Multilinear Regression: Auto dataset

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Import notebook functions

```
from notebookfuncs import *
```

Import standard libraries

```
import numpy as np
import pandas as pd

pd.set_option("display.max_rows", 1000)
pd.set_option("display.max_columns", 1000)
pd.set_option("display.width", 1000)
pd.set_option("display.max.colwidth", None)
import matplotlib.pyplot as plt
from matplotlib.pyplot import subplots
import seaborn as sns
import itertools
```

New imports

```
import statsmodels.api as sm
```

Import statsmodels.objects

```
from statsmodels.stats.outliers_influence import variance_inflation_factor as VIF
from statsmodels.stats.outliers_influence import summary_table
from statsmodels.stats.anova import anova_lm
import statsmodels.formula.api as smf
```

Import ISLP objects

```
import ISLP
from ISLP import models
from ISLP import load_data
from ISLP.models import ModelSpec as MS, summarize, poly
```

Import user functions

```
from userfuncs import *
```

Set level of significance (alpha)

```
LOS_Alpha = 0.01
```

```
0.01
```

```
Auto = load_data("Auto")
Auto = Auto.sort_values(by=["year"], ascending=True)
Auto.head()
Auto.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year', 'origin'],
```

```
Auto.shape
```

```
(392, 8)
```

```
Auto.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
count	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000
mean	23.445918	5.471939	194.411990	104.469388	2977.584184	15.541327	75.979592	1.576531
std	7.805007	1.705783	104.644004	38.491160	849.402560	2.758864	3.683737	0.805518
min	9.000000	3.000000	68.000000	46.000000	1613.000000	8.000000	70.000000	1.000000
25%	17.000000	4.000000	105.000000	75.000000	2225.250000	13.775000	73.000000	1.000000
50%	22.750000	4.000000	151.000000	93.500000	2803.500000	15.500000	76.000000	1.000000
75%	29.000000	8.000000	275.750000	126.000000	3614.750000	17.025000	79.000000	2.000000
max	46.600000	8.000000	455.000000	230.000000	5140.000000	24.800000	82.000000	3.000000

9. This question involves the use of multiple linear regression on the Auto data set.

(a) Produce a scatterplot matrix which includes all of the variables in the data set.

```
pd.plotting.scatter_matrix(Auto, figsize=(14, 14))
```

```
array([[<Axes: xlabel='mpg', ylabel='mpg'>,
        <Axes: xlabel='cylinders', ylabel='mpg'>,
        <Axes: xlabel='displacement', ylabel='mpg'>,
        <Axes: xlabel='horsepower', ylabel='mpg'>,
        <Axes: xlabel='weight', ylabel='mpg'>,
        <Axes: xlabel='acceleration', ylabel='mpg'>,
        <Axes: xlabel='year', ylabel='mpg'>,
        <Axes: xlabel='origin', ylabel='mpg'>],
       [<Axes: xlabel='mpg', ylabel='cylinders'>,
        <Axes: xlabel='cylinders', ylabel='cylinders'>,
        <Axes: xlabel='displacement', ylabel='cylinders'>,
        <Axes: xlabel='horsepower', ylabel='cylinders'>,
```

