ZONI FRANCESCO N° 323085

1

$$f(x) = 3\ln(1-x) + e^{3x} - 1 =$$

$$= 3\ln(1+(-x)) + e^{3x} - 1 =$$

$$= 3\left(-x - \frac{x^2}{2} - \frac{x^3}{3}\right) + \cancel{1} + 3x + \frac{9x^2}{2} + \frac{27x^3}{6} - \cancel{1} =$$

$$= \cancel{-3x} - \frac{3}{2}x^2 - x^3 + \cancel{3x} + \frac{9}{2}x^2 + \frac{\cancel{27}^{9}}{\cancel{6}_{2}}x^3 =$$

$$= 3x^2 + \frac{-2x^3+9x^3}{2} = \boxed{3x^2 + \frac{7}{2}x^3}$$

$$\lim_{x\to 0} \frac{f(x)-3x^2}{2x^3+x^6} = \frac{\cancel{3x^2} + \frac{7}{2}x^3 - \cancel{3x^2}}{2x^3+x^6} = \frac{\frac{7}{2}x^3 + o(x^3)}{2x^3 + o(x^3)} = \frac{\frac{7}{2}\cancel{x^3}}{2\cancel{x^3}} = \boxed{\frac{7}{4}}$$

3 SCELTA: B.

$$\sum_{n\geq 0} \left(\frac{2x^2}{x+1}\right)^n$$ la serie è convergente sse $\left|\frac{2x^2}{x+1}\right| < 1$

$$\begin{cases} \frac{2x^2}{x+1} < 1 & \text{a.} \\ \frac{2x^2}{x+1} > -1 & \text{b.} \end{cases}$$

a. $x < -1 \lor -\frac{1}{2} \leq x \leq 1$

b. $x > -1$

$-1 \quad -\frac{1}{2} \quad 1$

~~a. $\frac{2x^2}{x+1} < 1$ $N \geq 0$ $2x^2 \geq 0$ $x^2 \geq 0$ $\forall x$ $D > 0$ $x+1 > 0$ $x > -1$~~

~~$- \quad -1 \quad + \quad +$ $x < 1$~~

~~b. $\frac{2x^2}{x+1} > -1$ $N \geq 0$ $2x^2 \geq 0$~~

LA SERIE È CONVERGENTE SSE $-\frac{1}{2} \leq x \leq 1$

$$\sum_{N\geq 0}\left(\frac{2x^2}{x+1}\right)^n = \frac{1}{1-\frac{2x^2}{x+1}} = \frac{1}{\frac{x+1-2x^2}{x+1}} = \boxed{\frac{x+1}{1-2x^2+x}}$$

a. $\frac{2x^2}{x+1} < 1$

$\frac{2x^2}{x+1} - 1 < 0$

$\frac{2x^2-x-1}{x+1} < 0$

$N \geq 0$ $2x^2-x-1 \geq 0$

$\Delta = 1 - 4(2)(-1) = 9$

$x_1 = \frac{1+3}{4} = 1$

$x_2 = \frac{1-3}{4} = -\frac{1}{2}$

$+ \quad + \quad -\frac{1}{2} \quad 1$

$x \leq -\frac{1}{2} \lor x \geq 1$

~~b. $\frac{2x^2}{x+1} > -1$ $2x^2$~~

$D > 0$ $x+1 > 0$

$x > -1$

$- \quad -1 \quad + \quad -\frac{1}{2} \quad - \quad 1 \quad +$

$$\boxed{\begin{matrix} x < -1 \\ \lor \\ -\frac{1}{2} \leq x \leq 1 \end{matrix}}$$

b. $\frac{2x^2}{x+1} > -1$

$\frac{2x^2+x+1}{x+1} > 0$ $\quad N \geq 0 \quad 2x^2+x+1 \geq 0$

$\Delta = 1-4(2)(1) = -7$

NO INTERSEZIONI
CON ASSE X

$D > 0 \quad \boxed{x > -1}$

2

$f(x) = \frac{x^3}{x^2-1}$ $\quad \mathcal{D}: \text{dom} f = x^2-1 > 0 \quad x^2 > 1$

