CarSeats

February 21, 2025

1 Multilinear Regression: CarSeats dataset

1.1 Import notebook funcs

[1]:
```
from notebookfuncs import *
```

1.2 Import ISLP objects

[2]:
```
from ISLP import load_data
```

1.3 Import User Funactions

[3]:
```
from userfuncs import *
```

1.4 Load dataset

[4]:
```
Carseats = load_data("Carseats")
Carseats.head()
```

[4]:
```
   Sales  CompPrice  Income  Advertising  Population  Price ShelveLoc  Age  \
0   9.50        138      73           11         276    120       Bad   42
1  11.22        111      48           16         260     83      Good   65
2  10.06        113      35           10         269     80    Medium   59
3   7.40        117     100            4         466     97    Medium   55
4   4.15        141      64            3         340    128       Bad   38

   Education Urban   US
0         17   Yes  Yes
1         10   Yes  Yes
2         12   Yes  Yes
3         14   Yes  Yes
4         13   Yes   No
```

[5]:
```
Carseats.shape
```

[5]: (400, 11)

[6]:
```python
Carseats = Carseats.dropna()
Carseats.shape
```

[6]: (400, 11)

1.5 Display dataset stats

[7]:
```python
Carseats.describe()
```

[7]:
```
            Sales   CompPrice      Income  Advertising  Population  \
count  400.000000  400.000000  400.000000   400.000000  400.000000
mean     7.496325  124.975000   68.657500     6.635000  264.840000
std      2.824115   15.334512   27.986037     6.650364  147.376436
min      0.000000   77.000000   21.000000     0.000000   10.000000
25%      5.390000  115.000000   42.750000     0.000000  139.000000
50%      7.490000  125.000000   69.000000     5.000000  272.000000
75%      9.320000  135.000000   91.000000    12.000000  398.500000
max     16.270000  175.000000  120.000000    29.000000  509.000000

            Price         Age   Education
count  400.000000  400.000000  400.000000
mean   115.795000   53.322500   13.900000
std     23.676664   16.200297    2.620528
min     24.000000   25.000000   10.000000
25%    100.000000   39.750000   12.000000
50%    117.000000   54.500000   14.000000
75%    131.000000   66.000000   16.000000
max    191.000000   80.000000   18.000000
```

1.6 Set categorical types

[8]:
```python
Carseats["US"] = Carseats["US"].astype("category")
Carseats["Urban"] = Carseats["Urban"].astype("category")
```

1.7 Standardize variables

[9]:
```python
Carseats["Sales"] = standardize(Carseats["Sales"])
Carseats["Price"] = standardize(Carseats["Price"])
```

1.7.1 (a) Fit a multiple regression model to predict Sales using Price, Urban, and US.

[10]:
```python
formula = "Price + Urban + US"
perform_analysis("Sales", formula, Carseats)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.239
```

```
Model:                            OLS   Adj. R-squared:                  0.234
Method:                 Least Squares   F-statistic:                     41.52
Date:                Fri, 21 Feb 2025   Prob (F-statistic):           2.39e-23
Time:                        19:20:52   Log-Likelihood:                -512.88
No. Observations:                 400   AIC:                             1034.
Df Residuals:                     396   BIC:                             1050.
Df Model:                           3
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept       -0.2691      0.098     -2.734      0.007      -0.463      -0.076
Urban[T.Yes]    -0.0078      0.096     -0.081      0.936      -0.197       0.182
US[T.Yes]        0.4256      0.092      4.635      0.000       0.245       0.606
Price           -0.4566      0.044    -10.389      0.000      -0.543      -0.370
==============================================================================
Omnibus:                        0.676   Durbin-Watson:                   1.912
Prob(Omnibus):                  0.713   Jarque-Bera (JB):                0.758
Skew:                           0.093   Prob(JB):                        0.684
Kurtosis:                       2.897   Cond. No.                         4.27
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
             df      sum_sq    mean_sq           F        PR(>F)
Urban       1.0    0.095104   0.095104    0.123767  7.251713e-01
US          1.0   12.675983  12.675983   16.496407  5.877444e-05
Price       1.0   82.939070  82.939070  107.936143  1.609917e-22
Residual  396.0  304.289843   0.768409         NaN           NaN
```

[10]: <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x759c711dcfb0>

1.7.2 (b) Provide an interpretation of each coefficient in the model. Be careful—some of the variables in the model are qualitative!

- The coefficient of -0.0078 for Urban (True) indicates that -0.0078 of the SD of Sales can be explained by the level urban store as compared to a rural store. However, the p-value of 0.936 indicates that this difference is not significant and can be discounted or discarded.
- The coefficent of 0.4256 for US (True) indicates that 0.4256 of the typical deviation of Sales are explained by a US store as compared to a non-US store.
- The coefficient of -0.4566 for Price indicates that -0.4566 of the typical deviation of Sales is explained by one SD of change in the Price variable.
- We can also conclude that Price has the highest effect on Sales, the response variable, since the absolute value of its coefficient 0.4566 is the highest amongst all the coefficients.
- https://blogs.sas.com/content/iml/2023/07/17/standardize-reg-coeff-class.html
- https://www.statlect.com/fundamentals-of-statistics/

linear-regression-with-standardized-variables

1.7.3 (c) Write out the model in equation form, being careful to handle the qualitative variables properly.