```
 <Axes: xlabel='weight', ylabel='cylinders'>,
 <Axes: xlabel='acceleration', ylabel='cylinders'>,
 <Axes: xlabel='year', ylabel='cylinders'>,
 <Axes: xlabel='origin', ylabel='cylinders'>],
[<Axes: xlabel='mpg', ylabel='displacement'>,
 <Axes: xlabel='cylinders', ylabel='displacement'>,
 <Axes: xlabel='displacement', ylabel='displacement'>,
 <Axes: xlabel='horsepower', ylabel='displacement'>,
 <Axes: xlabel='weight', ylabel='displacement'>,
 <Axes: xlabel='acceleration', ylabel='displacement'>,
 <Axes: xlabel='year', ylabel='displacement'>,
 <Axes: xlabel='origin', ylabel='displacement'>],
[<Axes: xlabel='mpg', ylabel='horsepower'>,
 <Axes: xlabel='cylinders', ylabel='horsepower'>,
 <Axes: xlabel='displacement', ylabel='horsepower'>,
 <Axes: xlabel='horsepower', ylabel='horsepower'>,
 <Axes: xlabel='weight', ylabel='horsepower'>,
 <Axes: xlabel='acceleration', ylabel='horsepower'>,
 <Axes: xlabel='year', ylabel='horsepower'>,
 <Axes: xlabel='origin', ylabel='horsepower'>],
[<Axes: xlabel='mpg', ylabel='weight'>,
 <Axes: xlabel='cylinders', ylabel='weight'>,
 <Axes: xlabel='displacement', ylabel='weight'>,
 <Axes: xlabel='horsepower', ylabel='weight'>,
 <Axes: xlabel='weight', ylabel='weight'>,
 <Axes: xlabel='acceleration', ylabel='weight'>,
 <Axes: xlabel='year', ylabel='weight'>,
 <Axes: xlabel='origin', ylabel='weight'>],
[<Axes: xlabel='mpg', ylabel='acceleration'>,
 <Axes: xlabel='cylinders', ylabel='acceleration'>,
 <Axes: xlabel='displacement', ylabel='acceleration'>,
 <Axes: xlabel='horsepower', ylabel='acceleration'>,
 <Axes: xlabel='weight', ylabel='acceleration'>,
 <Axes: xlabel='acceleration', ylabel='acceleration'>,
 <Axes: xlabel='year', ylabel='acceleration'>,
 <Axes: xlabel='origin', ylabel='acceleration'>],
[<Axes: xlabel='mpg', ylabel='year'>,
 <Axes: xlabel='cylinders', ylabel='year'>,
 <Axes: xlabel='displacement', ylabel='year'>,
 <Axes: xlabel='horsepower', ylabel='year'>,
 <Axes: xlabel='weight', ylabel='year'>,
 <Axes: xlabel='acceleration', ylabel='year'>,
 <Axes: xlabel='year', ylabel='year'>,
 <Axes: xlabel='origin', ylabel='year'>],
[<Axes: xlabel='mpg', ylabel='origin'>,
 <Axes: xlabel='cylinders', ylabel='origin'>,
```

```
<Axes: xlabel='displacement', ylabel='origin'>,
<Axes: xlabel='horsepower', ylabel='origin'>,
<Axes: xlabel='weight', ylabel='origin'>,
<Axes: xlabel='acceleration', ylabel='origin'>,
<Axes: xlabel='year', ylabel='origin'>,
<Axes: xlabel='origin', ylabel='origin'>]], dtype=object)
```

(b) Compute the matrix of correlations between the variables using the DataFrame.corr() method.

```
Auto.corr()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
mpg	1.000000	-0.777618	-0.805127	-0.778427	-0.832244	0.423329	0.580541	0.565209
cylinders	-0.777618	1.000000	0.950823	0.842983	0.897527	-0.504683	-0.345647	-0.568932
displacement	-0.805127	0.950823	1.000000	0.897257	0.932994	-0.543800	-0.369855	-0.614535
horsepower	-0.778427	0.842983	0.897257	1.000000	0.864538	-0.689196	-0.416361	-0.455171
weight	-0.832244	0.897527	0.932994	0.864538	1.000000	-0.416839	-0.309120	-0.585005
acceleration	0.423329	-0.504683	-0.543800	-0.689196	-0.416839	1.000000	0.290316	0.212746
year	0.580541	-0.345647	-0.369855	-0.416361	-0.309120	0.290316	1.000000	0.181528
origin	0.565209	-0.568932	-0.614535	-0.455171	-0.585005	0.212746	0.181528	1.000000

(c) Use the sm.OLS() function to perform a multiple linear regression with mpg as the response and all other variables except name as the predictors. Use the summarize() function to print the results. Comment on the output. For instance:

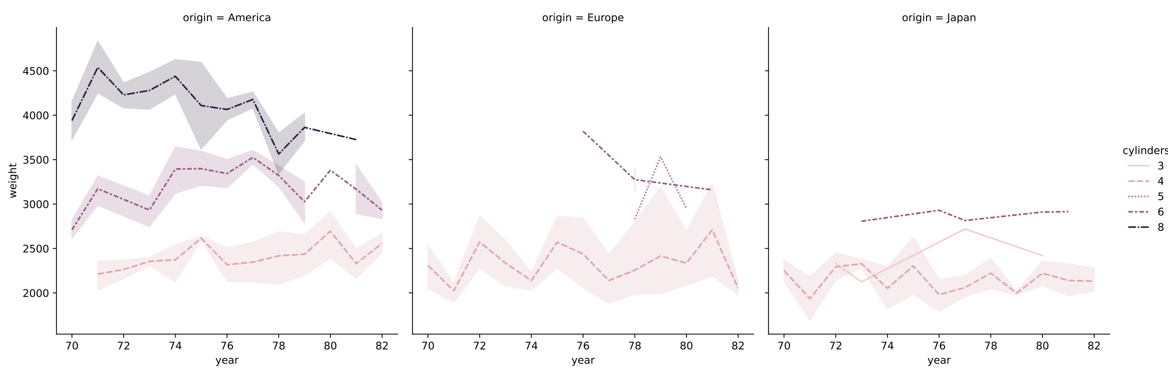
Convert year and origin columns to categorical types

```
Auto["origin"] = Auto["origin"].astype("category")
Auto["origin"] = Auto["origin"].cat.rename_categories(
    {1: "America", 2: "Europe", 3: "Japan"}
)
Auto["year"] = Auto["year"].astype("category")
Auto.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration
count	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000
mean	23.445918	5.471939	194.411990	104.469388	2977.584184	15.541327
std	7.805007	1.705783	104.644004	38.491160	849.402560	2.758864
min	9.000000	3.000000	68.000000	46.000000	1613.000000	8.000000
25%	17.000000	4.000000	105.000000	75.000000	2225.250000	13.775000
50%	22.750000	4.000000	151.000000	93.500000	2803.500000	15.500000
75%	29.000000	8.000000	275.750000	126.000000	3614.750000	17.025000
max	46.600000	8.000000	455.000000	230.000000	5140.000000	24.800000

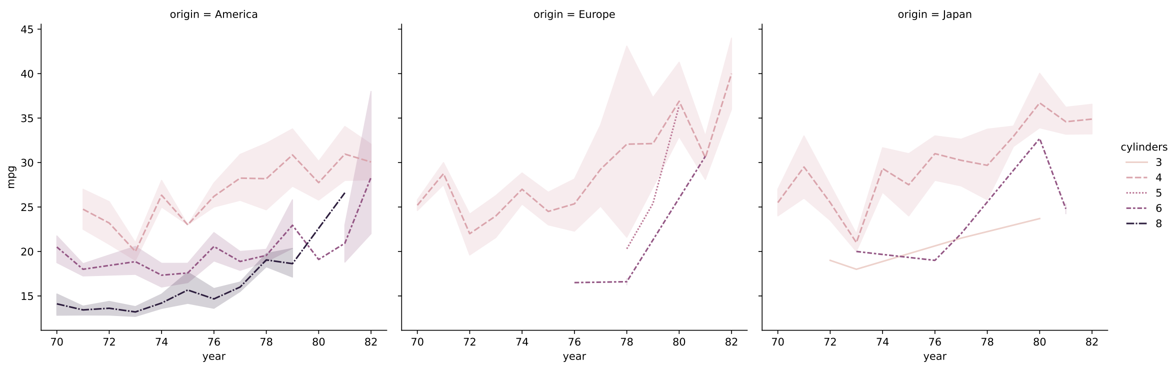
```
sns.relplot(
    Auto,
```

```
    x="year",
    y="weight",
    col="origin",
    hue="cylinders",
    style="cylinders",
    estimator="mean",
    kind="line",
)
```



The weight of the 8-cylinder American made models show a decline from the highs of 1972. It can also be seen that American made cars are heavier than their European and Japanese counterparts especially in the most common models with 4 cylinders.

