DEPARTMENT OF PHYSICS AND ASTRONOMY

THE UNIVERSITY OF SOUTHAMPTON

Investigating Single and Multi-Lane Traffic Flow Simulations via the Nagel-Schreckenberg (NaSch) Cellular Automata Model

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Abstract

Vehicular traffic plays a major part in many daily lives. This project aims to investigate common everyday examples in traffic using simulations in Python, specifically using the Nagel-Schrekenberg (NaSch) Cellular Automata model. The experiment deals with the simulation of traffic flow on multi-lane roads and how various factors: velocity limit, density, road length, driver eccentricity, lane switching rules and obstacles play into traffic congestion.

The maximum velocities were tested from range of 2 to 12, slowing probability (the probablilty to slow down by velocity 1) was tested from 0.2 to 0.8 and they were carried out on 1, 2, 3 and 4 lane configurations. Fundemental relationship diagrams, time-space plots and average velocity plots were produced for each simulation.

It is shown that in a multi lane configuration, the degrees of freedom increase as the number of lanes increase. At higher values of velocity (>6), there are diminishing returns when it comes to flow rate and density. Conversely, the decrease in driver eccentricity can greatly increase the flow rate, average velocity and the density of the lane where the peak flow rate occurs.

Vehicles also take a longer time to reach "equlibrium" as the maximum velocity increases due to the larger range of initial velocities. It is also shown that from interference of an obstacle, vehicles reach peak average velocity at the same rate regardless of the slowing probability, but take a longer time to reach peak average velocity as the maximum velocity is increased.

1 Introduction

There are various ways to model the intricacies traffic flow. These models are split into macroscopic and microscopic. The former does not distinguishes individual vehicles, whereas the latter does. A few examples will be stated below.

The macroscopic model can be seen as treating traffic as a one-dimensional compressible fluid [5], where traffic at position x and time t can be represented by the *spatial vehicle density*, $\rho(x,t)$ and average velocity, $v(x,t)$ Traffic can be described by the continuity equation of fluids [3], called the Lighthill-Whitham assumption:

$$\frac{\delta\rho(x,t)}{\delta t} + \frac{\delta q(x,t)}{\delta x} = 0 \tag{1}$$

The traffic current, $q(x,t)$ is given by $q(x,t) = \rho(x,t)v(x,t)$.

A more realistic description of traffic can be obtained by using an analouge of the Navier-Stokes equation for fluids, where the time-dependence of $v(x,t)$ can be considered [8], instead of the Lighthill-Whitham model.

Earlier microscopic models include the *optimal velocity model* [7] in 1961, where a driver adjusts the vehicle velocity according to the headway, which is the distance between the current vehicle and the vehicle in front of it. The equation of motion is:

$$\frac{dx_j(t+\tau)}{dt} = V(\Delta x_j(t)) \tag{2}$$

The equation can be expanded (via Taylor expansion) into [2]:

$$\frac{d^2x_j(t)}{dt^2} = (1/\tau)(V(\Delta x_j(t)) - \frac{dx_j(t)}{dt}) \tag{3}$$

Where x_j is the position of the vehicle j at time t, and τ is the "delay time", the time that the vehicle needs to reach the optimal velocity, $V(\Delta x_j(t))$. $\Delta x_j(t)$ is the headway of the vehicle, given by $x_{j+1}(t) - x_j(t)$. The drawback of this model is that as the delay time, τ increases, collision occurs, as a longer delay means a shorter reaction time. The optimal velocity model was adjusted to account for this in the *intelligent driver model* (IDM) [1] which brings the relative velocity of the vehicles into account.

The *Cellular Automata* (CA) model will be used in traffic flow simulation in this case as it is simple and can be modified accordingly to fit the requirements of the simulation. In the CA model, the road is made up of cells. In a single lane simulation, the vehicles are set to move at each time step depending on the distance between itself and the vehicle in front of it. If a vehicle occupies the cell directly in front of it, it will stop, otherwise it will move with the same velocity or increase its velocity, depending on the headway. All vehicles are updated in parallel.

The main equation of motion in the simplest CA model, the CA 184 [10][11] is given as:

$$x_j(t+1) = x_j + min[1, x_{j+1}(t) - x_j(t) - 1] \tag{4}$$

$x_j(t+1)$ is the location of the j^{th} vehicle at time $t+1$, which is determined by the vehicle in front of it, the $j+1^{th}$ vehicle, and the previous position of the j^{th} vehicle (at time $t-1$.

The CA model was then improved with the Nagel-Schreckenberg (NaSch) model[6] in 1992. The dynamics are:

$$x_j(t+1) = x_j(t) + max[0, min[v_{max}, x_{j+1}(t) - x_{j-1}(t) - 1, x_j(t) - x_j(t-1) + 1] - \xi_j(t)] \tag{5}$$

The equation of motion picks the smallest of these values:

- $v_m ax$ is the maximum allowed velocity
- $x_{j+1}(t) - x_j(t) - 1$ is the headway
- $x_j(t) - x_j(t-1) + 1$ is the difference between the vehicle's previous position and the vehicle's current position

If the value picked is lesser than 0, then $x_j(t+1) = x_j(t)$, and the vehicle stays in place.

$\xi_j(t)$ is a Boolean random variable that is 1 with probability p, and 0 with probability $1-p$. $\xi_j(t)$ in the following simulations is a random slowing of the vehicle, which will be elaborated on in the Methodology section.

The CA model of the *slow-start rule* is used to account for the inertia of the vehicle [4].

Section 2 will go over the methodology in how the simulation was carried out, and the various conditions imposed on the model. It will be followed by a presentation of the results obtained from the simulation in Section 3 and in Section 4, a rigorous analysis of the results.

2 Theoretical Background

The CA model will be used for its simplicity and ease to program. It has been studied and used extensively. The equation that the simulation will be using is the Nagel-Schrekenberg (NaSch) model [6].

The main condition that describes movement, as is stated in (5) is:

$$x_j(t+1) = x_j(t) + max[0, min[v_{max}, x_{j+1}(t) - x_{j-1}(t) - 1, x_j(t) - x_j(t-1) + 1] - \xi_j(t)] \quad (6)$$

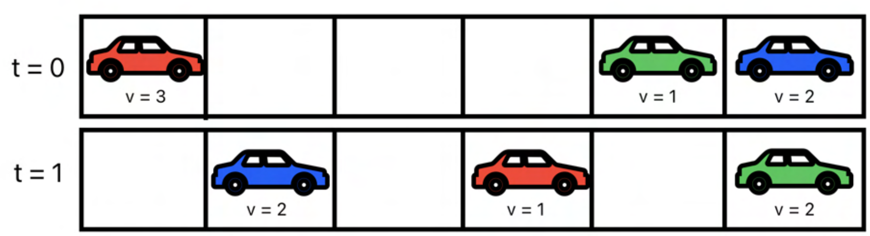


Figure 1: Graphical representation of how cars move on a single lane road according to the extended CA model.

This project will focus on expanding the CA model into a multiple lane simulation and altering various conditions to investigate the effects on traffic flow. There are a few important graphs to be familiar with when investigating traffic flow, namely the time-space and fundamental relationship graph. The time-space graph can be thought of as taking snapshots of the road at each time step and gives a visualisation of how vehicles move along the circular road. The fundamental density graph allows one to determine the range of densities for a road such that the flow rate (calculated by the number of vehicles passing through a point for the whole duration of the simulation) is the highest for the set conditions.

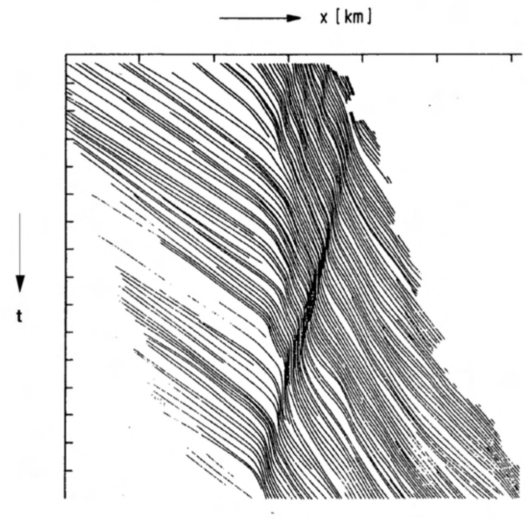


Figure 2: Time-Space plot for a road. The dark vertical line shows a backwards traffic jam. Taken from [9]

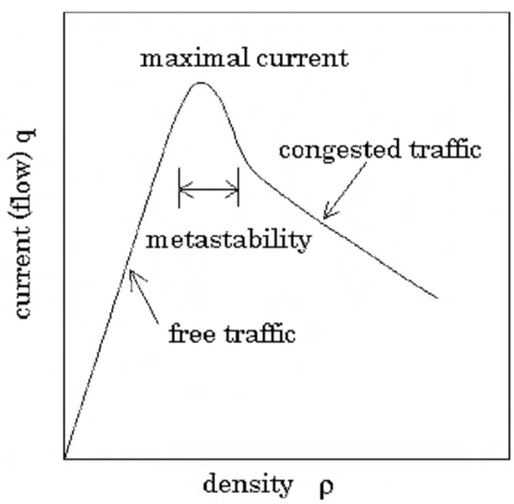


Figure 3: Fundamental relationship graph of a single lane road. Vehicles move freely at low densities, but are congested at higher densities. Taken from [5]

Another interesting metric to observe will be the average velocity at a certain point in the road, especially when obstacles are present. An average velocity against time graph can be plotted. This is because the time taken to recover to a stable average velocity can be investigated.

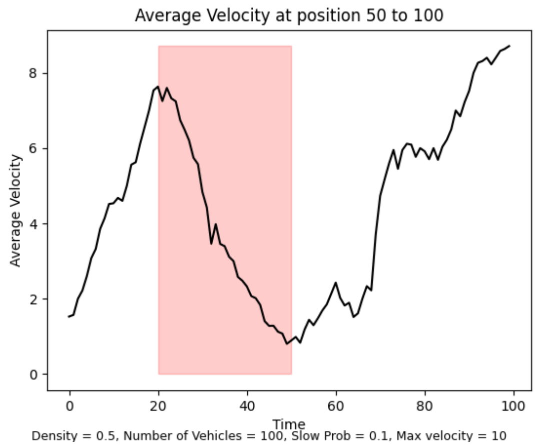


Figure 4: Average velocity plot of a single lane. The area highlighted in red is the time frame that obstacles are present.

3 Methodology

The simulation is done in Python 3.12.1 on Windows 11 with VSCode as the IDE. The code can be found over at the GitHub page.

In code, it is not practical to program a road such that 1 unit length of road in code is equal to 1 meter in practice, and have one cell take up a certain amount of unit length road. Although this would make data easier to interpret, this is taxing on the program. It is simpler to have one length of road be made of cells, and convert them into physical units. Since each car is moving a distance of a cell in each time step, it is convenient to say that the length of a cell is roughly equal to the length of a car. Of course, the length of the cars will affect the results significantly, as more or less cars can be fit into the same length of road, if cars are shorter or longer. The cell length is taken to be 4.5 meters, which is about the average length of a 4 seated sedan car. If each time step is taken to be around 15 seconds, a velocity of 1 will indicate a speed of 0.3 meters per second.

To convert "velocity" in code to a realistic value of velocity:

$$v_{real} = 0.3v_{code} \tag{7}$$

Where v_{code} is the velocity displayed in code (the number of cells moved per time step) and v_{real} is a more realistic number of velocity, measured in ms^{-1}.

The following simulations and results are all in cells for distance, and time steps for time. The velocity values are values in v_{code}

All vehicles are assigned random positions, velocities and lanes upon initialisation. The initial specified density is the density of each lane. The following sections introduce each aspect of the algorithm.

3.1 Lane Change Algorithm

If there are more than 1 lanes present in the road configuration, then vehicles will undergo lane switching. For a lane change to occur, the following conditions have to be met:

- Lane changing is probabilistic. Each car has a chance to check if it can switch lanes.
- Lane changing has to be advantageous to the driver - the lane change should allow itself to move through the road in a more efficient manner. This is implemented by checking the number of cars on the road. The driver would like to switch to a lane with lesser cars.
- The position that the car will switch to is not occupied by another car. A kindness condition is implemented to govern the minimum amount of free space that prompts a lane change.

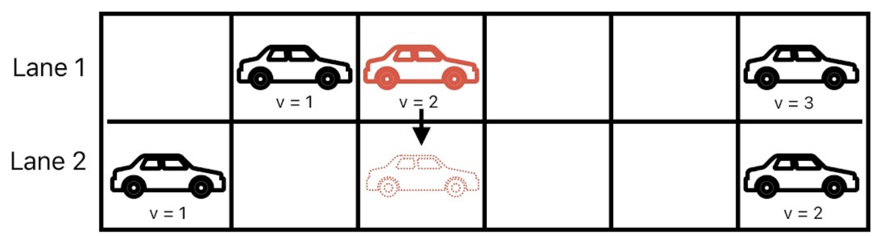


Figure 5: Graphical representation of a scenario where lane change can occur. The switching car is highlighted in red, and the new position is the red dotted outline of the car.

When switching lanes, there is a kindness condition which is not causing the vehicles behind it to slow down - the switching vehicle will consider the velocity of the car behind it and the car in front of it in the lane it will switch to. If the vehicle in the next lane has to slow down to accommodate for the car, it will not switch lanes.

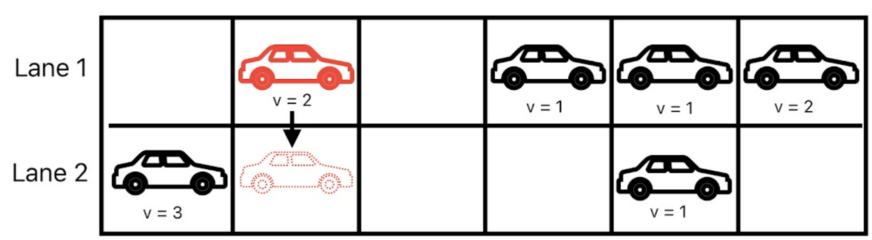


Figure 6: Graphical situation of an advantageous lane switch, but only for the driver doing the lane switch (car in red). The car in the next lane behind the dotted car has to slow down in the next time step to accommodate for the car coming into the lane.

The lane change algorithm comes in helpful when there are obstacles in a certain lane. After the vehicles switch lanes, they will begin to move, and then go through lane switching again, and so fourth. The following simulations follow the kindess lane switching model.

3.2 Moving in the lane

Vehicles will follow the standard moving equation in (5). Vehicles that are moving (non-zero velocity) will have a chance to be affected by a slowing of their velocity, when it is reduced by 1. The slowing down is indicative of driver concentration - a higher level of concentration will lead to a smaller slowing probability.

3.3 Obstacles

Obstacles are generated at certain positions for a certain length of time and are used to observe vehicle behaviour. Obstacles can be used to simulate islands, or actual obstacles - i.e. broken down car in the middle of the road, or a tree falling on the road. For simplicity, if the car's next position will cause a collision with the obstacle, it will decelerate to 0 velocity instantly in the next time step.

3.4 Areas of investigation

In the following section, the following scenarios are simulated.

- How increasing/decreasing vehicle maximum velocity affects the flow of traffic.
- What is the ideal per-lane density which gives maximum flow rate, with and without obstacle.
- How driver behaviour causes/mitigates traffic jams (the slowing probability which dictates randomness in driving), in headway distance and lane switching.
- How obstacles affect traffic flow - investigating the "backward wave" of vehicles (in terms of average velocities).
- How having multiple lanes affect the shape and peak of the fundamental relationship graph, and the physical meaning behind it.

In the 3 and 4 lane configuration, the difference between obstacle positions will be investigated - i.e. if there is a difference if the obstacle is in the first lane or the last lane.

4 Presentation of Results

There are 2 key graphs that are analysed throughout the simulations. The time-position graph and the fundamental relationship graph, as discussed earlier. Other graphs include average velocity plots, A standard simulation simulation consists of the following parameters:

- A road length of 1000
- A total simulation run-time of 1000 steps
- A per-lane density of 0.4
- A maximum vehicle velocity of 10
- A slow probability of 0.2
- If an obstacle is present, it will be present in lane 1, at position 200 with duration from 10 to 210, unless specified

In the following section, the result from altering variables (per-lane density, maximum velocity, slow probability) while keeping other variables constant will be investigated to find the most ideal conditions for a road with a specific number of lanes.

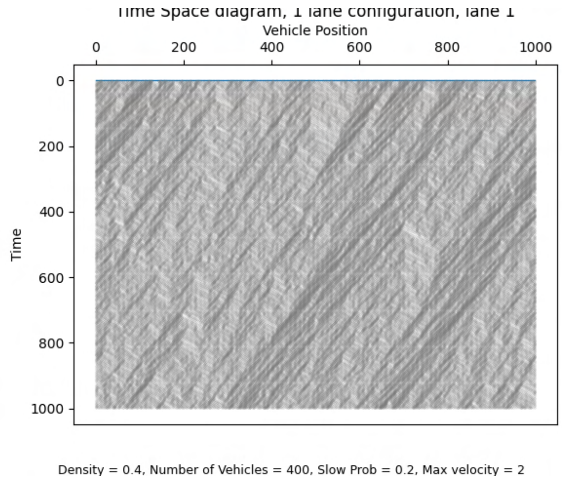
4.1 Single Lane Simulations

In the single lane simulation, results are very standard. Cars move with no obstructions, the factors that will affect the overall flow are:

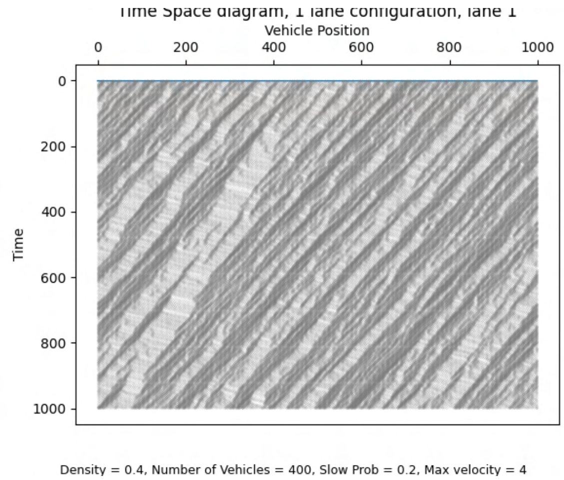
- The maximum velocity that the vehicle can travel at
- The slowing probability of the vehicle (how often the vehicle slows down, and by how much)
- The presence of obstacles
- Kindness of the drivers (headway distance)

4.1.1 Maximum Velocity

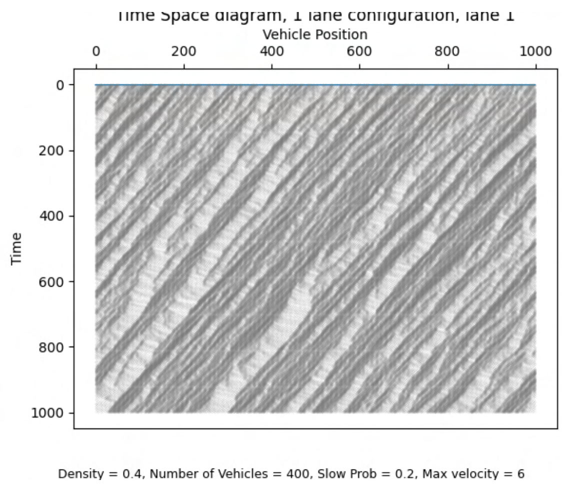
By modifying the maximum velocity, the time-space plots are obtained as follows:



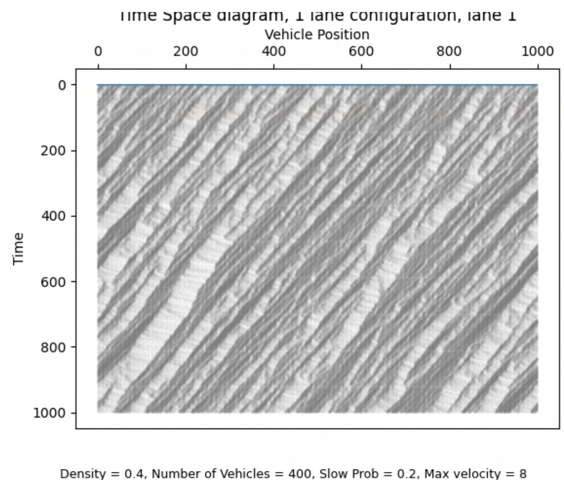
(a) Max velocity = 2



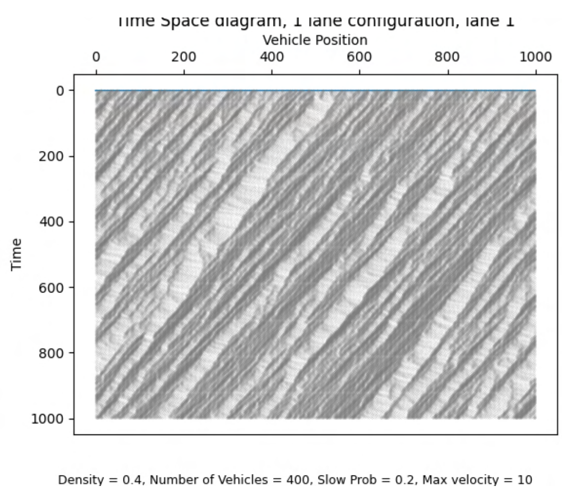
(b) Max velocity = 4



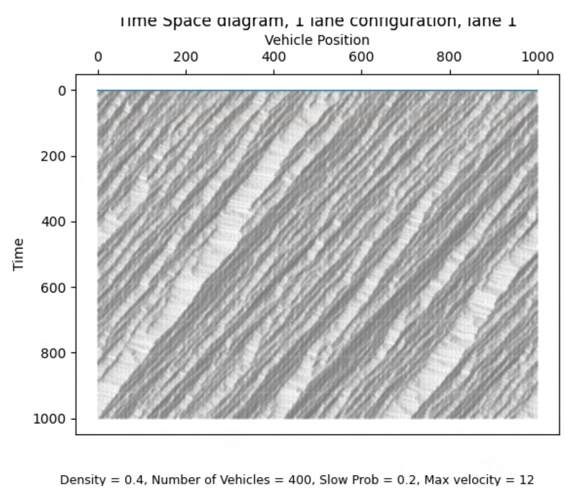
(c) Max velocity = 6



(d) Max velocity = 8

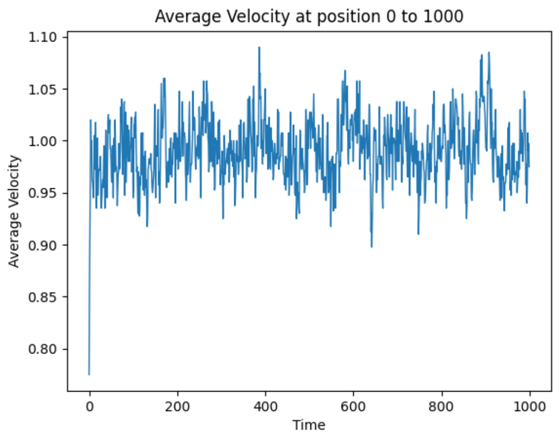


(e) Max velocity = 10

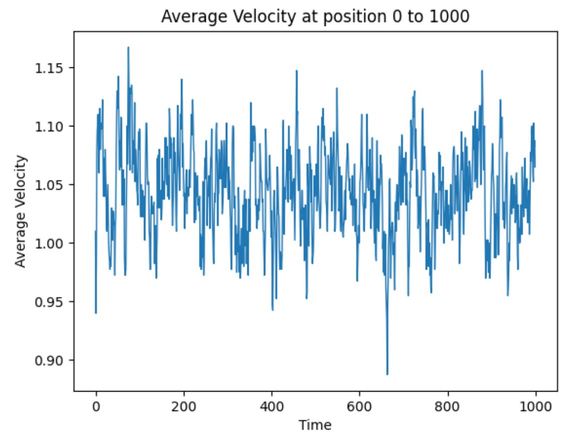


(f) Max velocity = 12

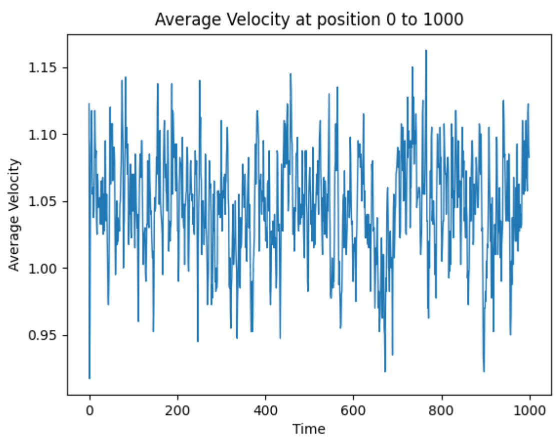
Figure 7: Time-space plot, maximum velocity iteration for a single lane configuration.



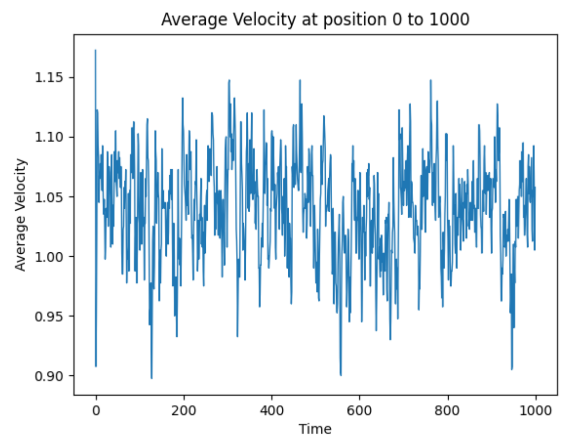
(a) Max velocity = 2



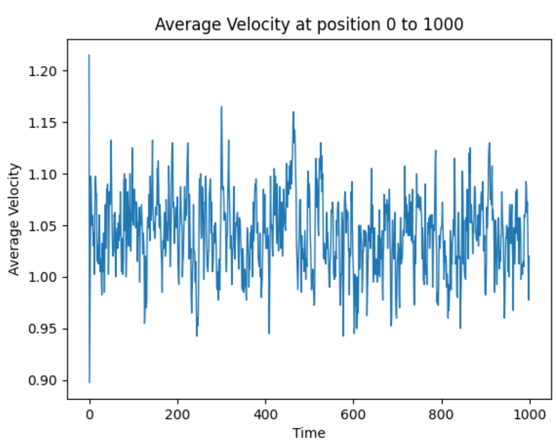
(b) Max velocity = 4



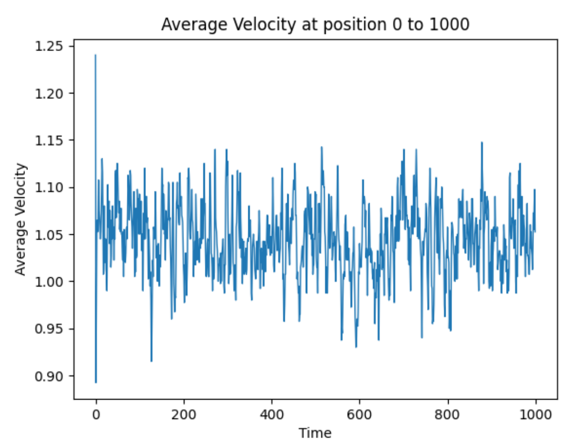
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 8: Average velocity plot, maximum velocity iteration for a single lane configuration.

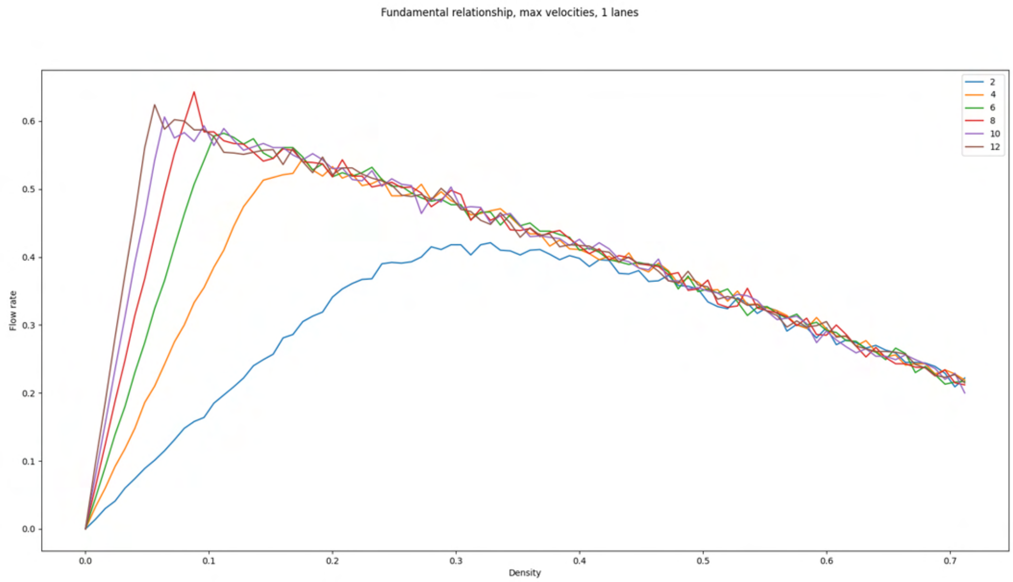
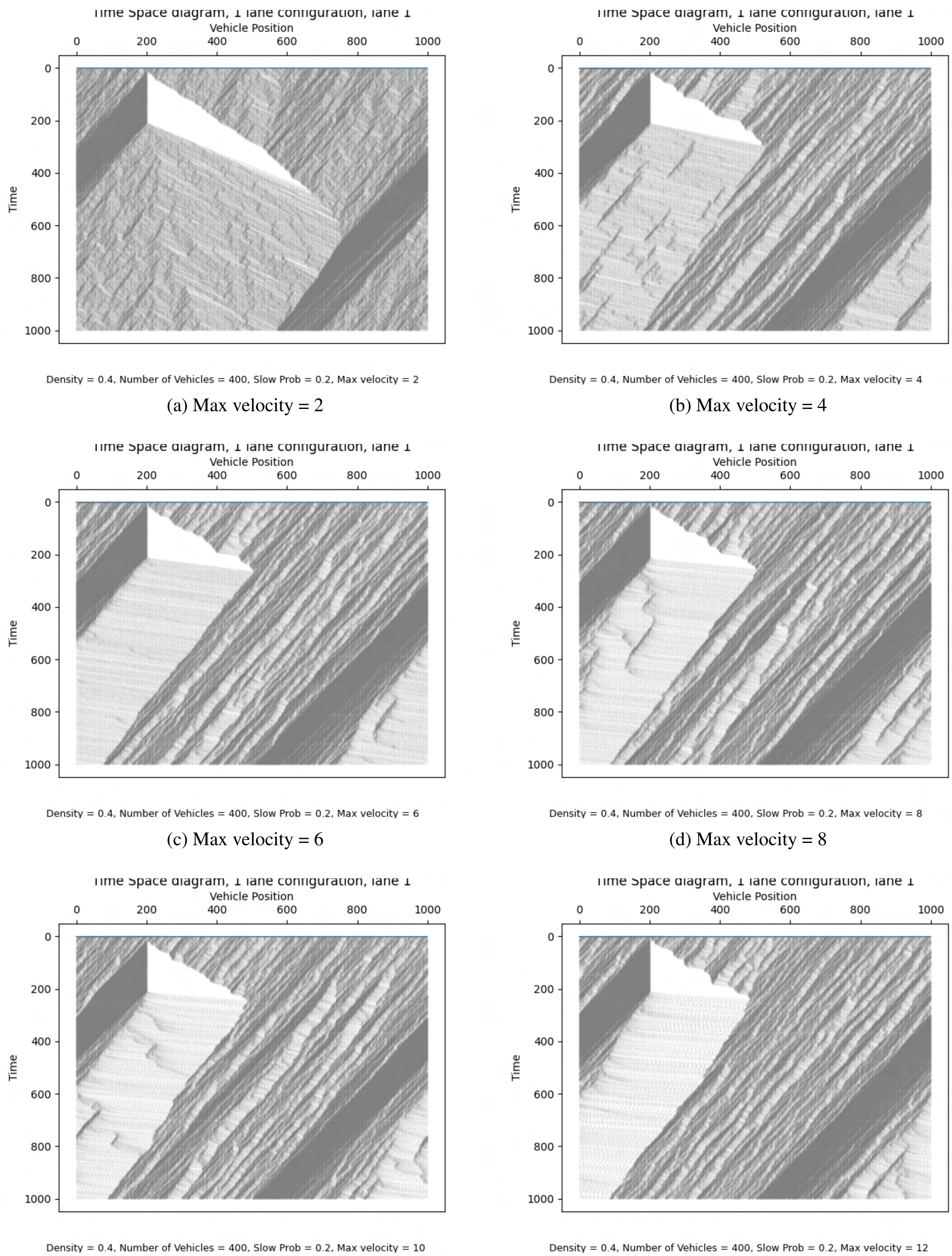


Figure 9: Fundamental relationship graph for a single lane as the maximum velocities are modified.

With obstacles, the time-space plots are obtained.



(a) Max velocity = 2

(b) Max velocity = 4

(c) Max velocity = 6

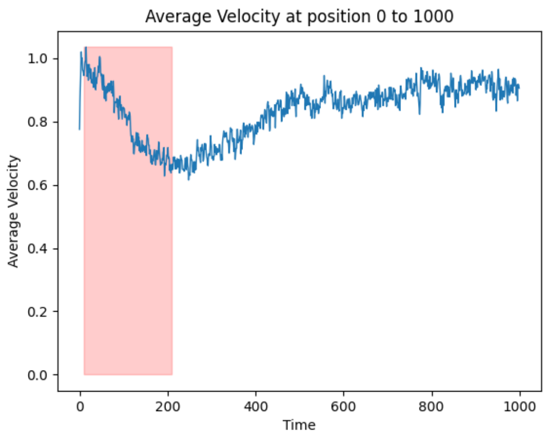
(d) Max velocity = 8

(e) Max velocity = 10

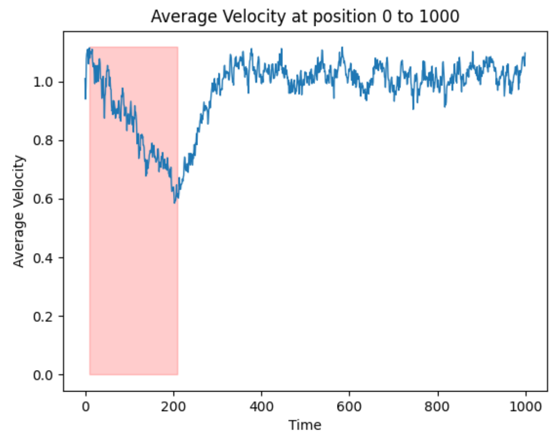
(f) Max velocity = 12

Figure 10: Time-space plot, maximum velocity iteration for a single lane configuration, with obstacle.

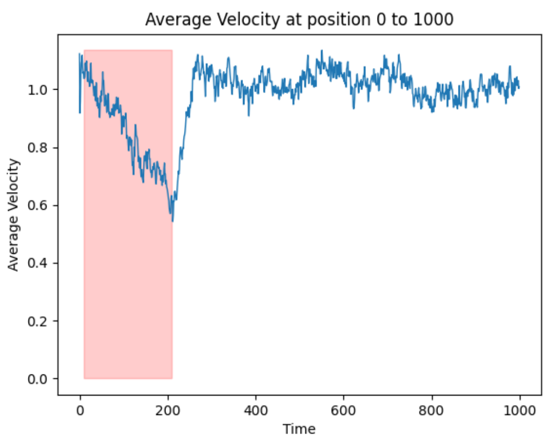
The average velocities are plotted below:



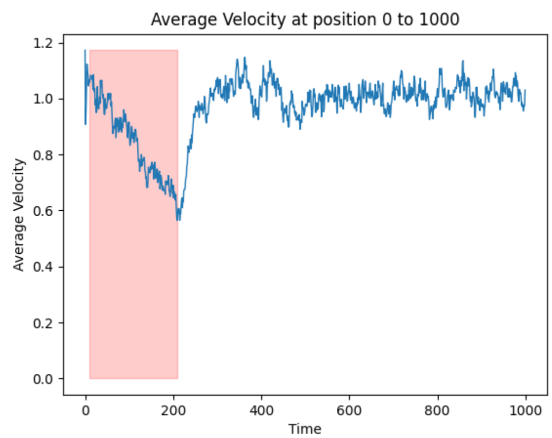
(a) Max velocity = 2



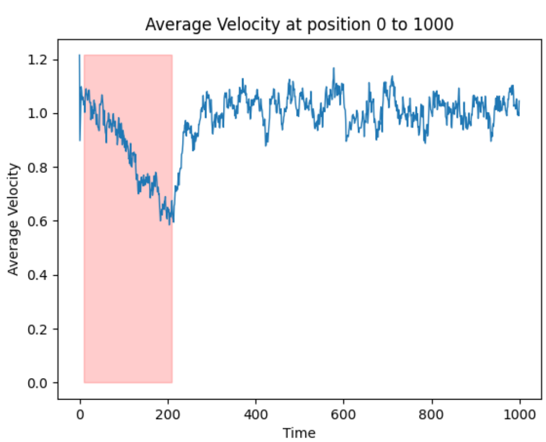
(b) Max velocity = 4



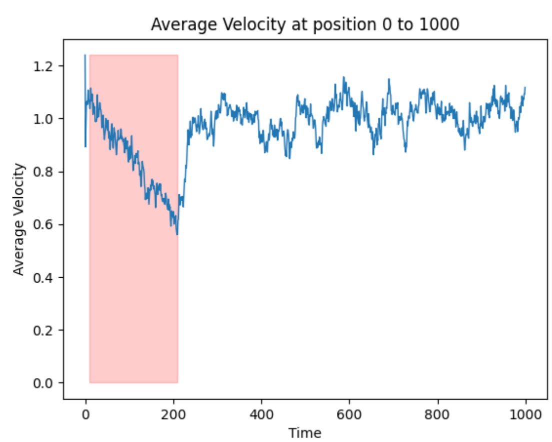
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 11: Average velocity plot, maximum velocity iteration for a single lane configuration, with obstacle.

The fundamental relationship is obtained.

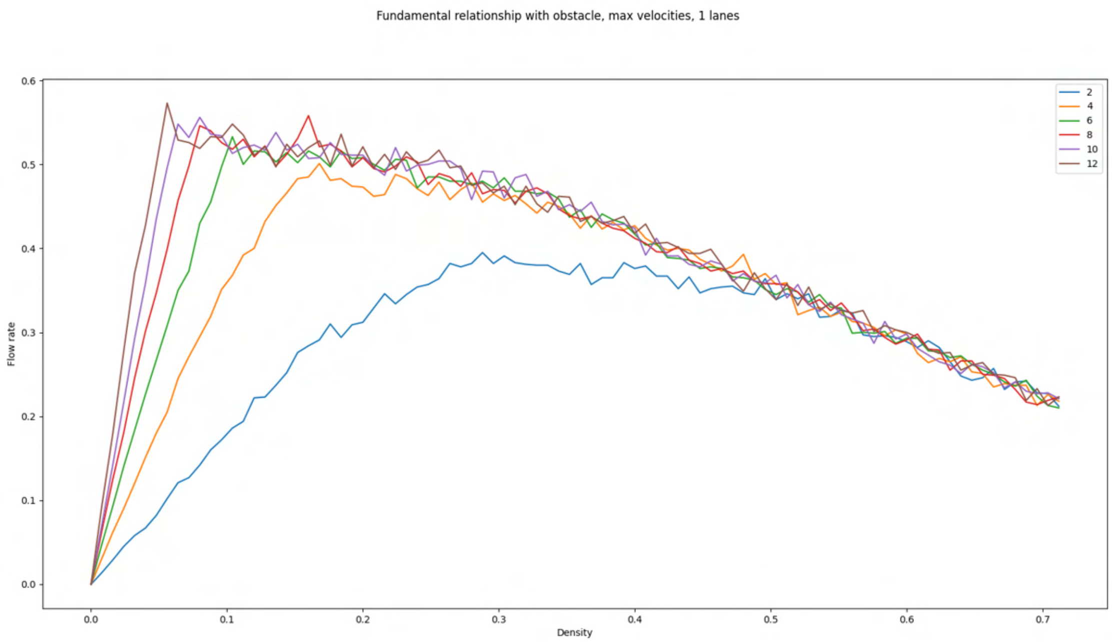
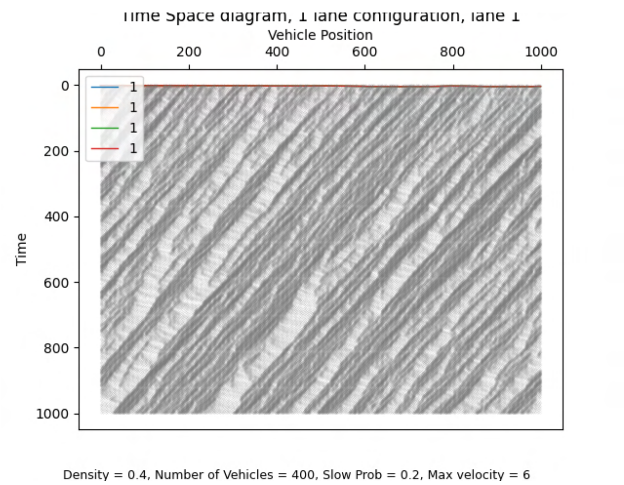


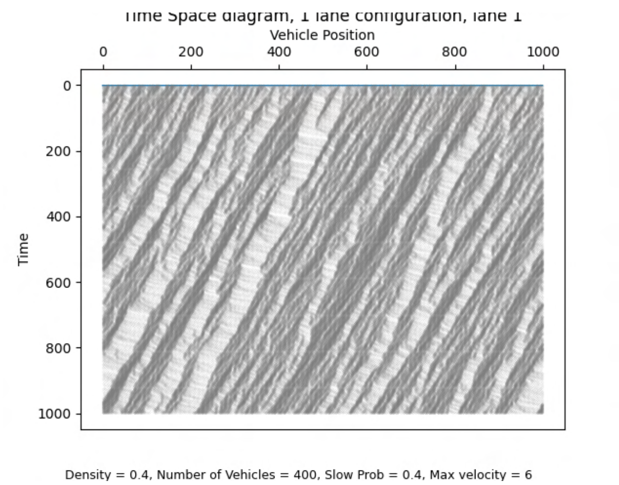
Figure 12: Fundamental relationship graph for a single lane as the maximum velocities are modified, with obstacle.