- The equation can be written out as follows:
- Sales = -0.0078 * Urban + 0.4256 * US -0.4566 * Price (Standardized) - 0.2691

1.7.4 (d) For which of the predictors can you reject the null hypothesis H0 : j = 0?

- The p-value for the Urban predictor is 0.936 which is much higher than our chosen level of significance 0.01. So we cannot reject the null Hypothesis in this case that its coefficient is zero.
- The p-values for US, Price and Intercept are zero. Hence, we reject the null hypothesis for them that their coefficients are zero.

1.7.5 (e) On the basis of your response to the previous question, fit a smaller model that only uses the predictors for which there is evidence of association with the outcome.

[11]:
```python
formula = "Price + US"
results = perform_analysis("Sales", formula, Carseats)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.239
Model:                            OLS   Adj. R-squared:                  0.235
Method:                 Least Squares   F-statistic:                     62.43
Date:                Fri, 21 Feb 2025   Prob (F-statistic):           2.66e-24
Time:                        19:20:52   Log-Likelihood:                -512.88
No. Observations:                 400   AIC:                             1032.
Df Residuals:                     397   BIC:                             1044.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -0.2743      0.074     -3.730      0.000      -0.419      -0.130
US[T.Yes]      0.4253      0.092      4.641      0.000       0.245       0.605
Price         -0.4567      0.044    -10.416      0.000      -0.543      -0.371
==============================================================================
Omnibus:                        0.666   Durbin-Watson:                   1.912
Prob(Omnibus):                  0.717   Jarque-Bera (JB):                0.749
Skew:                           0.092   Prob(JB):                        0.688
Kurtosis:                       2.895   Cond. No.                         3.12
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
```

```
specified.
             df      sum_sq    mean_sq           F        PR(>F)
US          1.0   12.544810  12.544810   16.366658  6.273751e-05
Price       1.0   83.160345  83.160345  108.495617  1.272157e-22
Residual  397.0  304.294845   0.766486         NaN           NaN
```

[11]: <statsmodels.regression.linear_model.RegressionResultsWrapper at 0x759be7d76270>

1.7.6 (f) How well do the models in (a) and (e) fit the data?

- Model(a) has an explanatory value R2 adjusted value of 0.234
- Model(e) has an explanatory value R2 adjusted value of 0.235

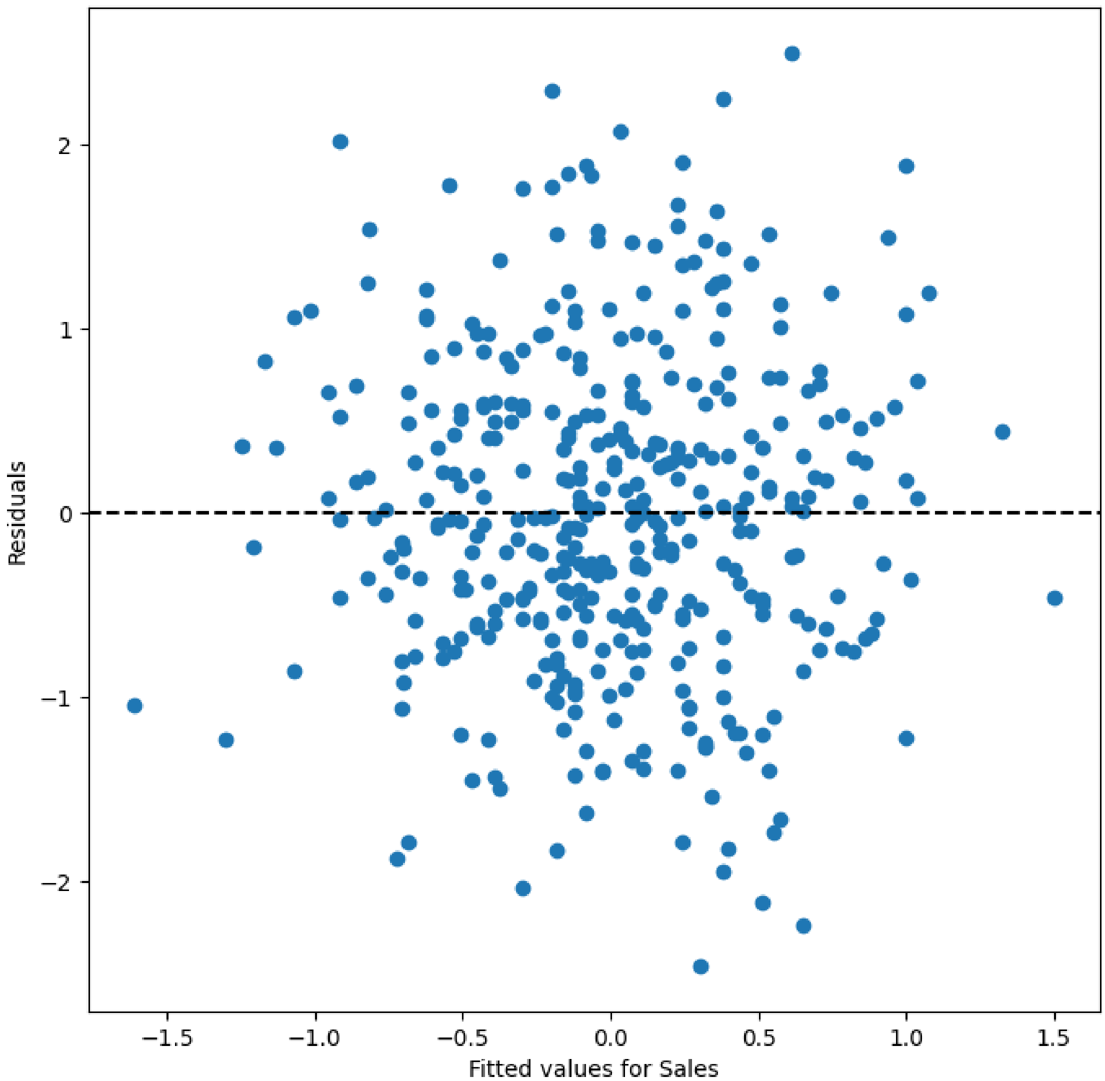
1.7.7 (g) Using the model from (e), obtain 95 % confidence intervals for the coefficient(s).