$x^2 = 1$

$x = \pm 1$ $\quad \mathcal{D}:]-\infty,-1[\cup]-1,1[\cup]1,+\infty[$

se $x=0 \quad y=0 \quad O(0,0)$

se $y=0 \quad \frac{x^3}{x^2-1} = 0 \quad x^2-1=0$

$x^2 = 1 \quad A(+1,0)$

$x = \pm 1 \quad B(-1,0)$

$f(x) > 0 \quad \frac{x^3}{x^2-1} > 0 \quad N \geq 0 \quad x^3 \geq 0$

$x \geq 0$

$D > 0 \quad x^2-1 > 0$

$x^2 > 1$

$x^2 = 1$

$x = \pm 1$

$x < -1$ ∨ $x > 1$

La funzione è positiva per:

$-1 < x \leq 0$
e
$x > 1$

Limiti

$\lim_{x \to -\infty} \frac{x^3}{x^2-1} = \frac{-\infty}{+\infty} = -\infty$ possibile asintoto obliquo....

$y = mx + q$

$m = \lim_{x \to -\infty} \frac{f(x)}{x} = \frac{x^3}{(x^2-1)x} = \frac{x^3}{x^3-x} = \frac{\cancel{x^3}}{\cancel{x^3}} - \frac{x^3}{x} = 1 \quad m = 1$

$q = \lim_{x \to -\infty} (f(x) - mx) = \lim_{x \to -\infty} \frac{x^3}{x^2-1} - x = \frac{x^3 - x(x^2-1)}{x^2-1} = \frac{x^3-x^3+x}{x^2-1} = \frac{x}{x^2-1} = \frac{-\infty}{+\infty}$

$y = x$ A. OB.

$\lim_{x \to 1^+} \frac{x^3}{x^2-1} = \frac{1^+}{0} = +\infty$ $\quad \lim_{x \to 1^-} \frac{x^3}{x^2-1} = -\infty$ $\quad x = 1$ A.V.

$\lim_{x \to -1^+} \frac{x^3}{x^2-1} = \frac{-1^+}{0} = +\infty$ $\quad \lim_{x \to -1^-} \frac{x^3}{x^2-1} = -\infty$ $\quad x = -1$ A.V.

$f'(x) = D\left(\frac{x^3}{x^2-1}\right) = \frac{3x^2(x^2-1) - x^3(2x)}{(x^2-1)^2} = \frac{3x^4 - 3x^2 - 2x^4}{x^4+1-2x^2} = \frac{x^4-3x^2}{x^4-2x^2+1}$ DERIVATA PRIMA

$f'(x) > 0 \quad \frac{x^4-3x^2}{x^4-2x^2+1} > 0$

N≥0 $\quad x^4 - 3x^2 \geq 0$

$x^2(x^2-3) \geq 0$

$x^2 \geq 0$

D>0 $\quad x^4 - 2x^2 + 1 > 0$

oppure

$(x^2-1)^2 > 0$

$x^2 - 1 \neq 0$

$x^2 \neq 1 \quad x \neq -1$ e 1

$f'(x) > 0 \quad \frac{x^4-3x^2}{(x^2-1)^2} \quad x^4-3x^2 \geq 0$

$x^2(x^2-3) \geq 0$

$x^2 - 3 \geq 0$

$x^2 \geq 3$

$x \leq -\sqrt{3} \vee x \geq \sqrt{3}$

$x^2 = 3 \quad x = \pm\sqrt{3}$

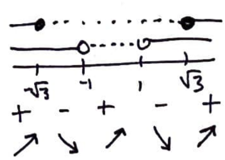
$x = -\sqrt{3}$ p.to di min locale

$x = \sqrt{3}$ p.to di max locale

$(x^2-1)^2 > 0$

$x^2 - 1 > 0$

$x^2 > 1 \quad x < -1 \vee x > 1$



$x = -\sqrt{3} \quad y = \frac{(-\sqrt{3})^4 - 3(\sqrt{3})^2}{(-\sqrt{3})^4 - 2(\sqrt{3})^2 + 1}$