4.1.2 Slowing Probabilities

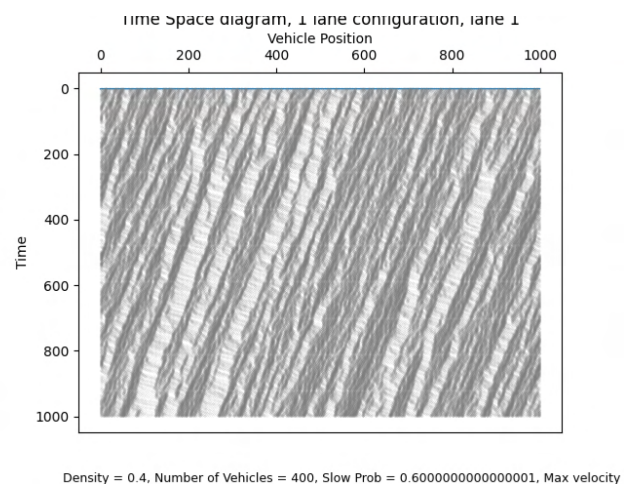
In a similar manner, the slow probabilities were altered and the time-space graphs are plotted.



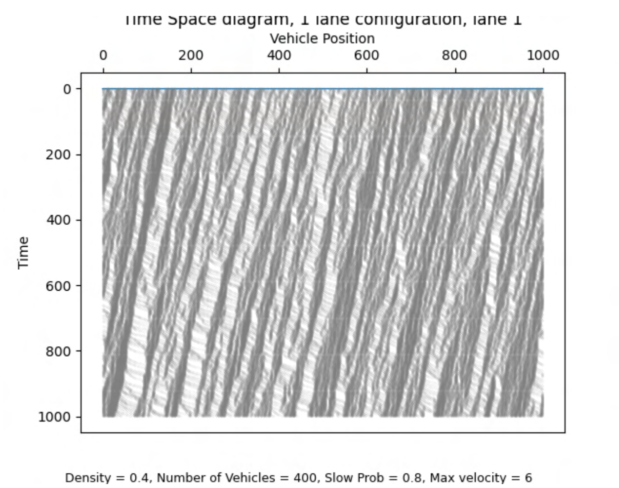
(a) Slow probability = 0.2



(b) Slow probability = 0.4



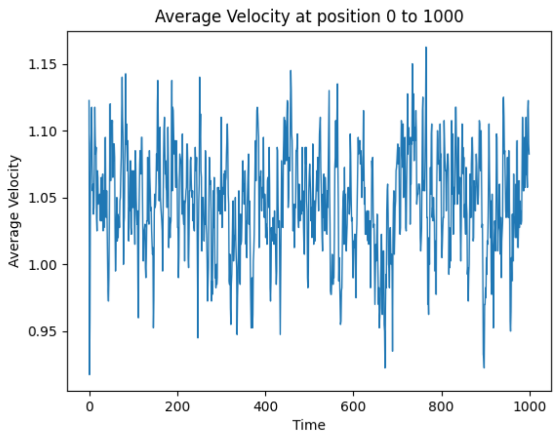
(c) Slow probability = 0.6



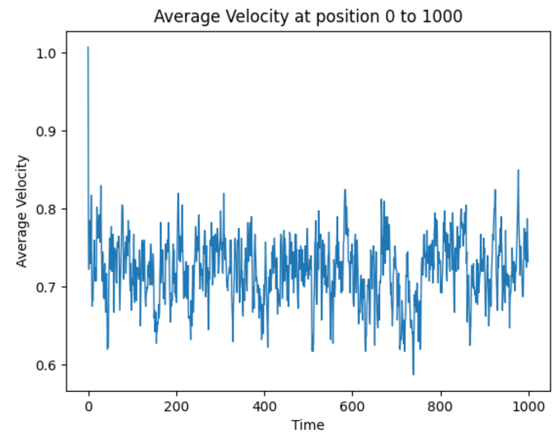
(d) Slow probability = 0.8

Figure 13: Time-space plot, slowing probability iteration for a single lane configuration.

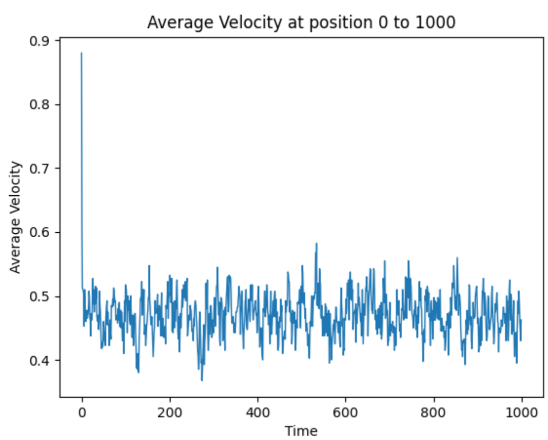
The average velocities are:



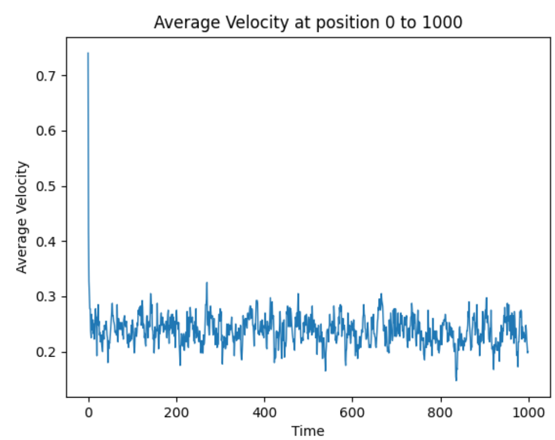
(a) Slow probability = 0.2



(b) Slow probability = 0.4



(c) Slow probability = 0.6



(d) Slow probability = 0.8

Figure 14: Average velocity plot, slowing probability iteration for a single lane configuration.

The fundamental relationship is obtained.

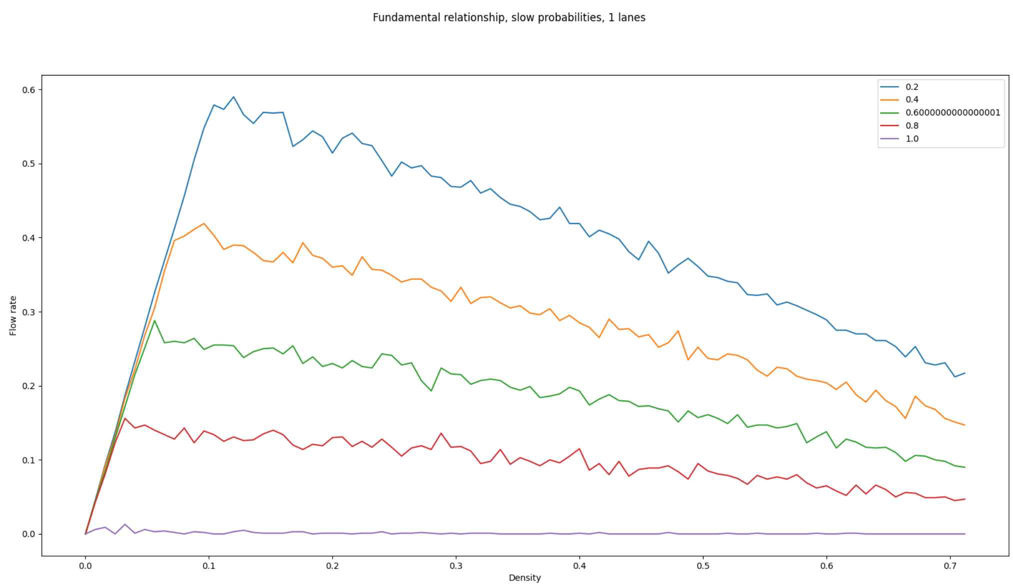
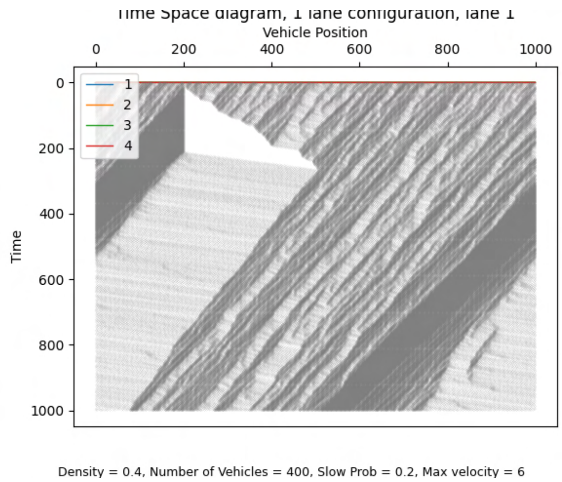
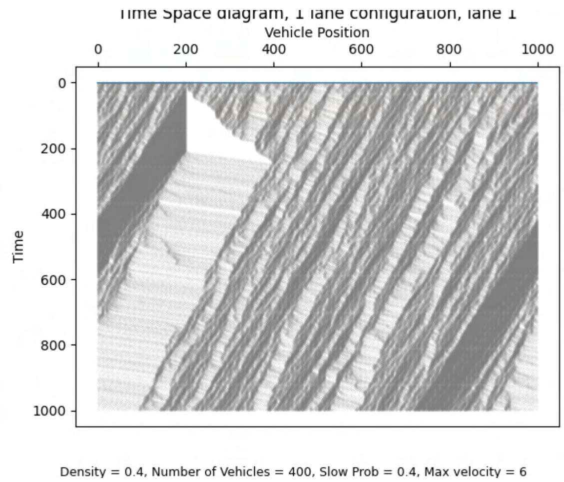


Figure 15: Fundamental relationship graph for a single lane as the slow probability is modified.

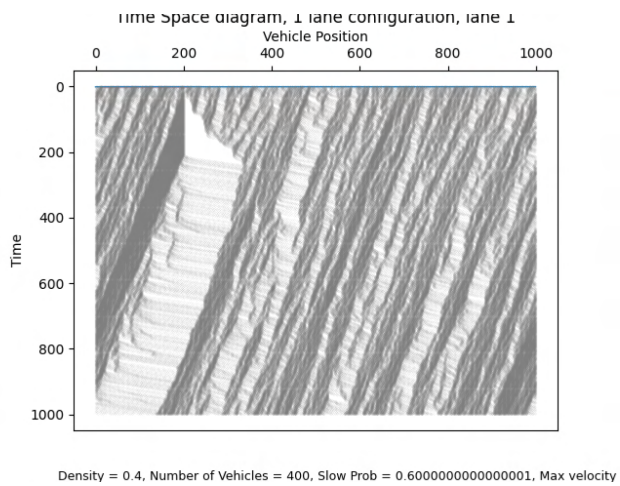
With obstacle implementation,



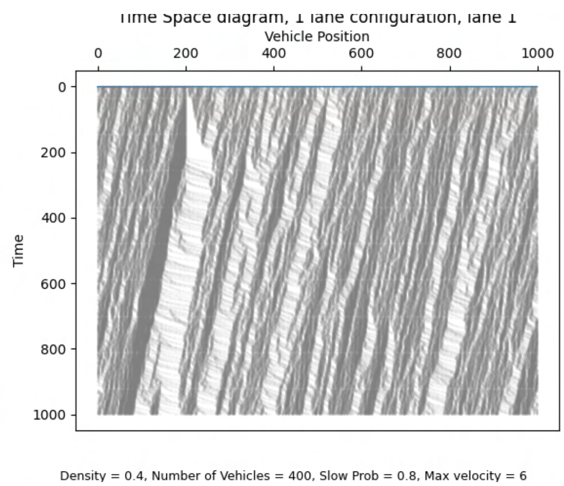
(a) Slow probability = 0.2



(b) Slow probability = 0.4



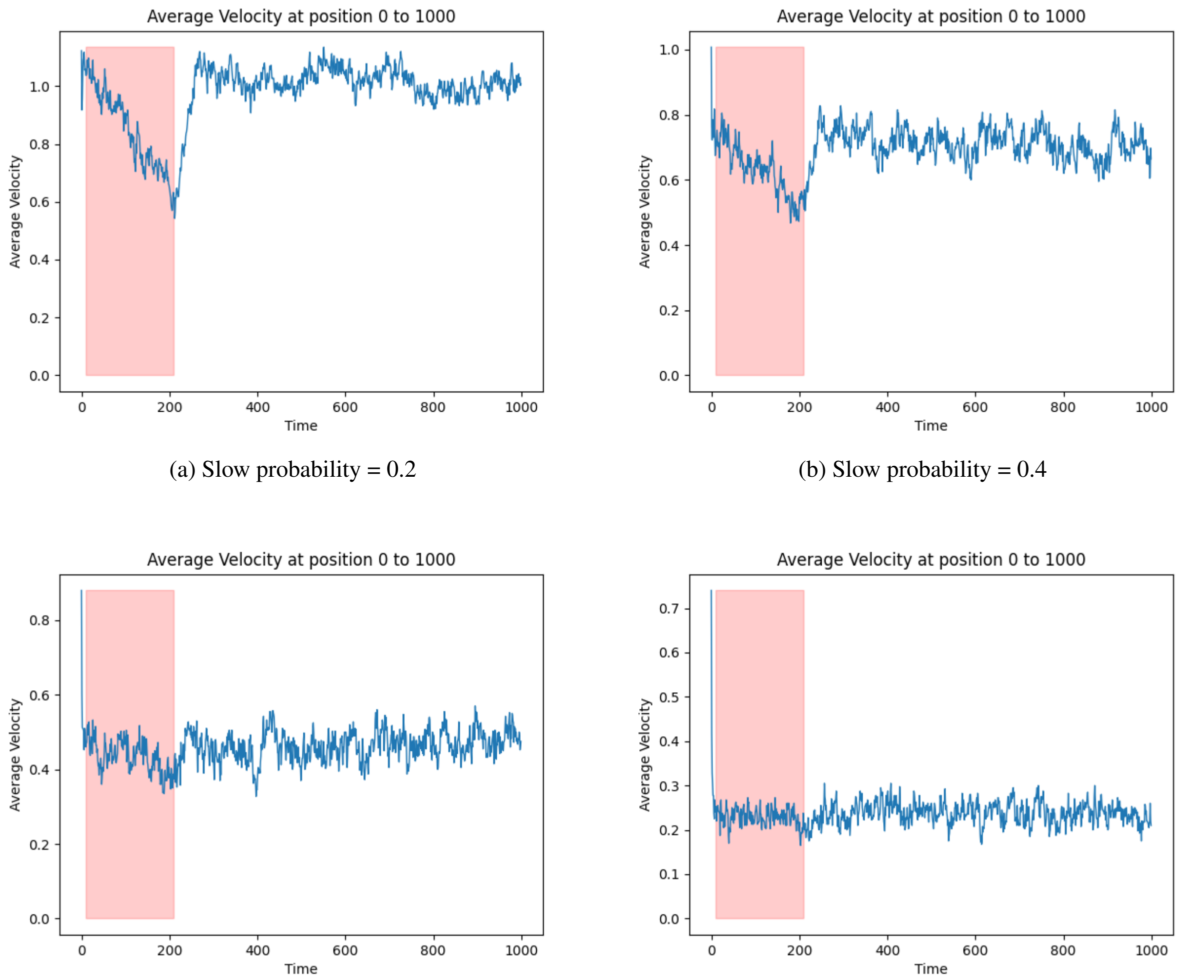
(c) Slow probability = 0.6



(d) Slow probability = 0.8

Figure 16: Time-space plot, slowing probability iteration for a single lane configuration with obstacle.

The average velocities are:



(a) Slow probability = 0.2

(b) Slow probability = 0.4

(c) Slow probability = 0.6

(d) Slow probability = 0.8

Figure 17: Average velocity plot, slowing probability iteration for a single lane configuration with obstacle.

The fundamental relationship is obtained.

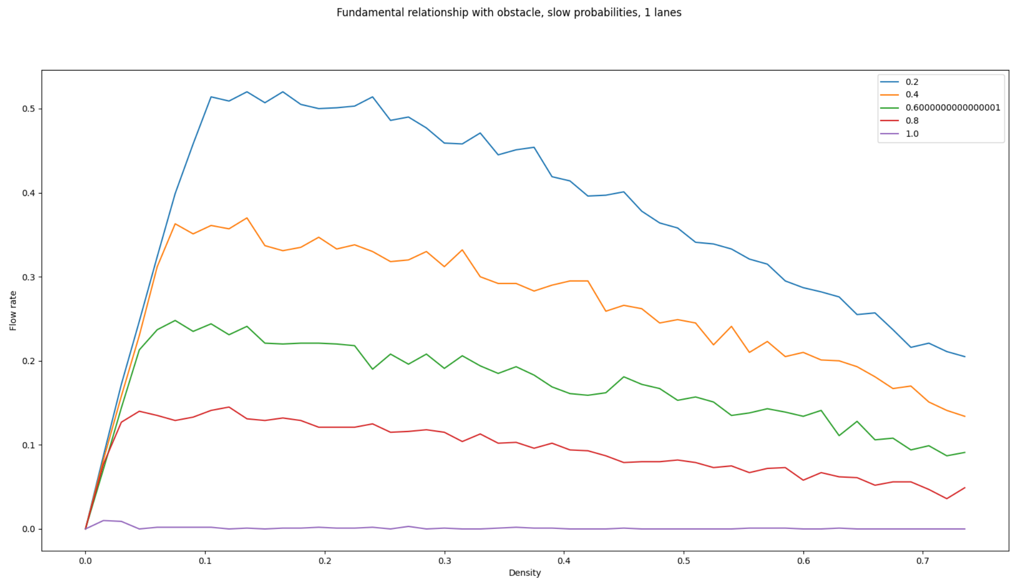


Figure 18: Fundamental relationship graph for a single lane as the slow probability is modified, with obstacle.

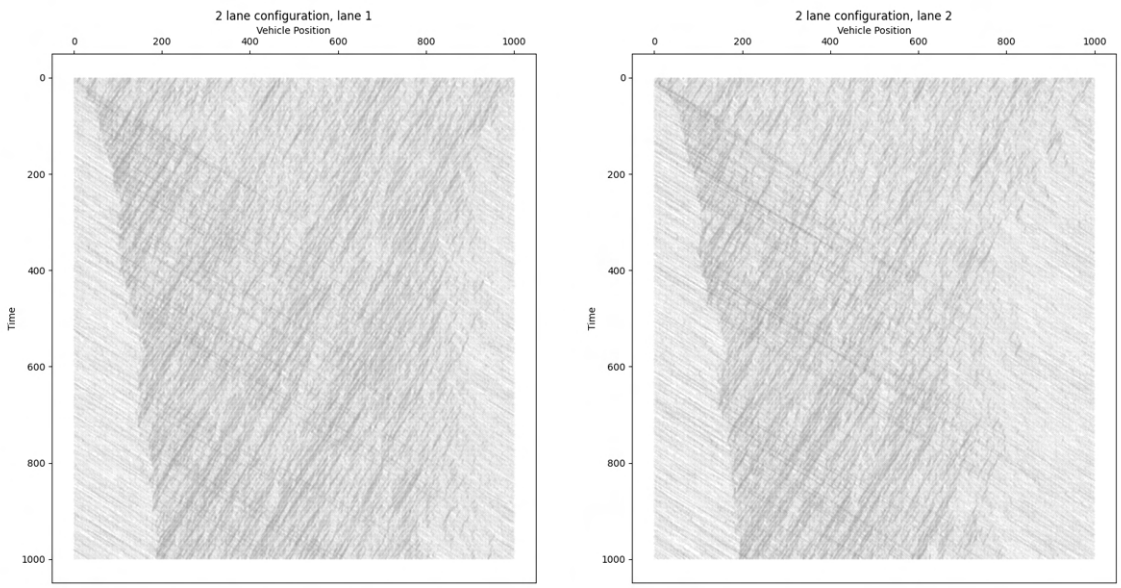
4.2 Double Lane Simulations

4.2.1 Standard Simulations

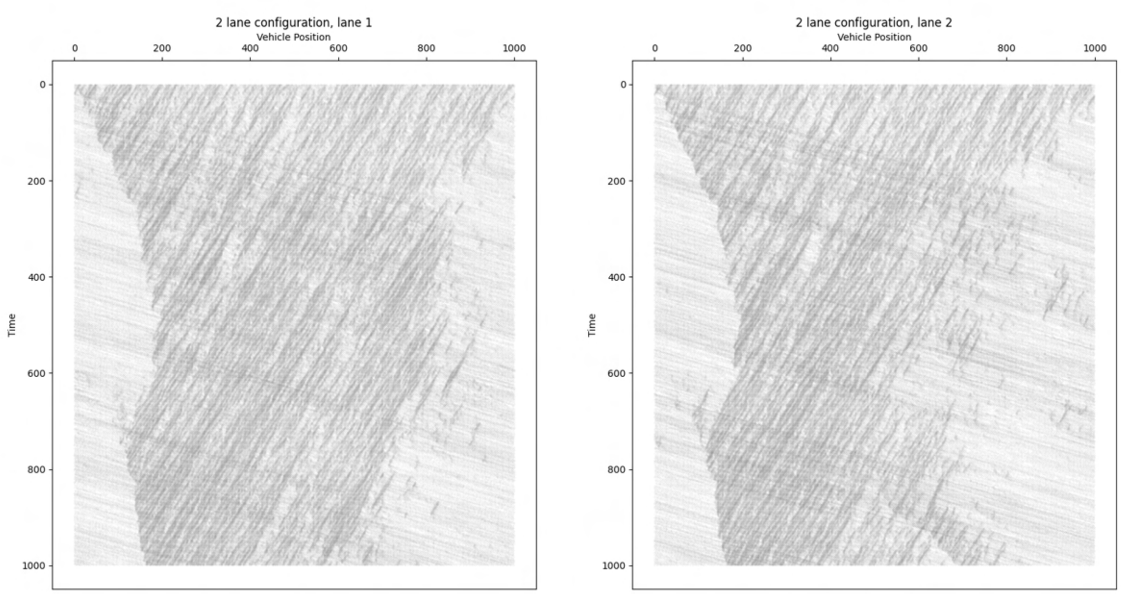
In a 2 lane simulation, cars only can switch between lanes.

4.2.2 Maximum Velocity

Likewise, the maximum velocities are changed and the behaviour is observed.

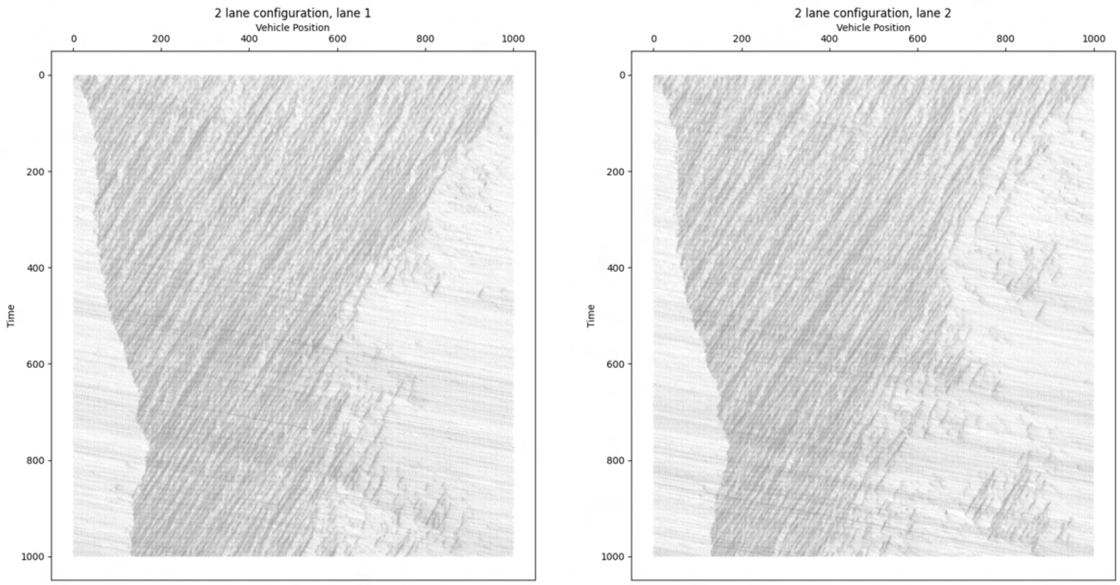


(a) Max velocity = 2

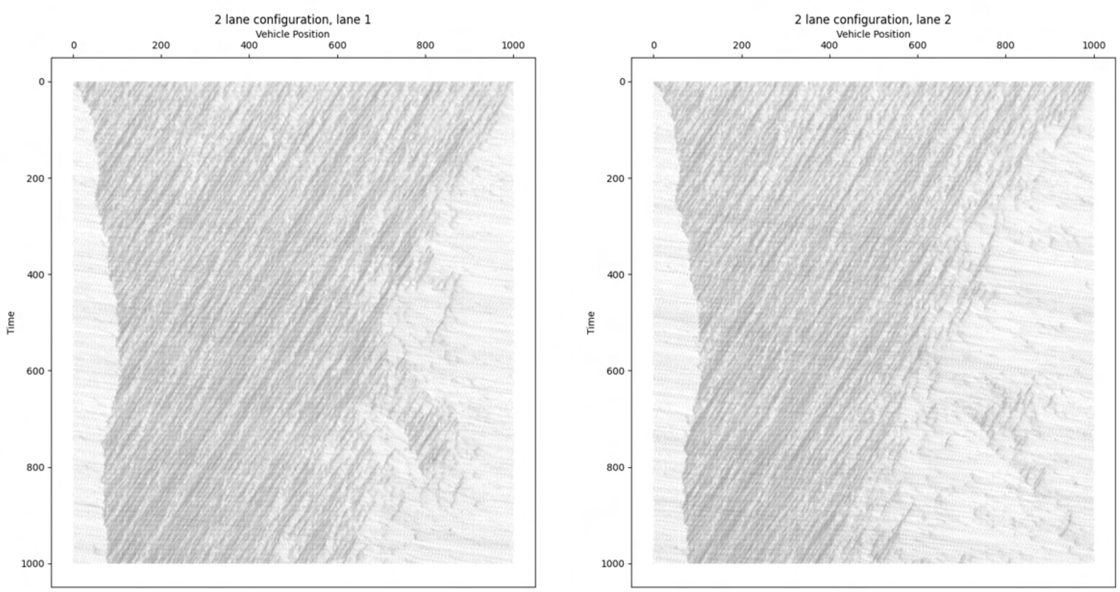


(b) Max velocity = 4

Figure 19: Time-space plot, maximum velocity iteration for a double lane configuration.(1)

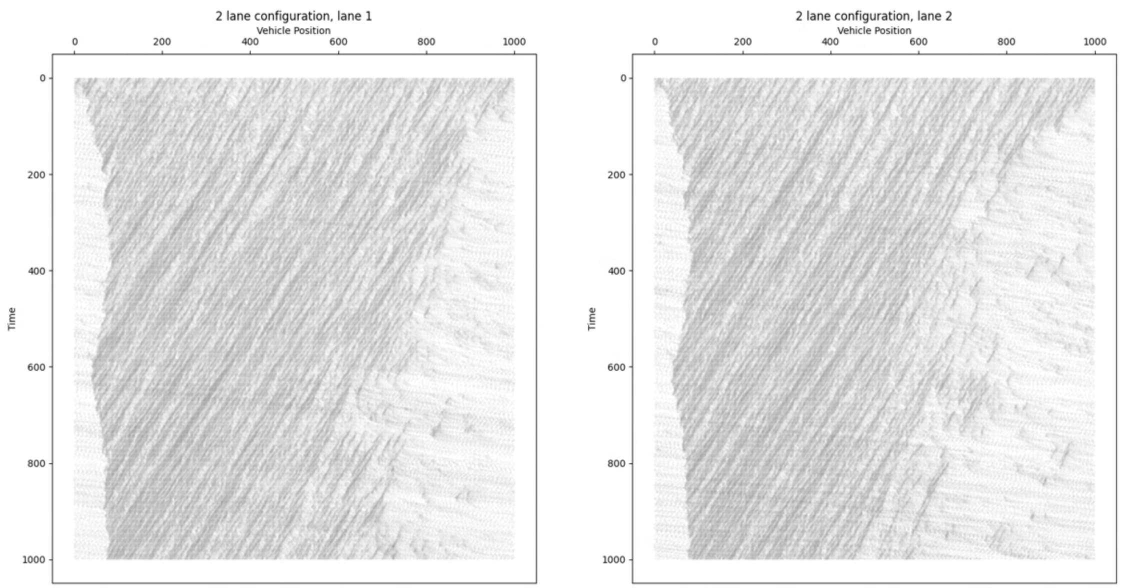


(a) Max velocity = 6

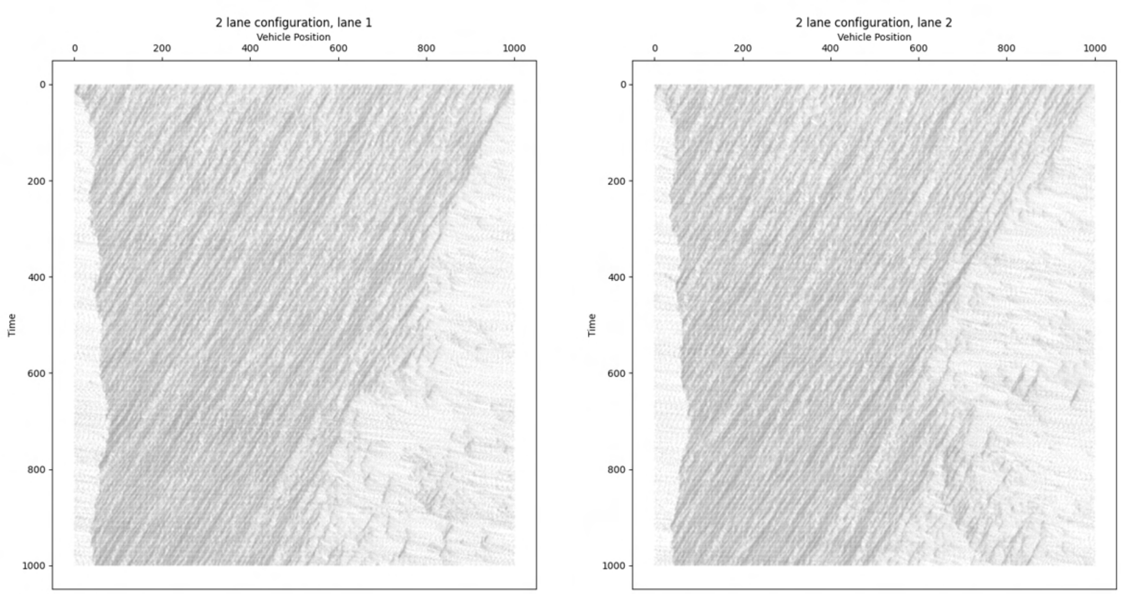


(b) Max velocity = 8

Figure 20: Time-space plot, maximum velocity iteration for a double lane configuration.(2)

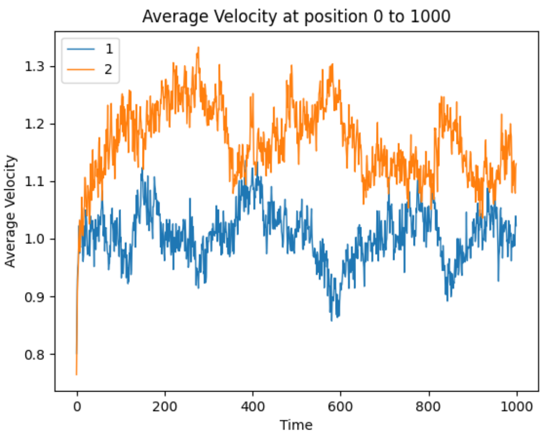


(a) Max velocity = 10

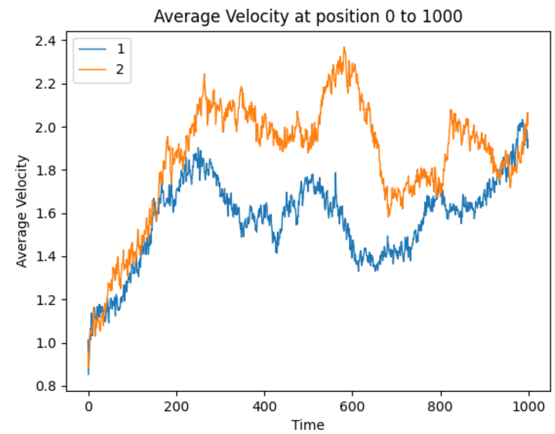


(b) Max velocity = 12

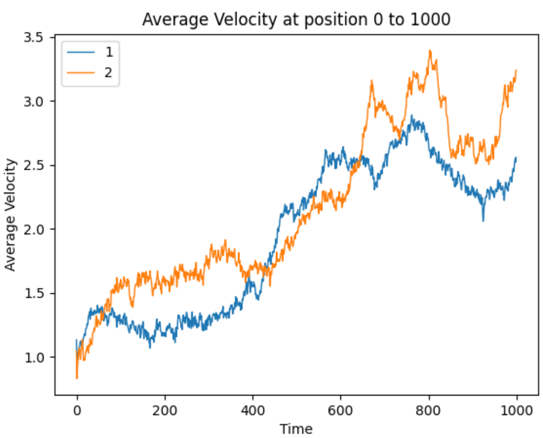
Figure 21: Time-space plot, maximum velocity iteration for a double lane configuration.(3)



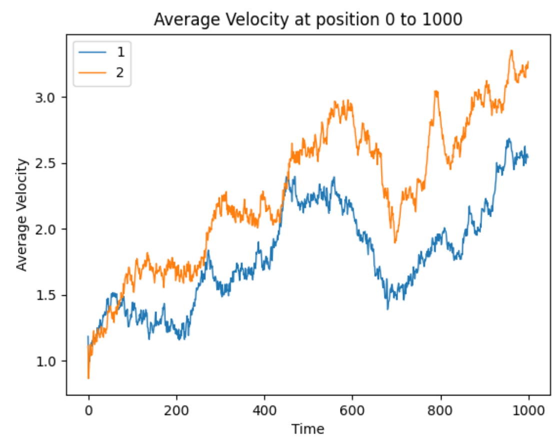
(a) Max velocity = 2



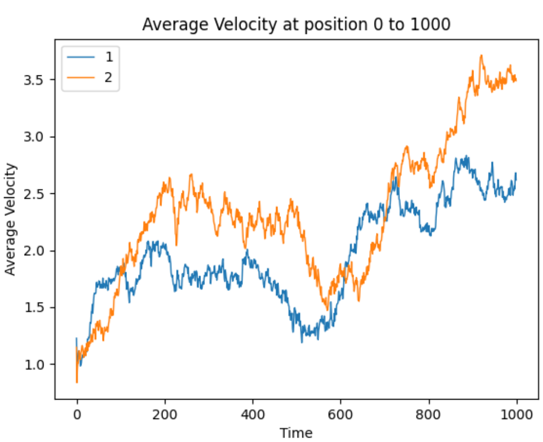
(b) Max velocity = 4



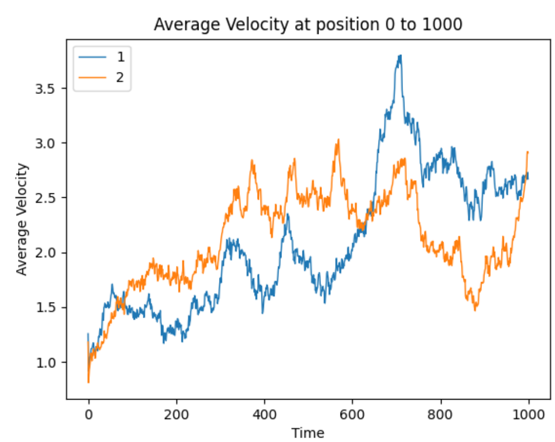
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 22: Average velocity plot, maximum velocity iteration for a double lane configuration.

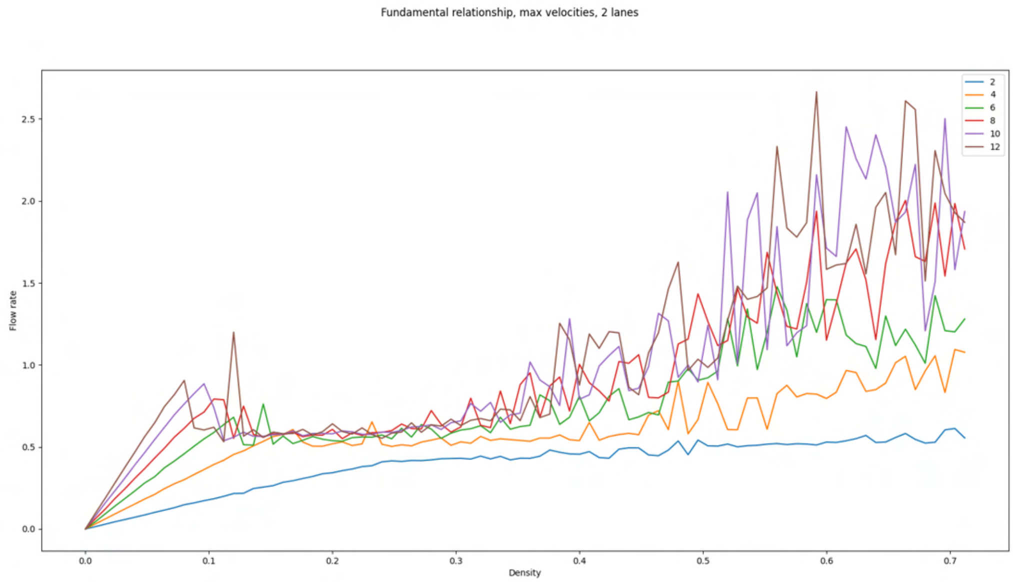
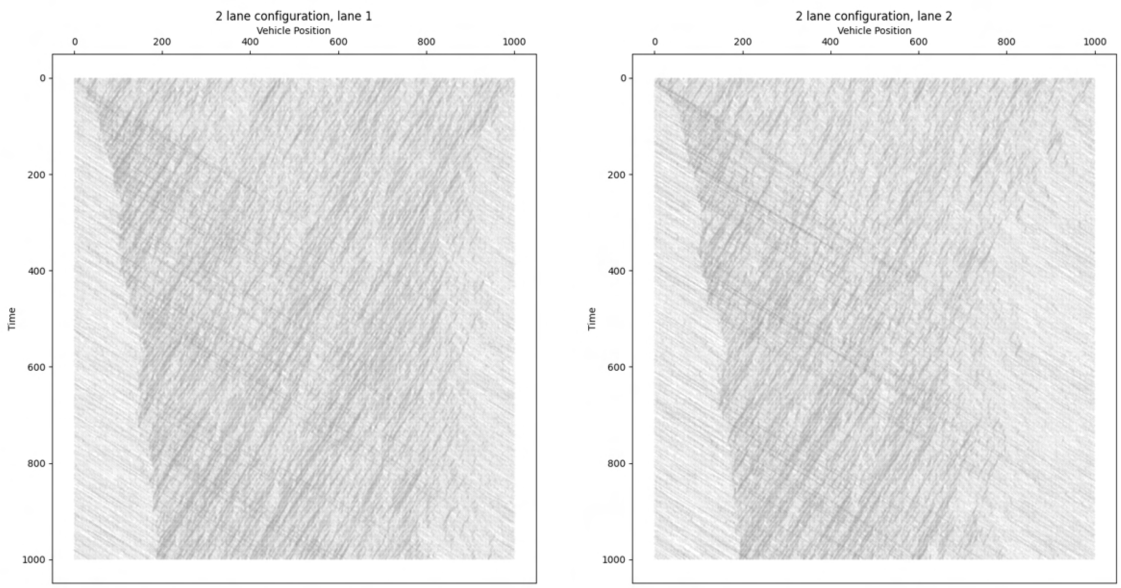
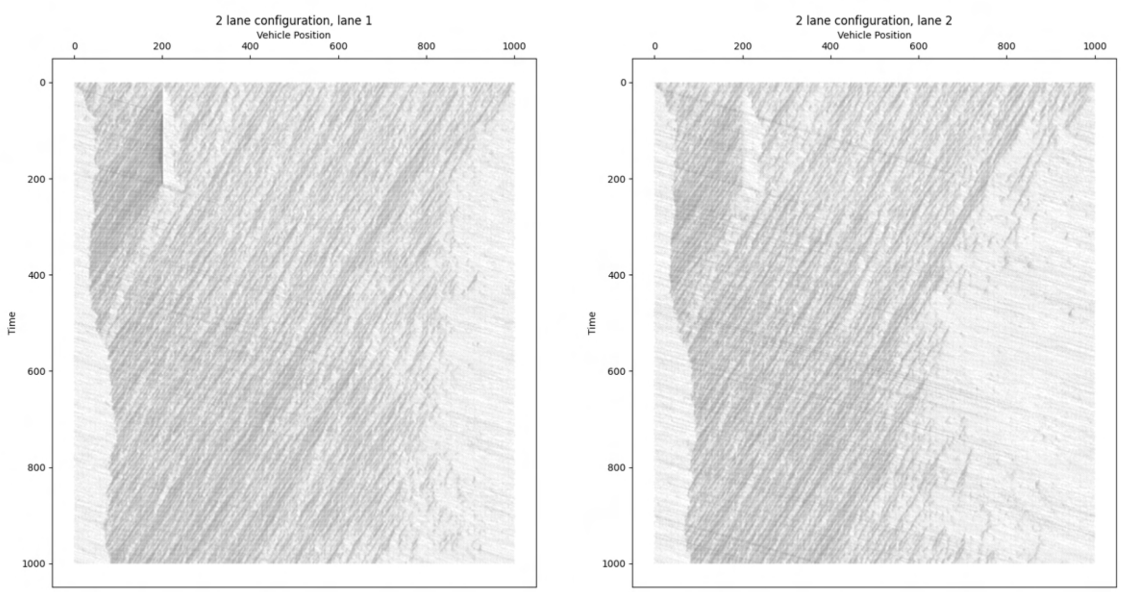


Figure 23: Fundamental relationship graph for double lane as the maximum velocities are modified.

With obstacles, the time-space plots are also obtained.