From the summary analysis, it can be seen that the 95% confidence limits for the three terms are as follows: + Intercept (-0.419, -0.130) + US[T.Yes] (0.245, 0.605) + Price (-0.543, -0.371) + None of them include zero in their range unlike that for Urban[T.Yes] in Model(a) which is another indicator that the coefficient is not significant.

1.7.8 (h) Is there evidence of outliers or high leverage observations in the model from (e)?

We can check for presence of outliers by plotting the residuals plot and seeing if there are any outliers.

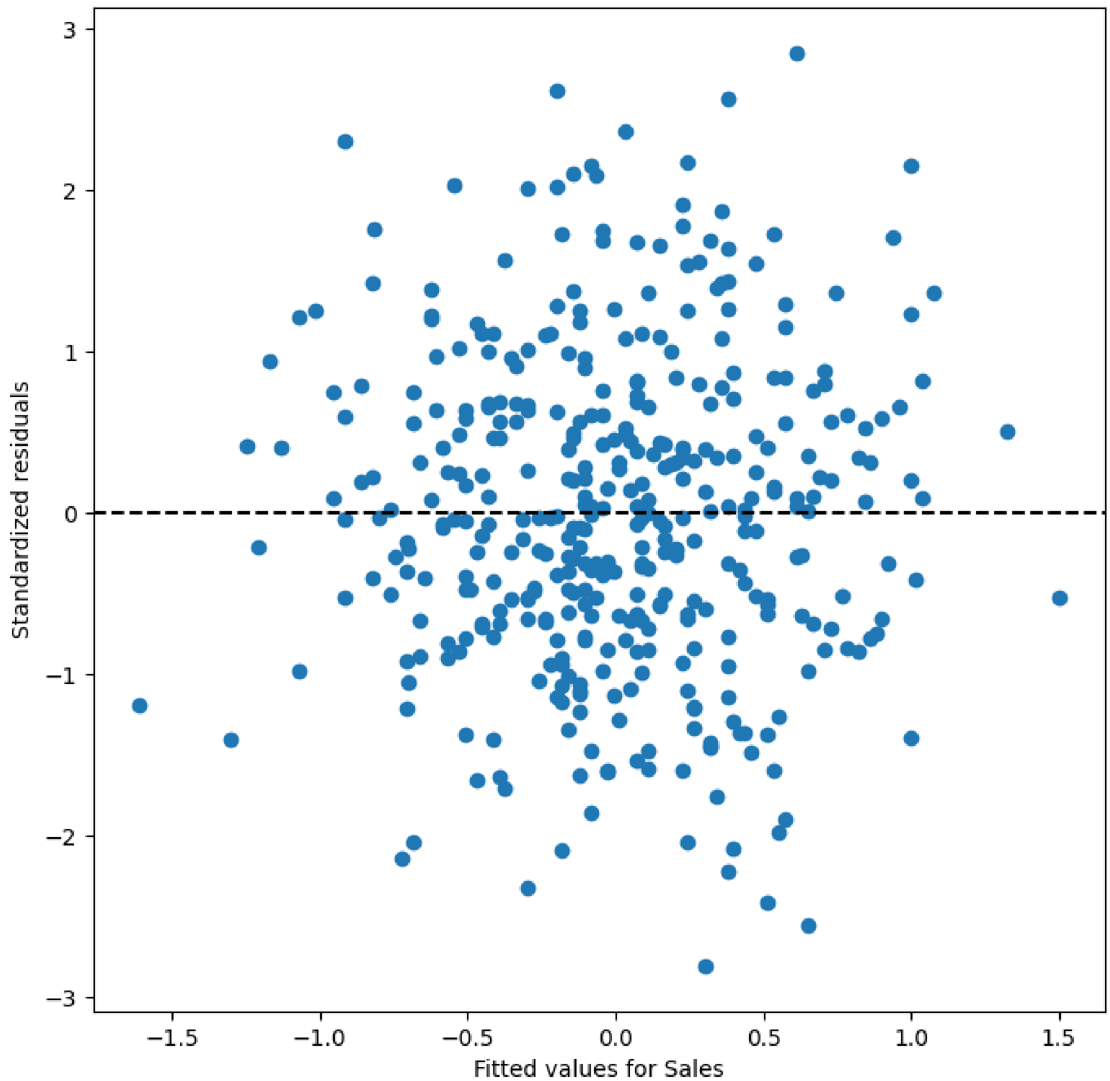
[12]:
```
display_residuals_plot(results)
```



