right|\right) + a^{\frac{7}{3}}(4\cdot 6\cdot 6^{\frac{5}{3}}2^{\frac{5}{3}}3^{\frac{4}{3}})\right)}{\quad}\right)$$

I Will Simplify before Writing.

Continuing on overleaf

$$= \frac{\partial A}{\partial t} + \sqrt{\frac{g}{f}}\left(\frac{\sqrt{\sin(\alpha)}\left(\underbrace{28\sqrt[3]{6^7a^{11}A}}_{\gamma_1} + 1008a^3 c\right)A^2 + \left(\left(\underbrace{\sqrt[3]{3^{11}4^4a^7}}_{\gamma_2}\ln\left(\frac{c+\beta}{c-\beta}\right) + \underbrace{a^{7/3}(4\cdot 6\cdot 6^{5/3}\cdot 2^{5/3}3^{4/3})}_{\gamma_6}\right)A^{\frac{2}{3}} + \left(\underbrace{2\cdot 3^{8/3}\sqrt[3]{4\cdot 6a}}_{\gamma_3}\right.\right.}{\quad}\right.$$

B1 B2 B3 D3 B3 D4 B5 B6 D6 B5 B7

$$0=\frac{\partial A}{\partial t}+\sqrt{\frac{g}{f}}\frac{\partial}{\partial s}\left(A^{3/2}\sqrt{\sin(\alpha)}\sqrt{\frac{1}{\frac{1}{4a}\ln\left(\left|\frac{\sqrt{1+(6a^2A)^{2/3}}+\sqrt[3]{6a^2A}}{\sqrt{1+(6a^2A)^{2/3}}-\sqrt[3]{6a^2A}}\right|\right)+\sqrt[3]{\frac{3A}{4a}}\sqrt{1+(6a^2A)^{2/3}}}}\right)$$

$$=\frac{\partial A}{\partial t}+\sqrt{\frac{g}{f}\sin(\alpha)}\frac{\partial A}{\partial s}\frac{\left((\beta_1A^{2/3}+\beta_2A^{1/3}C+\beta_3)A+\beta_4CA^{2/3}\right)\ln\left(\left|\frac{C+B}{C-B}\right|\right)+(\beta_5A^{1/3}+1008a^3C)A^2+(\beta_6A^{2/3}+\beta_7C\sqrt[3]{A}+\beta_8)A^3}{\beta_9(B+C)^2CA^{1/6}\left(DC+\frac{1}{4a}\ln\left(\left|\frac{C+B}{C-B}\right|\right)\right)^{3/2}}$$

$$V(A)=\sqrt{\frac{g}{f}\sin(\alpha)}\;\frac{\left((\beta_1A^{2/3}+\beta_2A^{1/3}C+\beta_3)A+\beta_4CA^{2/3}\right)\ln\left(\left|\frac{C+B}{C-B}\right|\right)+(\beta_5A^{1/3}+1008a^3C)A^2+(\beta_6A^{2/3}+\beta_7C\sqrt[3]{A}+\beta_8)A}{\beta_9(B+C)^2CA^{1/6}\left(DC+\frac{1}{4a}\ln\left(\left|\frac{C+B}{C-B}\right|\right)\right)^{3/2}}$$

Where:

$B=\sqrt[3]{6a^2A}$

$C=\sqrt{1+(6a^2A)^{2/3}}$

$D=\sqrt[3]{\frac{3A}{4a}}$

$\beta_1=\sqrt[3]{3^{11}4^4a^7}$

$\beta_2=2\sqrt[3]{4\cdot3^86^2a^5}$

$\beta_3=2\sqrt[3]{3^84\cdot6}\,a$

$\beta_4=\sqrt[3]{3^84a}$

$\beta_5=28\sqrt[3]{6^7a^{11}}$

$\beta_6=a^{7/3}(46\cdot6^{5/3}-2\cdot3^{5/3}4^{4/3})$

$\beta_7=a^{5/3}(32\cdot6^{4/3}-3^{2/3}4^{4/3}6^{2/3})$

$\beta_8=a(96-2\cdot3^{2/3}4^{1/3}6^{1/3})=84a$

$\beta_9=2\cdot3^{5/3}4^{4/3}a^{4/3}$