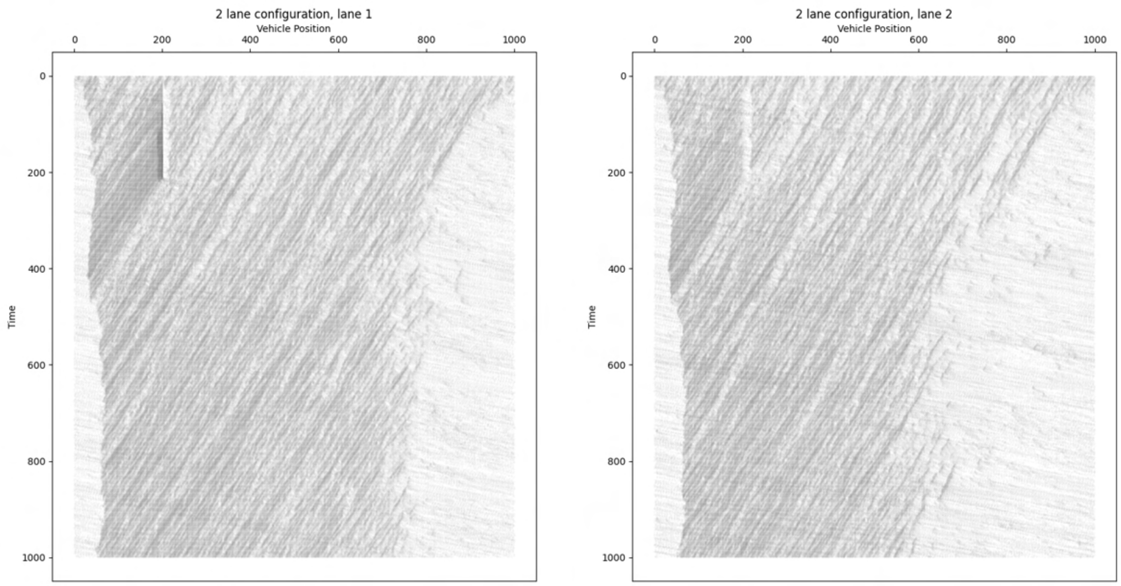


(a) Max velocity = 2

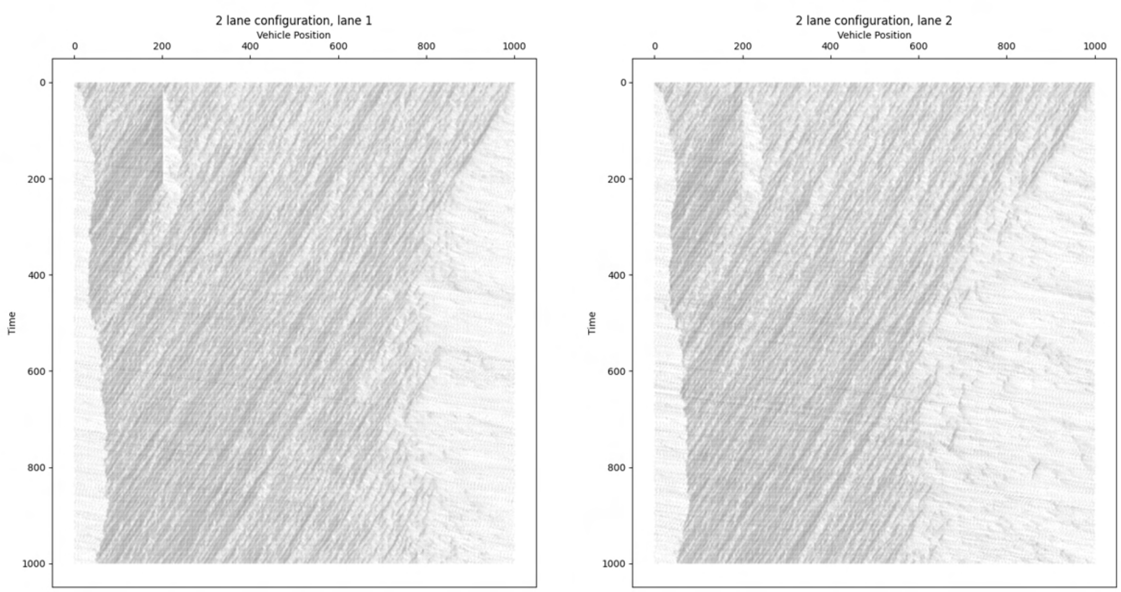


(b) Max velocity = 4

Figure 24: Time-space plot, maximum velocity iteration for a double lane configuration obstacle in lane 1.(1)

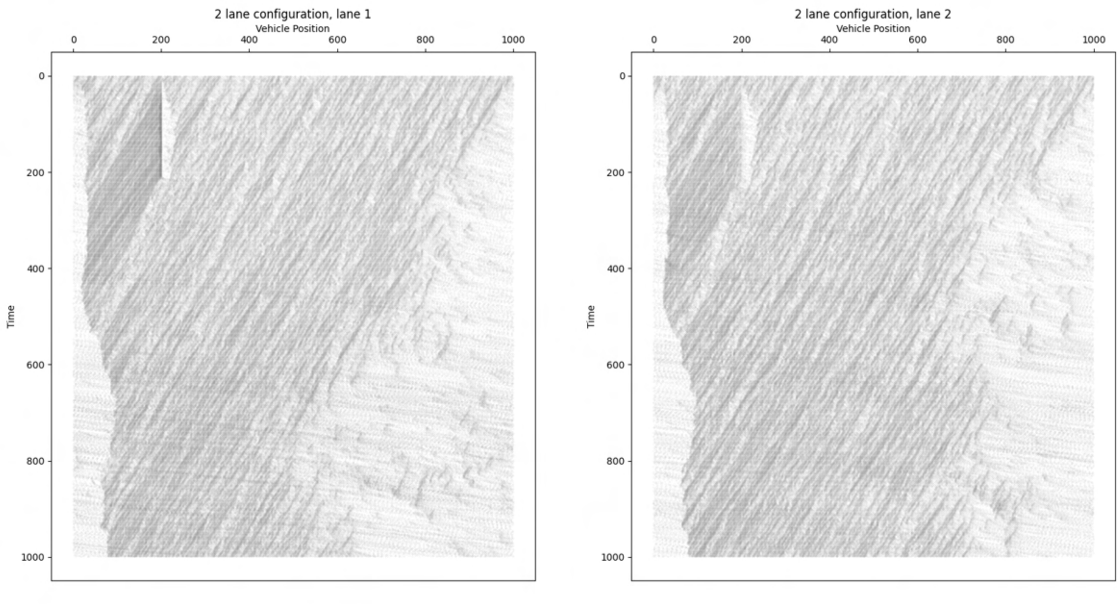


(a) Max velocity = 6

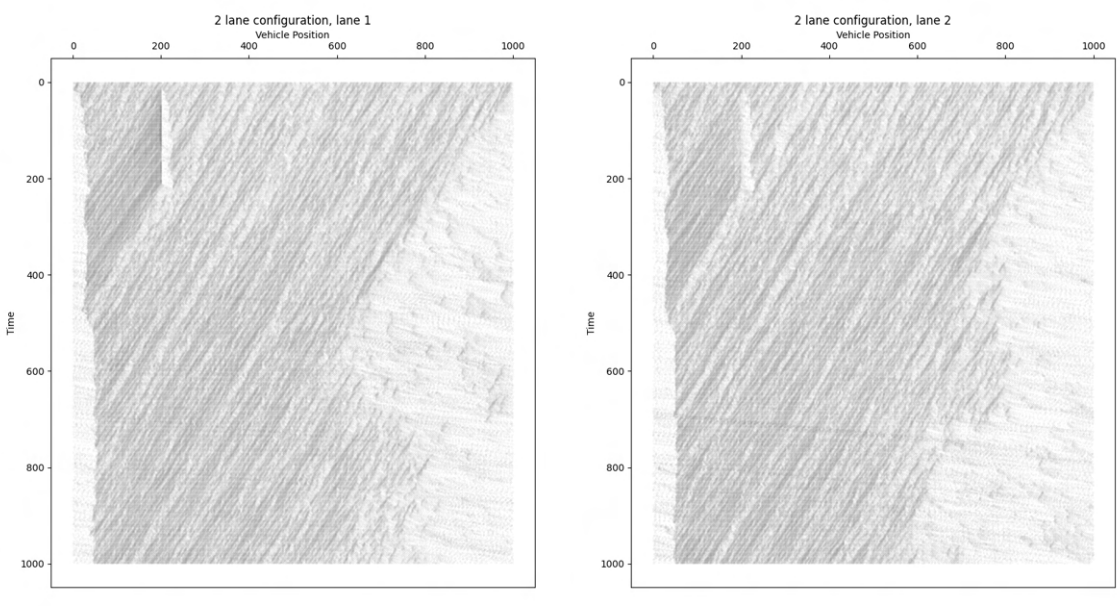


(b) Max velocity = 8

Figure 25: Time-space plot, maximum velocity iteration for a double lane configuration obstacle in lane 1.(2)



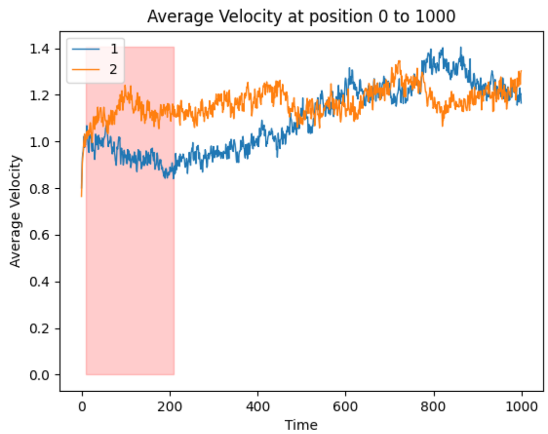
(a) Max velocity = 10



(b) Max velocity = 12

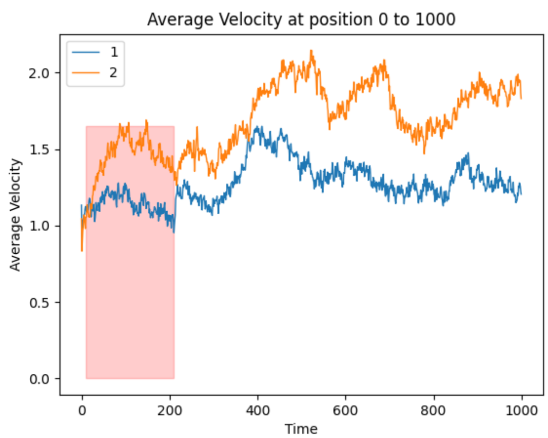
Figure 26: Time-space plot, maximum velocity iteration for a double lane configuration with obstacle in lane 1.(3)

The average velocities are plotted below:

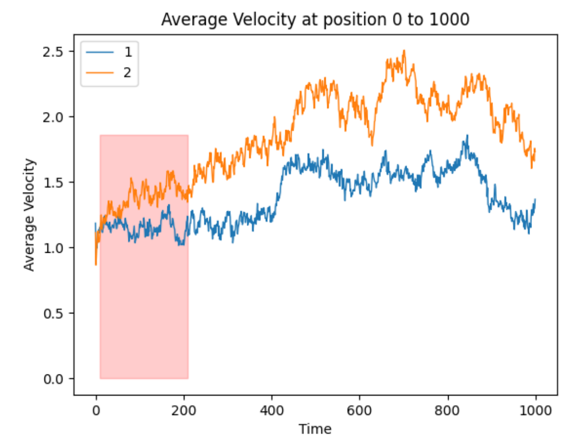


(a) Max velocity = 2

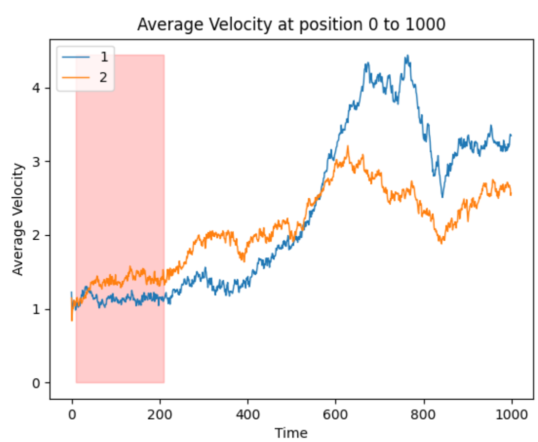
(b) Max velocity = 4



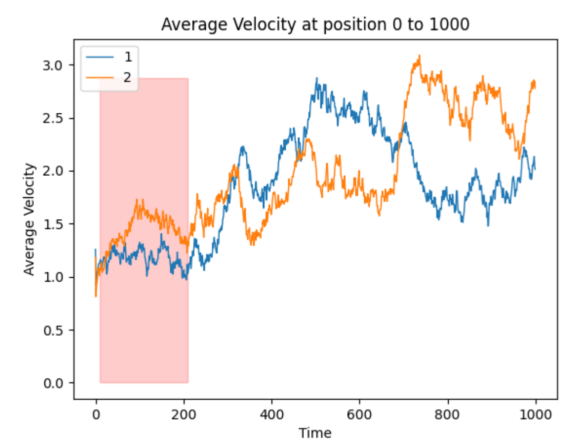
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 27: Average velocity plot, maximum velocity iteration for a double lane configuration with obstacle in lane 1.

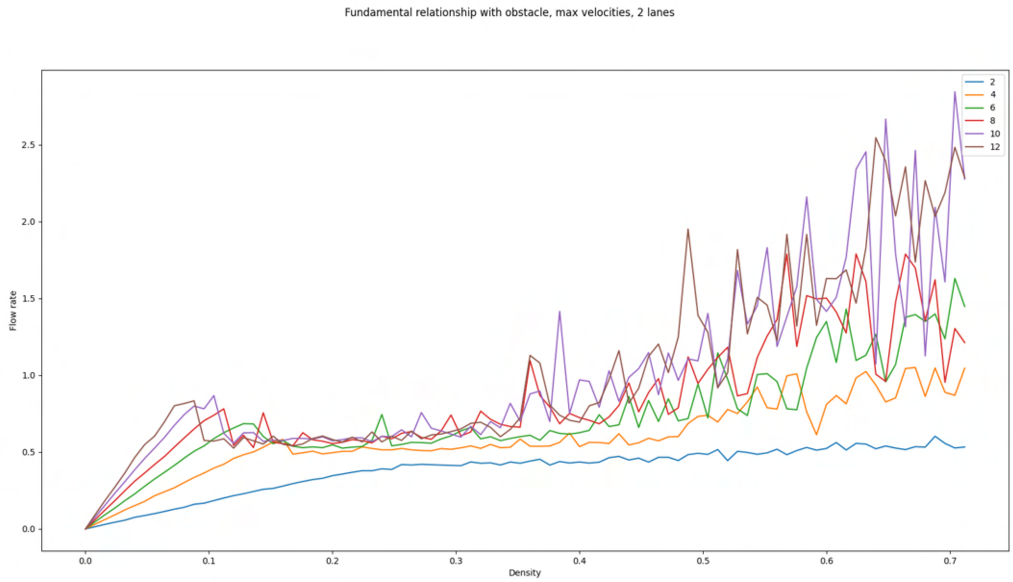
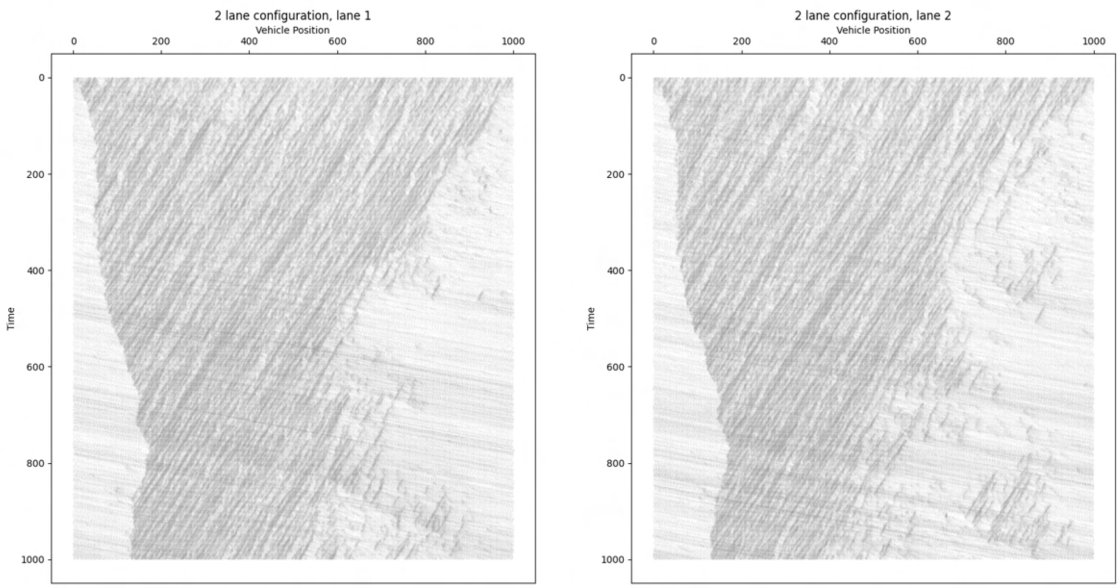
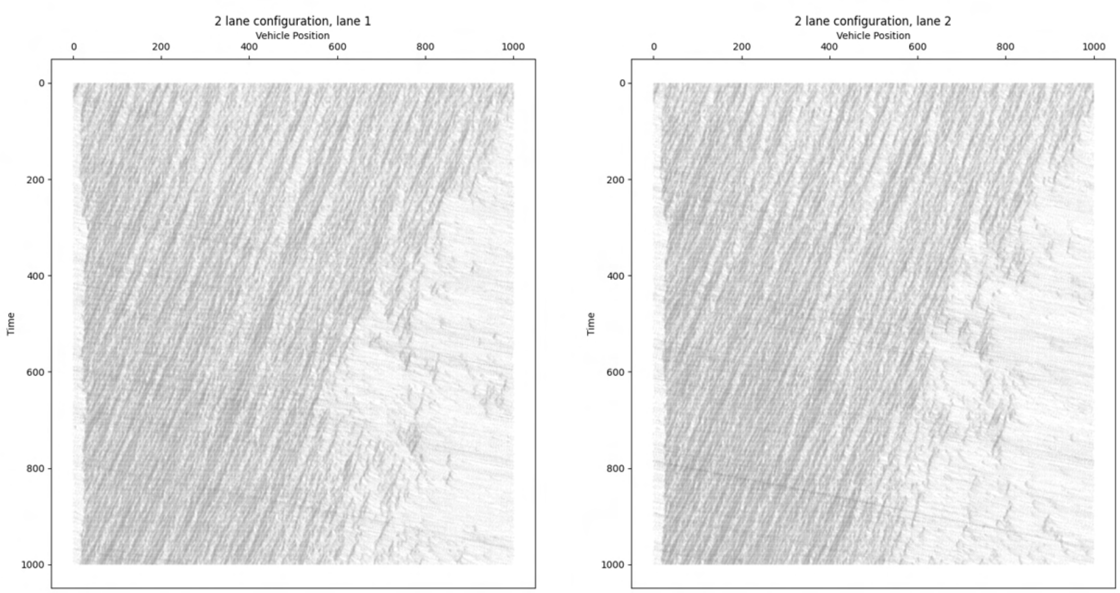


Figure 28: Fundamental relationship graph for double lane as the maximum velocities are modified with obstacle in lane 1.

4.2.3 Slowing Probabilities

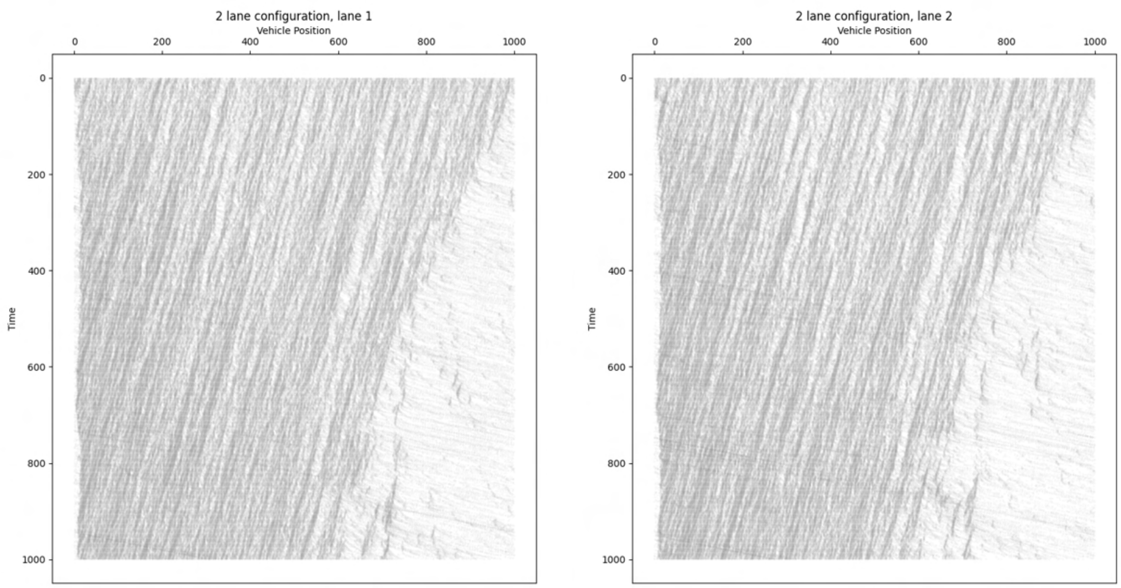


(a) Slow prob = 0.2

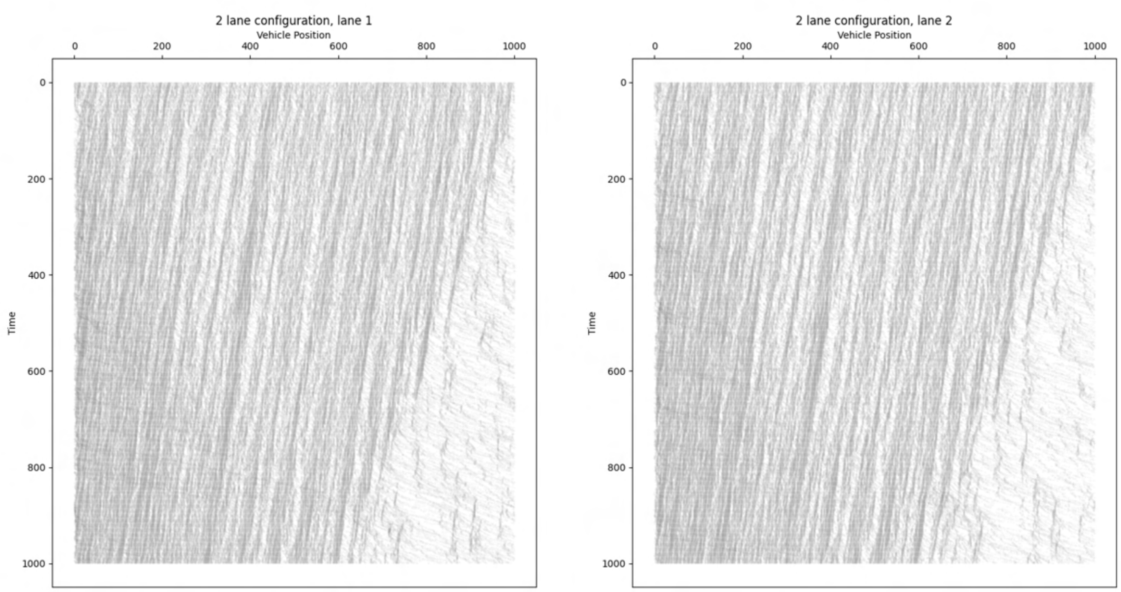


(b) Slow prob = 0.4

Figure 29: Time-space plot, slow probability iteration for a double lane configuration.(1)



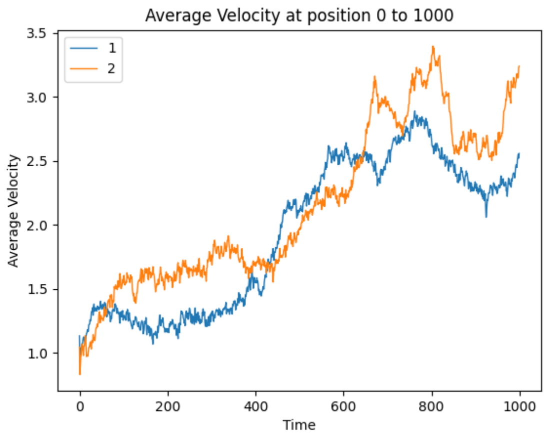
(a) Slow prob = 0.6



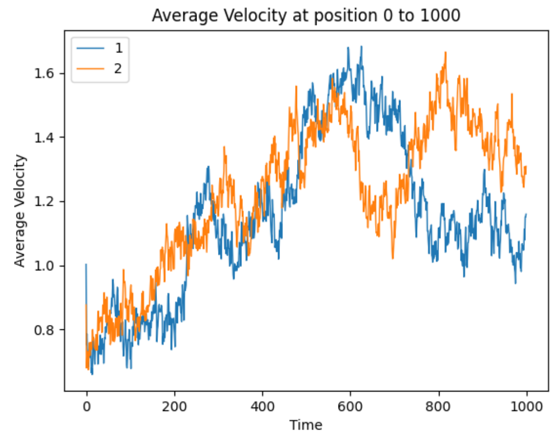
(b) Slow prob = 0.8

Figure 30: Time-space plot, slow probability iteration for a double lane configuration.(2)

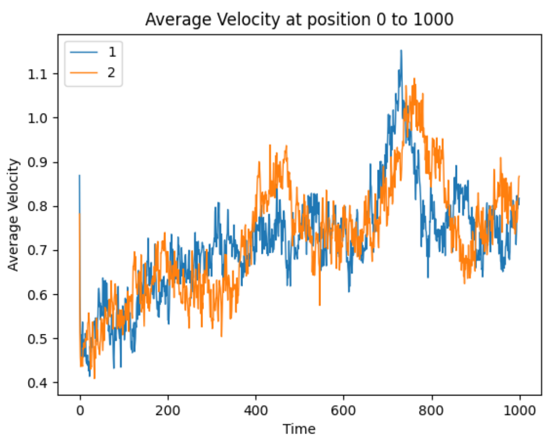
The average velocities are plotted below:



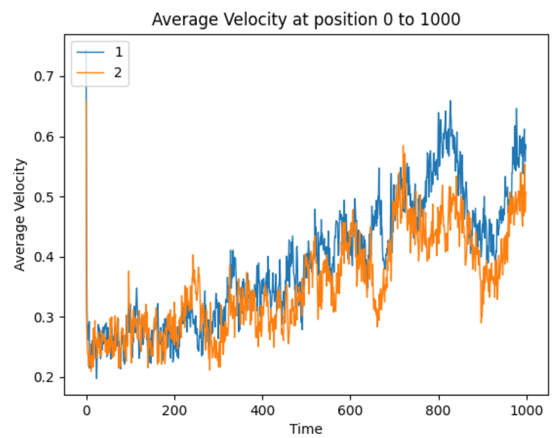
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 31: Average velocity plot, slow probability iteration for a double lane configuration.

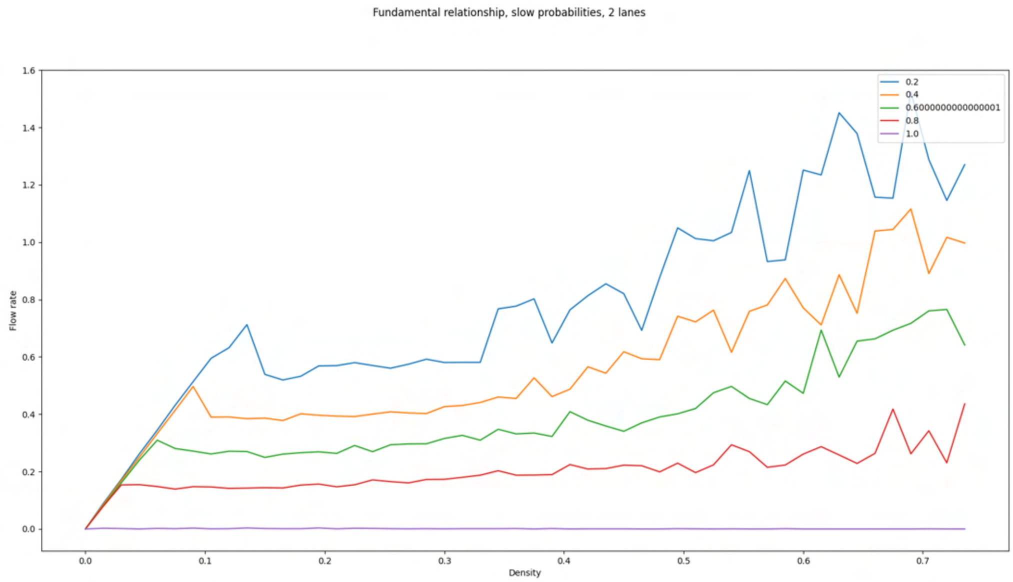
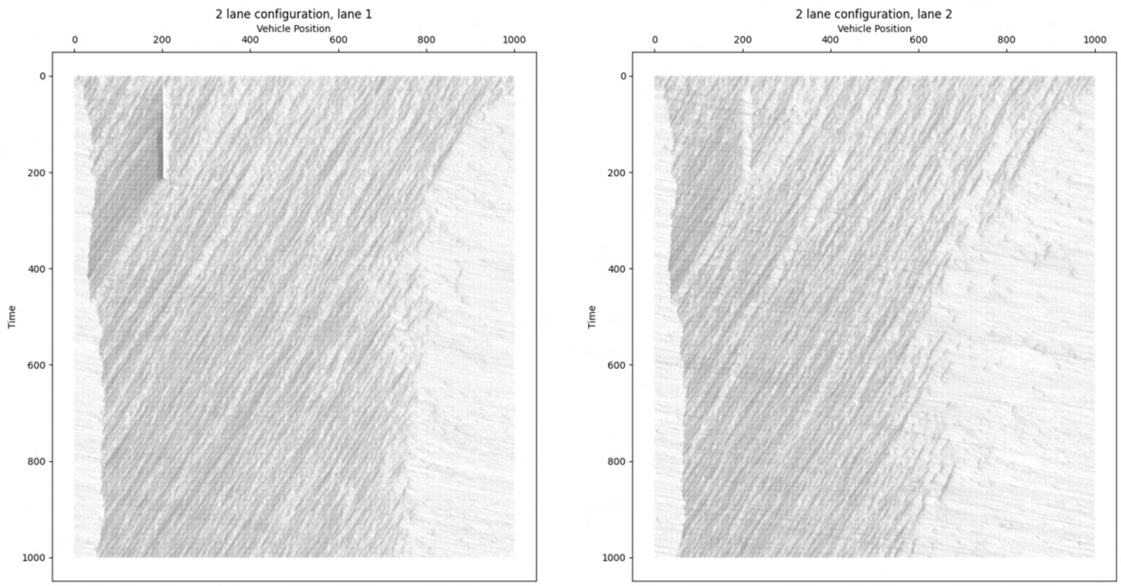
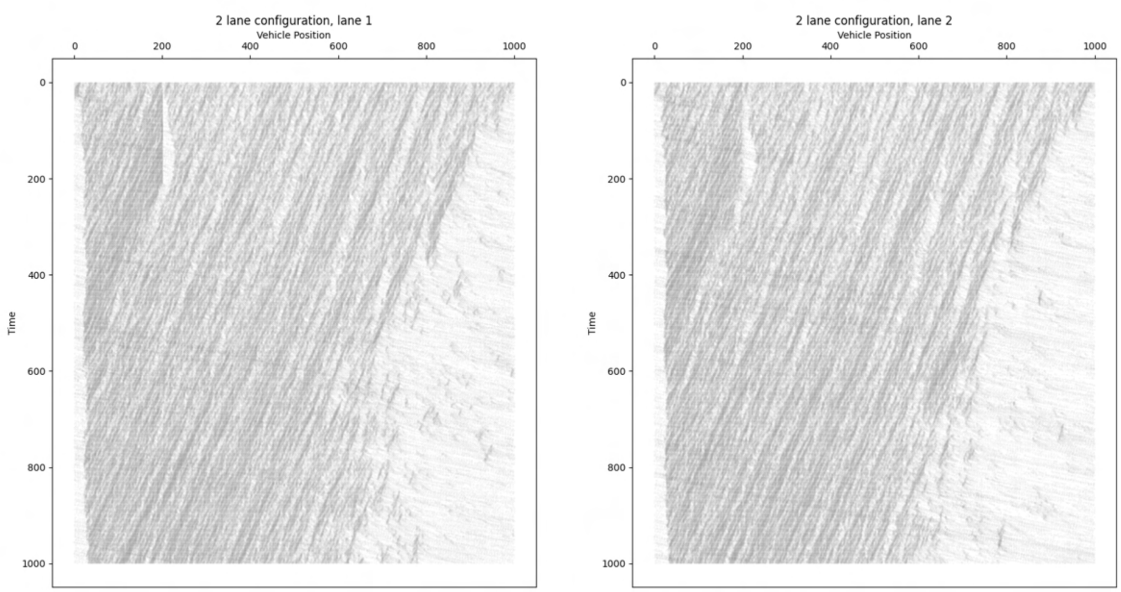


Figure 32: Fundamental relationship graph for double lane as the slow probability is modified

With obstacles, the time-space plots are also obtained.

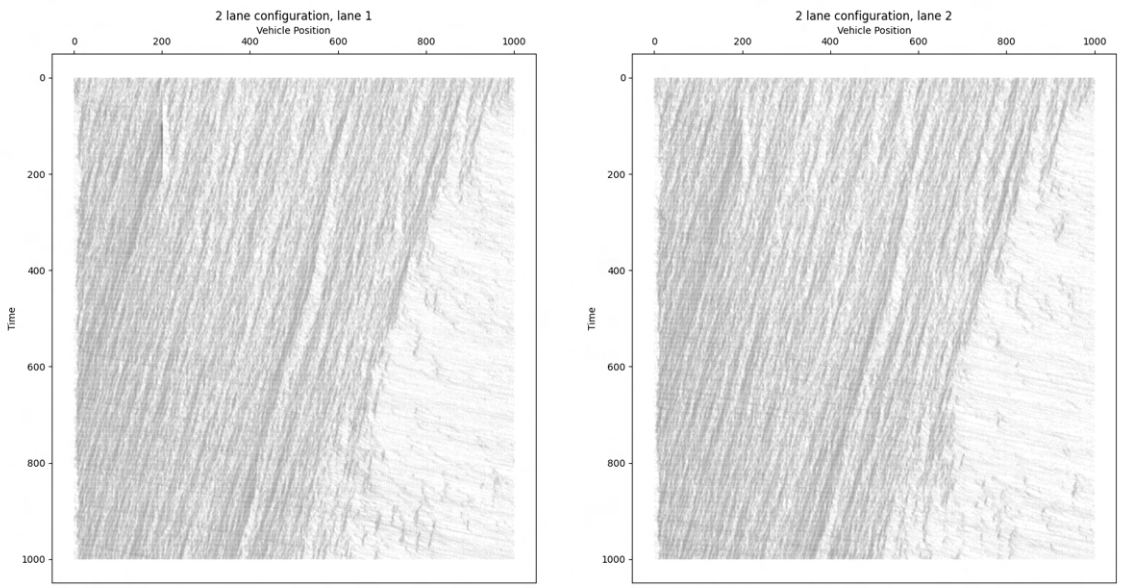


(a) Slow prob = 0.2

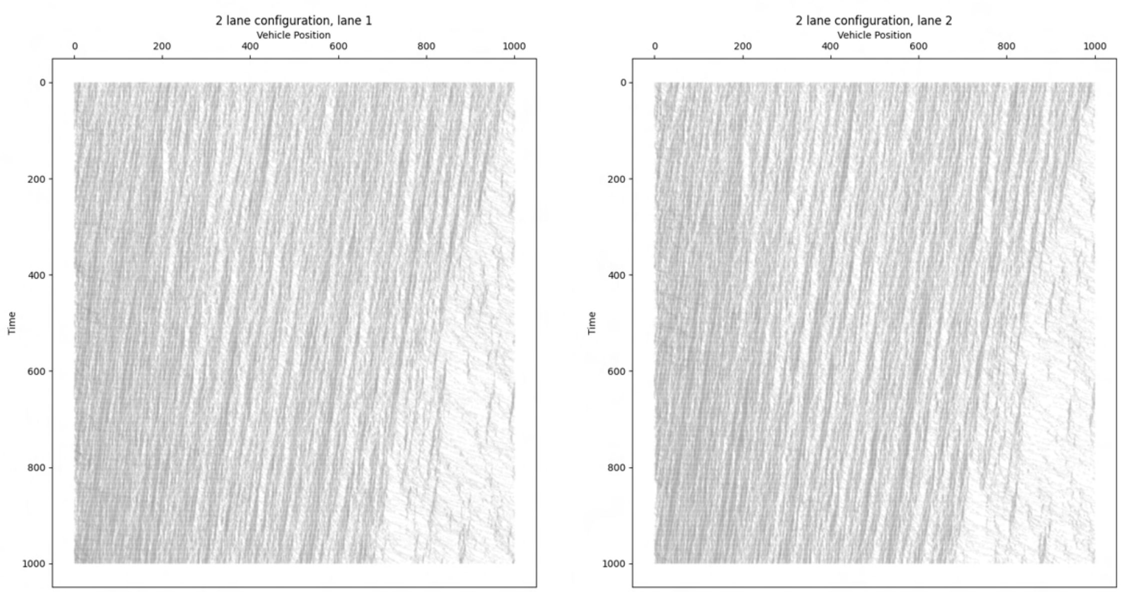


(b) Slow prob = 0.5

Figure 33: Time-space plot, slow probability iteration for a double lane configuration obstacle in lane 1.(1)



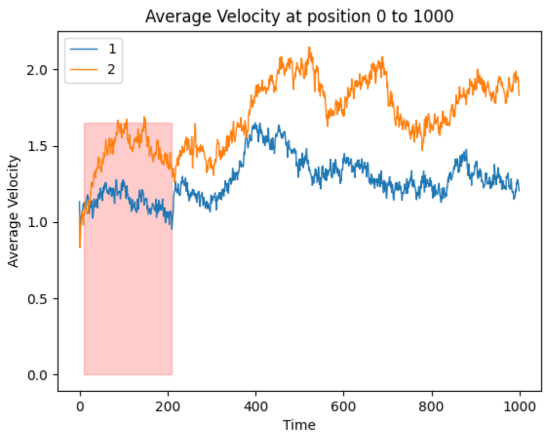
(a) Slow prob = 0.6



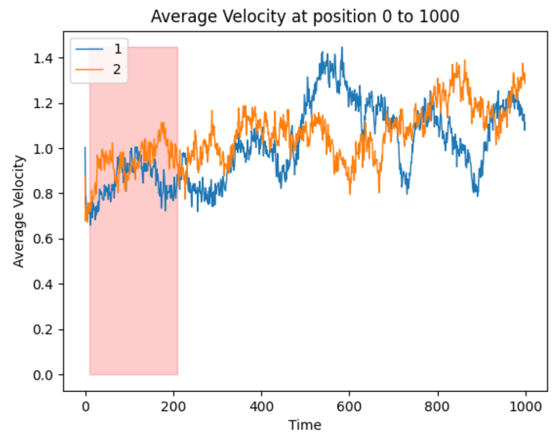
(b) Slow prob = 0.8

Figure 34: Time-space plot, slow probability iteration for a double lane configuration obstacle in lane 1.(2)

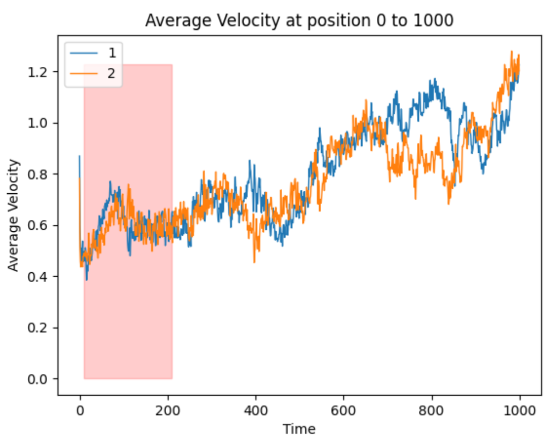
The average velocities are plotted below:



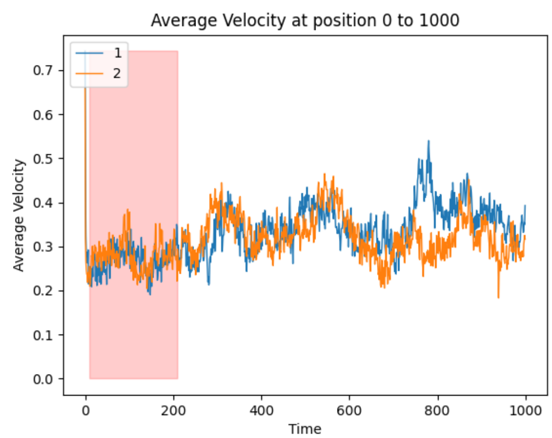
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 35: Average velocity plot, slow probability iteration for a double lane configuration with obstacle in lane 1.

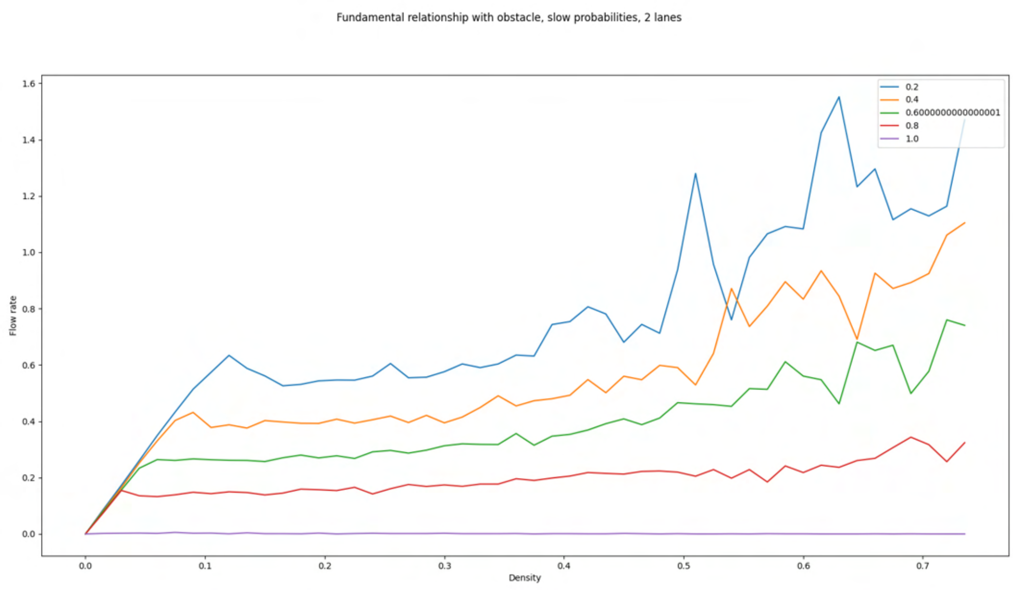


Figure 36: Fundamental relationship graph for double lane as the slow probability is modified with obstacle in lane 1.

4.3 Triple Lane Simulations

In a 3 lane simulation, vehicles in the 2nd lanes are switched to the lane with the lesser number of vehicles.

4.3.1 Maximum Velocity

The maximum velocities are changed and the behaviour is observed.