- From the plot above, there doesn't appear to be any obvious outliers.

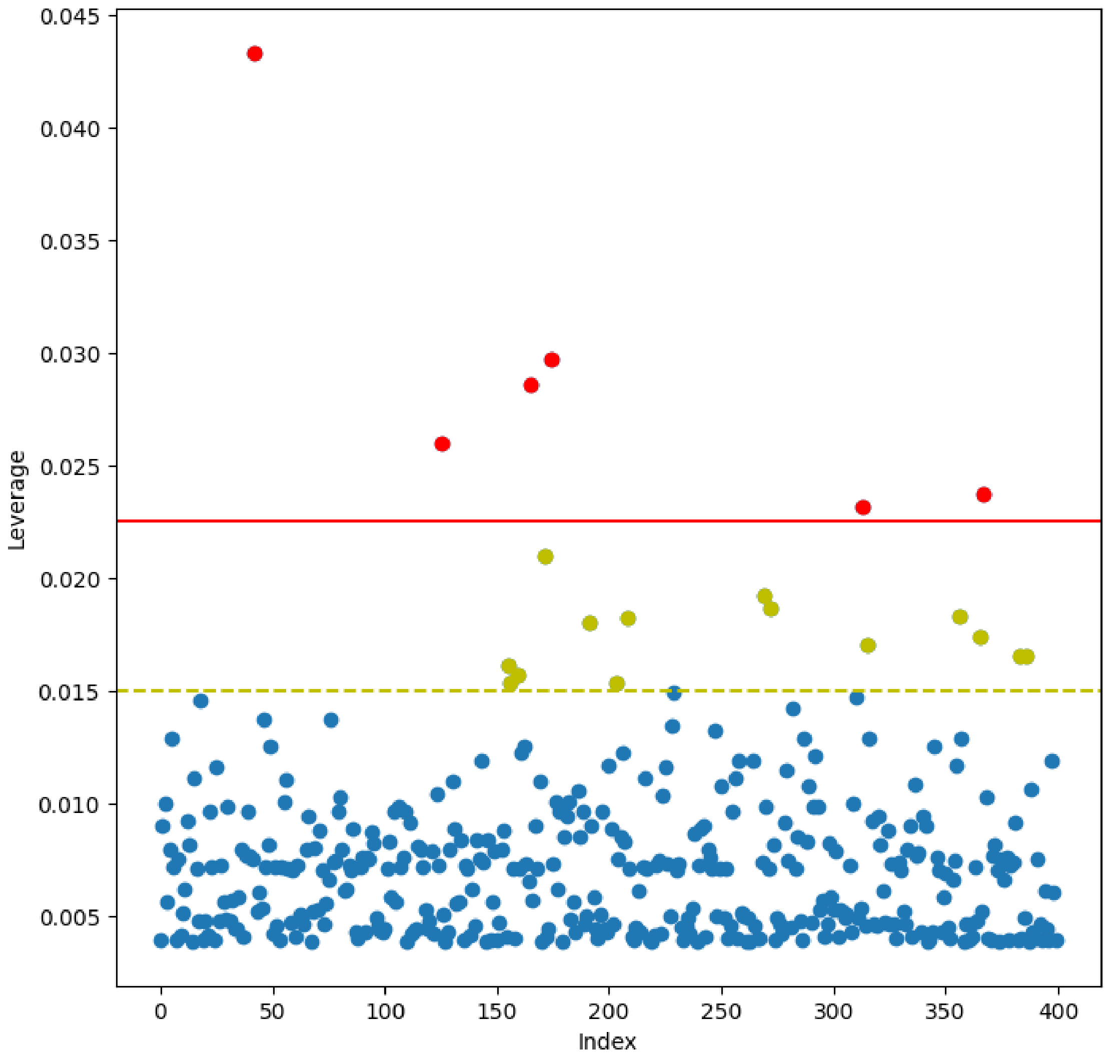
We can plot studentized residuals to see whether there are any visible there.