```
sns.relplot(
    Auto,
    x="year",
    y="mpg",
    col="origin",
    hue="cylinders",
    style="cylinders",
    estimator="mean",
    kind="line",
)
```



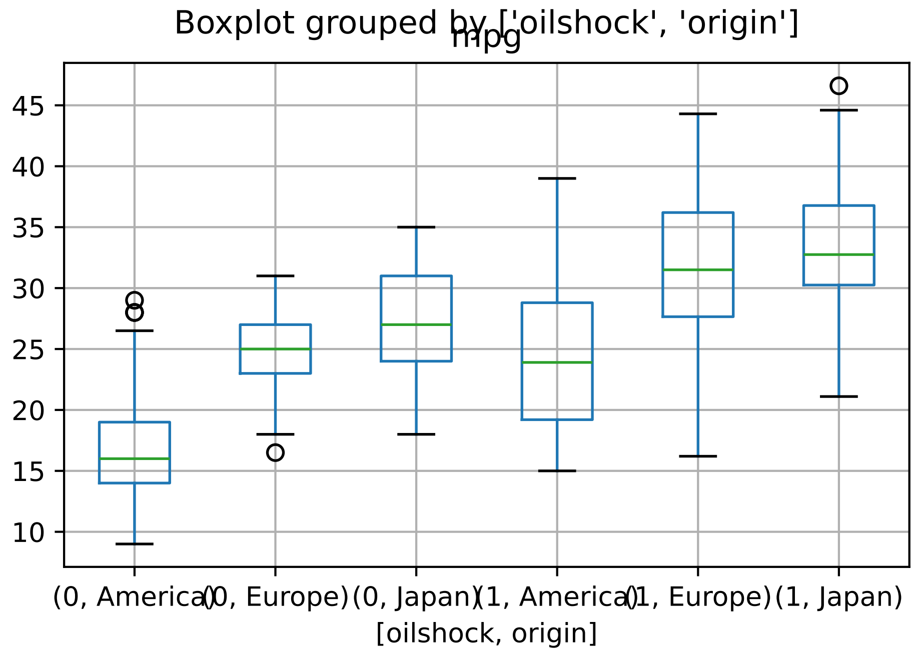
It can be seen that after the oil shock of 1973 and the regulations and actions taken by the US government, the mileage for American made cars rose across all models. This was, however, matched by the European and Japanese models which were already lighter and more fuel efficient.

Encode categorical variables as dummy variables dropping the first to remove multicollinearity.

```
def categorize_for_oil_shock(row):
    # we add 3 years because it takes approximately that long for car manufacturers
     ↪  to introduce a new model
    if row["year"] in (70, 71, 72, 73, 74, 75, 76):
        return 0
    return 1


Auto["oilshock"] = Auto.apply(categorize_for_oil_shock, axis=1)
```

```
Auto.boxplot(column="mpg", by=["oilshock", "origin"])
```




```python
Auto_os = Auto.drop(["year"], axis=1)
Auto_os.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'origin', 'oilshoc
```

```python
# standardizing dataframes
Auto_os["oilshock"] = Auto_os["oilshock"].astype("category")
Auto_os = Auto_os.apply(standardize)
Auto_os.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration
count	3.920000e+02	3.920000e+02	3.920000e+02	3.920000e+02	3.920000e+02	3.920000e+02
mean	1.812609e-16	-1.087565e-16	-7.250436e-17	-1.812609e-16	-3.625218e-17	-8.519262e-16
std	1.001278e+00	1.001278e+00	1.001278e+00	1.001278e+00	1.001278e+00	1.001278e+00
min	-1.853218e+00	-1.451004e+00	-1.209563e+00	-1.520975e+00	-1.608575e+00	-2.736983e+00
25%	-8.269250e-01	-8.640136e-01	-8.555316e-01	-7.665929e-01	-8.868535e-01	-6.410551e-01
50%	-8.927701e-02	-8.640136e-01	-4.153842e-01	-2.853488e-01	-2.052109e-01	-1.499869e-02
75%	7.125143e-01	1.483947e+00	7.782764e-01	5.600800e-01	7.510927e-01	5.384714e-01
max	2.970359e+00	1.483947e+00	2.493416e+00	3.265452e+00	2.549061e+00	3.360262e+00

```python
Auto_os = pd.get_dummies(
    Auto_os, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
```

```
Auto_os.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'oilshock', 'origi
```

```
y = Auto_os["mpg"]
```

```
name
chevrolet chevelle malibu           -0.698638
hi 1200d                            -1.853218
dodge d200                          -1.596645
chevy c20                           -1.724931
ford f250                           -1.724931
amc gremlin                         -0.313779
bmw 2002                             0.327654
saab 99e                             0.199368
audi 100 ls                          0.071081
volkswagen 1131 deluxe sedan         0.327654
datsun pl510                         0.455941
ford maverick                       -0.313779
amc hornet                          -0.698638
plymouth duster                     -0.185492
peugeot 504                          0.199368
buick estate wagon (sw)             -1.211785
toyota corona mark ii                0.071081
plymouth satellite                  -0.698638
amc rebel sst                       -0.955212
ford torino                         -0.826925
ford galaxie 500                    -1.083498
chevrolet impala                    -1.211785
buick skylark 320                   -1.083498
pontiac catalina                    -1.211785
amc ambassador dpl                  -1.083498
dodge challenger se                 -1.083498
plymouth 'cuda 340                  -1.211785
chevrolet monte carlo               -1.083498
plymouth fury iii                   -1.211785
amc hornet sportabout (sw)          -0.698638
chevrolet vega (sw)                 -0.185492
pontiac firebird                    -0.570352
ford mustang                        -0.698638
mercury capri 2000                  -0.057205
toyota corolla 1200                  0.969088
peugeot 304                          0.840801
datsun 1200                          1.482234
volkswagen model 111                 0.455941
plymouth cricket                     0.327654
```