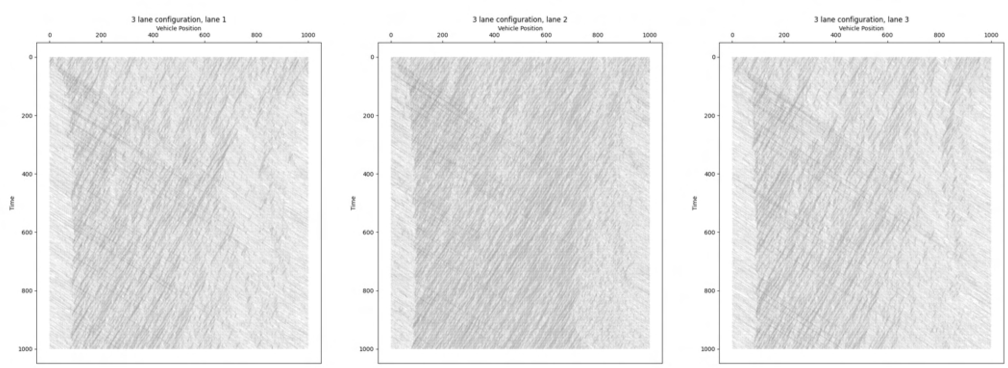


Figure 37: Time-space plot, maximum velocity iteration for a triple lane configuration, max velocity = 2

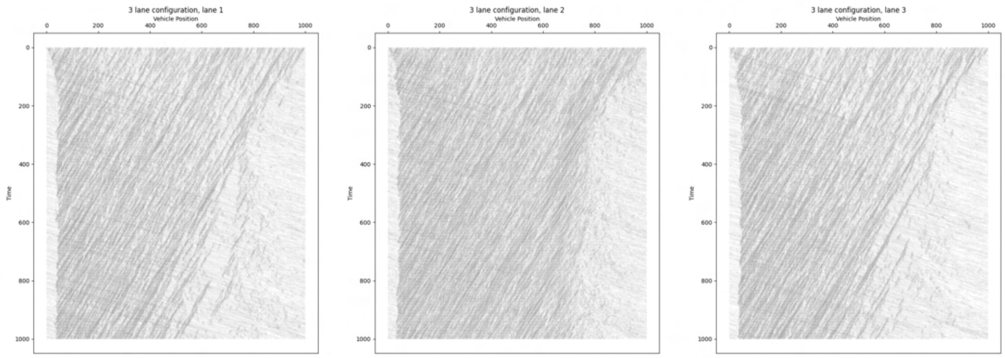


Figure 38: Time-space plot, maximum velocity iteration for a triple lane configuration, max velocity = 4

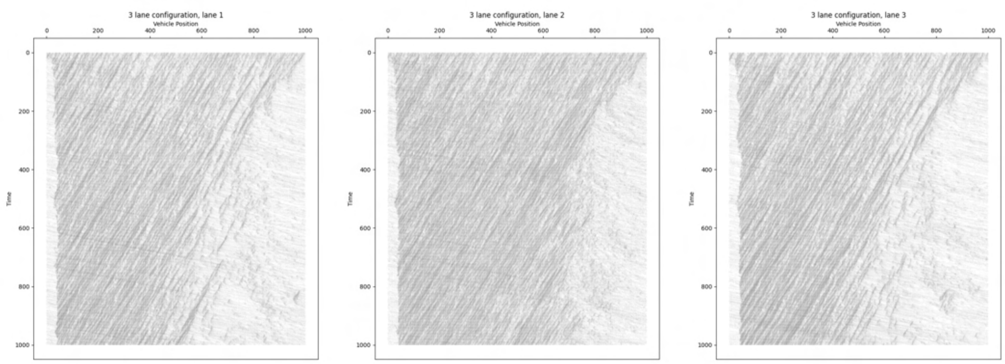


Figure 39: Time-space plot, maximum velocity iteration for a triple lane configuration, max velocity = 6

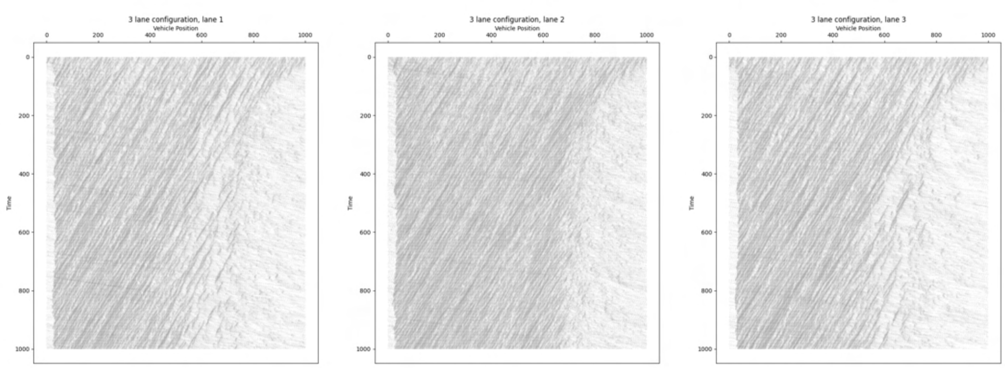


Figure 40: Time-space plot, maximum velocity iteration for a triple lane configuration, max velocity = 8

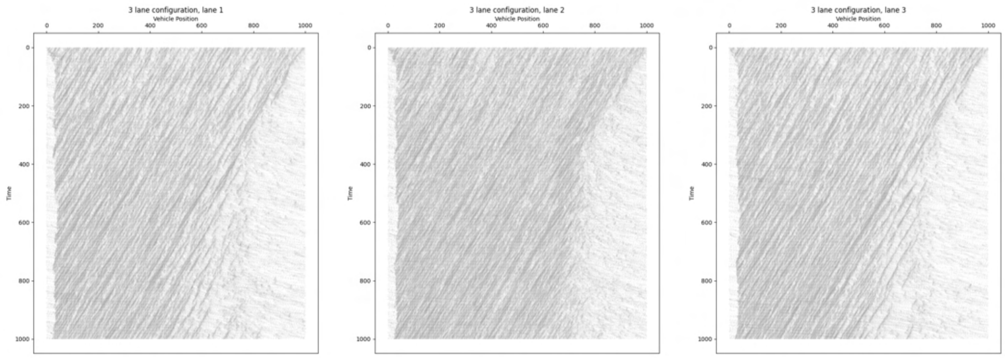


Figure 41: Time-space plot, maximum velocity iteration for a triple lane configuration, max velocity = 10

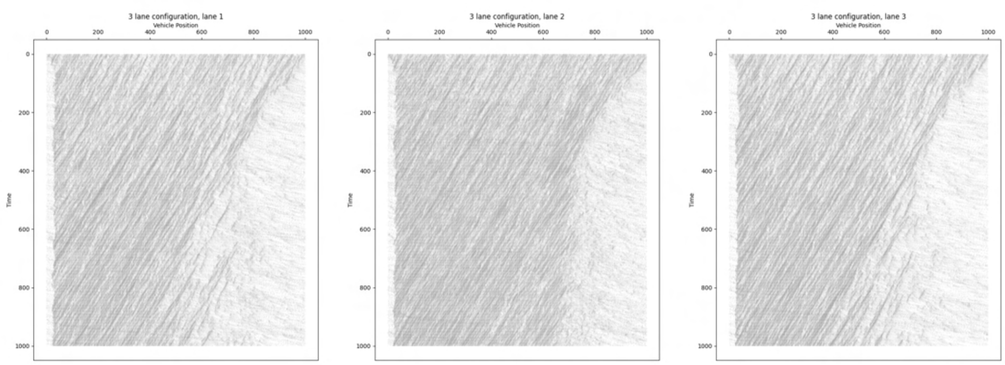
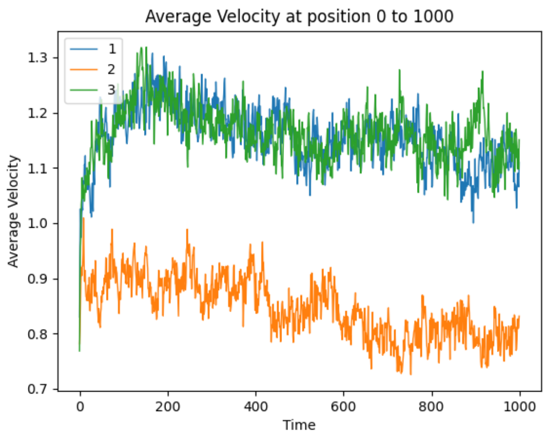
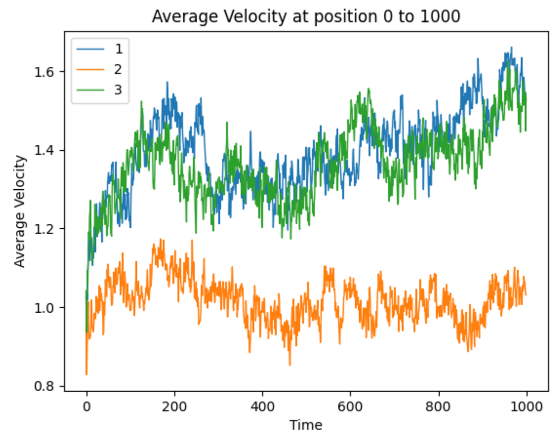


Figure 42: Time-space plot, maximum velocity iteration for a triple lane configuration, max velocity = 12

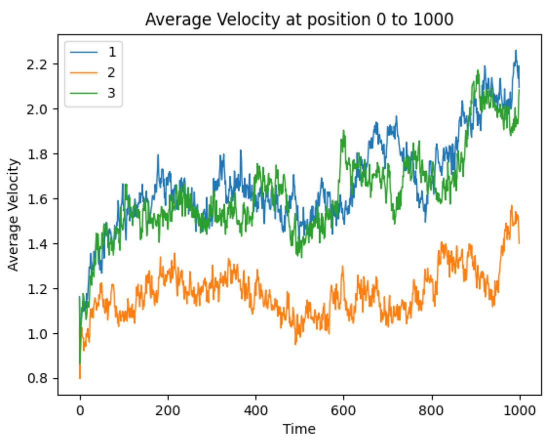
The average velocities are plotted below:



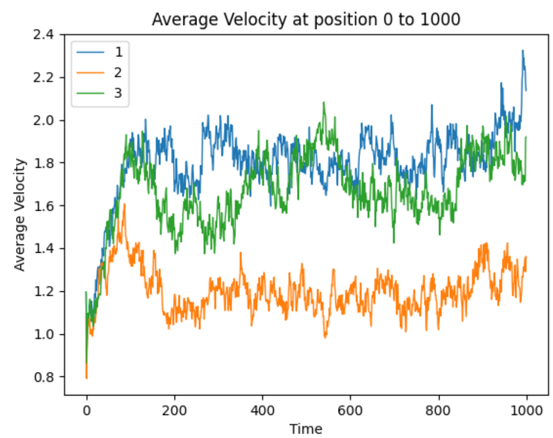
(a) Max velocity = 2



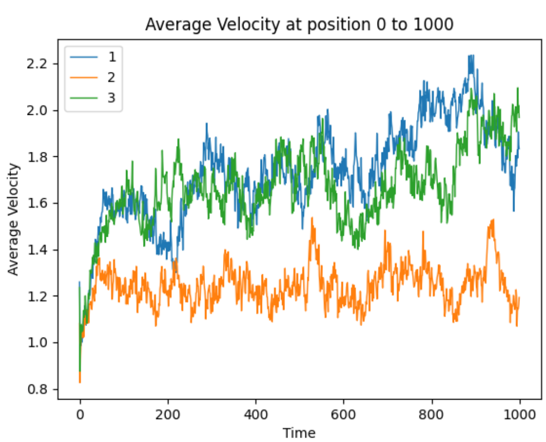
(b) Max velocity = 4



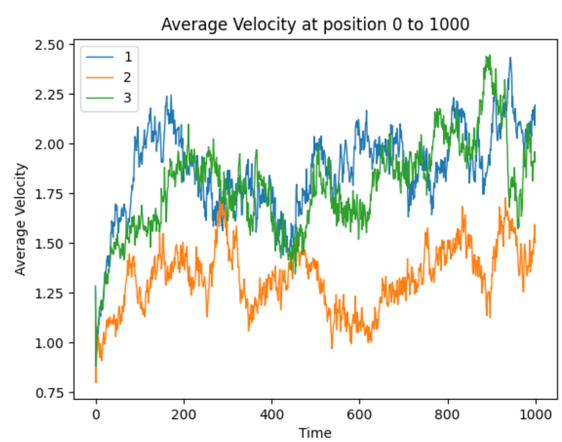
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 43: Average velocity plot, maximum velocity iteration for a triple lane configuration.

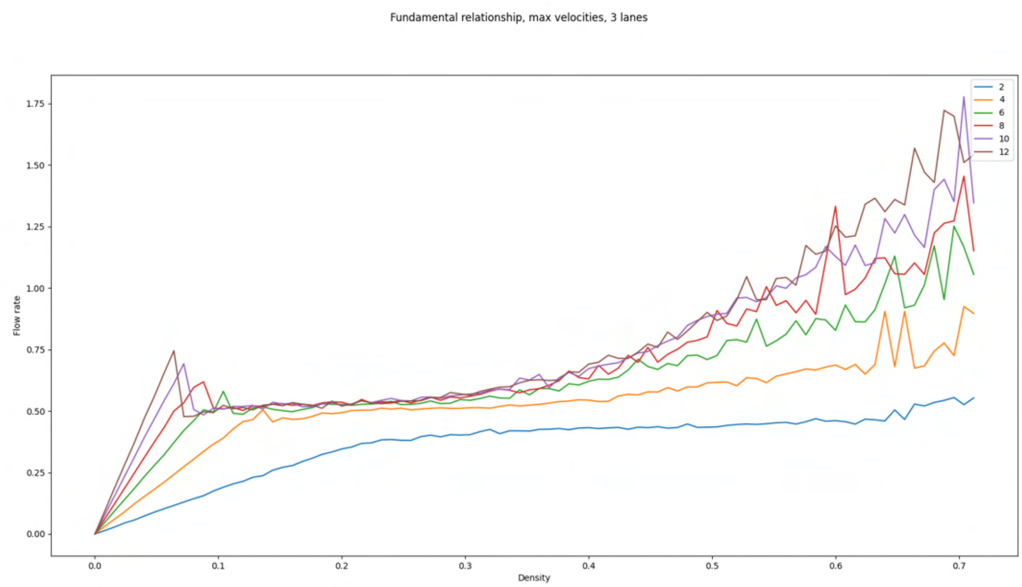


Figure 44: Fundamental relationship graph for triple lanes as the maximum velocities are modified.

With an obstacle in the first lane, the time-space plots are also obtained.

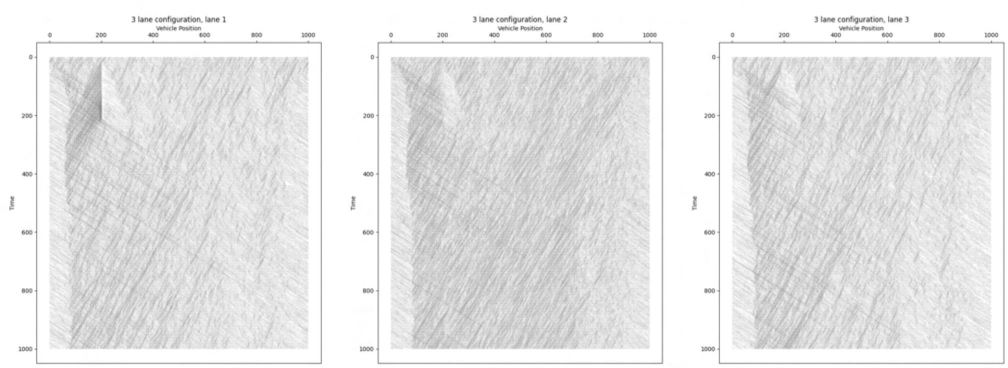


Figure 45: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle, max velocity = 2

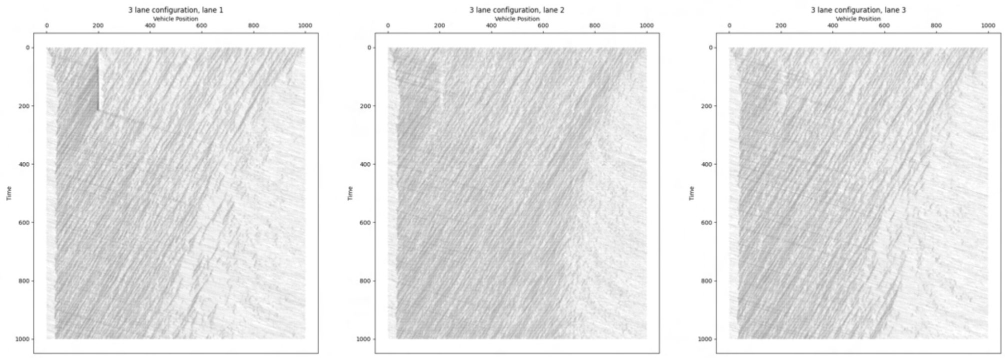


Figure 46: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle, max velocity = 4

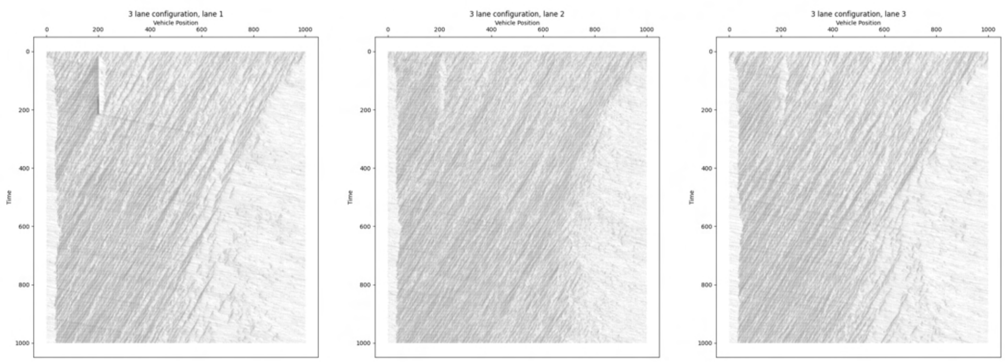


Figure 47: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle, max velocity = 6

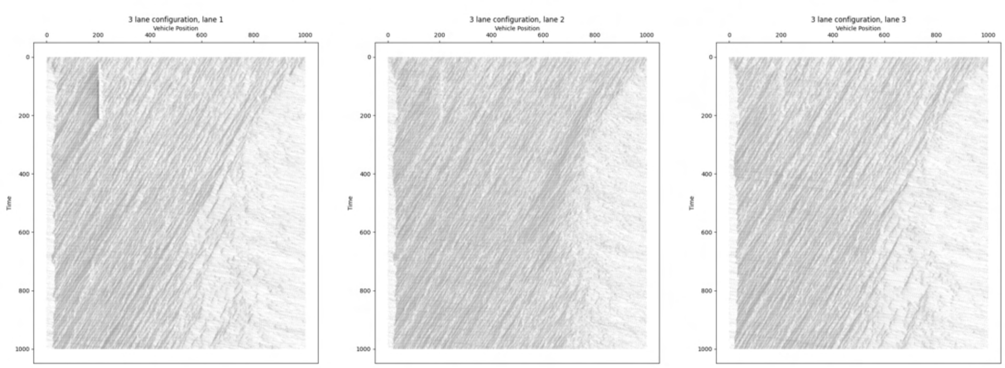


Figure 48: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle, max velocity = 8

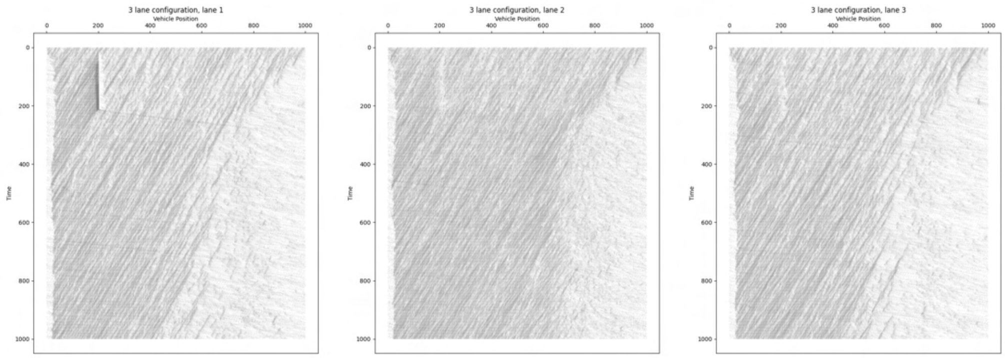


Figure 49: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle, max velocity = 10

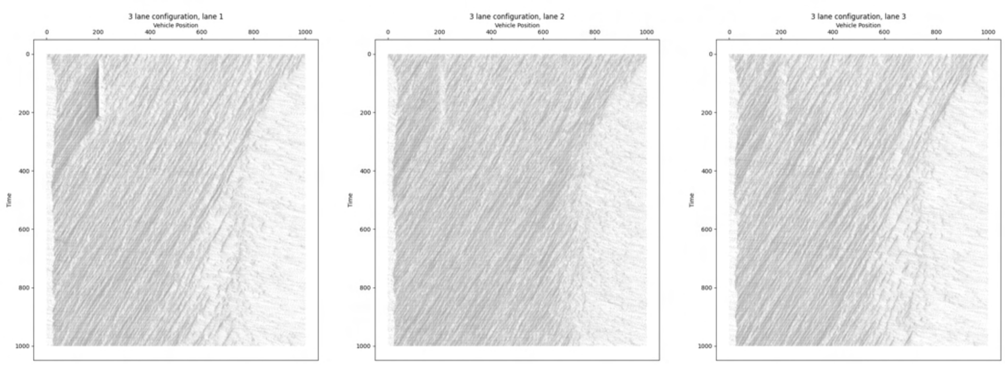
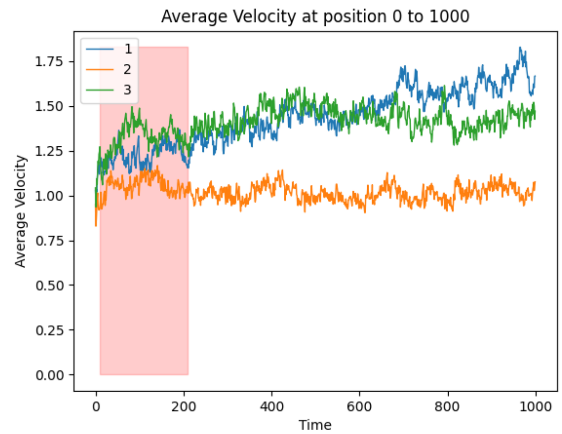


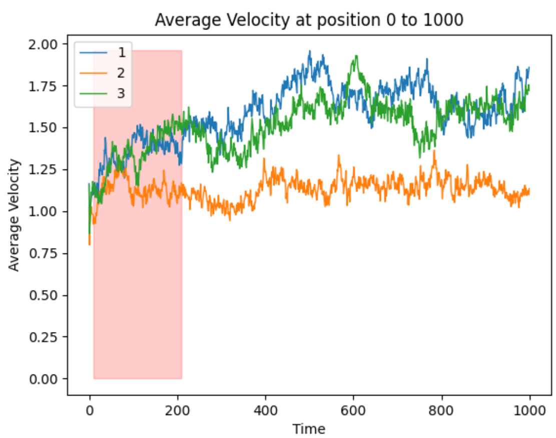
Figure 50: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle, max velocity = 12

The average velocities are plotted below:

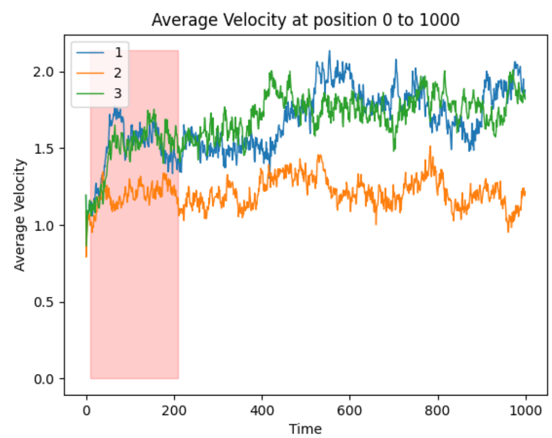
(a) Max velocity = 2



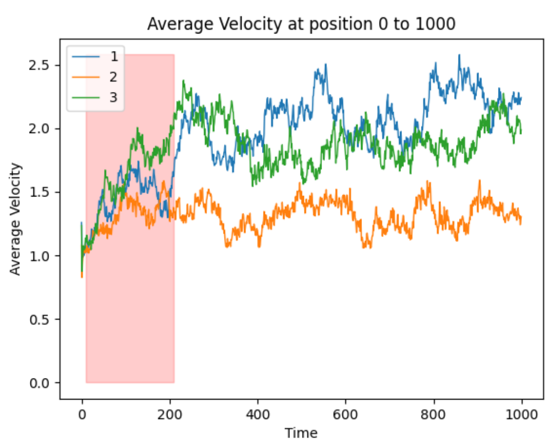
(b) Max velocity = 4



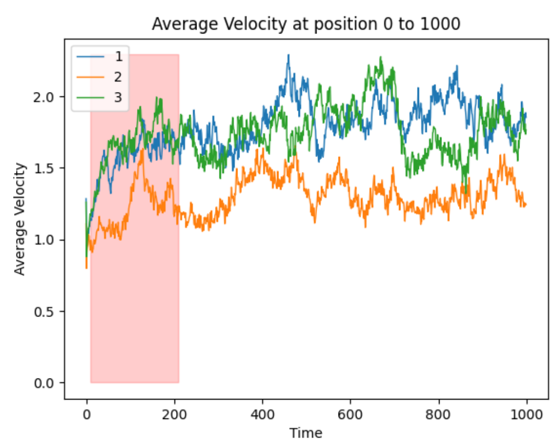
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 51: Average velocity plot, maximum velocity iteration for a triple lane configuration with obstacle in the first lane.

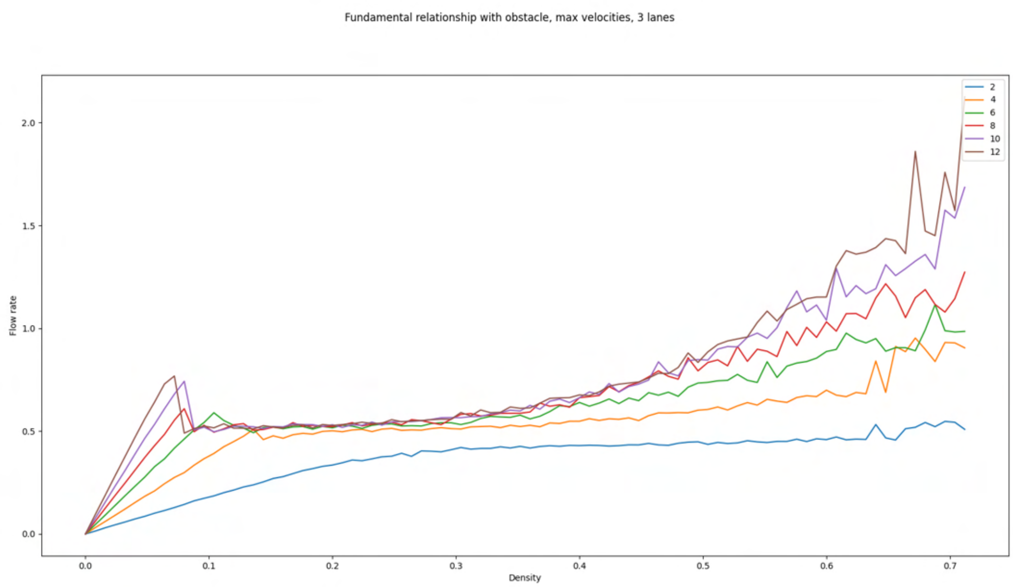


Figure 52: Fundamental relationship graph for triple lanes as the slow probability is modified with obstacle in lane 1.

Since this is a 3 lane configuration, it makes sense to consider the scenario where the obstacle is in the 2nd lane instead of the 1st.

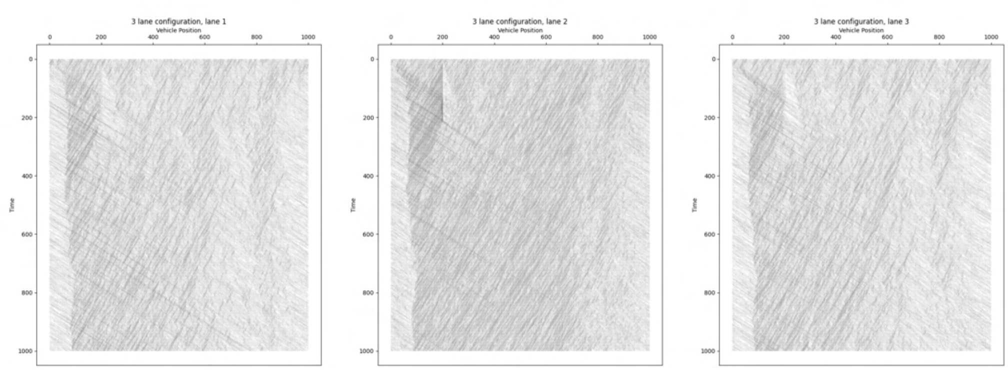


Figure 53: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle in lane 2, max velocity = 2

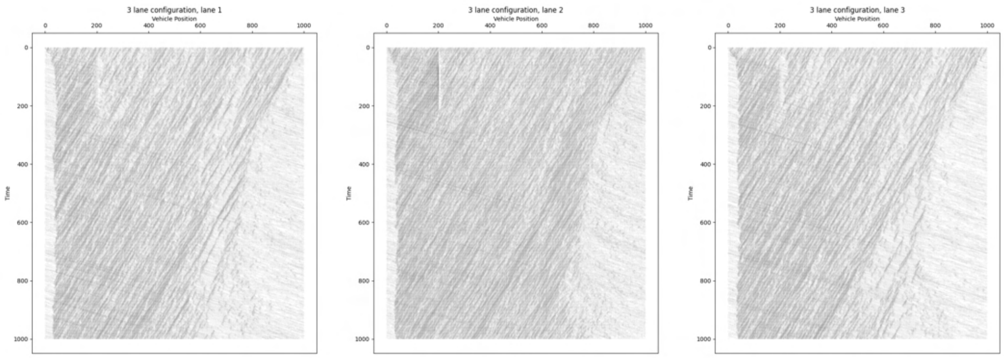


Figure 54: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle in lane 2, max velocity = 4

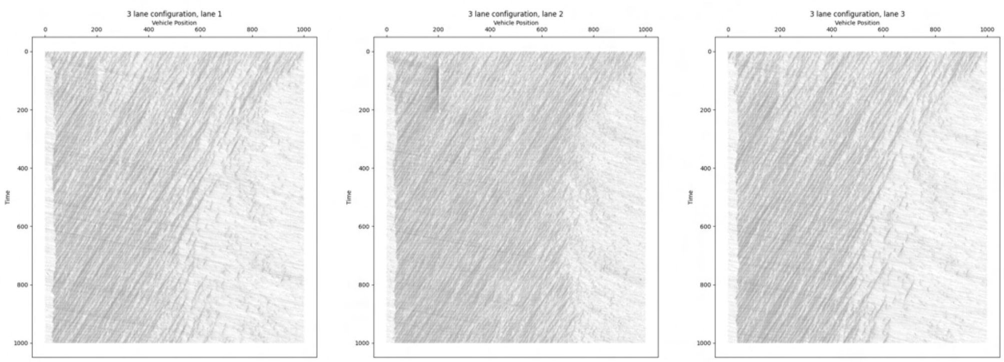


Figure 55: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle in lane 2, max velocity = 6

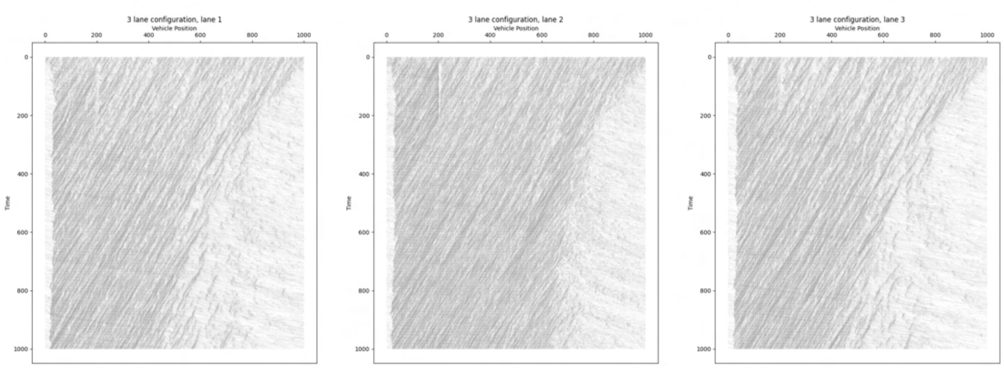


Figure 56: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle in lane 2, max velocity = 8

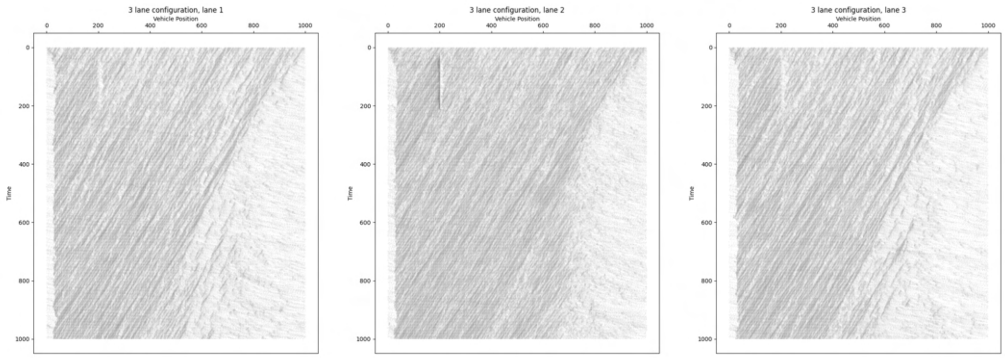


Figure 57: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle in lane 2, max velocity = 10

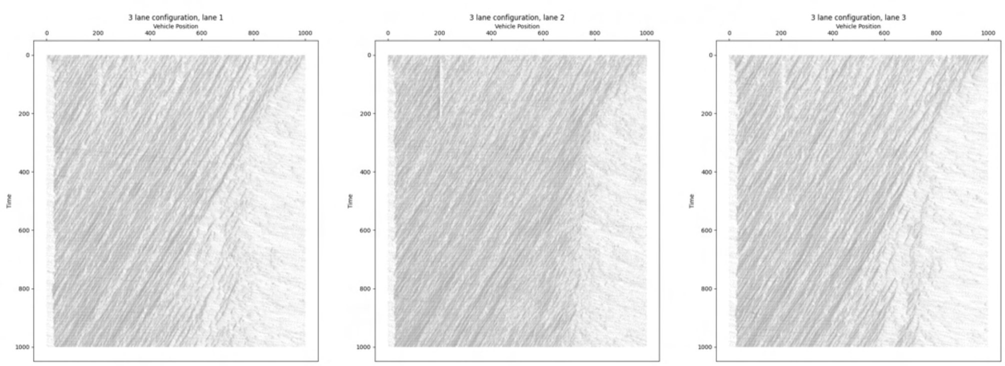
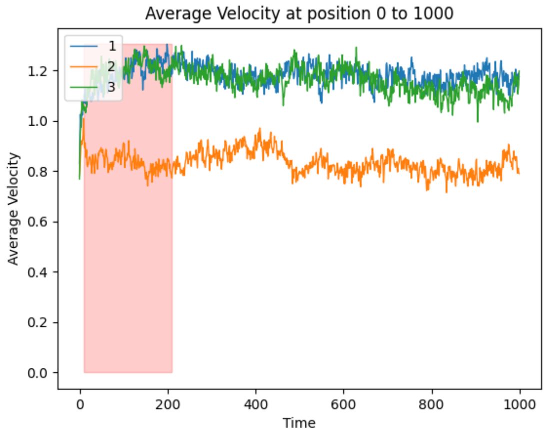


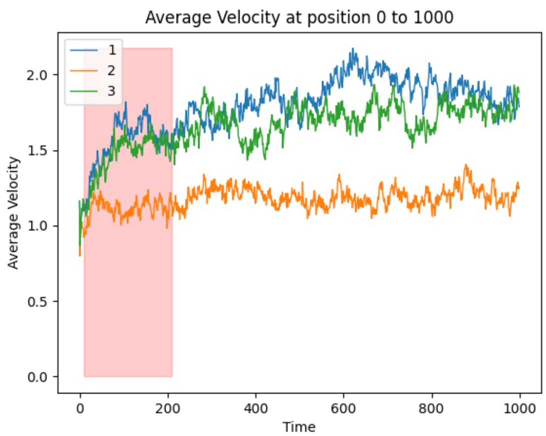
Figure 58: Time-space plot, maximum velocity iteration for a triple lane configuration with obstacle in lane 2, max velocity = 12

The average velocities are plotted below:

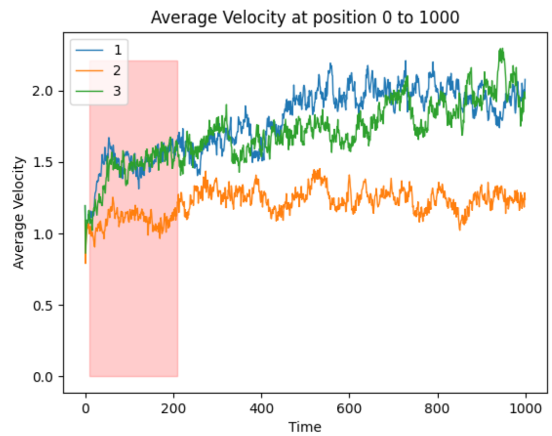


(a) Max velocity = 2

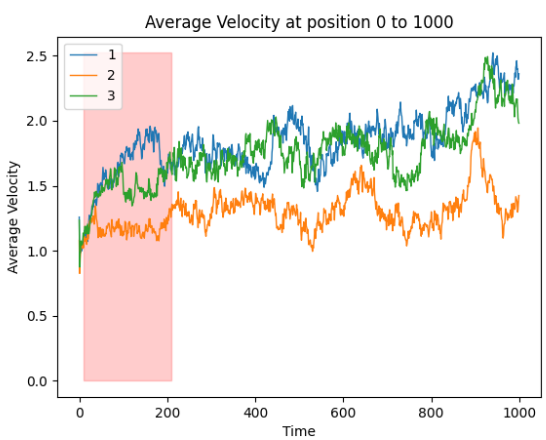
(b) Max velocity = 4



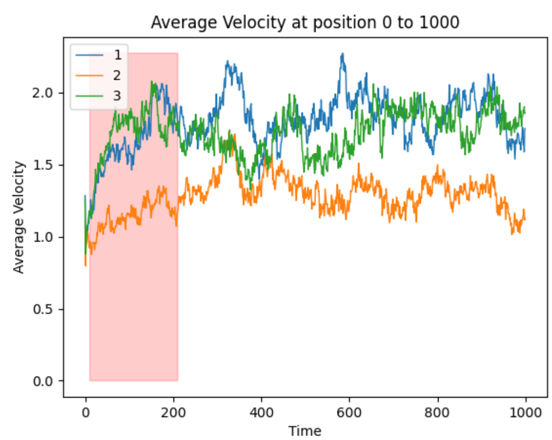
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 59: Average velocity plot, maximum velocity iteration for a triple lane configuration with obstacle in the second lane.

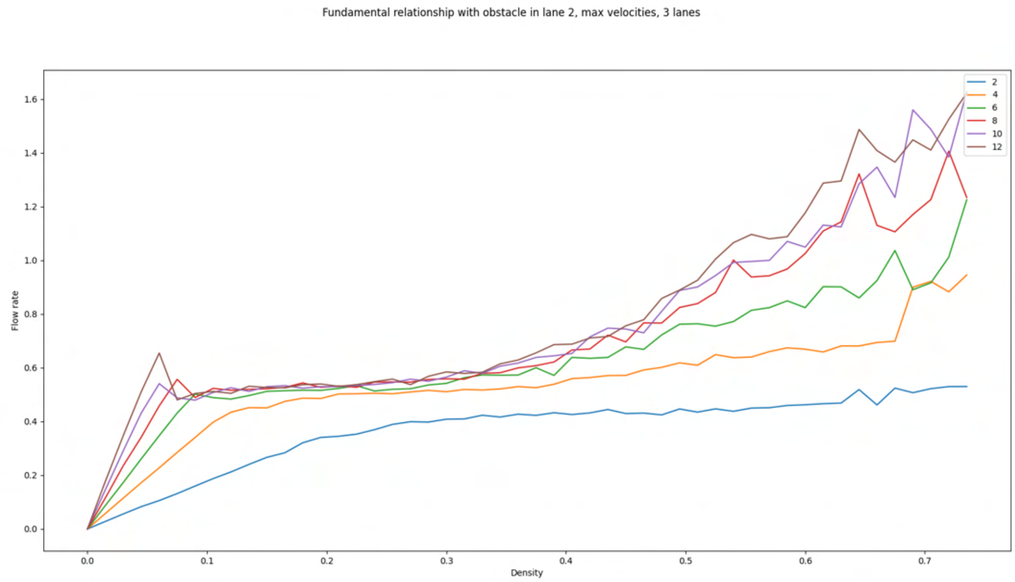


Figure 60: Fundamental relationship graph for triple lanes as the slow probability is modified with obstacle in lane 2.