[13]:
```
display_studentized_residuals(results)
```



- From the above plot, no observation lies outside the (-3,3) range. Hence, we can safely conclude that there are no evident outliers in the dataset.

[14]:
```
display_hat_leverage_cutoffs(results)
```

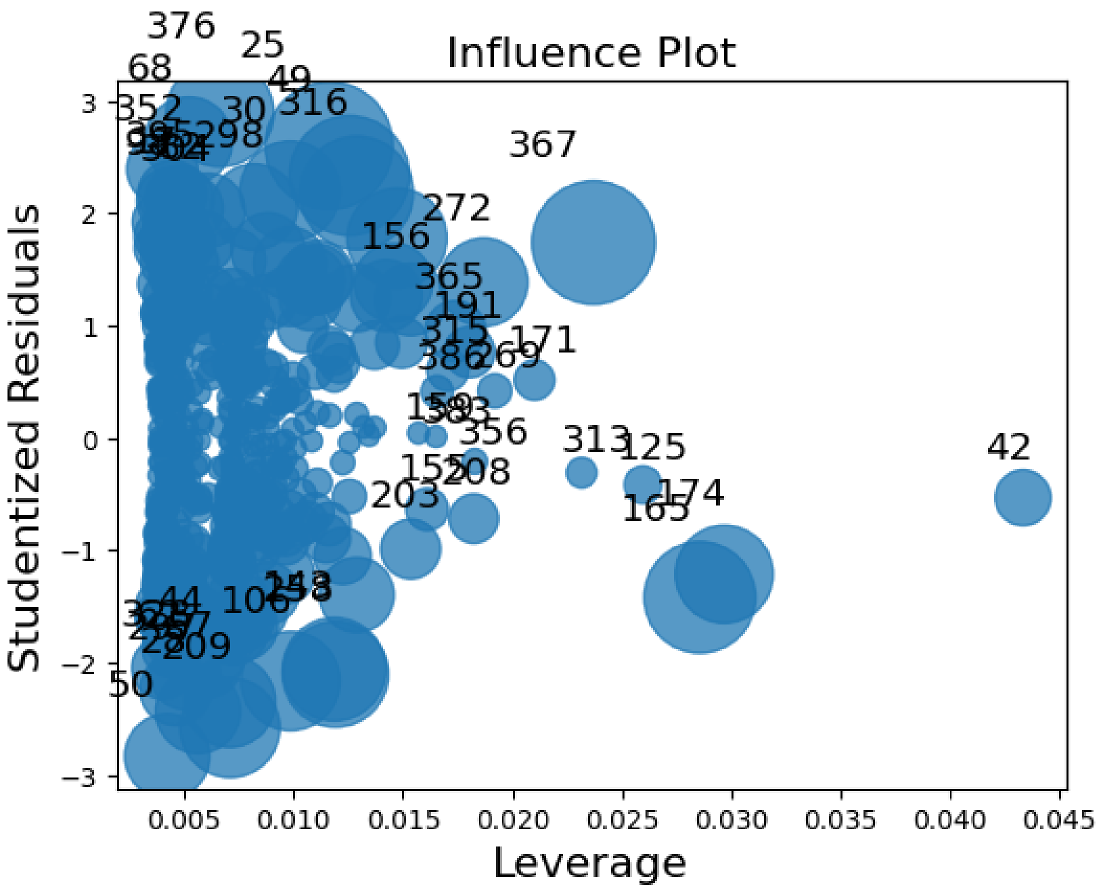


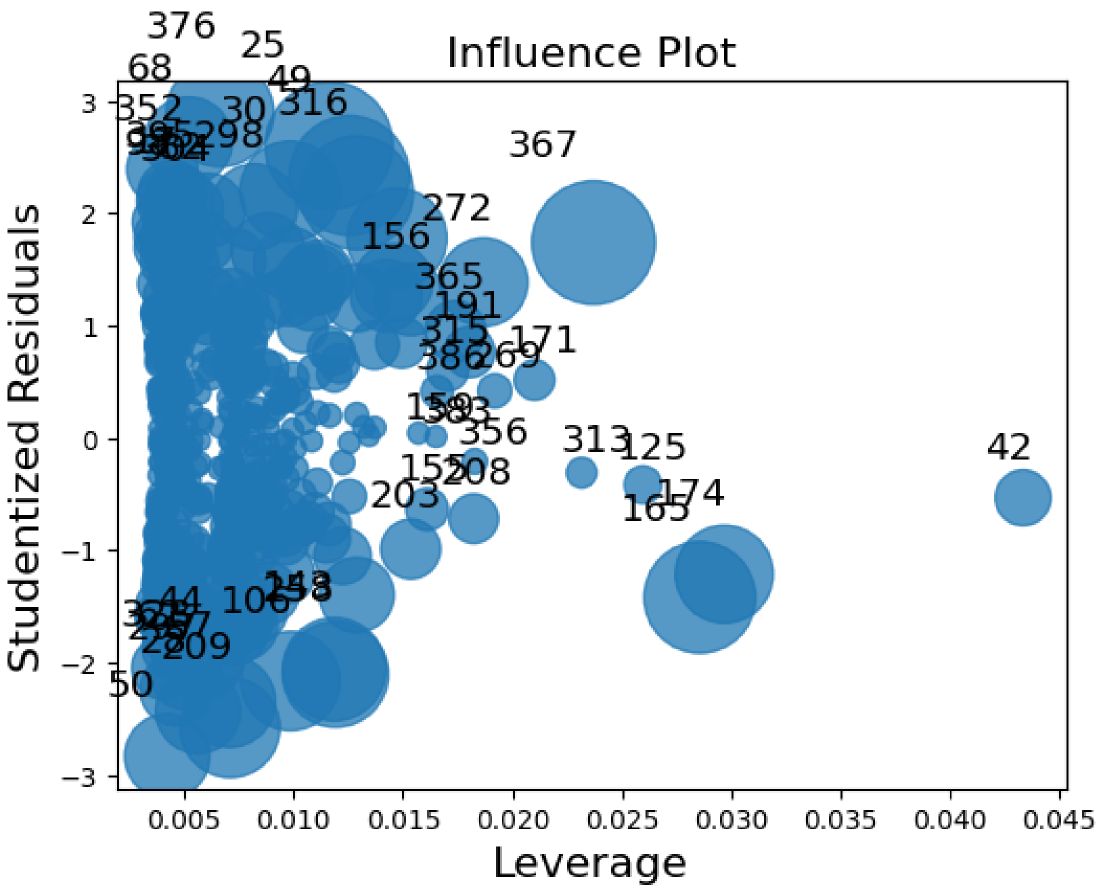
- We can see from the above graph that we have a few leverage points that exceed the cutoff of 3 * average leverage value. These are plotted in red.