pontiac safari (sw)	-1.340071
opel 1900	0.584228
ford country squire (sw)	-1.340071
fiat 124b	0.840801
plymouth fury iii	-1.211785
dodge monaco (sw)	-1.468358
chevrolet vega 2300	0.584228
toyota corona	0.199368
amc gremlin	-0.570352
plymouth satellite custom	-0.955212
datsun pl510	0.455941
ford torino 500	-0.570352
amc matador	-0.698638
chevrolet impala	-1.211785
pontiac catalina brougham	-1.211785
ford galaxie 500	-1.211785
chevrolet chevelle malibu	-0.826925
chevrolet chevelle concours (sw)	-1.340071
plymouth satellite custom (sw)	-1.211785
volvo 145e (sw)	-0.698638
volkswagen 411 (sw)	-0.185492
peugeot 504 (sw)	-0.313779
ford pinto (sw)	-0.185492
datsun 510 (sw)	0.584228
toyouta corona mark ii (sw)	-0.057205
dodge colt (sw)	0.584228
amc matador (sw)	-1.083498
toyota corolla 1600 (sw)	0.455941
renault 12 (sw)	0.327654
mazda rx2 coupe	-0.570352
ford gran torino (sw)	-1.340071
oldsmobile delta 88 royale	-1.468358
chrysler newport royal	-1.340071
toyota corona hardtop	0.071081
volkswagen type 3	-0.057205
chevrolet vega	-0.442065
ford pinto runabout	-0.313779
chevrolet impala	-1.340071
dodge colt hardtop	0.199368
plymouth fury iii	-1.083498
ford galaxie 500	-1.211785
amc ambassador sst	-0.826925
mercury marquis	-1.596645
buick lesabre custom	-1.340071
pontiac catalina	-1.211785
fiat 124 sport coupe	0.327654

amc gremlin	-0.698638
toyota carina	-0.442065
chevrolet vega	-0.313779
datsun 610	-0.185492
maxda rx3	-0.698638
ford pinto	-0.570352
mercury capri v6	-0.313779
chevrolet monte carlo s	-1.083498
saab 99le	0.071081
fiat 128	0.712514
opel manta	0.071081
audi 100ls	-0.442065
volvo 144ea	-0.570352
dodge dart custom	-1.083498
toyota mark ii	-0.442065
oldsmobile omega	-1.596645
oldsmobile vista cruiser	-1.468358
pontiac grand prix	-0.955212
plymouth custom suburb	-1.340071
amc hornet	-0.698638
chevrolet impala	-1.596645
buick century 350	-1.340071
amc matador	-1.211785
chevrolet malibu	-1.340071
dodge coronet custom	-1.083498
mercury marquis brougham	-1.468358
chevrolet caprice classic	-1.340071
ford ltd	-1.340071
plymouth fury gran sedan	-1.211785
ford gran torino	-1.211785
buick electra 225 custom	-1.468358
amc ambassador brougham	-1.340071
plymouth valiant	-0.698638
chevrolet nova custom	-0.955212
ford country	-1.468358
ford maverick	-0.698638
plymouth duster	-0.057205
volkswagen super beetle	0.327654
chrysler new yorker brougham	-1.340071
audi fox	0.712514
volkswagen dasher	0.327654
opel manta	0.327654
toyota corona	0.969088
datsun 710	1.097374
subaru	0.327654
fiat 128	0.071081