4.3.2 Slowing Probabilities

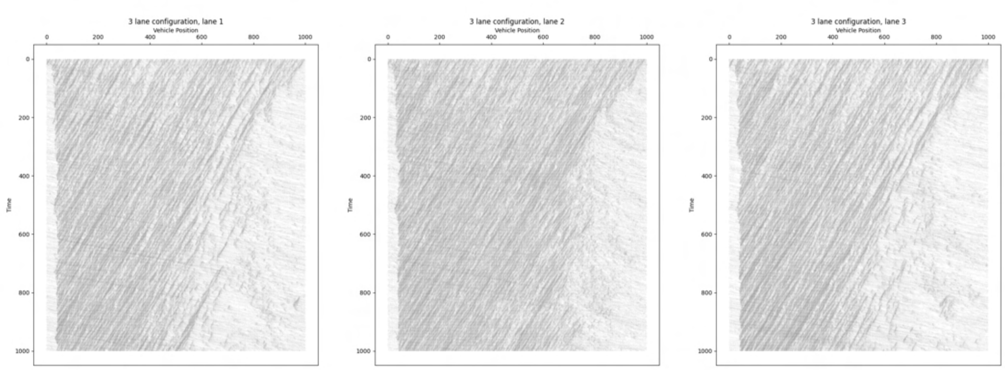


Figure 61: Time-space plot, slow probability iteration for a triple lane configuration, slow probability = 0.2

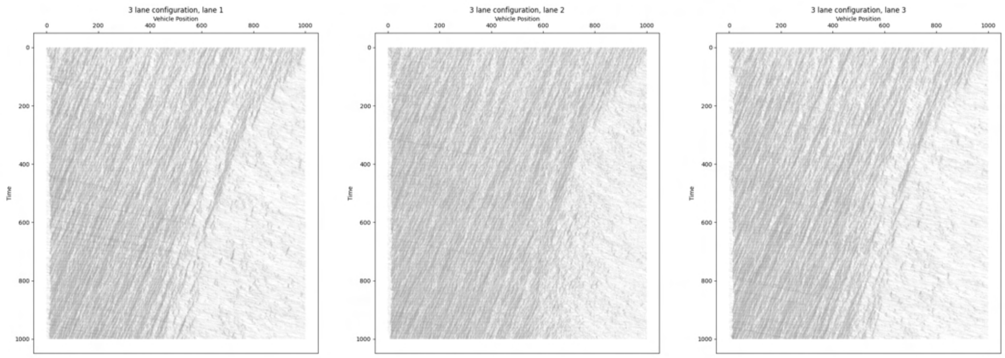


Figure 62: Time-space plot, slow probability iteration for a triple lane configuration, slow probability = 0.4

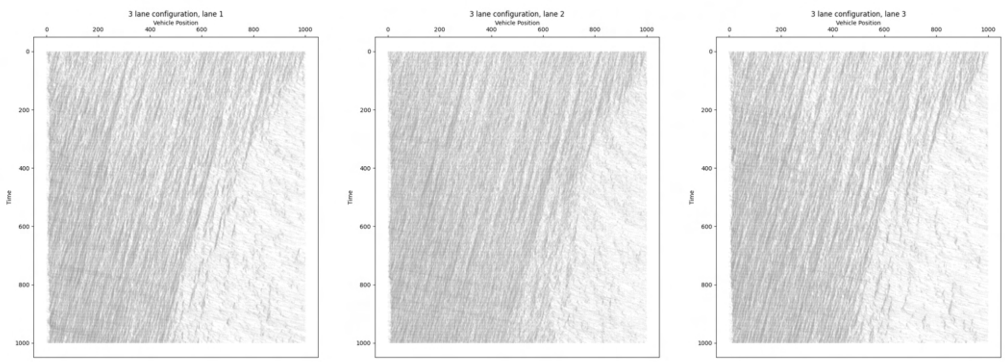


Figure 63: Time-space plot, slow probability iteration for a triple lane configuration, slow probability = 0.6

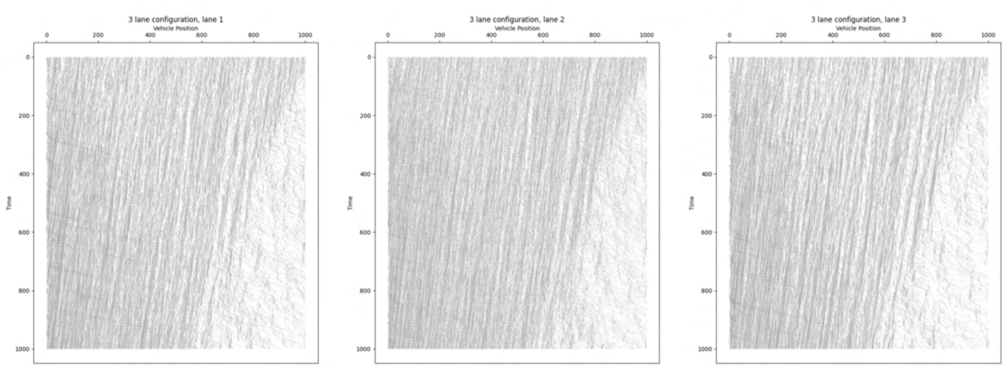
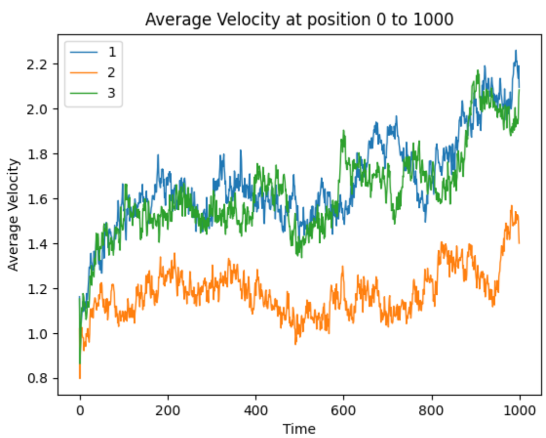
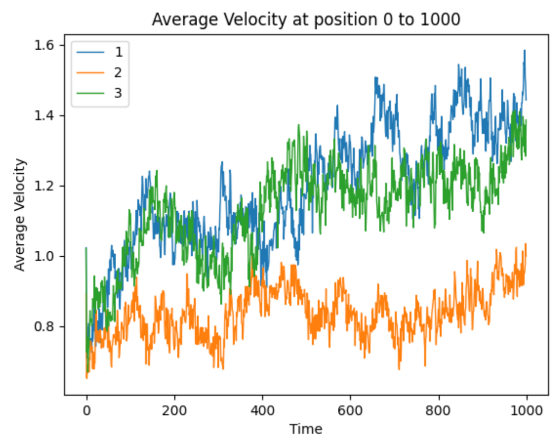


Figure 64: Time-space plot, slow probability iteration for a triple lane configuration, slow probability = 0.8

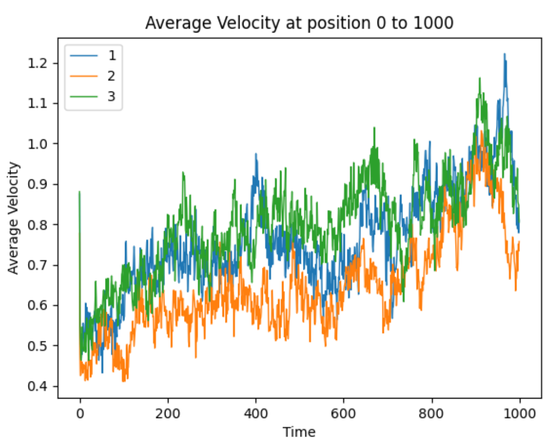
The average velocities are plotted below:



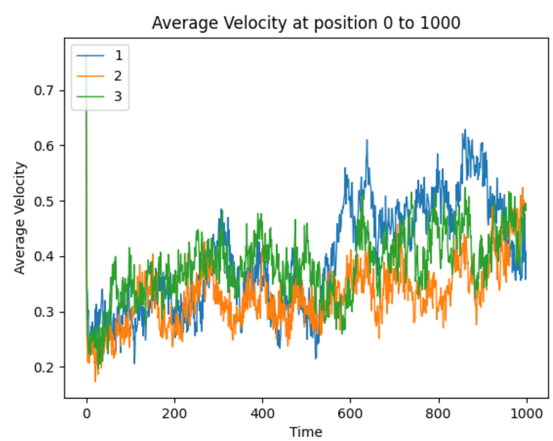
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 65: Average velocity plot, slow probability iteration for a triple lane configuration.

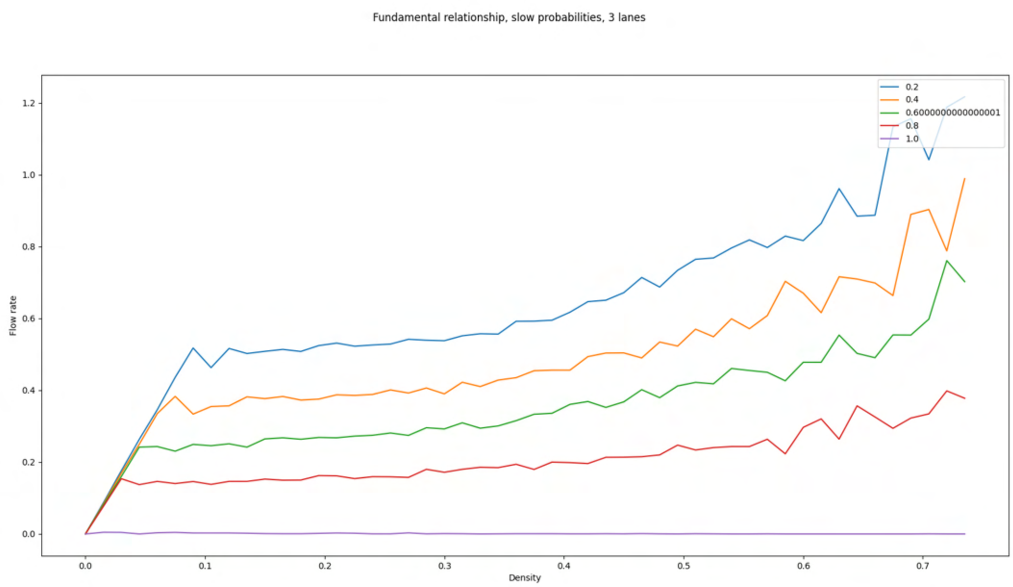


Figure 66: Fundamental relationship graph for triple lanes as the slow probability is modified.

With obstacles, the time-space plots are also obtained.

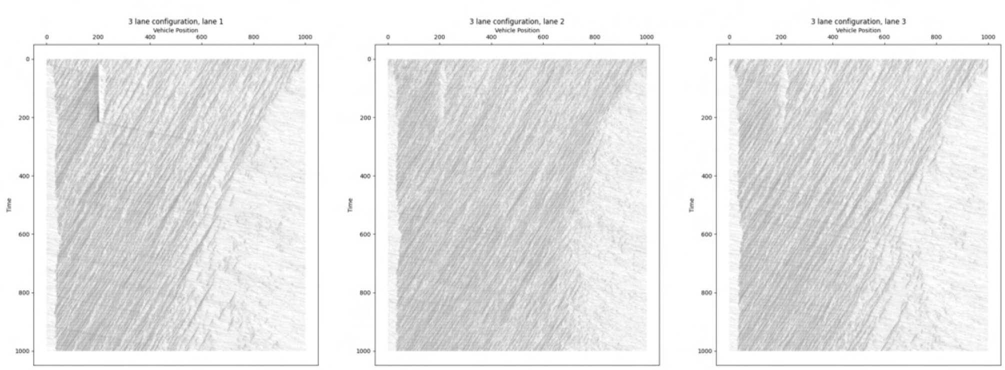


Figure 67: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 1, slow probability = 0.2

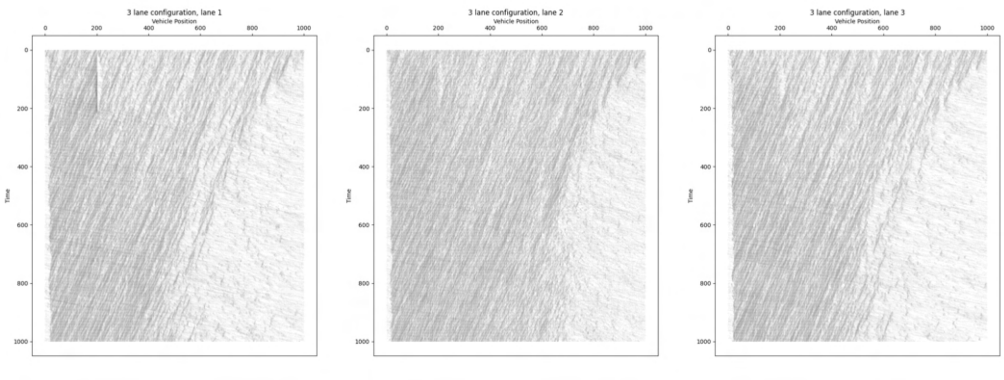


Figure 68: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 1, slow probability = 0.4

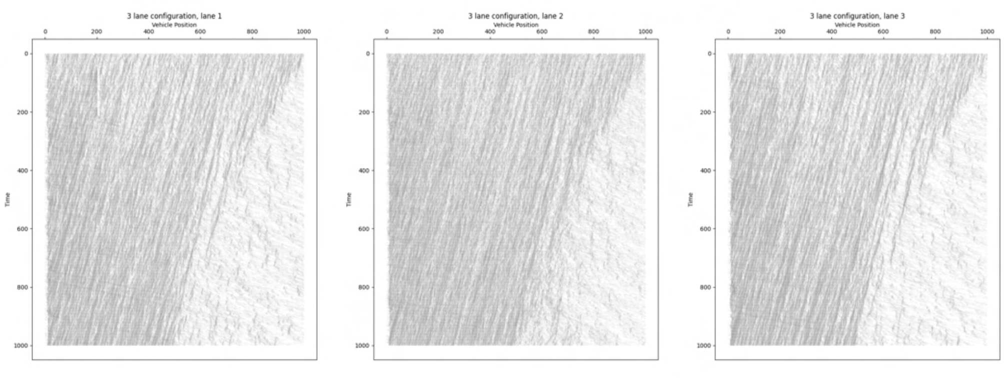


Figure 69: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 1, slow probability = 0.6

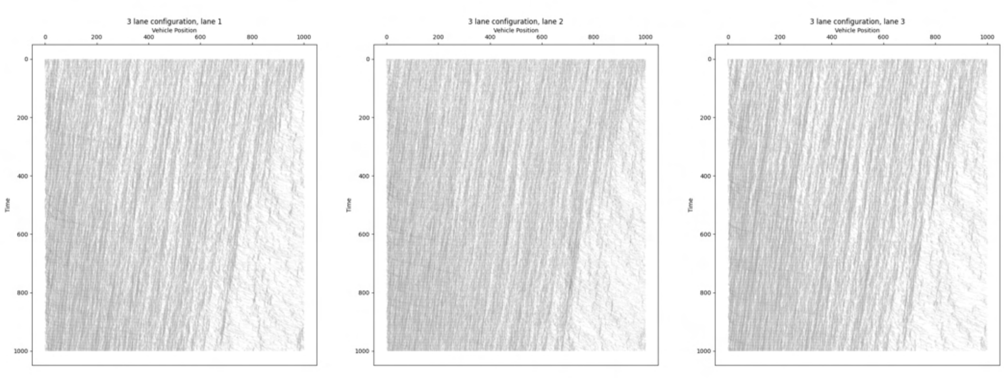
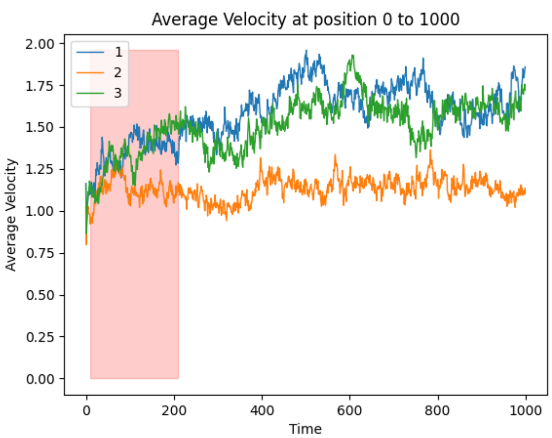
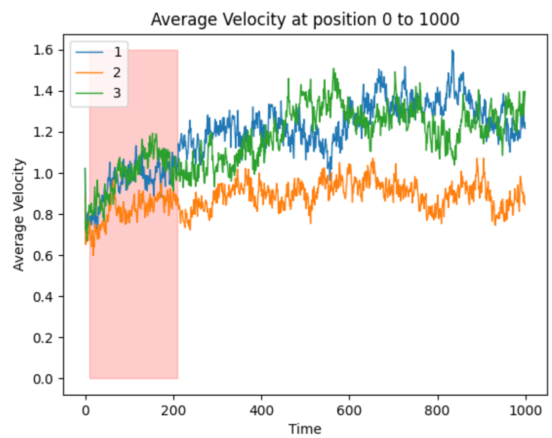


Figure 70: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 1, slow probability = 0.8

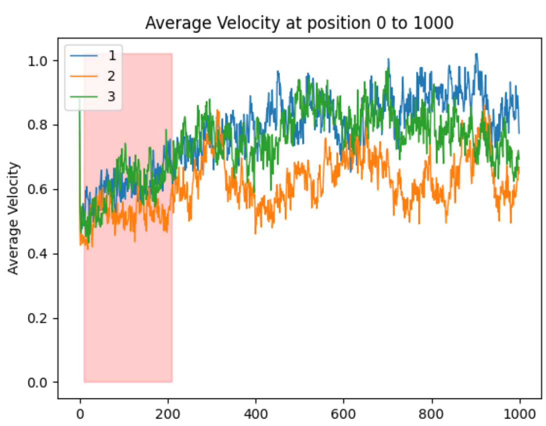
The average velocities are plotted below:



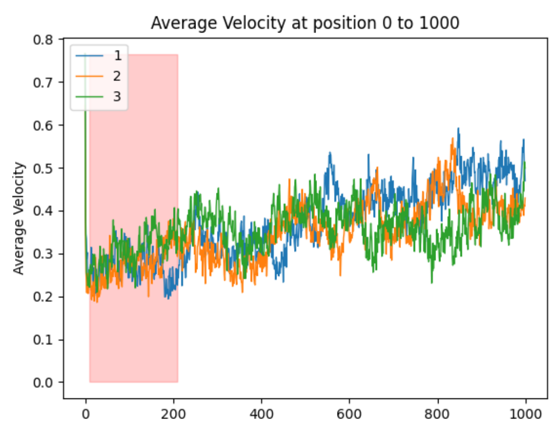
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 71: Average velocity plot, slow probability iteration for a triple lane configuration.

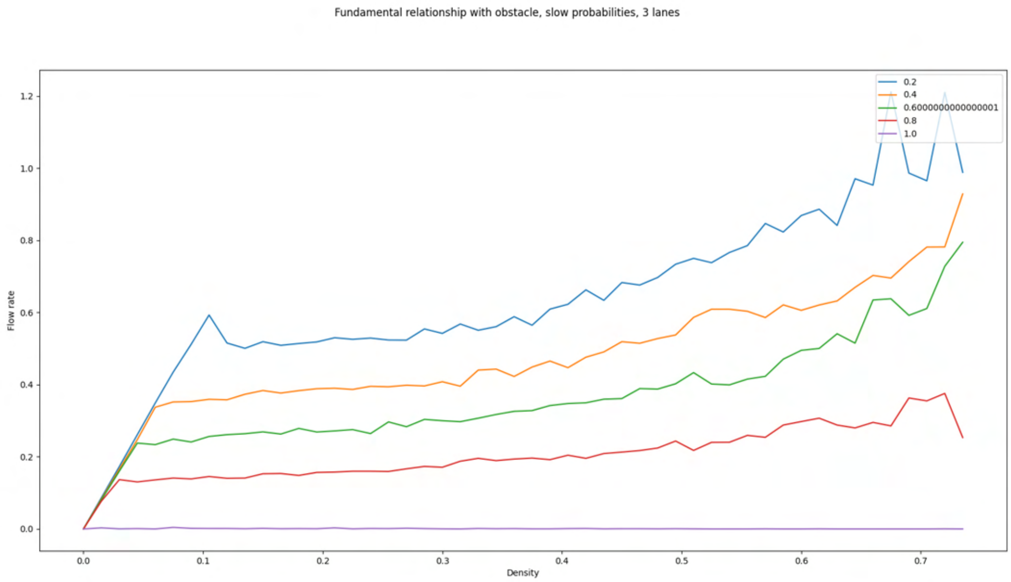


Figure 72: Fundamental relationship graph for triple lanes as the slow probability is modified with obstacle in lane 1.

Likewise,

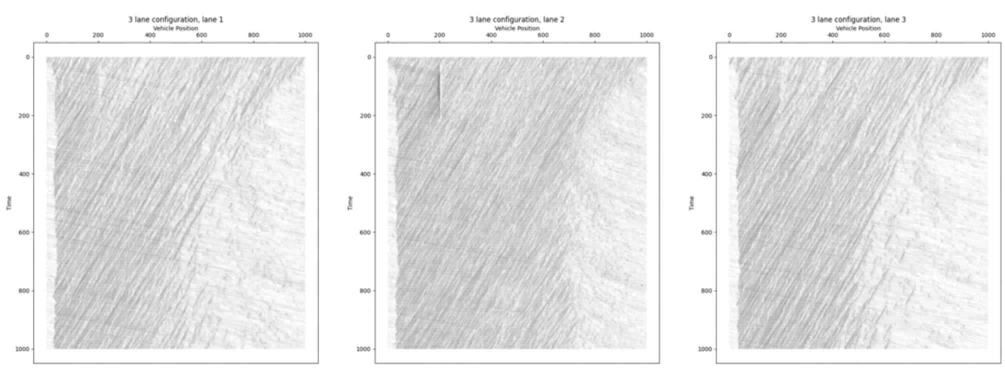


Figure 73: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 2, slow probability = 0.2

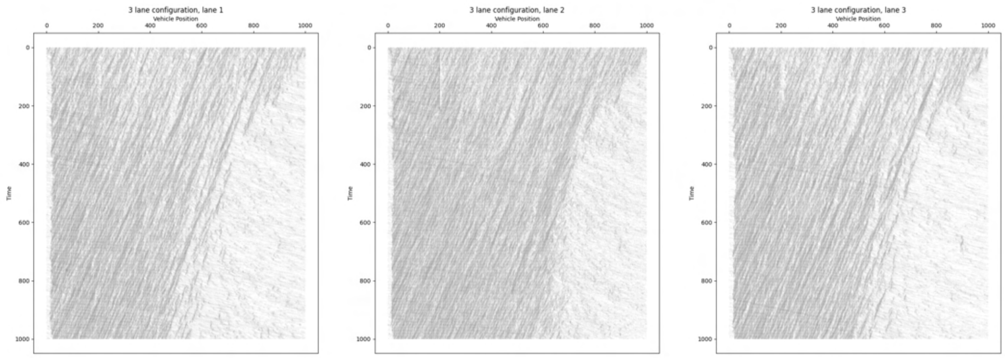


Figure 74: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 2, slow probability = 0.4

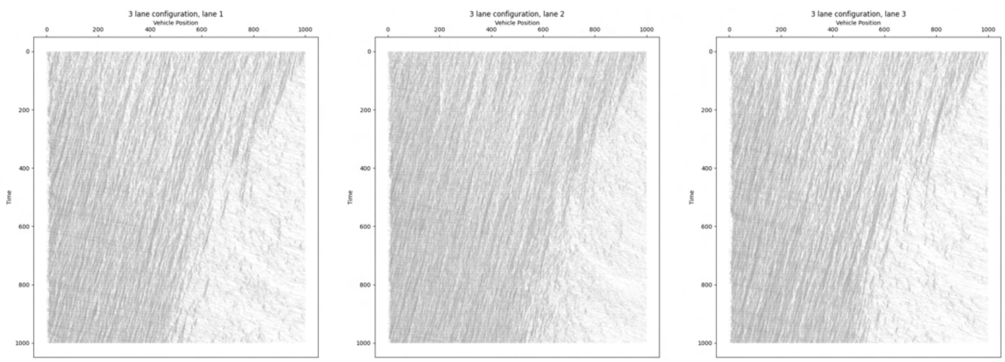


Figure 75: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 2, slow probability = 0.6

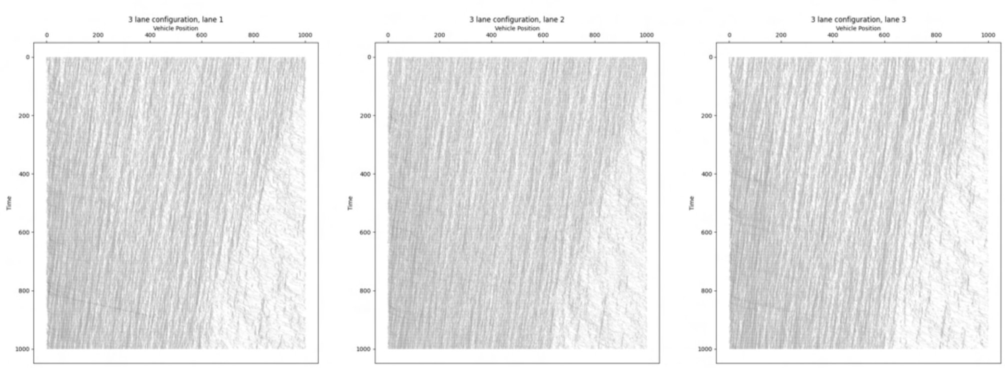
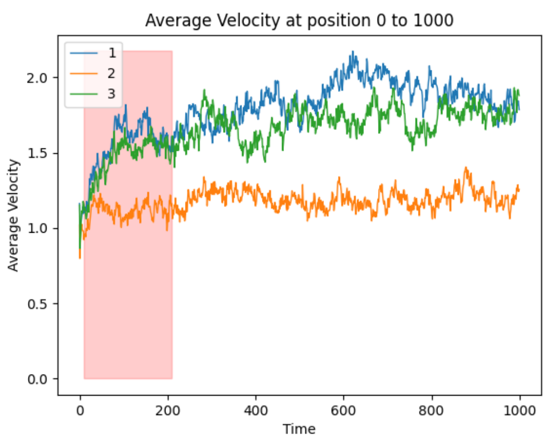
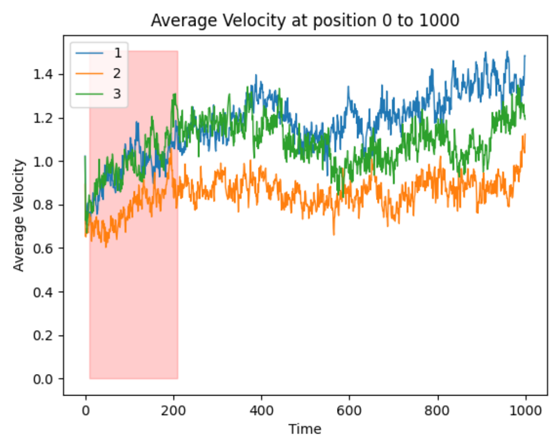


Figure 76: Time-space plot, slow probability iteration for a triple lane configuration with obstacle in lane 2, slow probability = 0.8

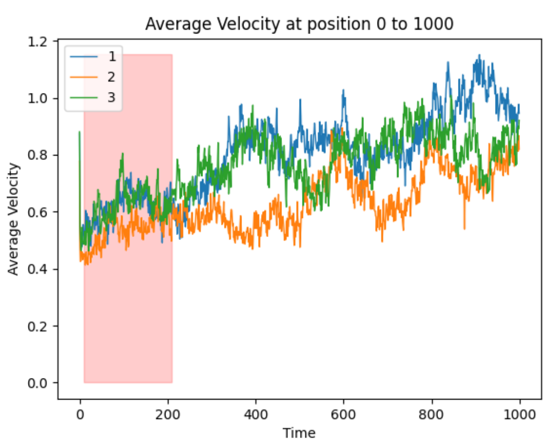
The average velocities are plotted below:



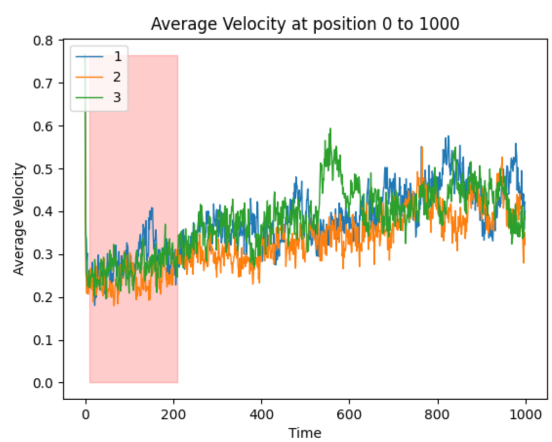
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 77: Average velocity plot, slow probability iteration for a triple lane configuration with obstacle in lane 2.

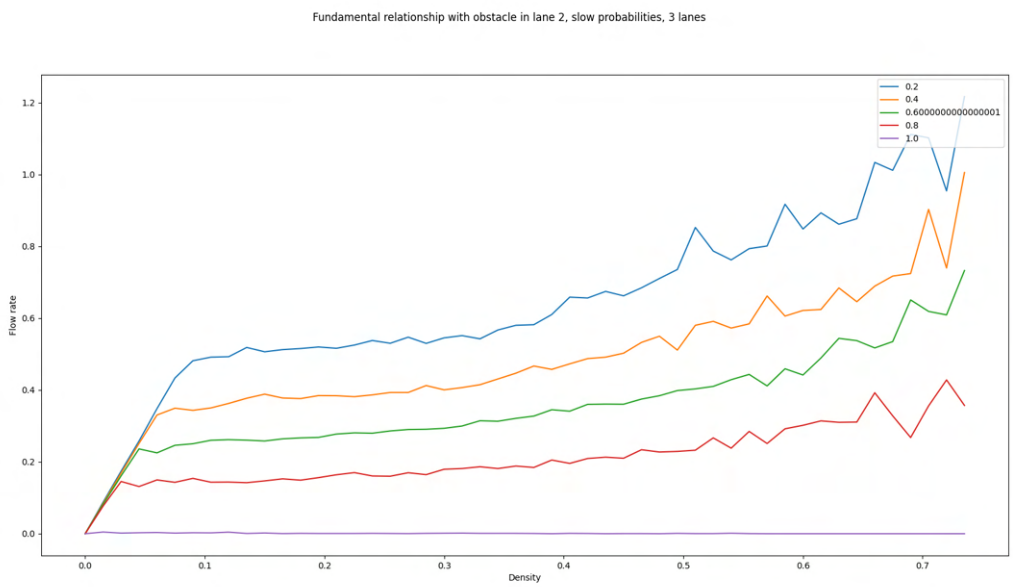


Figure 78: Fundamental relationship graph for triple lanes as the slow probability is modified with obstacle in lane 2.

4.4 Quadruple Lane Simulations

In a 4 lane simulation, vehicles in the 2nd and 3rd lane are switched to the lane with the lesser number of vehicles. The same simulations are run.

4.4.1 Maximum Velocity

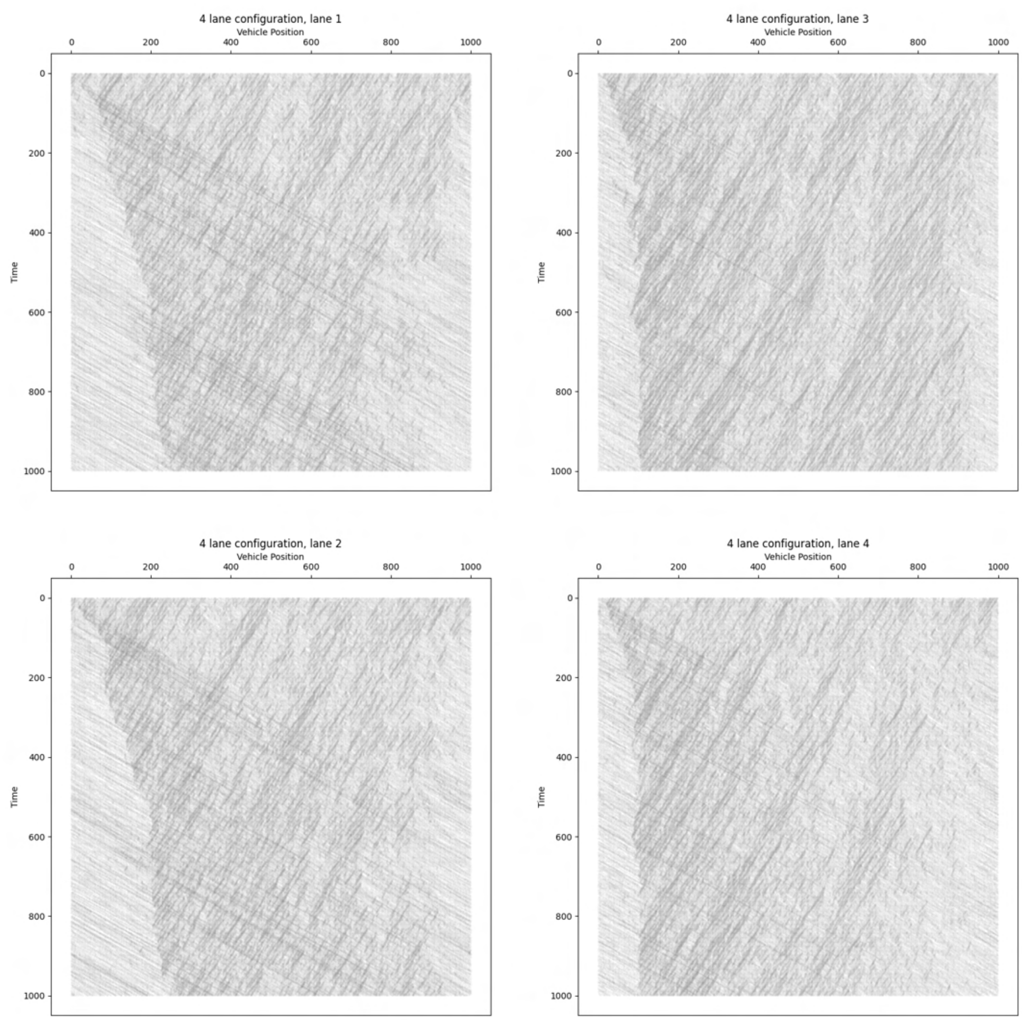


Figure 79: Time-space plot, maximum velocity iteration for a quadruple lane configuration, max velocity = 2

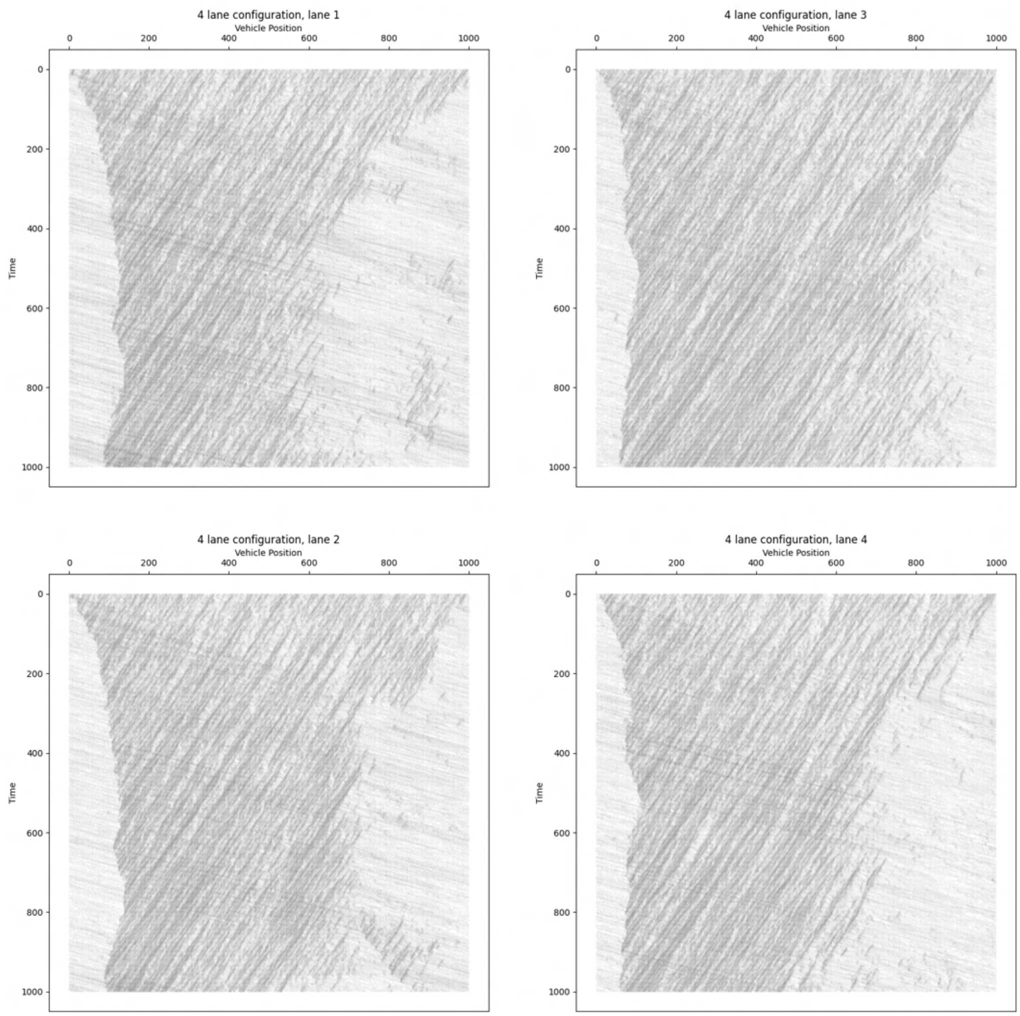


Figure 80: Time-space plot, maximum velocity iteration for a quadruple lane configuration, max velocity = 4

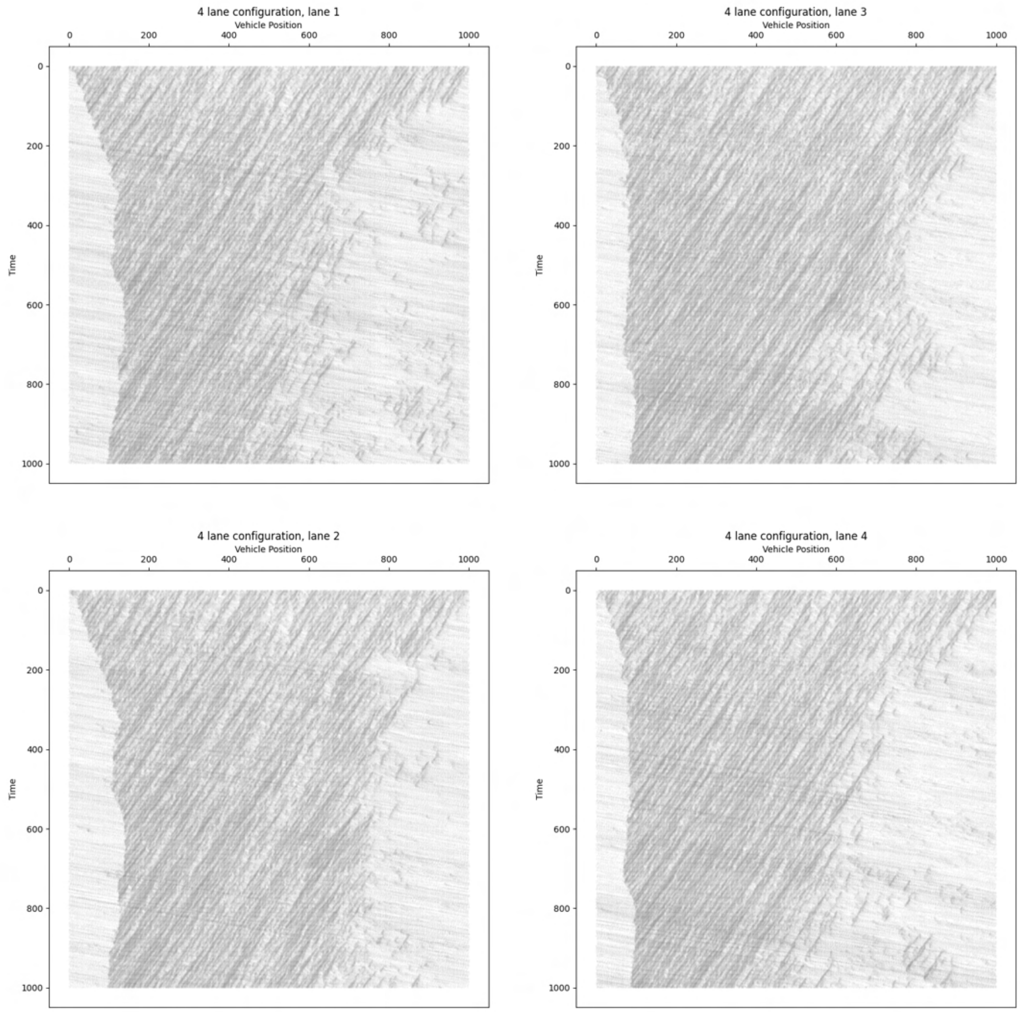


Figure 81: Time-space plot, maximum velocity iteration for a quadruple lane configuration, max velocity = 6

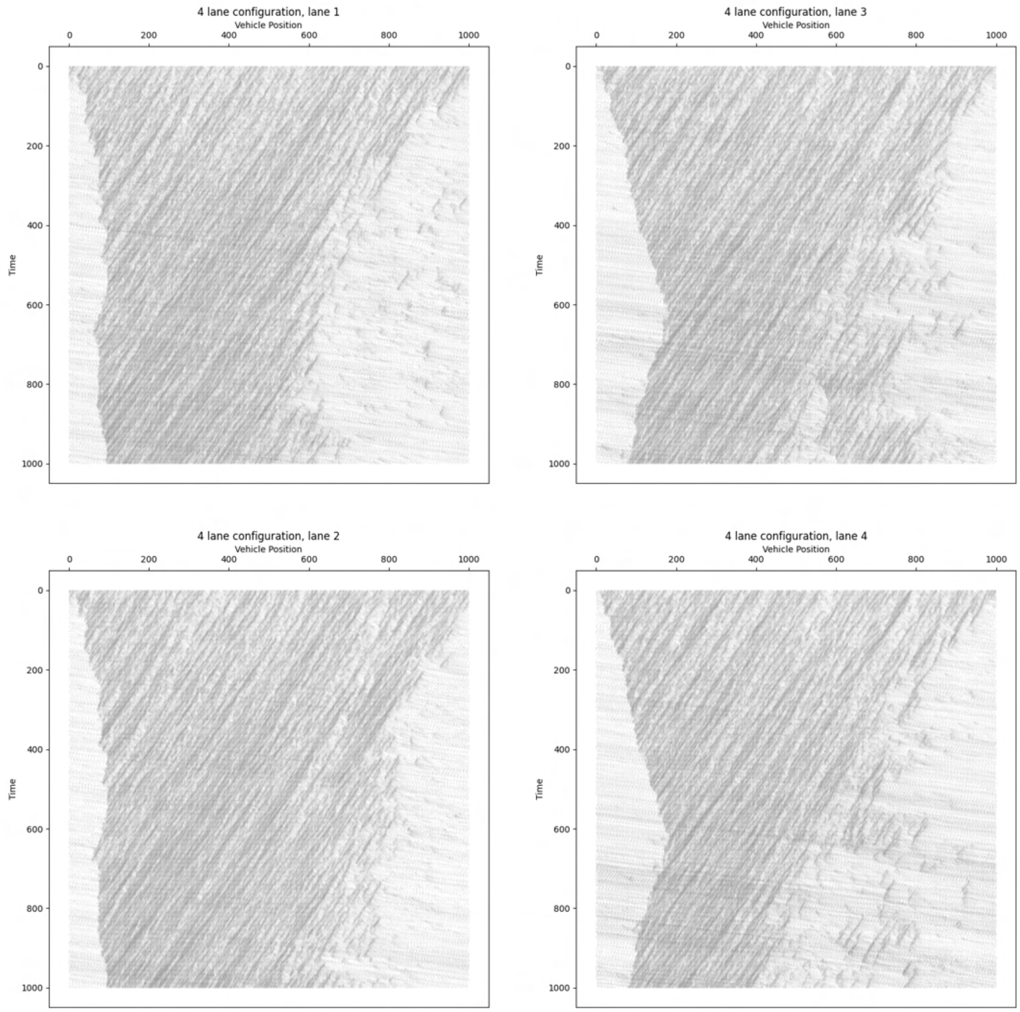


Figure 82: Time-space plot, maximum velocity iteration for a quadruple lane configuration, max velocity = 8