- The ones in yellow exceed the less conservative estimate of 2 * average leverage value
- We could also use more conservative estimates for the cutoff of either 4 * average leverage value or 5 * average cutoff value

References: - https://online.stat.psu.edu/stat501/lesson/11/11.2

[15]:
```
display_cooks_distance_plot(results)
```

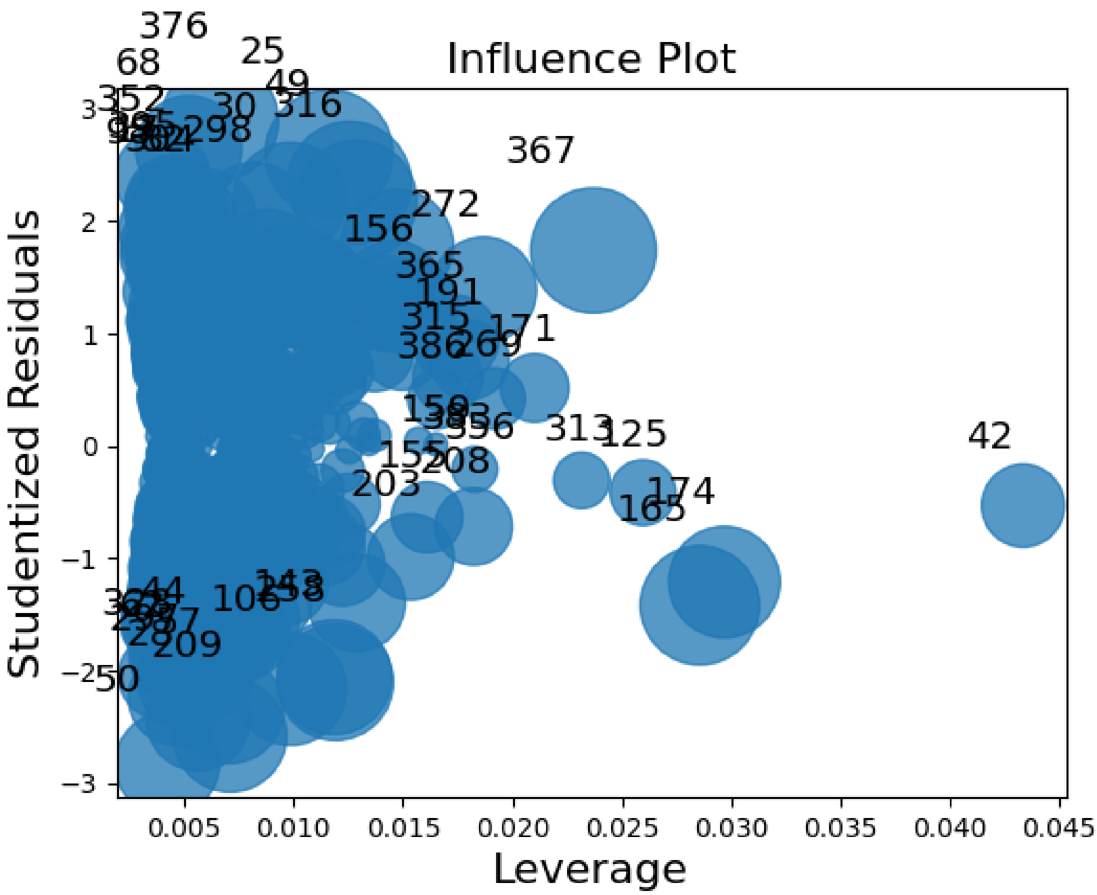
[15]:

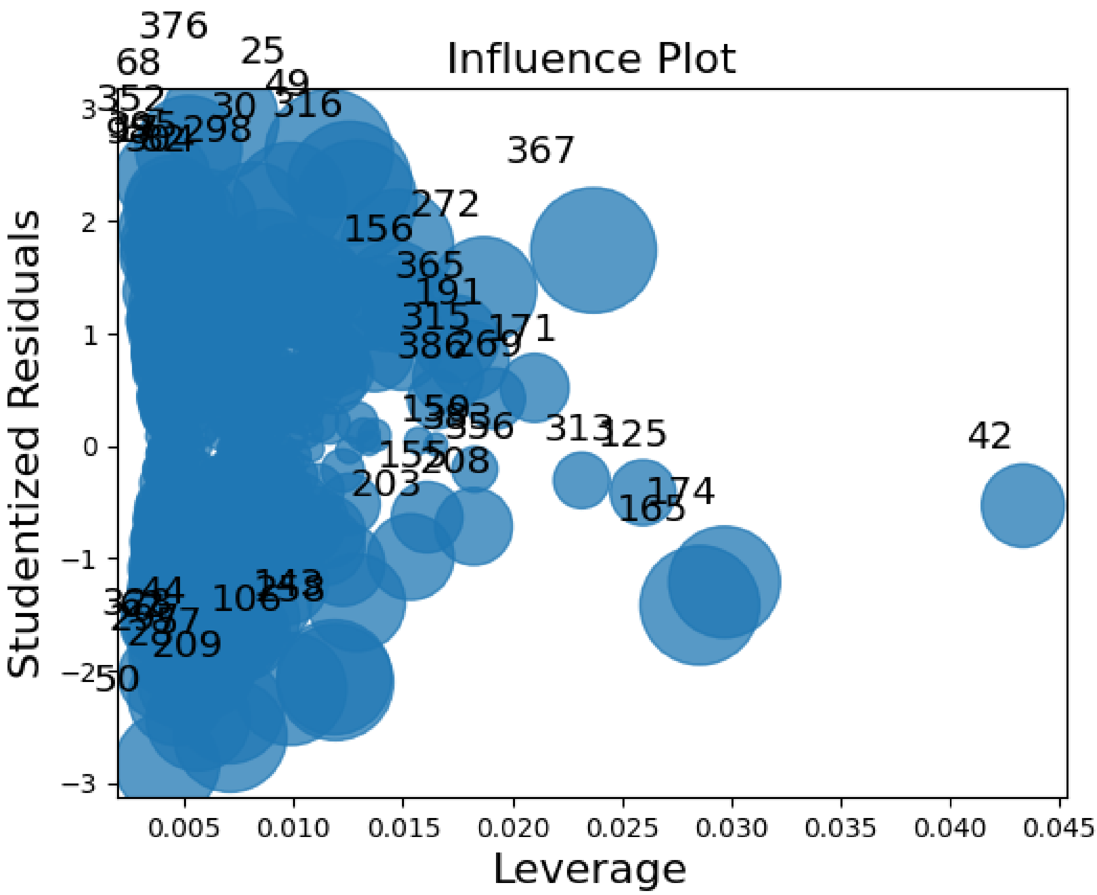

Influence Plot
Studentized Residuals
Leverage
3
2
1
0
−1
−2
−3
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040
0.045
376
25
68
49
352
30
316
298
367
272
156
365
191
315
171
386
269
356
313
125
42
203
208
174
165
106
209
50




```
[16]: display_DFFITS_plot(results)
```

[16]:


Influence Plot
Studentized Residuals
Leverage
3
2
1
0
−1
−2
−3
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040
0.045
376
68
25
49
352
30
316
298
367
272
156
365
191
315
171
386
269
356
313
125
42
208
203
174
165
106
209
50




```
[17]: display_hat_leverage_plot(results)
```