$= \frac{9-9}{9-6+1} =$

$$f''(x) = D\left(\frac{x^4-3x^2}{(x^2-1)^2}\right) = \cancel{\frac{(4x^3-6x)(x^2-1)^2-(x^4-3x^2)(x^2-1)^3\cdot 2x}{(x^2-1)^4}} =$$

$$= \cancel{(4x^3-6x)(x^4-2x^2+1)}$$

$$= \frac{(4x^3-6x)(x^4+1+2x^2)-(x^4-3x^2)(4x^3+4x)}{(x^2-1)^4} =$$

$$= \frac{4x^7+4x^3+8x^5-6x^5-6x-12x^3-(4x^7+4x^5-12x^5-12x^3)}{(x^2-1)^4} =$$

$$= \frac{\cancel{4x^7}-8x^3+2x^5-6x-\cancel{4x^7}-4x^5+12x^5+12x^3}{(x^2-1)^4} =$$

$$= \frac{10x^5+12x^3-6x}{(x^2-1)^4} =$$ DERIVATA SECONDA

$f''(x) > 0 \qquad \frac{10x^5+12x^3-6x}{(x^2-1)^4} > 0$

$N \geq 0$

$10x^5+12x^3-6x \geq 0$

$2x(5x^4+6x^2-3) \geq 0$

$2x \geq 0$

$\boxed{x \geq 0}$

$(5x^4+6x^2-3) \geq 0$

$5x^4+6x^2 \geq 3$

$x^2(5x^2+6) \geq 3$

$x^2 \geq 3 \qquad x^2 = 3 \qquad x \leq -\sqrt{3} \lor x \geq \sqrt{3}$

$5x^2+6 \geq 3$

$5x^2 \geq -3$

$x^2 \geq -\frac{3}{5}$

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

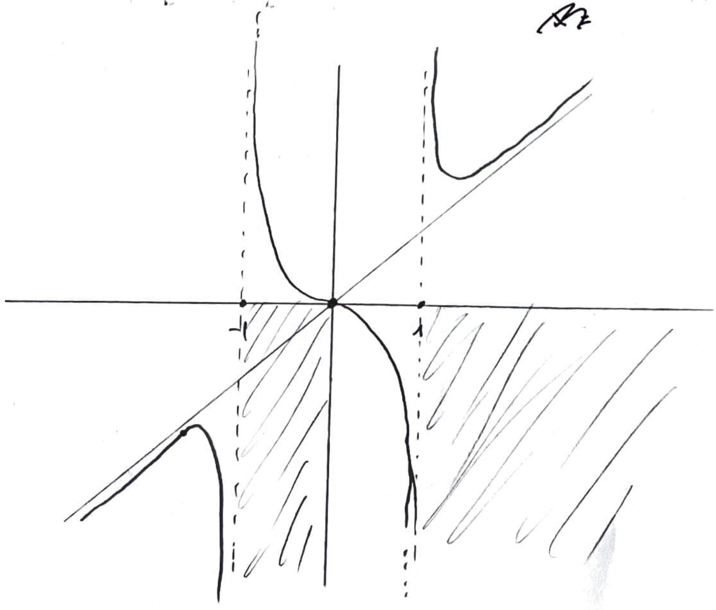
$- \quad + \quad - \quad +$

$\cap \quad \cup \quad \cap \quad \cup$

$x=0$ p.to di flesso

$x=0$ p.to di flesso $(0,0)$

$y = \frac{0}{1} = 0$



$f(x) = \frac{x^3}{x^2-1}$ $\quad x_0 = 0$ $\quad y - f(x_0) = f'(x_0)(x - x_0)$

$f(x_0) = 0$

La retta tangente è l'asse delle x

$f'(x_0) = \frac{0}{0} = 0$

$$\text{Area} = \int_{-\frac{1}{4}}^{-\frac{1}{2}} f(x)\,dx = \int_{-\frac{1}{4}}^{-\frac{1}{2}} \frac{x^3}{x^2-1}\,dx = \int_{-\frac{1}{4}}^{-\frac{1}{2}} \frac{\cancel{x^3}}{\cancel{x^2}}\,dx - \int_{-\frac{1}{4}}^{-\frac{1}{2}} x^3\,dx = \left[x\right]_{-\frac{1}{4}}^{-\frac{1}{2}} - \left[\frac{x^4}{4}\right]_{-\frac{1}{4}}^{-\frac{1}{2}} =$$

$$= \left[-\frac{1}{2} + \frac{1}{4}\right] - \left[\frac{\left(-\frac{1}{2}\right)^4}{4} - \left(\frac{\left(-\frac{1}{4}\right)^4}{4}\right)\right] = \frac{-4+2}{8} - \left(\frac{1}{64} - \frac{1}{1024}\right) = -\frac{1}{4} - \frac{16-1}{1024} =$$

$$= -\frac{1}{4} - \frac{14}{1024} = \frac{-256-14}{1024} = -\frac{270}{1024} = \boxed{-\frac{135}{512}}$$