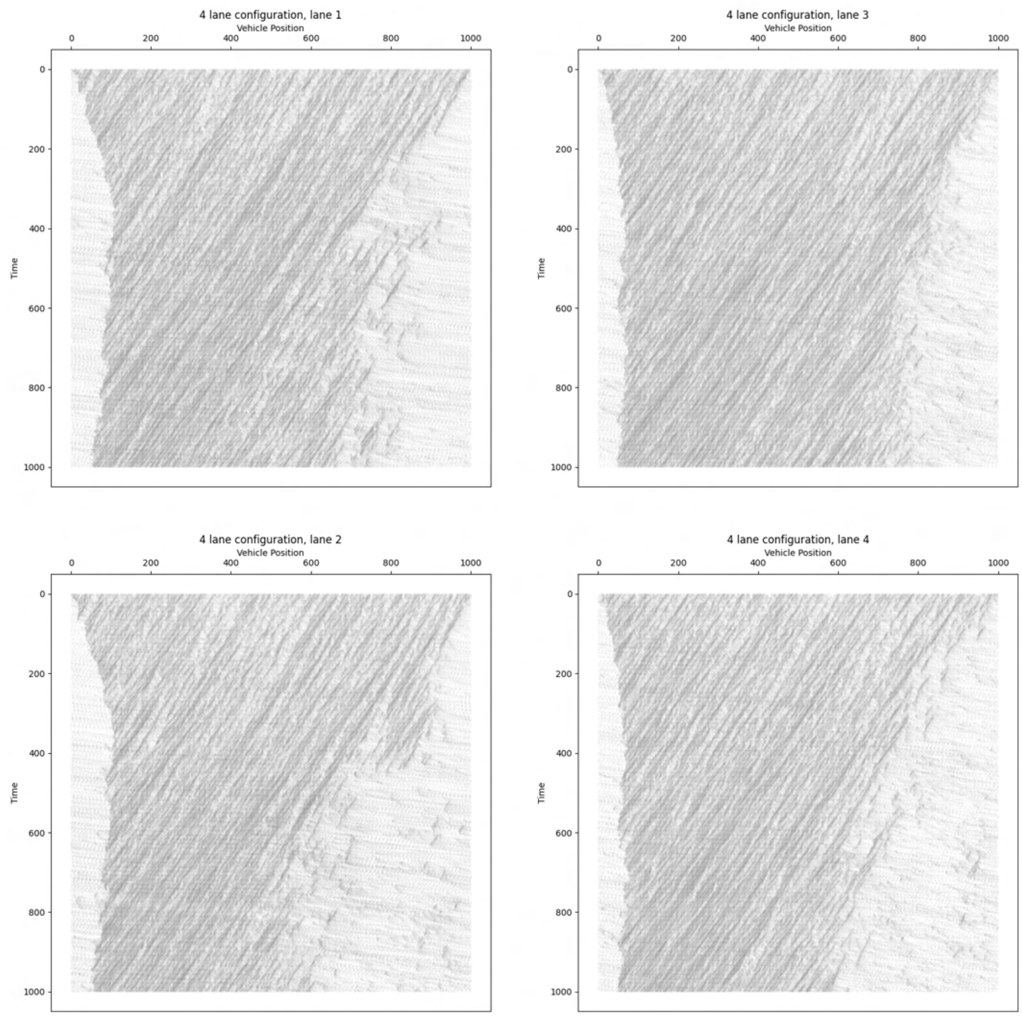


Figure 83: Time-space plot, maximum velocity iteration for a quadruple lane configuration, max velocity = 10

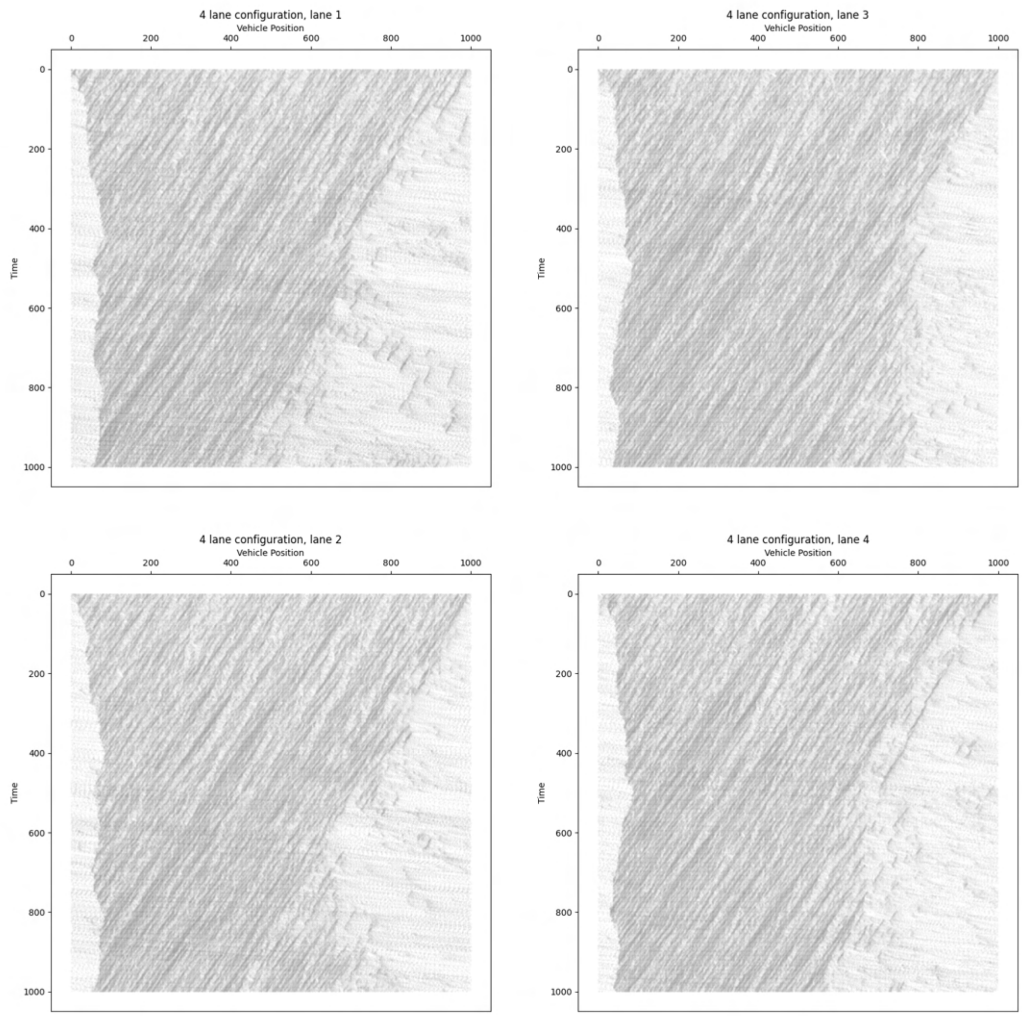
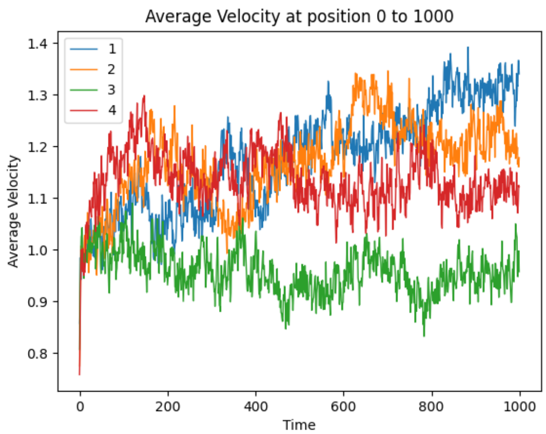
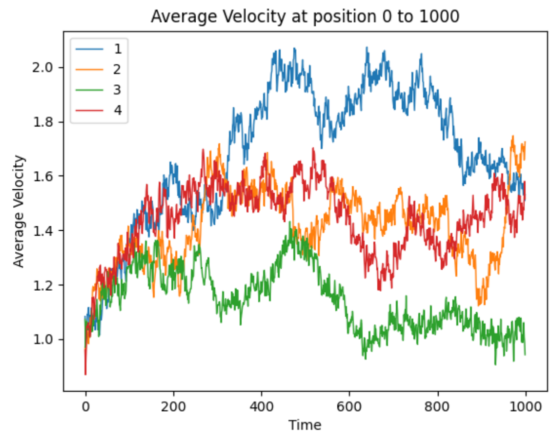


Figure 84: Time-space plot, maximum velocity iteration for a quadruple lane configuration, max velocity = 12

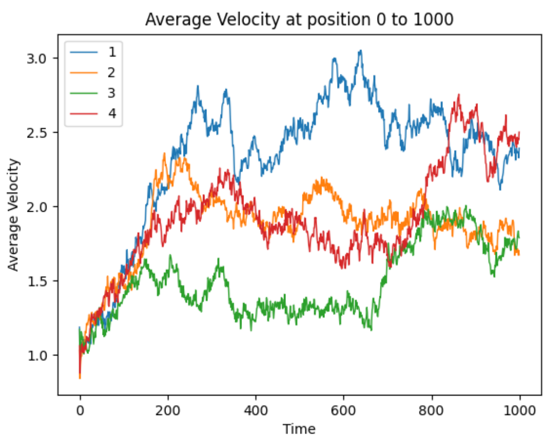
The average velocities are plotted below:



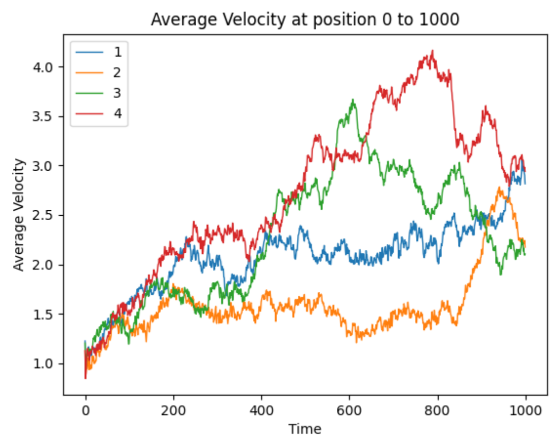
(a) Max velocity = 2



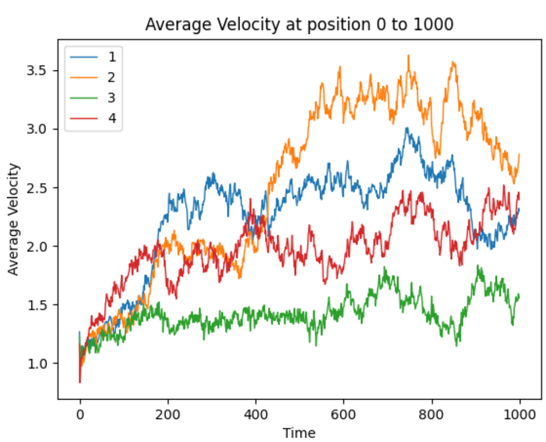
(b) Max velocity = 4



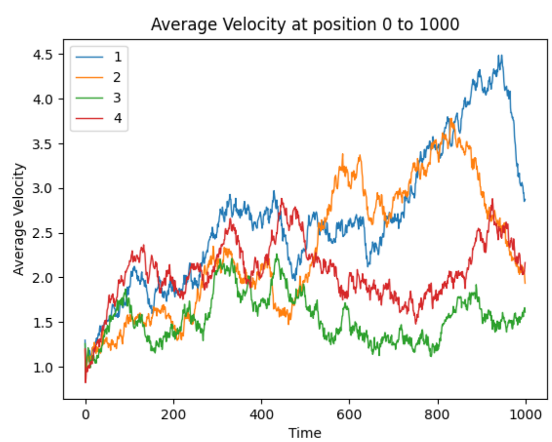
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 85: Average velocity plot, maximum velocity iteration for a quadruple lane configuration.

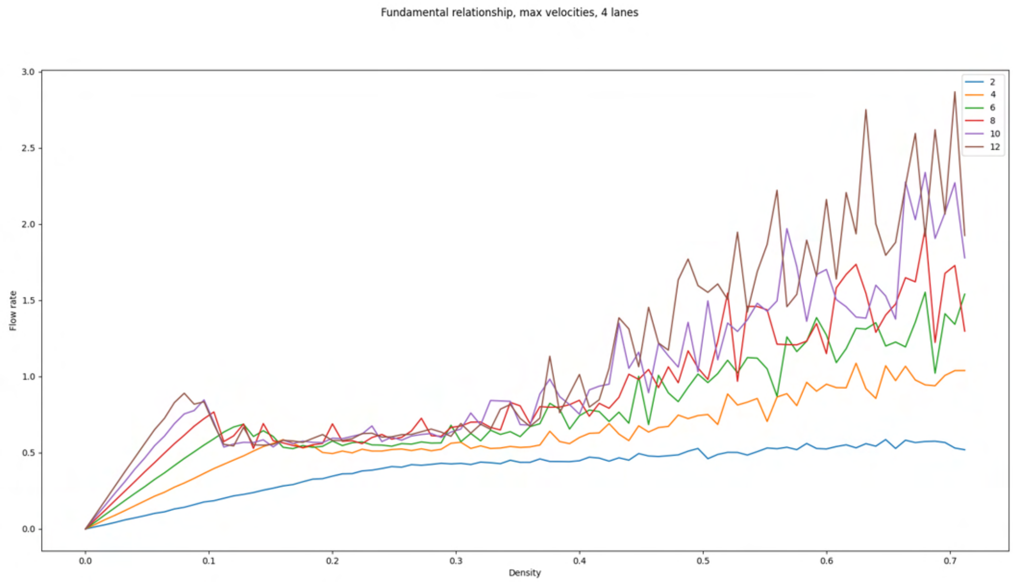


Figure 86: Fundamental relationship graph for quadruple lanes as the velocity is modified.

With an obstacle in the first lane, the time-space plots are also obtained.

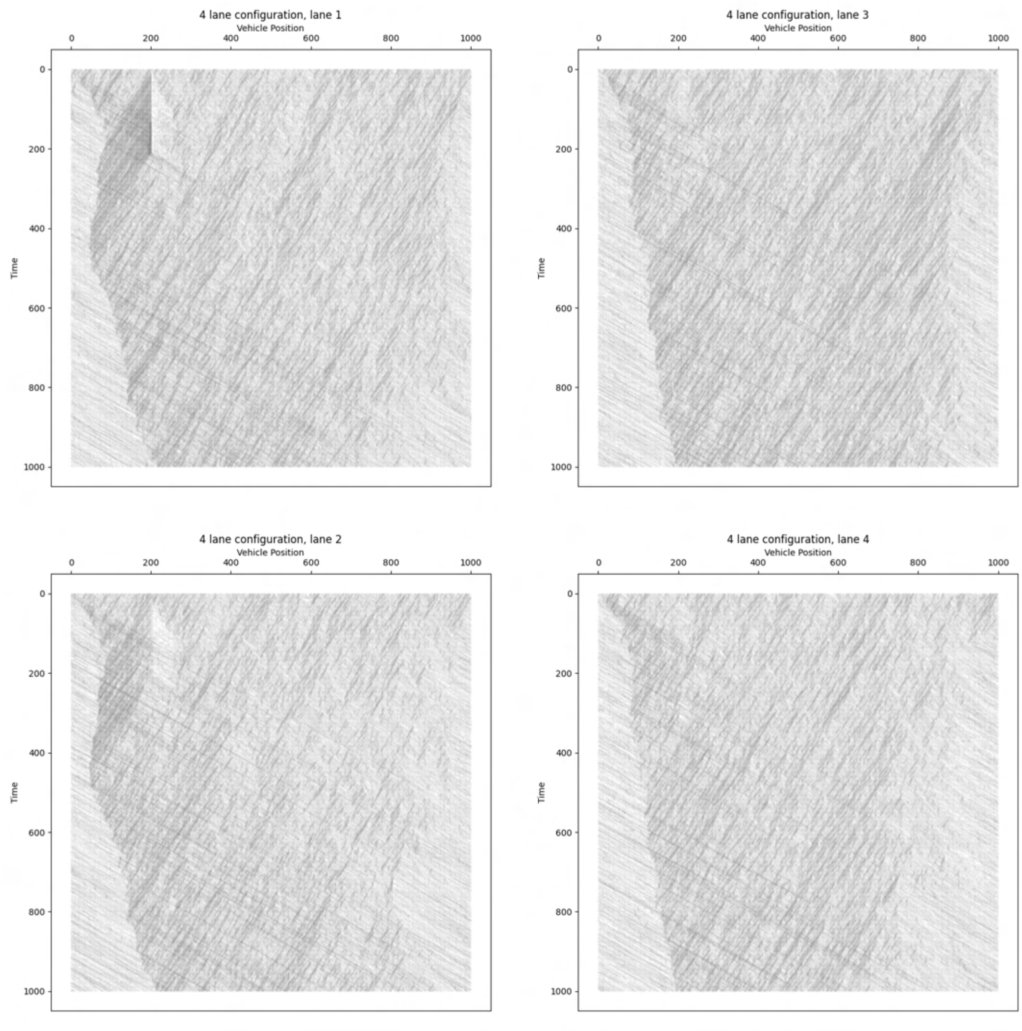


Figure 87: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle, max velocity = 2

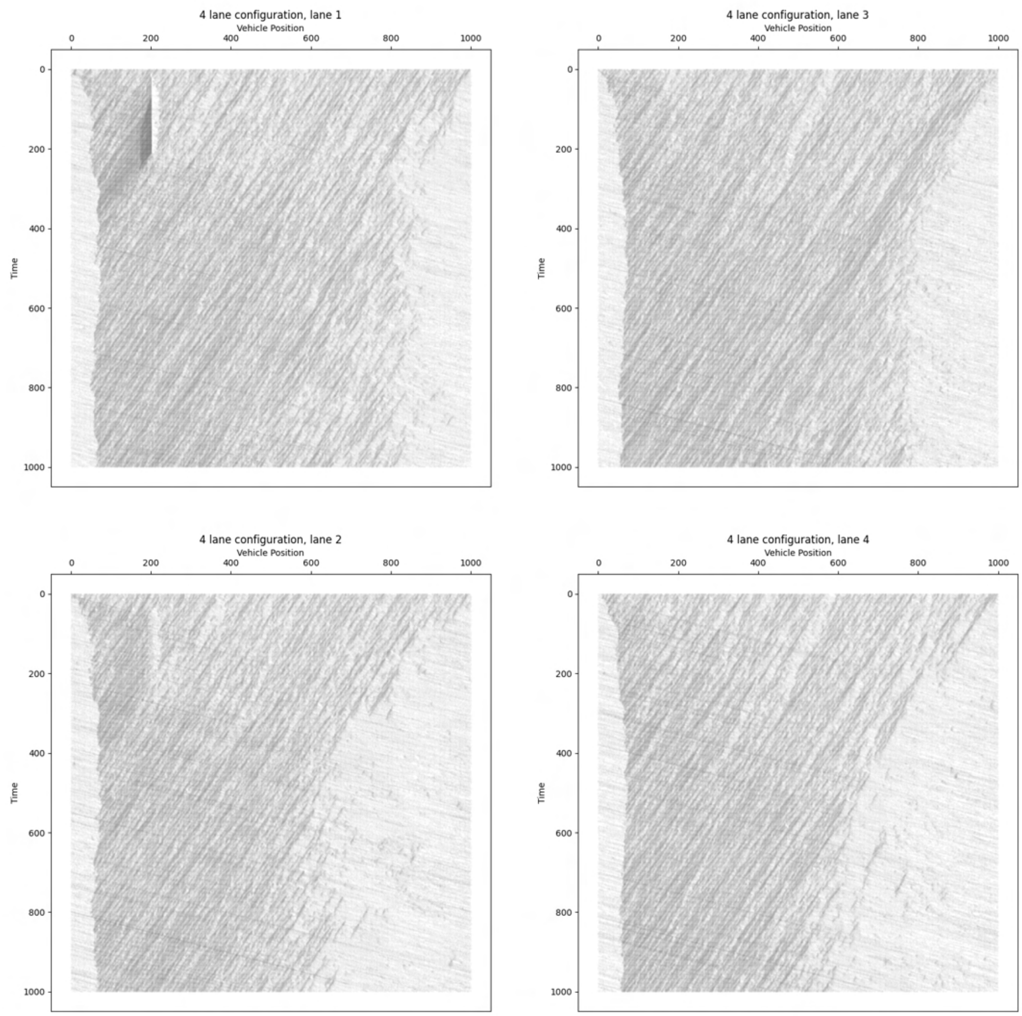


Figure 88: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle, max velocity = 4

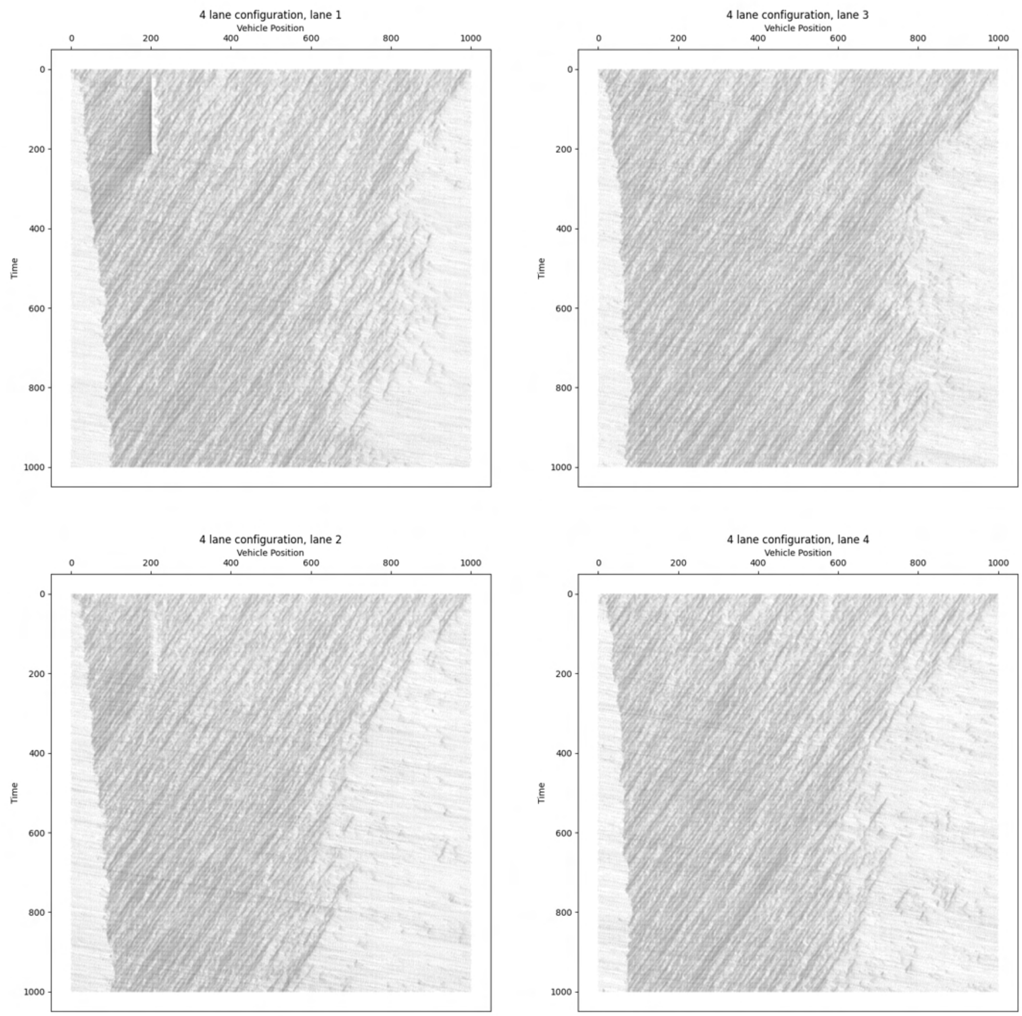


Figure 89: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle, max velocity = 6

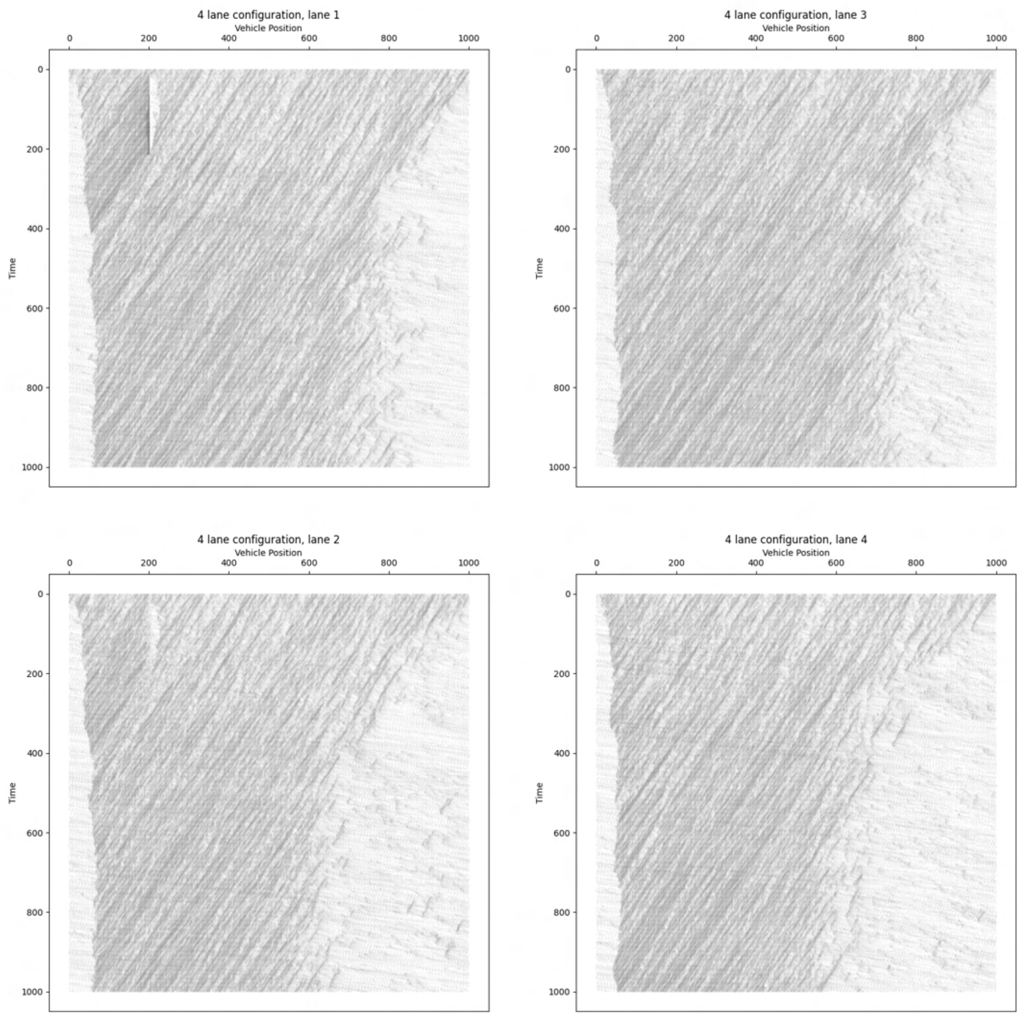


Figure 90: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle, max velocity = 8

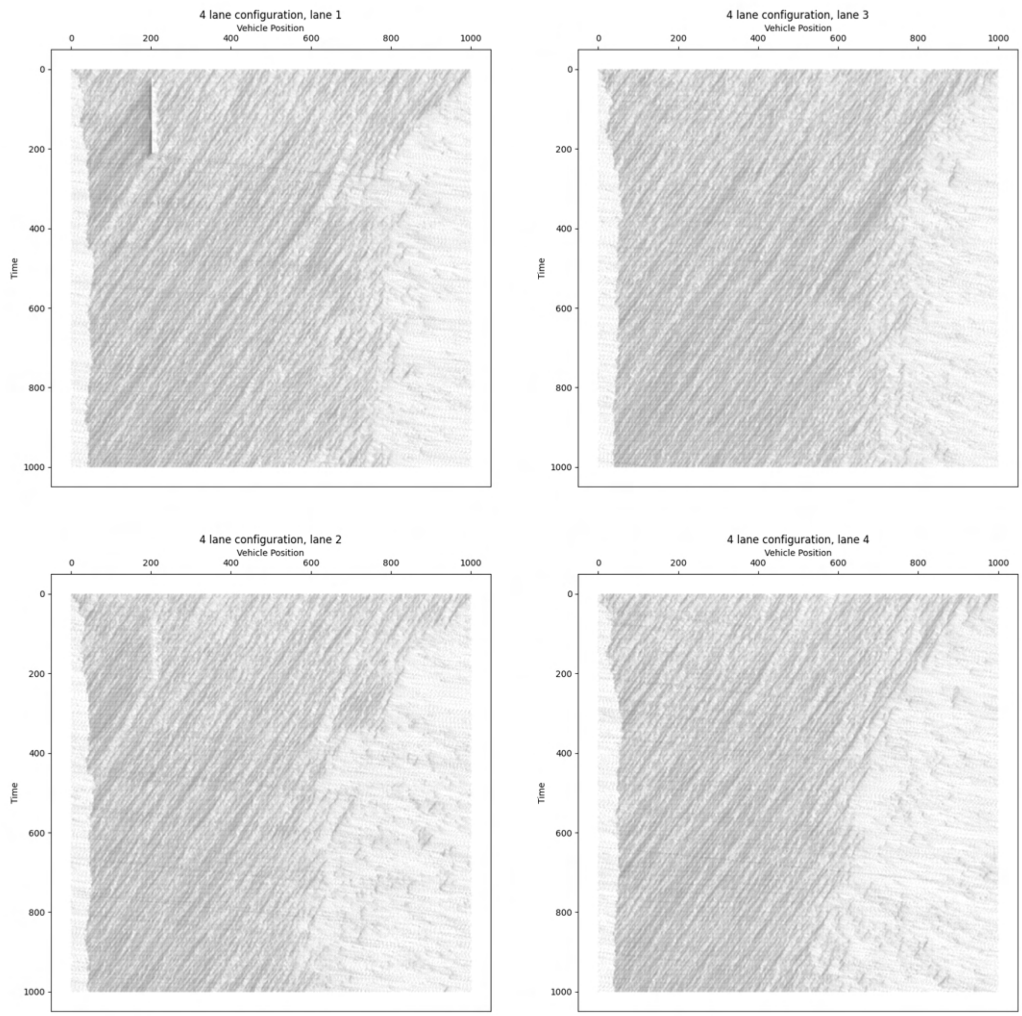


Figure 91: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle, max velocity = 10

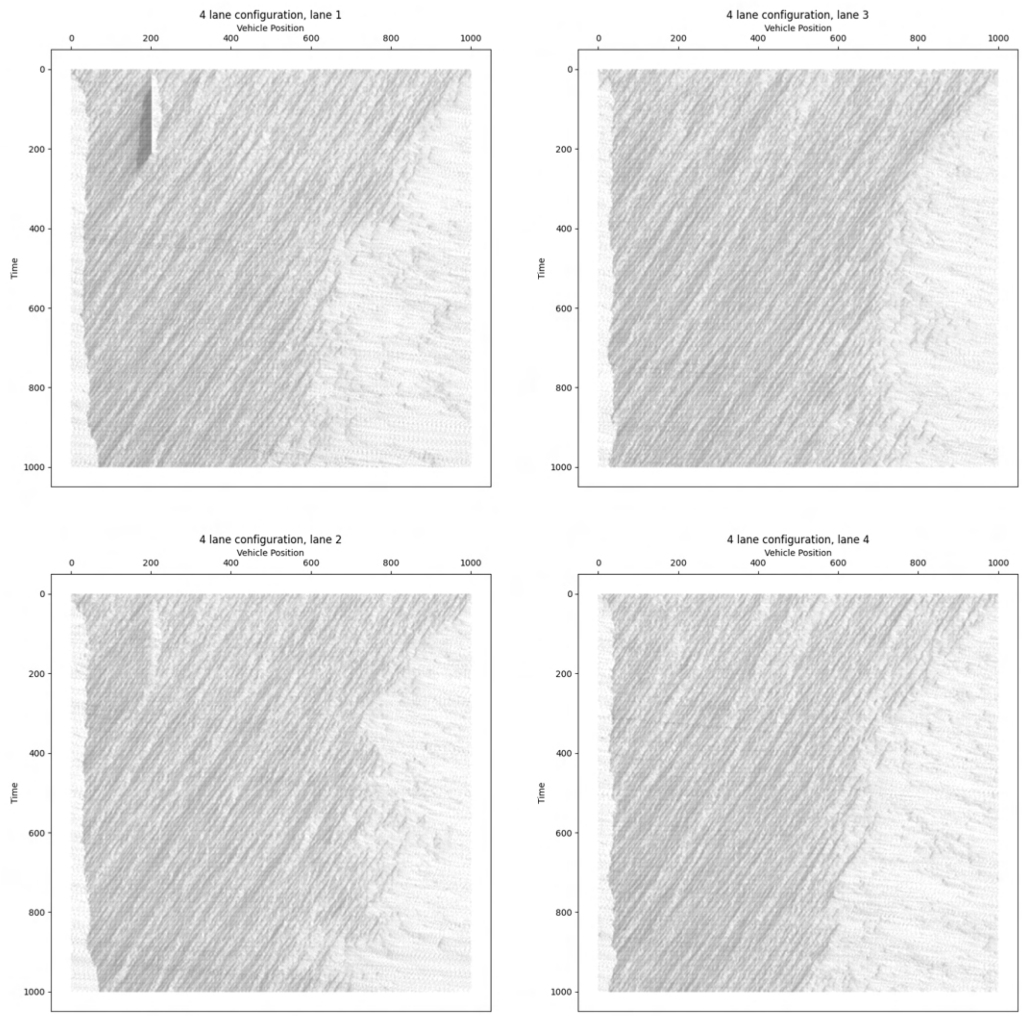
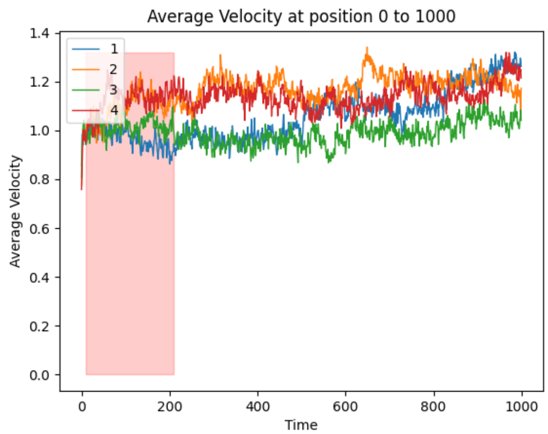
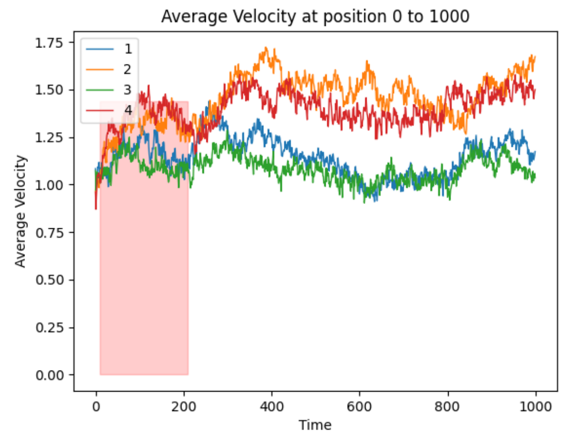


Figure 92: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle, max velocity = 12

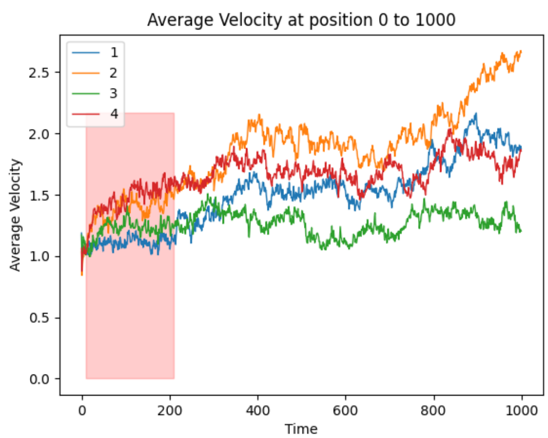
The average velocities are plotted below:



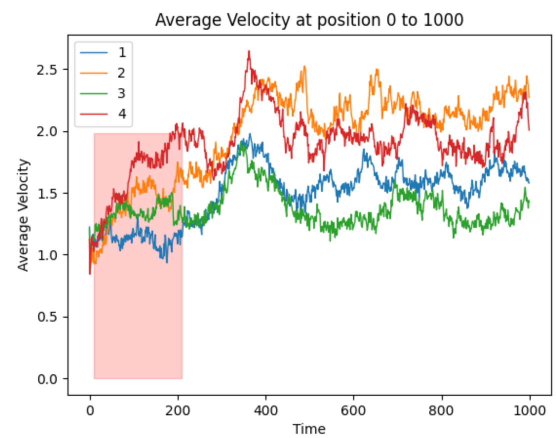
(a) Max velocity = 2



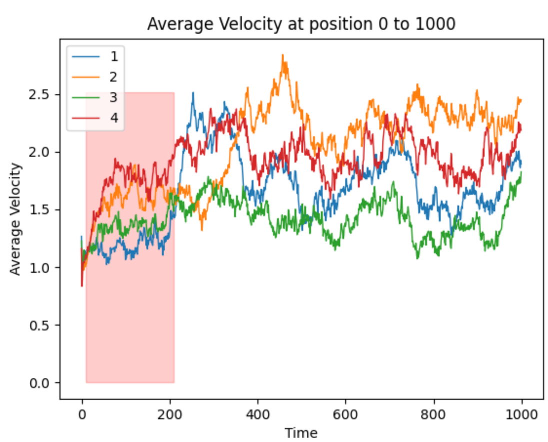
(b) Max velocity = 4



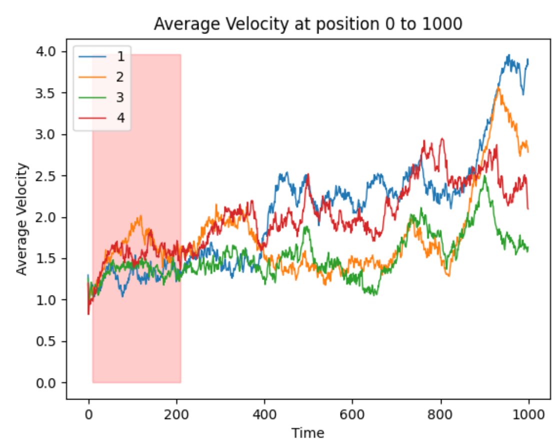
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 93: Average velocity plot, maximum velocity iteration for a quadruple lane configuration with obstacle in the first lane.

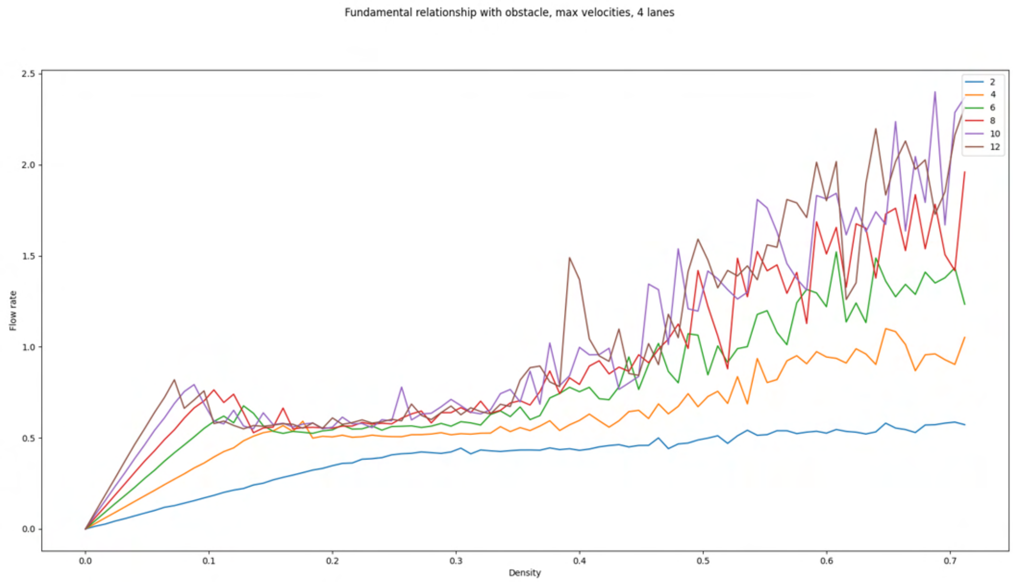


Figure 94: Fundamental relationship graph for quadruple lanes as the velocity is modified with obstacle in lane 1.

Since this is a 4 lane configuration, it makes sense, similar to the triple lane configuration, to consider the scenario where the obstacle is in the 2nd lane instead of the 1st.