- Looking at all three studentized plots for leverage, it can be concluded that even if there are a few outliers, none wield a significant influence on the regression since the points with high leverage values have low studentized residual values.

```
[18]: inf_df, _ = get_influence_points(results)
inf_df
print(_)
```

```
n = 400.0, p = 3
Average Hat Leverage: 0.0075000000000000015
Hat Leverage  Cutoff = 2 * Average Hat Leverage = 0.015000000000000003
DFBetas Cutoff = 3 / sqrt(n) = 0.15
DFFITS Cutoff = 2 * sqrt(p/n) = 0.17320508075688773
Cooks Distance Cutoff = 1.0
Cooks Distance p-value Cutoff = 0.05
Studentized Residuals Cutoff = 3.0
Studentized Residuals p-value Cutoff = 0.01
{'n': 400.0, 'p': 3, 'average_hat': 0.0075000000000000015,
```

```
'hat_leverage_cutoff': 0.015000000000000003, 'dfbetas_cutoff': 0.15,
'dffits_cutoff': 0.17320508075688773, 'studentized_residuals_cutoff': 3.0,
'studentized_residuals_pvalue_cutoff': 0.01, 'cooks_d_cutoff': 1.0,
'cooks_d_pvalue_cutoff': 0.05}
```

1.7.9 For a more conservative cutoff values for hat_diag, we have the following infuence point(s):

[19]:
```python
inf_df[inf_df["hat_diag"] > (3 * np.mean(inf_df["hat_diag"]))]
```

[19]:
```
Empty DataFrame
Columns: [dfb_Intercept, dfb_US[T.Yes], dfb_Price, cooks_d, hat_diag,
student_resid, dffits, student_resid_pvalue, hat_influence, cooks_d_pvalue]
Index: []
```

1.7.10 Using DFFITS cutoff, we have the following influential points

[20]:
```python
inf_df[inf_df["dffits"] > 2 * np.sqrt(len(results.params) / results.nobs)]
```

[20]:
```
     dfb_Intercept  dfb_US[T.Yes]  dfb_Price   cooks_d  hat_diag  \
25        0.210972      -0.165700  -0.176952  0.026109  0.011622
68        0.004485       0.092526   0.096454  0.011988  0.005202
376      -0.007088       0.116257  -0.152448  0.018282  0.006637

     student_resid    dffits  student_resid_pvalue  hat_influence  \
25        2.599652  0.281894              0.004840       0.030212
68        2.642364  0.191069              0.004280       0.013744
376       2.891521  0.236355              0.002023       0.019192

     cooks_d_pvalue
25         0.994295
68         0.998202
376        0.996634
```

1.7.11 Using Cooks Distance, we have the following influential points

[21]:
```python
inf_df[inf_df["cooks_d"] > 1.0]
```

[21]:
```
Empty DataFrame
Columns: [dfb_Intercept, dfb_US[T.Yes], dfb_Price, cooks_d, hat_diag,
student_resid, dffits, student_resid_pvalue, hat_influence, cooks_d_pvalue]
Index: []
```

1.7.12 Using Cooks Distance p-values, we have the following influential points

[22]:
```python
inf_df[inf_df["cooks_d_pvalue"] < 0.05]
```

[22]: Empty DataFrame
Columns: [dfb_Intercept, dfb_US[T.Yes], dfb_Price, cooks_d, hat_diag,
student_resid, dffits, student_resid_pvalue, hat_influence, cooks_d_pvalue]
Index: []

1.7.13 Using DFBeta for intercept, we have the following influential points

```
[23]: inf_df[inf_df["dfb_Intercept"] > (3 / np.sqrt(results.nobs))]
```

```
[23]:     dfb_Intercept  dfb_US[T.Yes]  dfb_Price   cooks_d  hat_diag  \
25        0.210972        -0.1657  -0.176952  0.026109  0.011622

    student_resid    dffits  student_resid_pvalue  hat_influence  \
25       2.599652  0.281894               0.00484       0.030212

    cooks_d_pvalue
25        0.994295
```

1.7.14 Using DFBeta for US, we have the following influential points

```
[24]: inf_df[inf_df["dfb_US[T.Yes]"] > (3 / np.sqrt(results.nobs))]
```

[24]: Empty DataFrame
Columns: [dfb_Intercept, dfb_US[T.Yes], dfb_Price, cooks_d, hat_diag,
student_resid, dffits, student_resid_pvalue, hat_influence, cooks_d_pvalue]
Index: []

1.7.15 Using DFBeta for Price, we have the following influential points

```
[25]: inf_df[inf_df["dfb_Price"] > (3 / np.sqrt(results.nobs))]
```

[25]: Empty DataFrame
Columns: [dfb_Intercept, dfb_US[T.Yes], dfb_Price, cooks_d, hat_diag,
student_resid, dffits, student_resid_pvalue, hat_influence, cooks_d_pvalue]
Index: []

```
[26]: allDone()
```

<IPython.lib.display.Audio object>