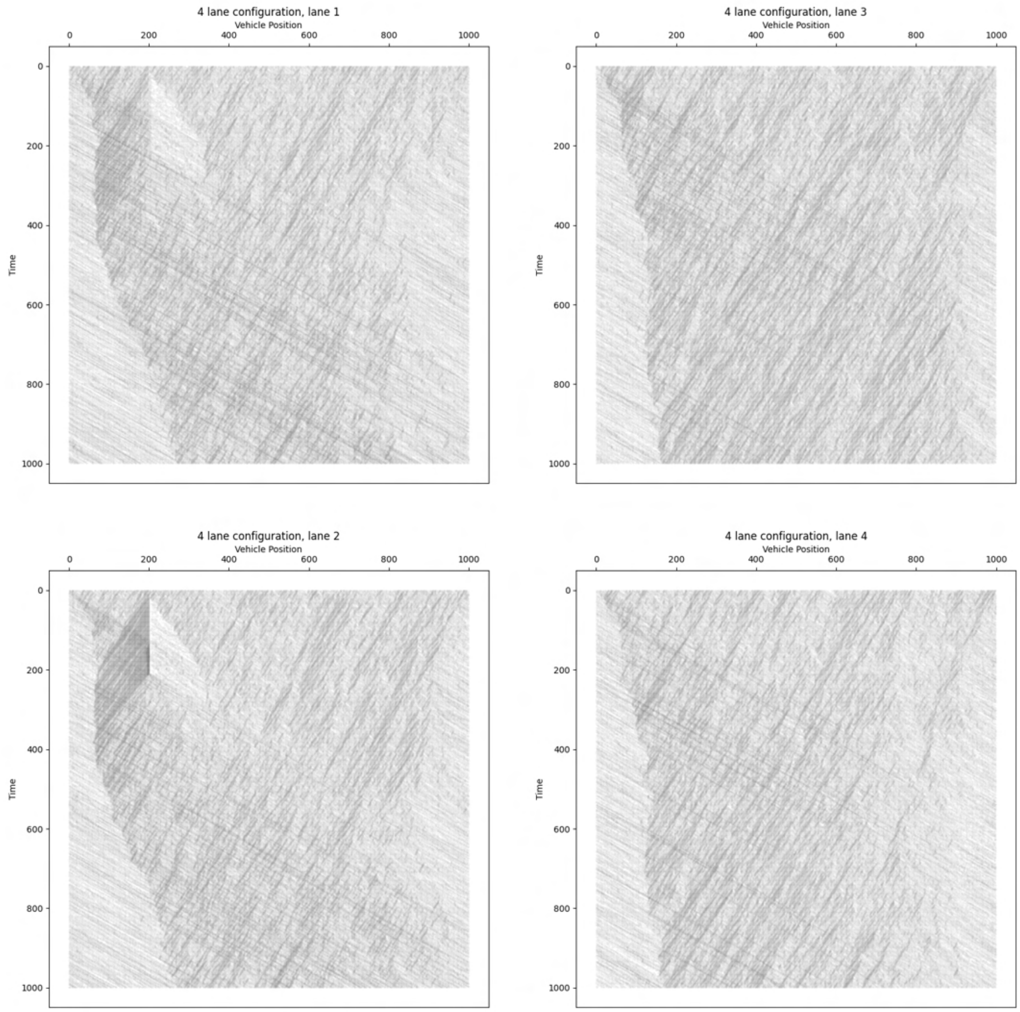


Figure 95: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle in lane 2, max velocity = 2

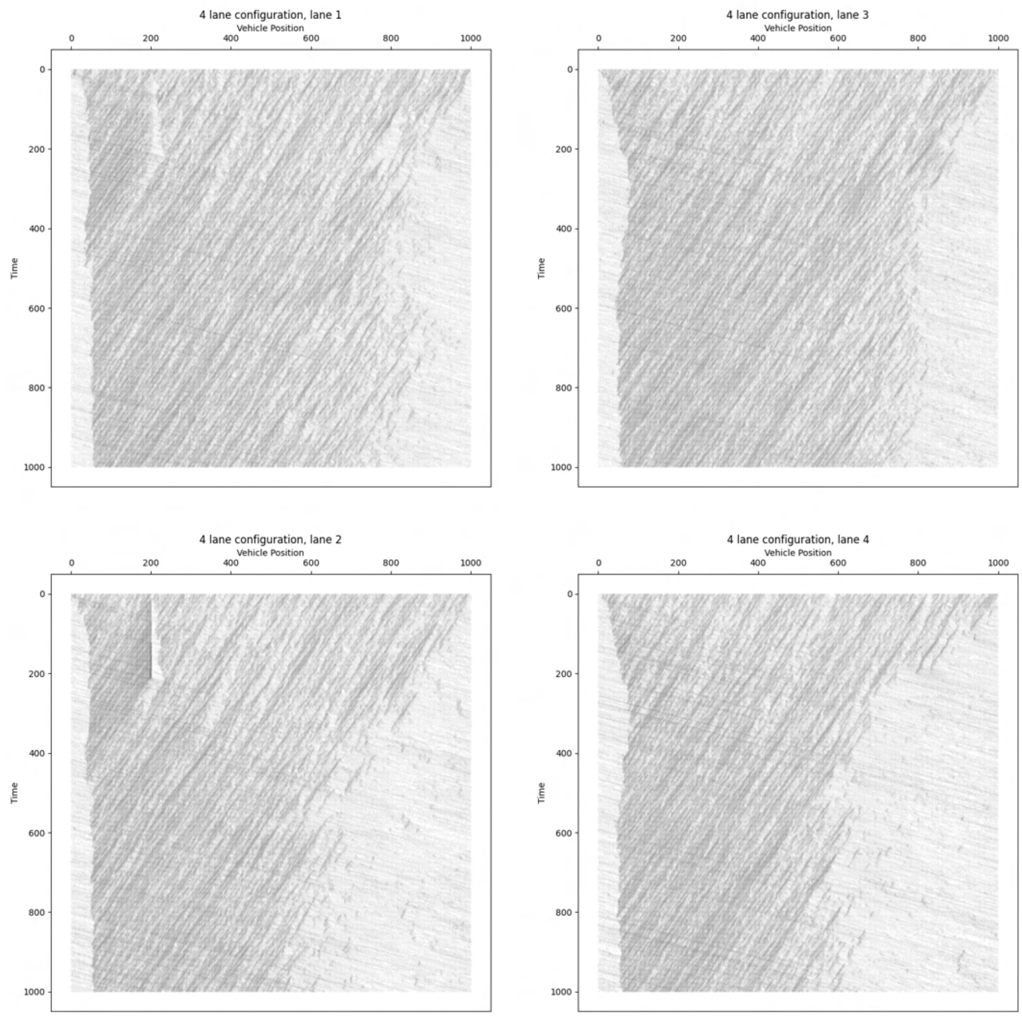


Figure 96: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle in lane 2, max velocity = 4

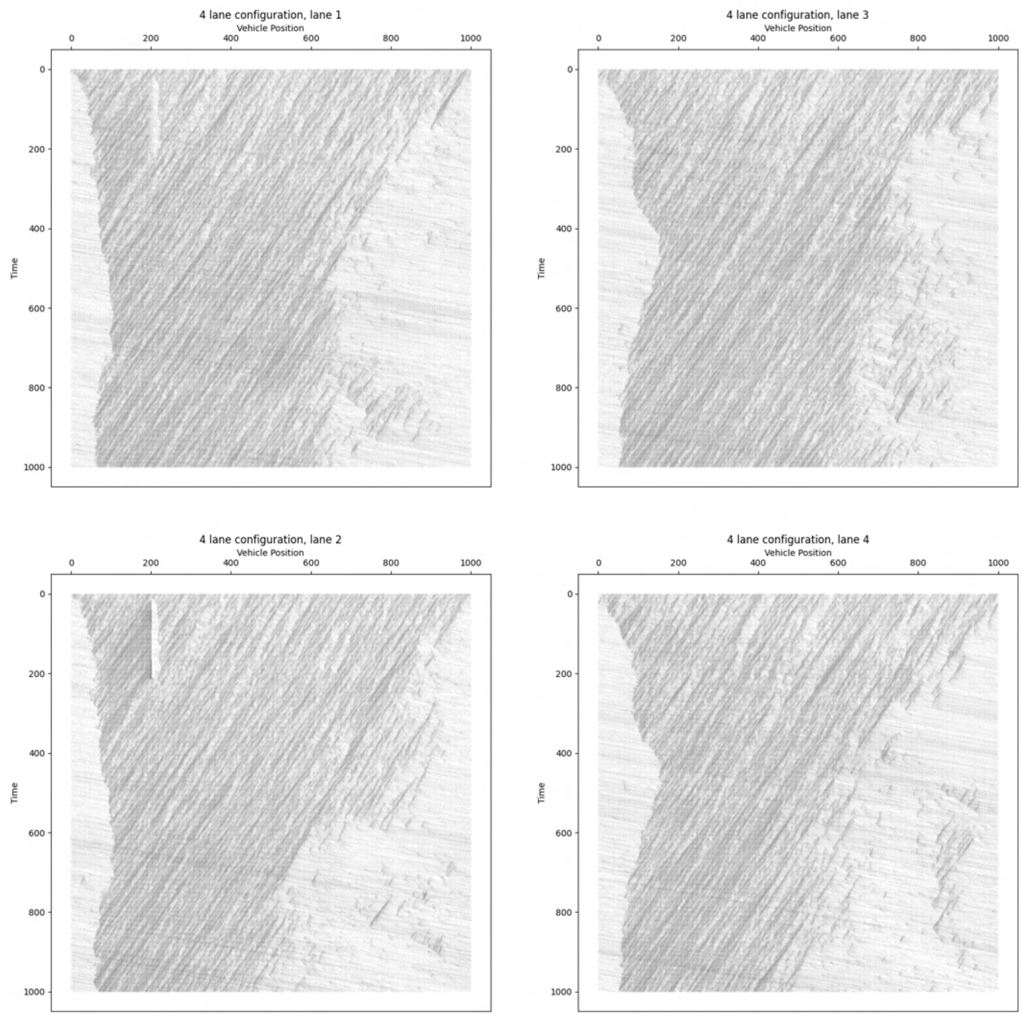


Figure 97: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle in lane 2, max velocity = 6

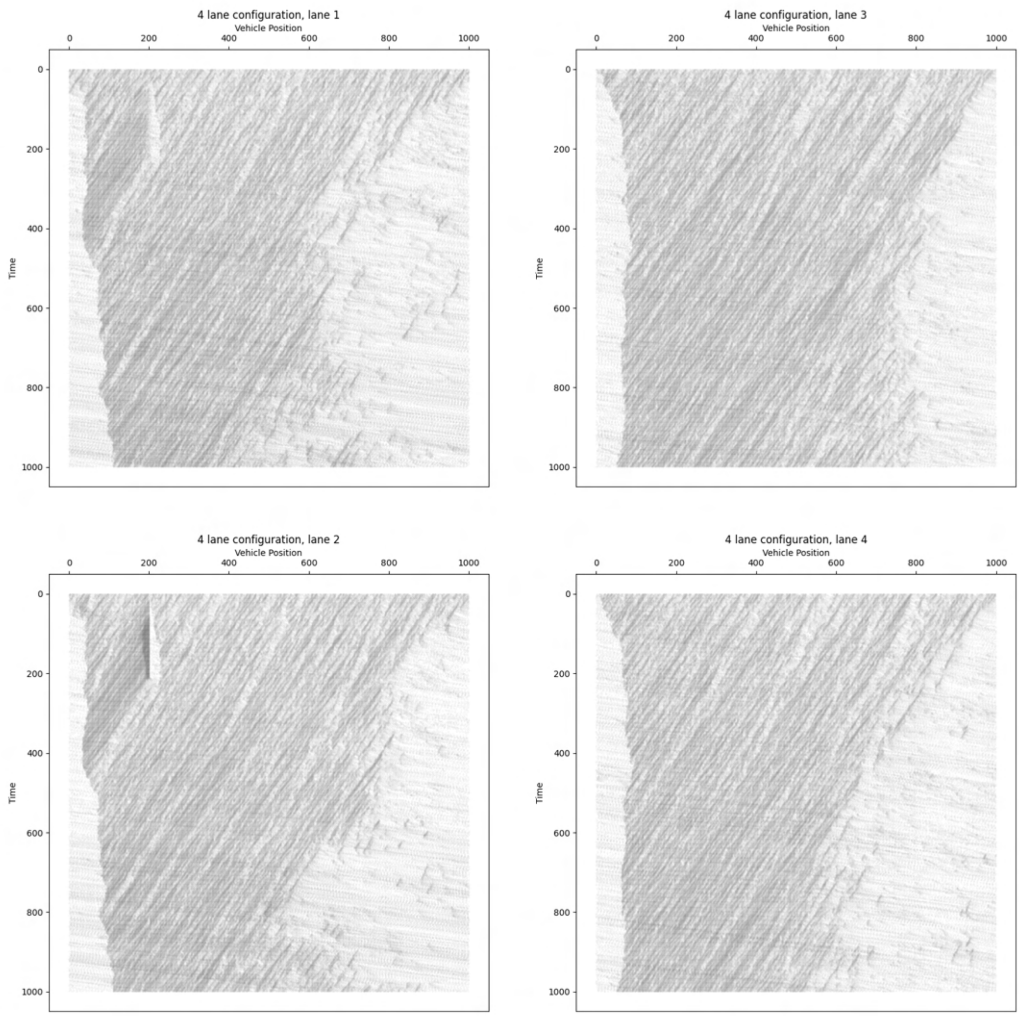


Figure 98: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle in lane 2, max velocity = 8

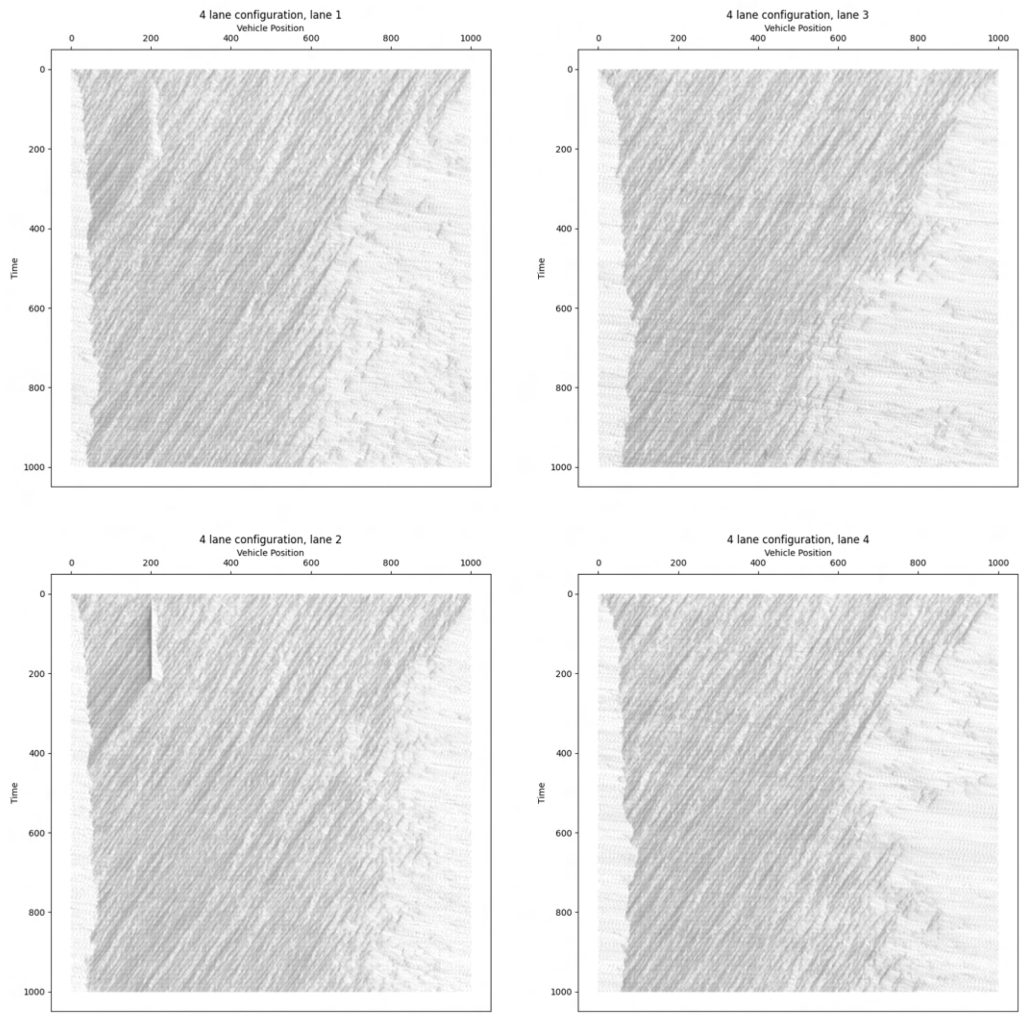


Figure 99: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle in lane 2, max velocity = 10

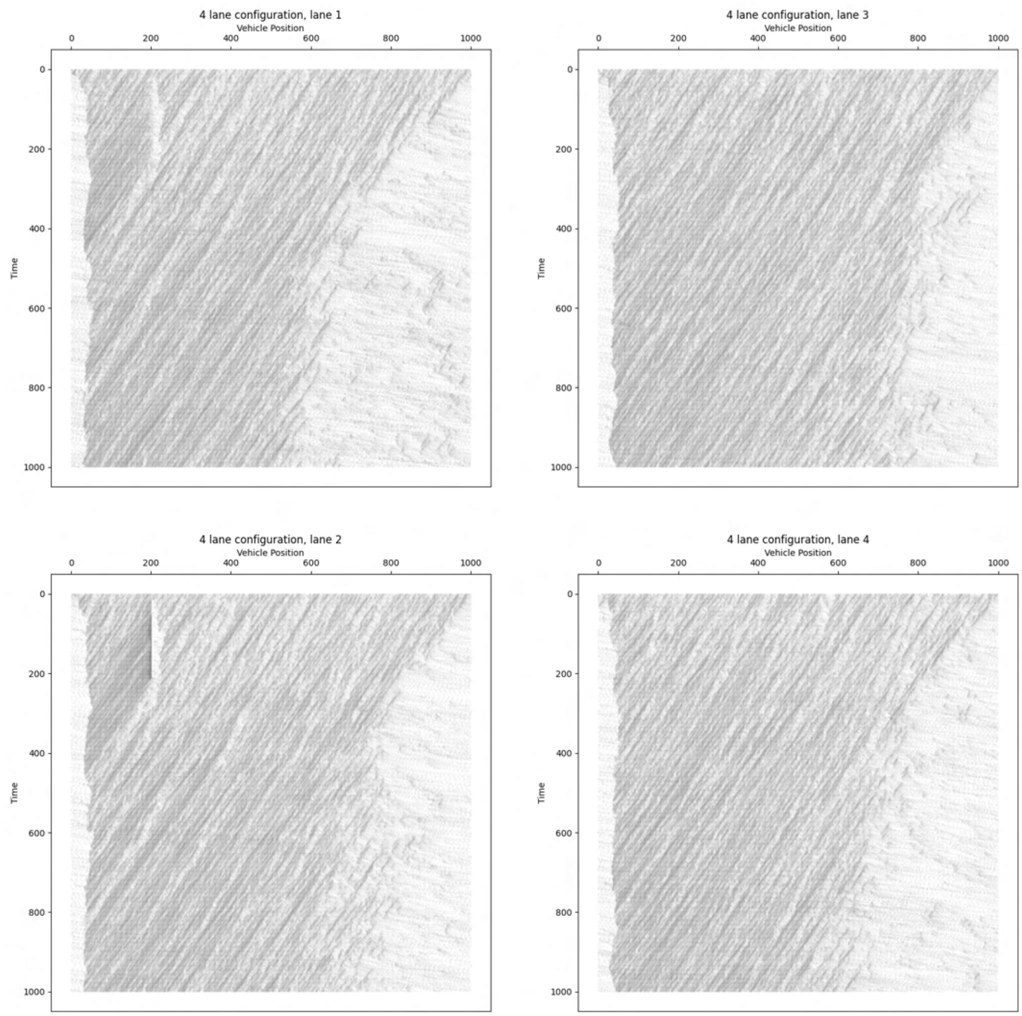
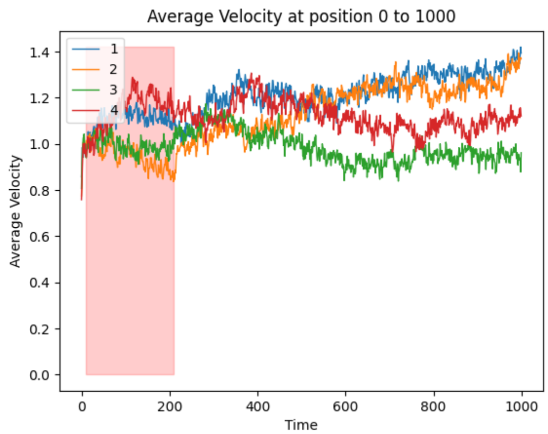
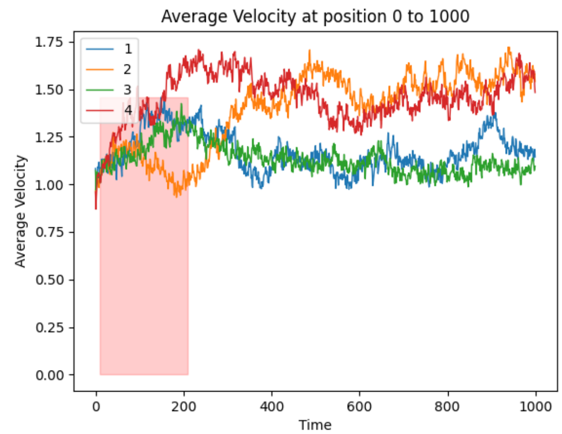


Figure 100: Time-space plot, maximum velocity iteration for a quadruple lane configuration with obstacle in lane 2, max velocity = 12

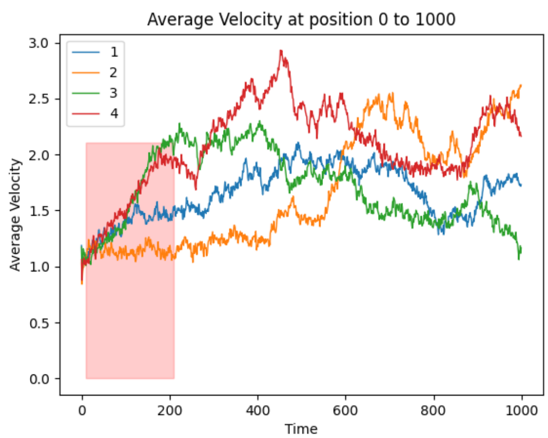
The average velocities are plotted below:



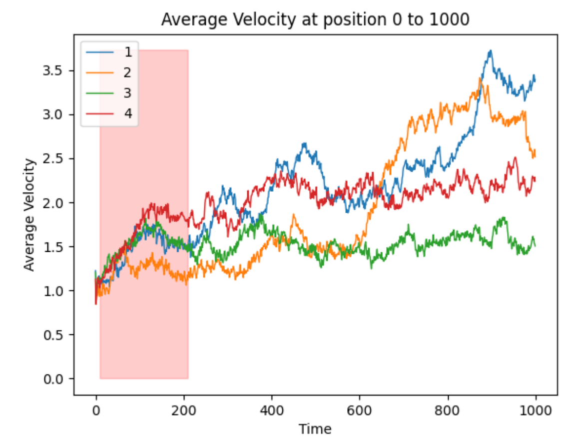
(a) Max velocity = 2



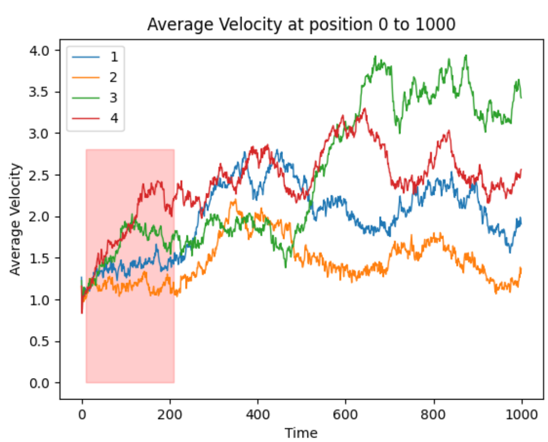
(b) Max velocity = 4



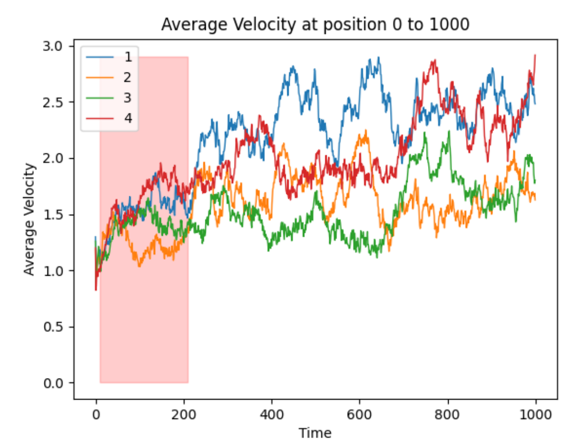
(c) Max velocity = 6



(d) Max velocity = 8



(e) Max velocity = 10



(f) Max velocity = 12

Figure 101: Average velocity plot, maximum velocity iteration for a quadruple lane configuration with obstacle in the second lane.

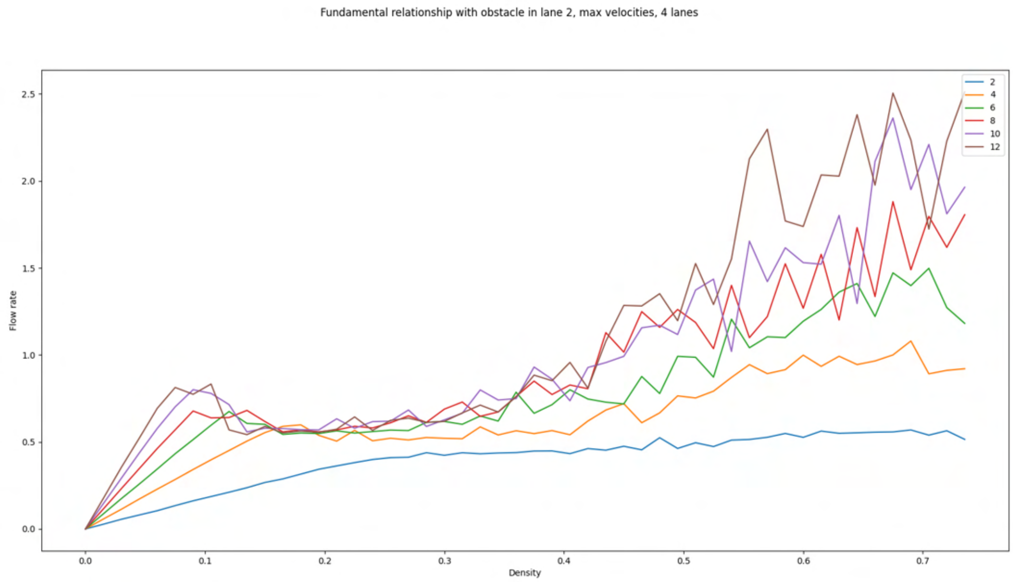


Figure 102: Fundamental relationship graph for quadruple lanes as the slow probability is modified with obstacle in lane 2.

4.4.2 Slowing Probabilities

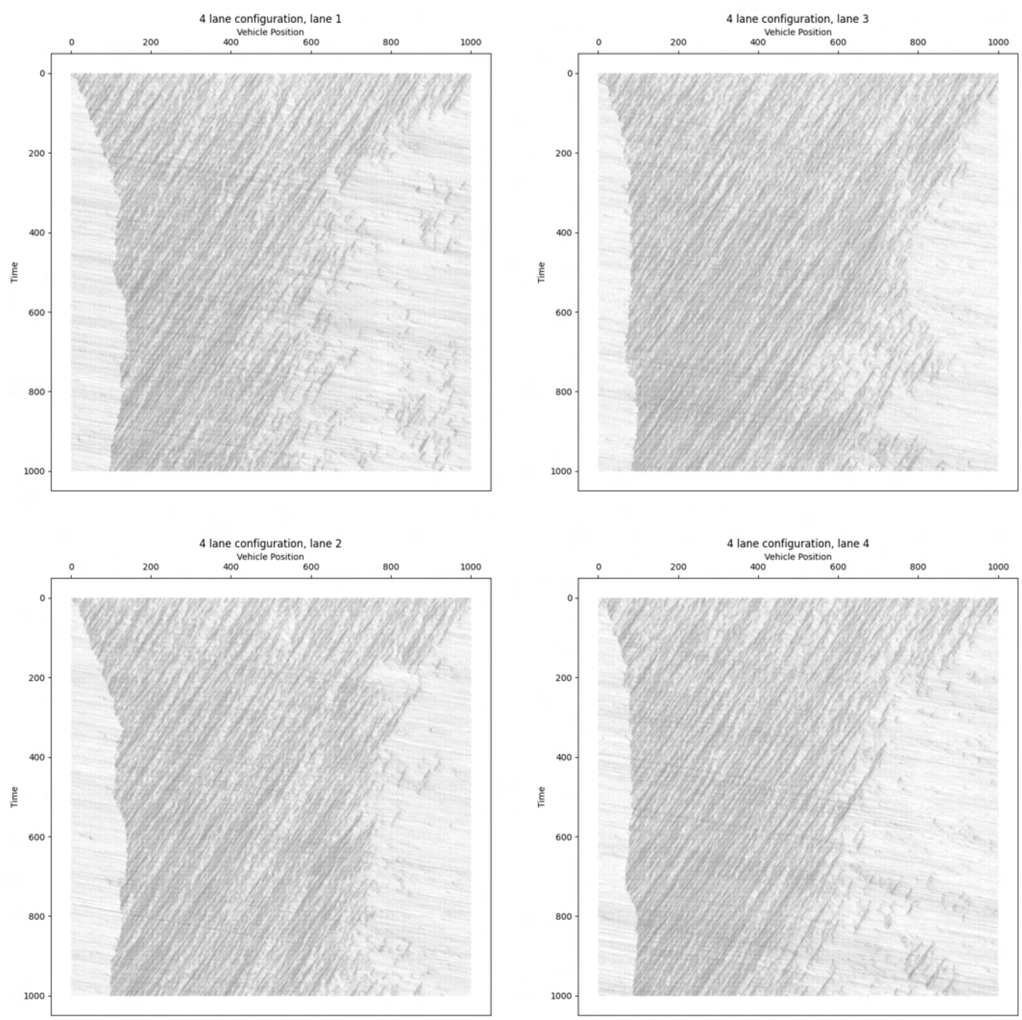


Figure 103: Time-space plot, slow probability iteration for a quadruple lane configuration, slow probability = 0.2

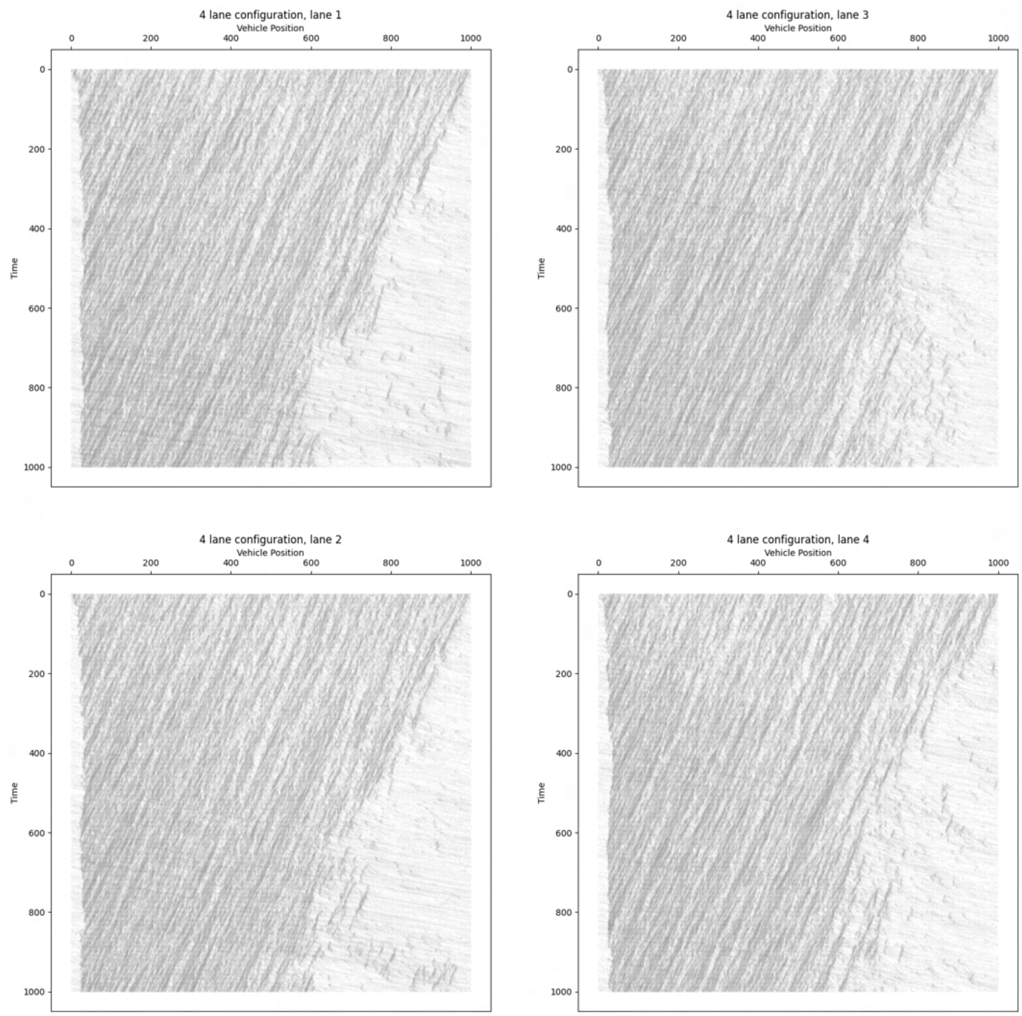


Figure 104: Time-space plot, slow probability iteration for a quadruple lane configuration, slow probability = 0.4

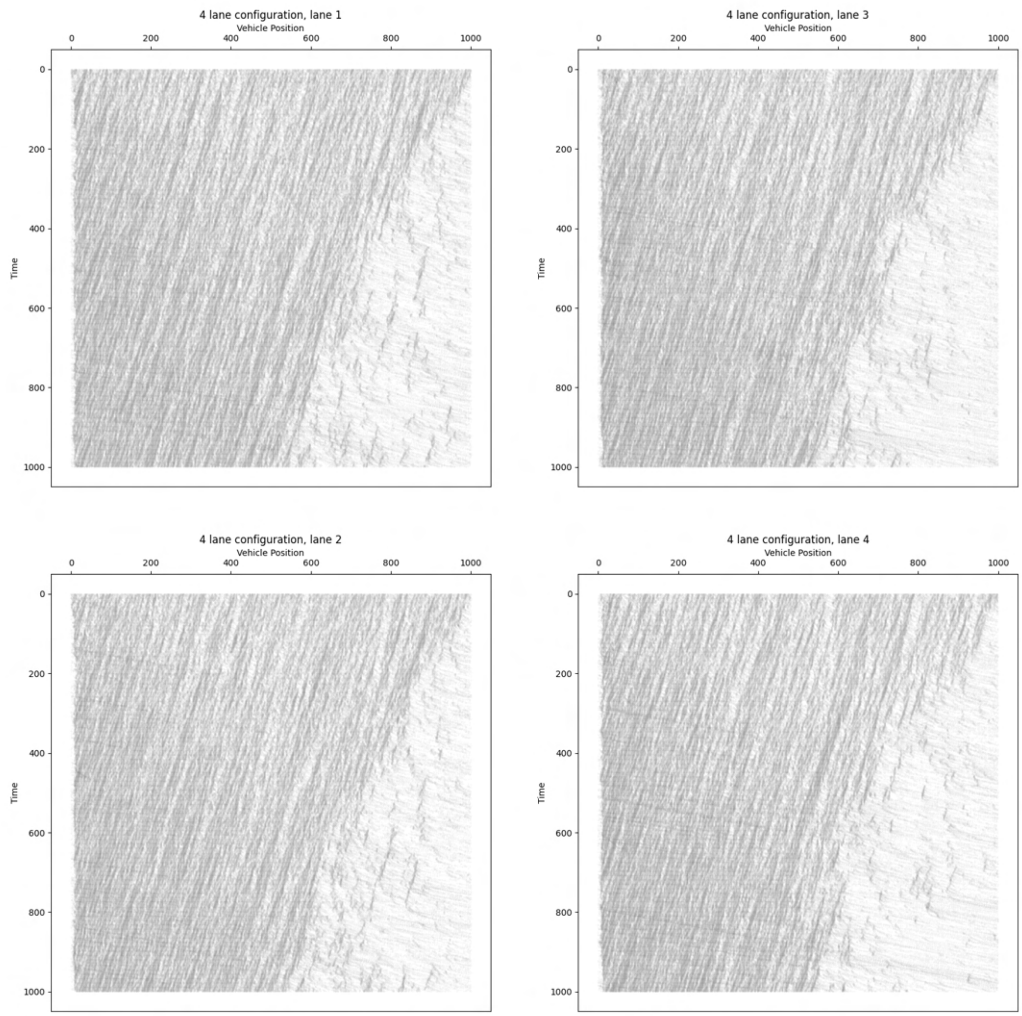


Figure 105: Time-space plot, slow probability iteration for a quadruple lane configuration, slow probability = 0.6

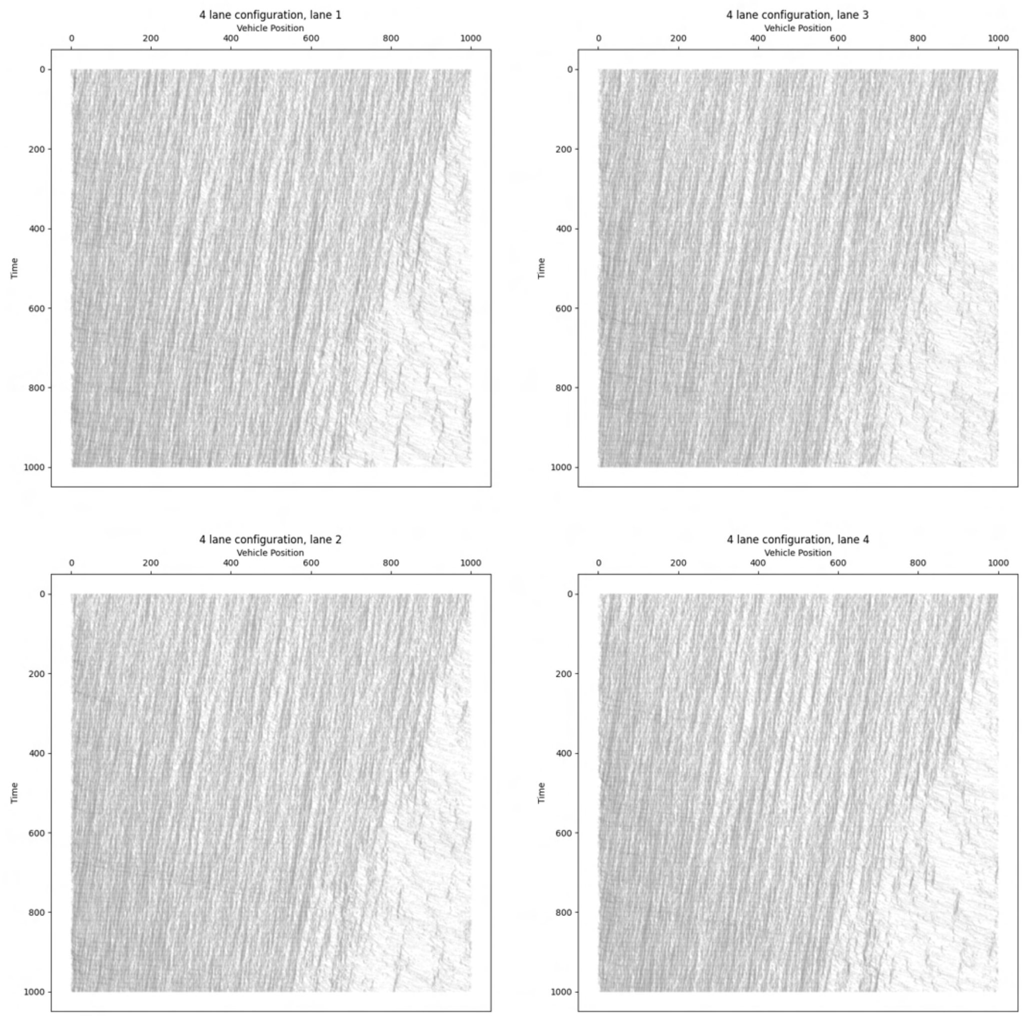
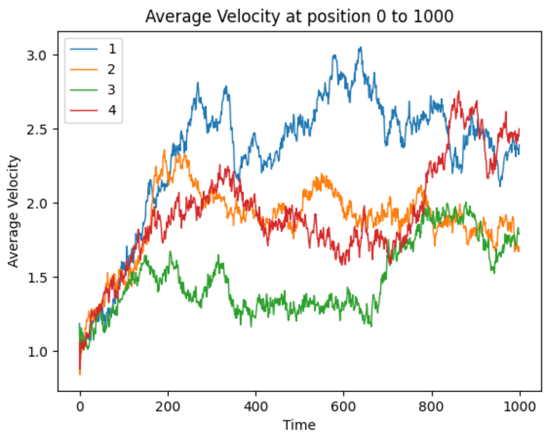
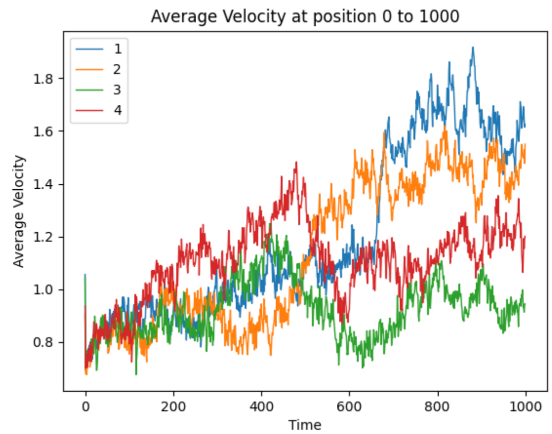


Figure 106: Time-space plot, slow probability iteration for a quadruple lane configuration, slow probability = 0.8

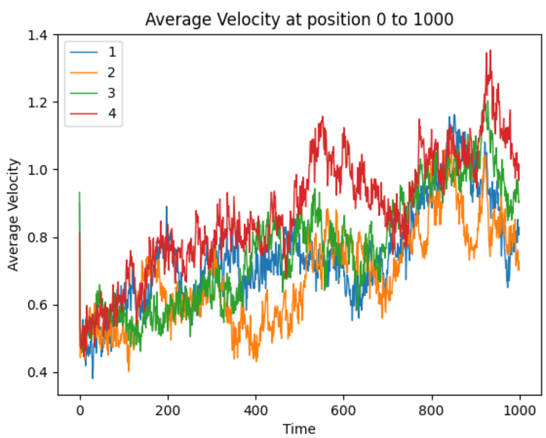
The average velocities are plotted below:



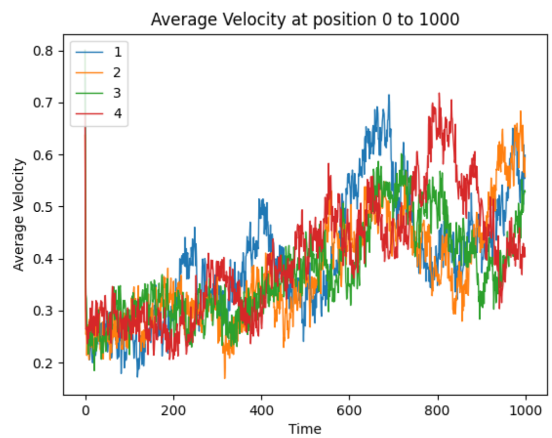
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 107: Average velocity plot, slow probability iteration for a quadruple lane configuration.

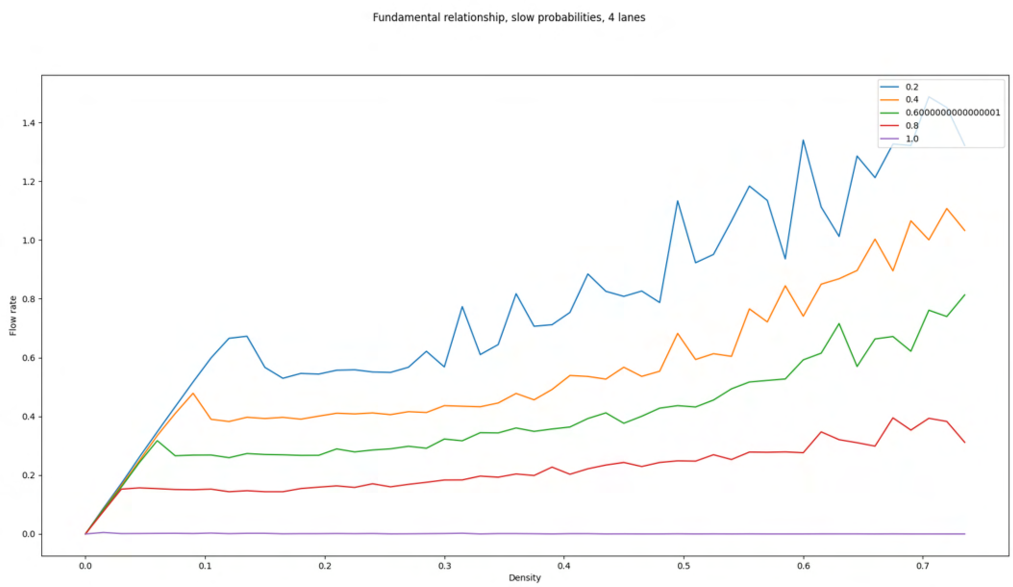


Figure 108: Fundamental relationship graph for quadruple lanes as the slow probability is modified.

With obstacles, the time-space plots are also obtained.

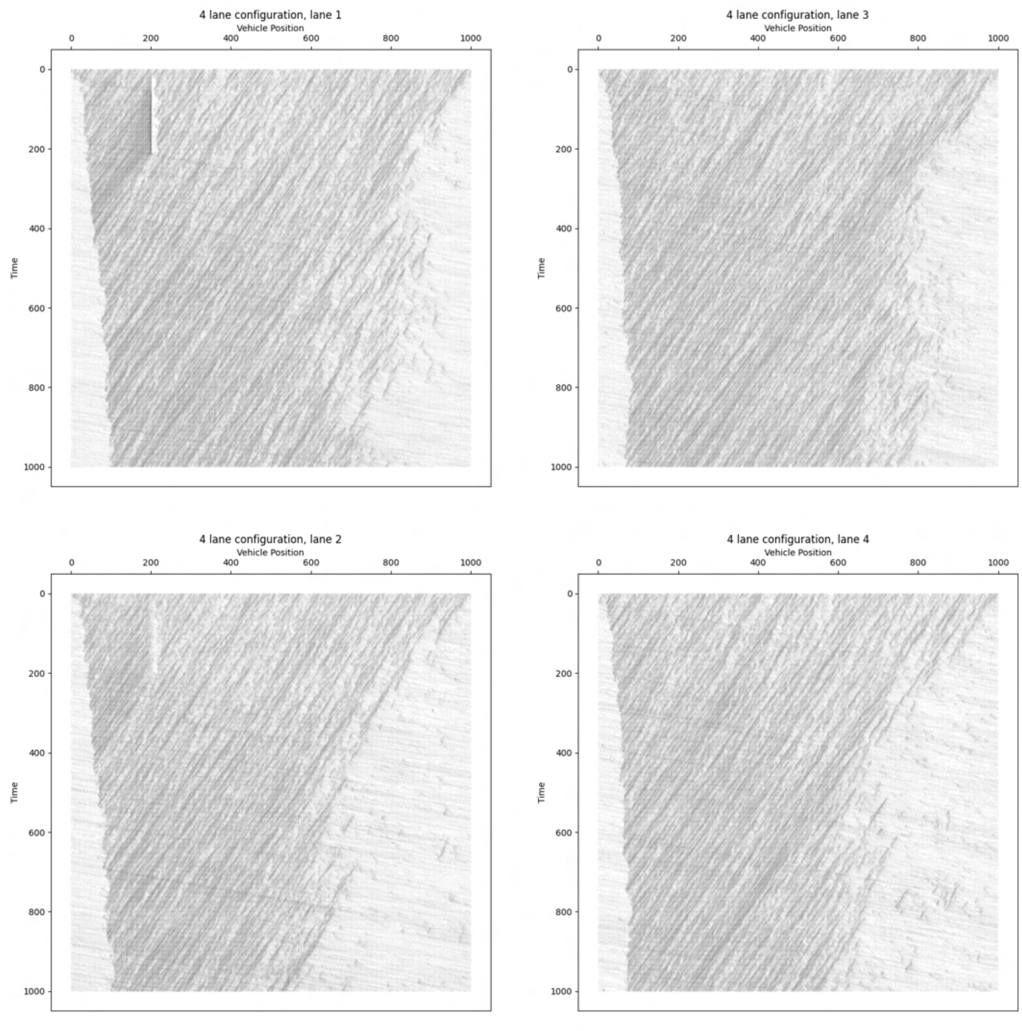


Figure 109: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 1, slow probability = 0.2

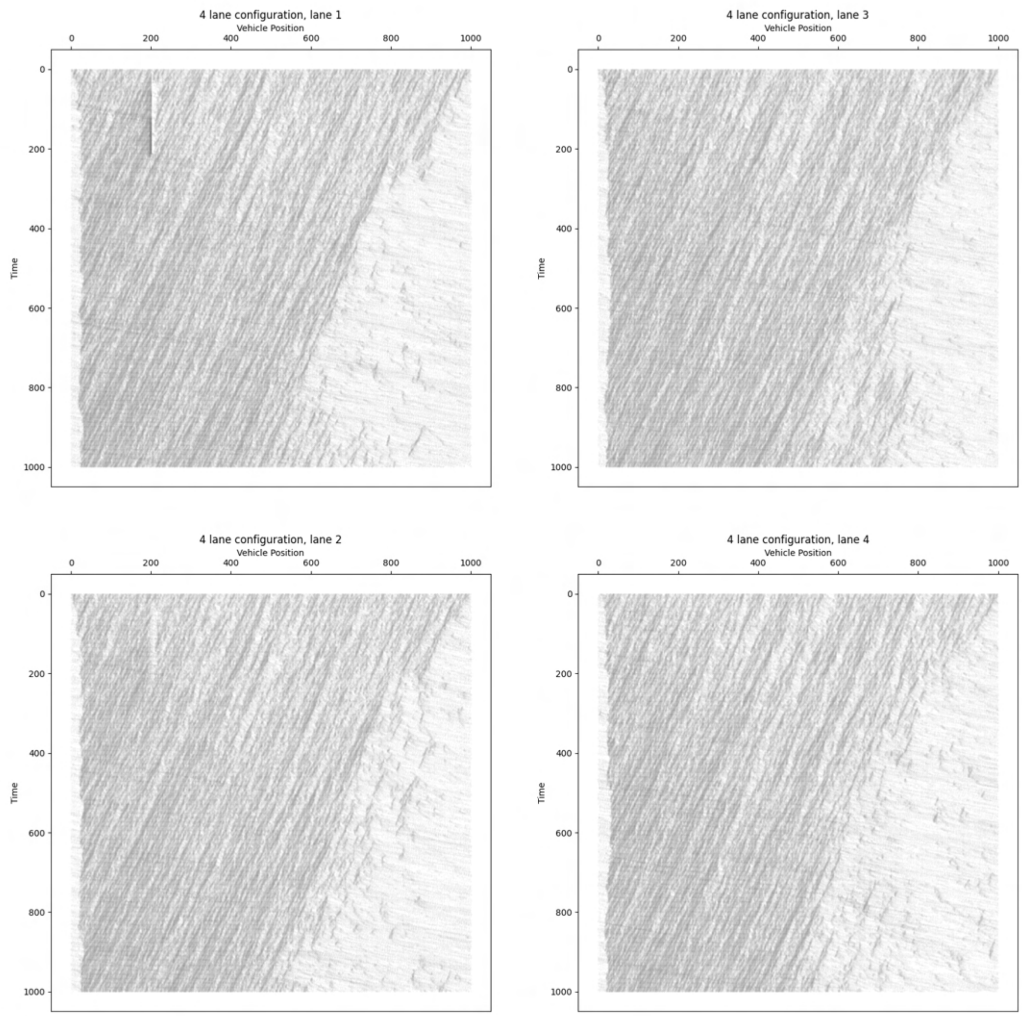


Figure 110: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 1, slow probability = 0.4

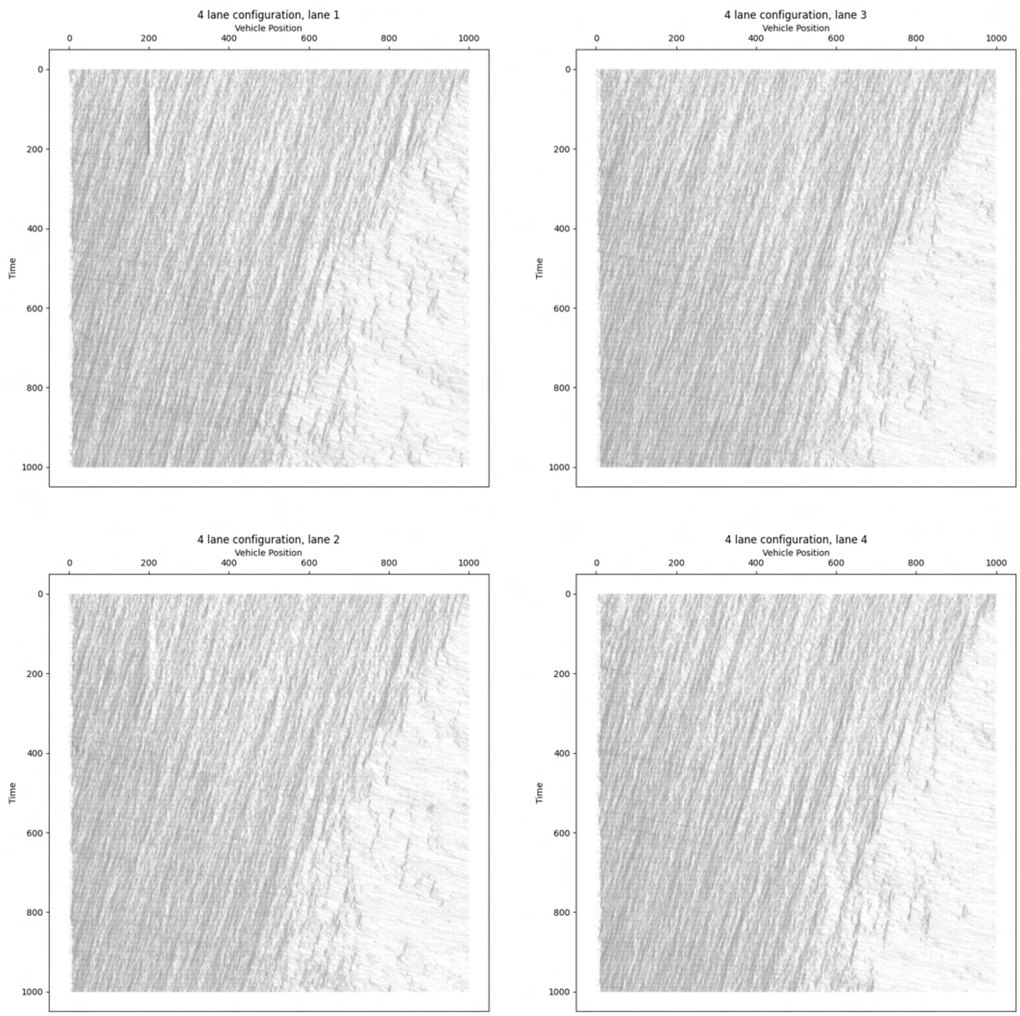


Figure 111: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 1, slow probability = 0.6

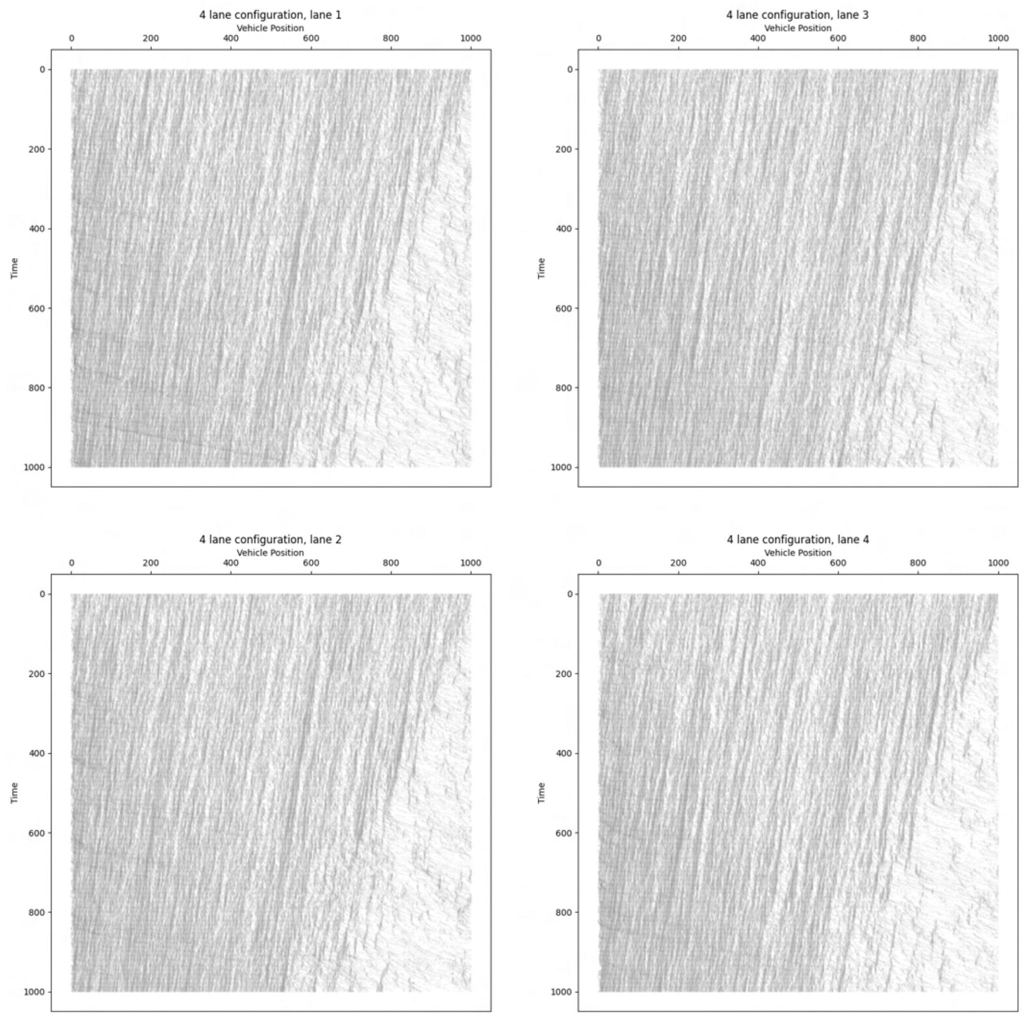
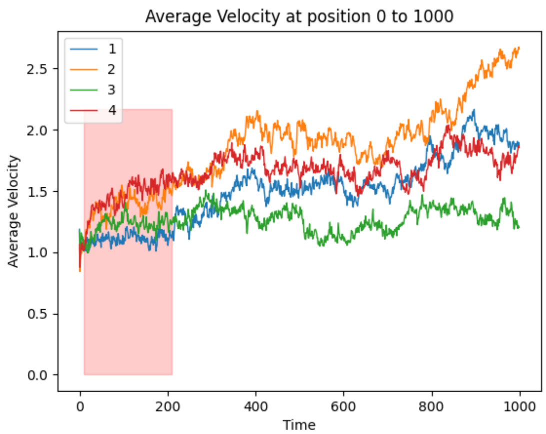
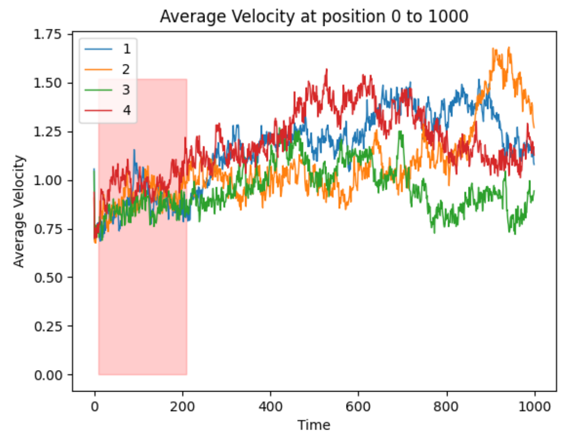


Figure 112: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 1, slow probability = 0.8

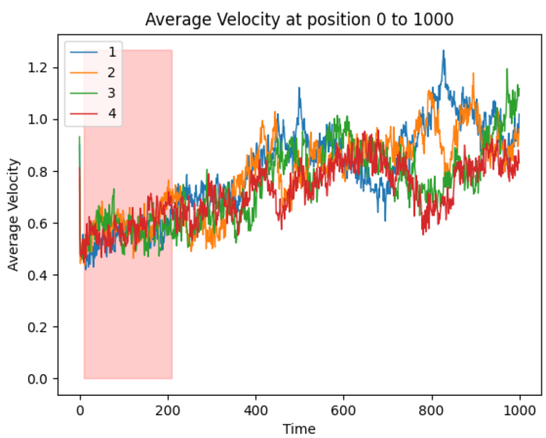
The average velocities are plotted below:



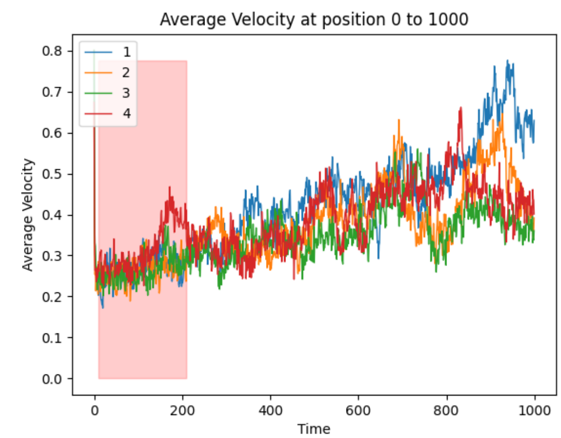
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 113: Average velocity plot, slow probability iteration for a quadruple lane configuration.

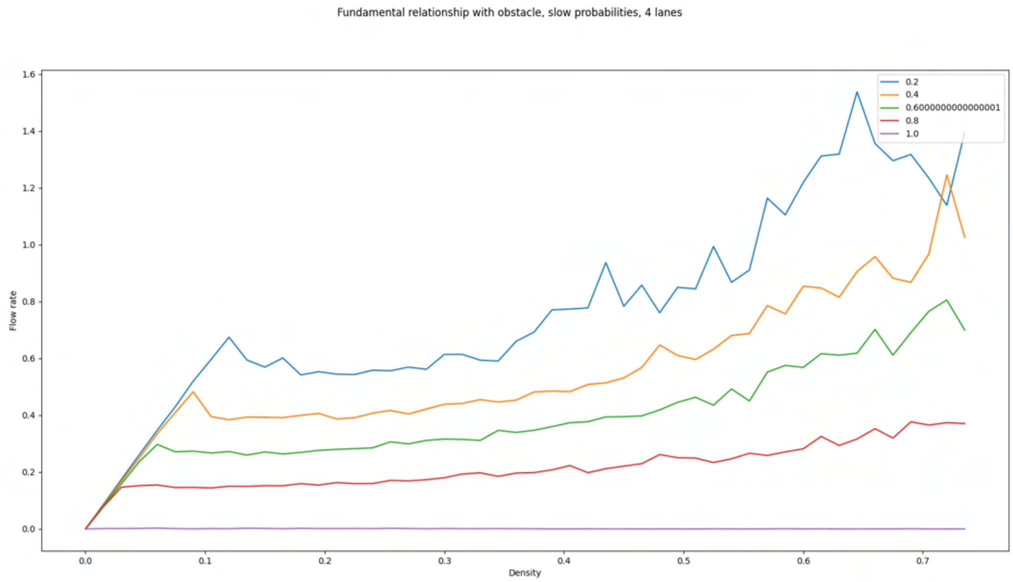


Figure 114: Fundamental relationship graph for quadruple lanes as slow probability is modified.

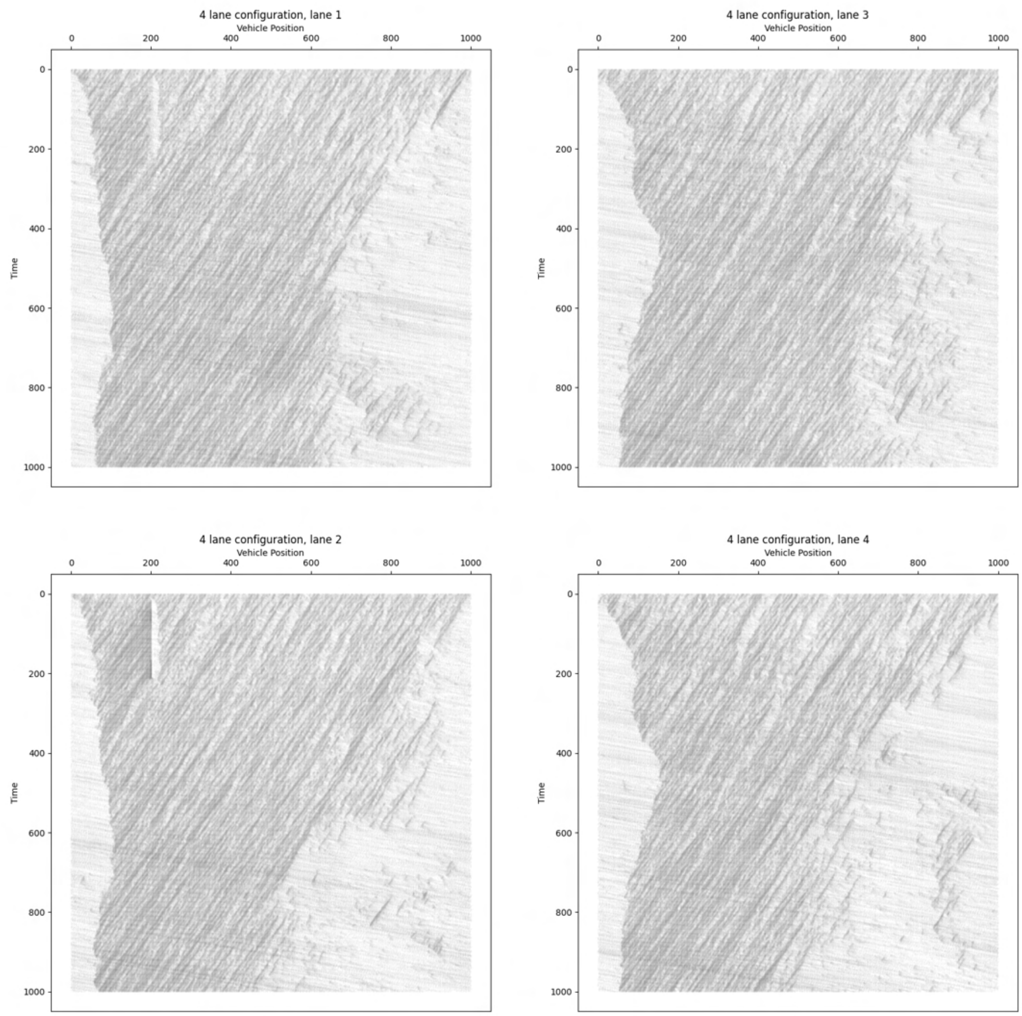


Figure 115: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 2, slow probability = 0.2

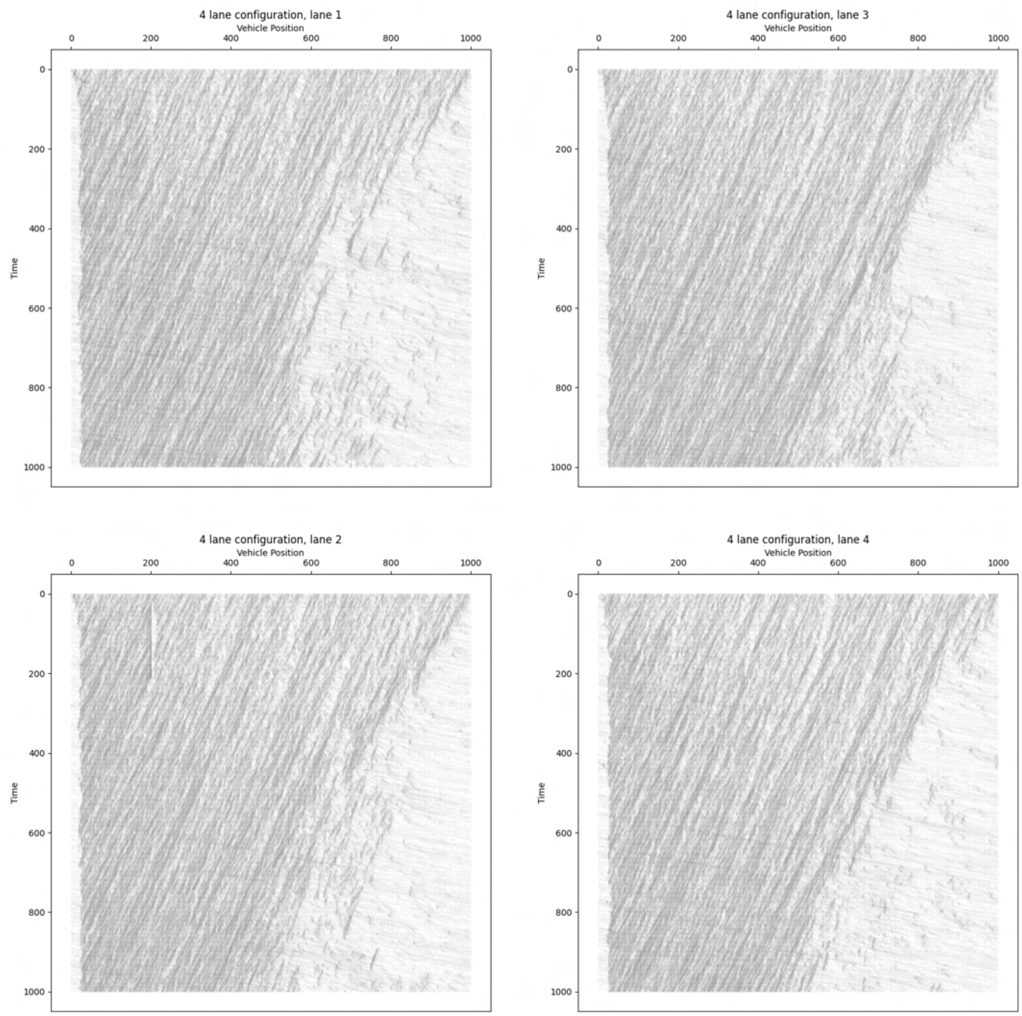


Figure 116: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 2, slow probability = 0.4

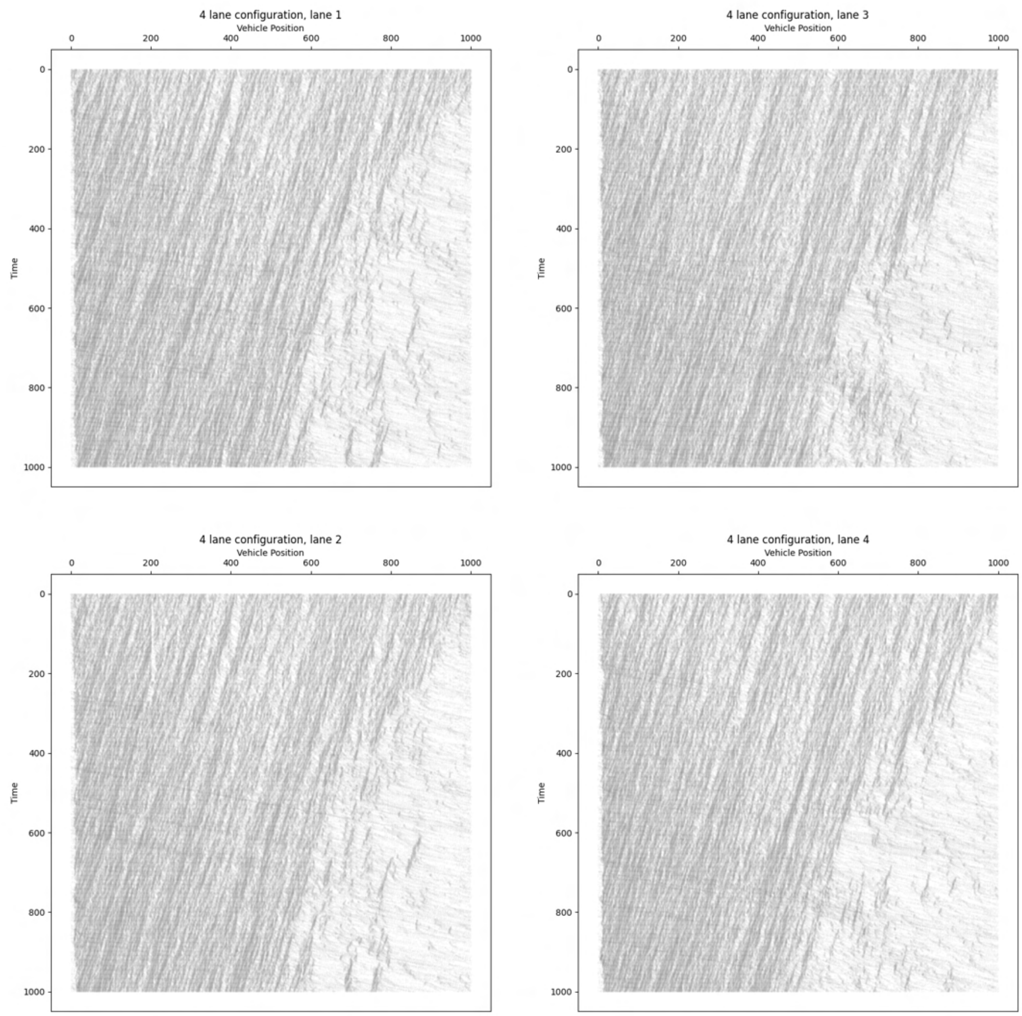


Figure 117: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 2, slow probability = 0.6

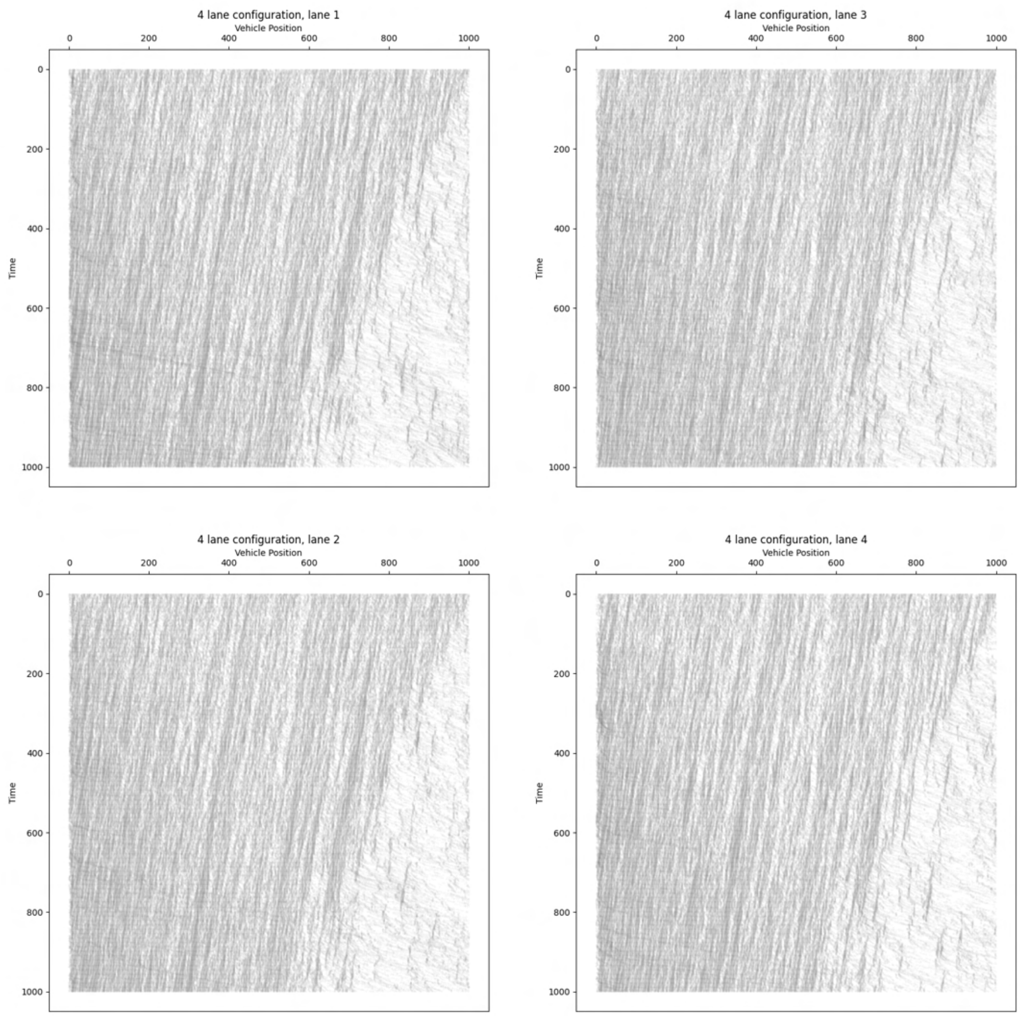
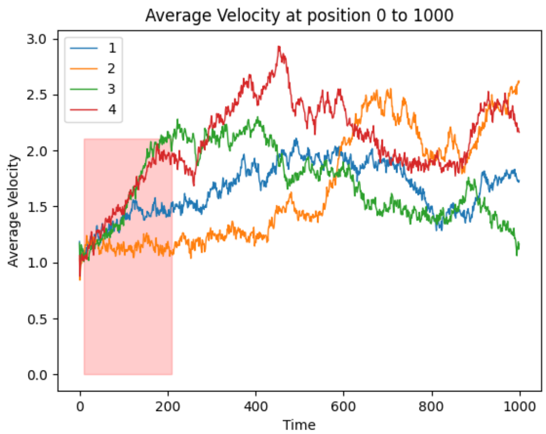
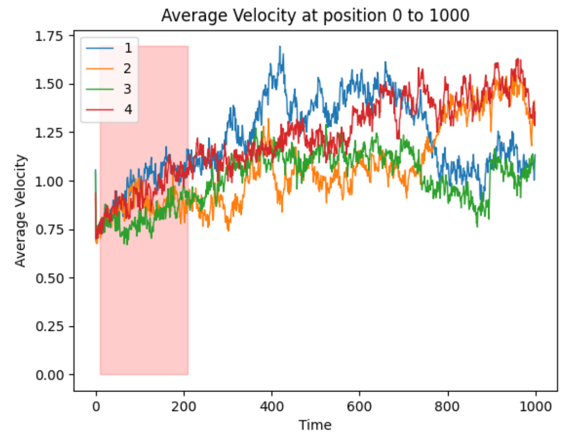


Figure 118: Time-space plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 2, slow probability = 0.8

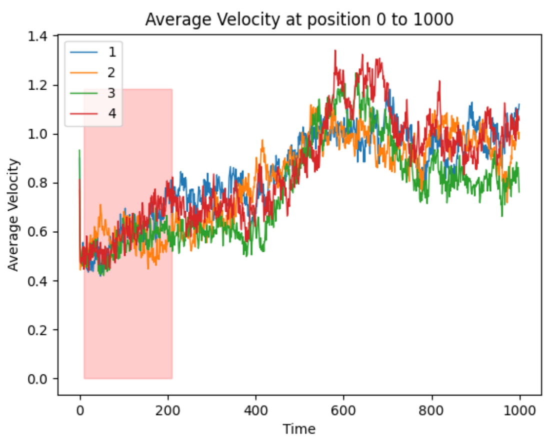
The average velocities are plotted below:



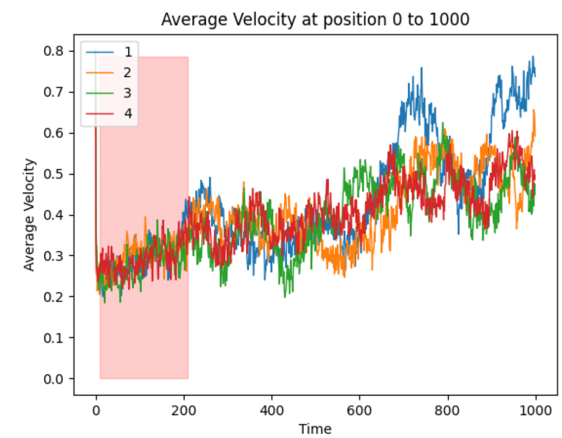
(a) Slow prob = 0.2



(b) Slow prob = 0.4



(c) Slow prob = 0.6



(d) Slow prob = 0.8

Figure 119: Average velocity plot, slow probability iteration for a quadruple lane configuration with obstacle in lane 2.

Figure 120: Fundamental relationship graph as the maximum velocities are modified.

5 Analysis

5.1 Single Lane

5.1.1 Velocities

The fundamental relationship graphs (Fig. 9 and 12) show that as the maximum velocity increases, the peak is shifted backwards, which is not ideal as it means that less cars can be on the road at the same time, but also the maximum flow rate has been increased. It is then important to find the balance between flow rate and density. There is one value of maximum velocity that shows the best compromise between density and flow rate, which is when the maximum velocity is set to 4. There are diminishing returns as the maximum velocity is increased.

With an obstacle in place it can be said that from the average velocity graphs, the average velocity did not increase as much, although the maximum velocity allowed was increased (Fig. 11. This holds true even if there is no obstacle (Fig. 8). The obstacle had not much impact at low velocities (2 - 4) but had a big impact at high velocities, as the range of velocities possible has increased. The recovery time (time taken to reach peak average velocity) was roughly equal across the high velocities, which was about 50 time steps.

5.1.2 Slow Probabilities

In Fig.15 and Fig.18, as the slowing probability increased, the flow rate decreases. Therefore it is safe to conclude that driver eccentricity has to be reduced to ensure optimal traffic flow. Fig. 14. There is also decrease in average velocity as the slow probability increased (Fig. 14 and Fig. 17).

With an obstacle, in the average velocity graph, as the slow probability increases, the average velocity of the road is decreased (Fig. 17), and the recovery effect was not observable at a higher probability ($p > 0.4$). The recovery time (for $p < 0.6$) was roughly the same at 50 time steps.

5.2 Multi Lane

5.2.1 Fundamental Relationship Graphs

The fundamental relationship graphs exhibit odd behaviour at higher densities. This is likely due to how the algorithm handles the flow rate count. At high densities, each lane has difficulty moving. However, some cars will find a way to swap in near the point of reference (the point where the flow rate is calculated), causing a count in the flow rate. The eccentricity at the tail end should not be taken into consideration during analysis. The early peak is the main point.

5.2.2 Obstacles

In the multi lane (number of lanes > 1) scenario, it is interesting to see the "imprints" of the effect of an obstacle on the lane that the obstacle is not on. This is due to the influx of vehicles switching from the other lane as the "next empty lane" is beneficial for lane switching. Having the obstacle in the middle in a 3 lane configuration makes the imprint that the obstacle has on the lane smaller (compare Fig. 47 and Fig. 55), which looks like a step in the right direction.

However, in the average velocity plots (Fig. 51 and Fig. 59), having the obstacle in the middle lane or side lane did not show any noticeable effect in the average velocity of the lanes. This could be due a hint to an error in the algorithm, or it is due to the number of cars on the road, the maximum velocity can only get so high. If the obstacle duration was adjusted to be longer, or simulation time increased, a slightly clearer picture may be painted.

5.2.3 Velocities

In the average velocity graphs, a trend can be noticed: with/without an obstacle present, the average velocity of the road increases as time goes on, until it reaches the peak average velocity. The peak is shifted further and increased in value as the max velocity is increased. This would be due to vehicles having a larger headroom, thus taking more time to reach the limit.

This effect is very noticeable in the 2 and 4 lane simulation (Fig. **??** and Fig. **??**). In higher number of lanes, there are some lanes that have a low average velocity compared to the other lanes (see Fig. 43 and Fig. **??** again), mainly being the lowest lane (the last lane in the simulation), the reason being that vehicles sometimes may end up stuck in a more congested lane, creating an imbalance in the average velocities across the lanes. This could be a limitation in the algorithm in choosing which lane to switch to.

In the time-space plots, something prevalent is that vehicles are clumped together in the beginning, and then slowly disperse and move relatively freely compared to the start of the simulation. The vehicles take a longer time to "disperse" (when the dark patch becomes lighter) as the maximum velocity is increased as the vehicles are started off with a bigger range, so it takes time to reach an equilibrium state (i.e. when the vehicles are "dispersed")

5.2.4 Slow Probabilities

In the average velocity graphs, the average velocity is decreased as the probability is increased. The effect of reaching maximum velocity is still observable. In the fundamental relationship graph (Fig. 32, Fig. 66, Fig. 108) shows a steady relationship between the decrease in slowing probability and the increase in flow rate and density. This is also reflected in the average velocity plots, where a decrease in slowing probability lead to an increase in average velocity (Fig. 31, Fig. 65, Fig. 107).

6 Conclusions and Future Work

Through the simulations done, clearly it is shown that as the number of lanes increase and if vehicles are allowed to switch lanes, the degrees of freedom increase. This leads to a very interesting relationship between the maximum velocities allowed on a road and the overall flow rate of the road.

It is found that at higher values of velocity (>6), there are diminishing returns when it comes to flow rate and density. The flow rate does not see much of an increase but the density at which the maximum flow rate occurs is decreased. On the other hand, the reduction of slowing probability - driver eccentricity can greatly increase the flow rate, average velocity and the density of the lane where the peak flow rate occurs.

Vehicles also take a longer time to reach "equlibrium" as the maximum velocity increases due to the larger range of initial velocities. It is also shown that vehicles recover (if recovery is observed) at the same rate regardless of the slowing probabilities, but take a longer time to reach maximum average velocity as the maximum velocity is increased.

There were some eccentricities with the fundamental density plots at the higher densities, as it should follow the shape of a single lane's fundamental relationship graph (Fig. 9). These could indicate something wrong with the algorithm, or something interesting which is for a later day. Other than that, the triple lane simulation with the obstacle in the 2nd lane gave unexpected results. Initially it was predicted to have a greater flow rate when the obstacle was placed in the centre lane, since having 2 lanes to switch to meant that the lane the obstacle was in would not be too congested, but it gave the same number of flow rate.

Future work will also involve expanding this simple circular road algorithm into a network of roads, involving shoulders, on-ramps, traffic lights, and roundabouts for a more realistic approach.

7 Bibliography

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8 Appendix

The code used to obtain the simulations can be accessed through: https://github.com/linsuong/traffic_flow_sim/tree/main