fiat 124 tc	0.327654
honda civic	0.071081
fiat x1.9	0.969088
amc matador (sw)	-1.211785
dodge colt	0.584228
ford gran torino (sw)	-1.211785
ford gran torino	-0.955212
buick century luxus (sw)	-1.340071
dodge coronet custom (sw)	-1.211785
plymouth duster	-0.442065
amc hornet	-0.570352
chevrolet nova	-1.083498
ford pinto	0.327654
datsun b210	0.969088
chevrolet vega	0.199368
chevrolet chevelle malibu classic	-0.955212
amc matador	-0.955212
plymouth satellite sebring	-0.698638
toyota corolla 1200	1.097374
datsun 710	0.071081
pontiac astro	-0.057205
amc gremlin	-0.442065
toyota corona	0.071081
volkswagen dasher	0.199368
ford pinto	-0.698638
saab 99le	0.199368
amc pacer	-0.570352
audi 100ls	-0.057205
peugeot 504	-0.057205
volvo 244dl	-0.185492
honda civic cvcc	1.225661
ford pinto	-0.057205
volkswagen rabbit	0.712514
toyota corolla	0.712514
plymouth valiant custom	-0.570352
chevrolet monza 2+2	-0.442065
ford mustang ii	-1.340071
chevrolet nova	-0.698638
mercury monarch	-1.083498
pontiac catalina	-0.955212
chevrolet bel air	-1.083498
plymouth grand fury	-0.955212
ford maverick	-1.083498
buick century	-0.826925
chevroelt chevelle malibu	-0.955212
amc matador	-1.083498

plymouth fury	-0.698638
buick skyhawk	-0.313779
ford ltd	-1.211785
ford pinto	0.391798
pontiac ventura sj	-0.634495
amc pacer d/l	-0.762782
volkswagen rabbit	0.776658
datsun b-210	1.097374
toyota corolla	0.584228
volvo 245	-0.442065
ford f108	-1.340071
peugeot 504	-0.570352
toyota mark ii	-0.570352
mercedes-benz 280s	-0.891068
cadillac seville	-0.891068
chevy c10	-1.340071
dodge d100	-1.340071
ford granada ghia	-0.698638
plymouth volare premier v8	-1.340071
dodge aspen se	-0.442065
vw rabbit	0.712514
opel 1900	0.199368
honda civic	1.225661
fiat 131	0.584228
capri ii	0.199368
dodge colt	0.327654
renault 12tl	0.455941
dodge coronet brougham	-0.955212
amc matador	-1.019355
chevrolet chevelle malibu classic	-0.762782
plymouth valiant	-0.185492
chevrolet nova	-0.185492
ford maverick	0.071081
amc hornet	-0.121349
chevrolet chevette	0.712514
chevrolet woody	0.135225
ford gran torino	-1.147642
ford mustang ii 2+2	0.263511
volkswagen rabbit custom	0.712514
pontiac sunbird coupe	0.135225
toyota corolla liftback	0.327654
chevrolet chevette	0.904944
bmw 320i	-0.249635
subaru dl	0.840801
volkswagen dasher	0.904944
datsun 810	-0.185492

mazda rx-4	-0.249635
ford thunderbird	-0.955212
dodge colt m/m	1.289804
chrysler cordoba	-1.019355
chevrolet monte carlo landau	-1.019355
plymouth arrow gs	0.263511
buick opel isuzu deluxe	0.840801
renault 5 gtl	1.610521
datsun f-10 hatchback	1.289804
pontiac grand prix lj	-0.955212
oldsmobile cutlass supreme	-0.826925
chevrolet caprice classic	-0.762782
mercury cougar brougham	-1.083498
chevrolet concours	-0.762782
buick skylark	-0.377922
plymouth volare custom	-0.570352
ford granada	-0.634495
dodge monaco brougham	-1.019355
honda accord cvcc	1.033231
datsun 510	0.481598
toyota corona	0.520084
chevrolet chevette	0.840801
buick regal sport coupe (turbo)	-0.737124
ford futura	-0.685810
dodge omni	0.956259
dodge magnum xe	-0.762782
toyota celica gt liftback	-0.300950
peugeot 604sl	-0.929554
oldsmobile starfire sx	0.045424
datsun 200-sx	0.058253
audi 5000	-0.403579
volvo 264gl	-0.826925
saab 99gle	-0.236807
volkswagen scirocco	1.033231
honda accord lx	0.776658
plymouth sapporo	-0.031548
chevrolet monte carlo landau	-0.544694
mazda glc deluxe	1.200003
dodge aspen	-0.621666
volkswagen rabbit custom diesel	2.521356
ford fiesta	1.623349
datsun b210 gx	2.046695
honda civic cvcc	1.623349
amc concord d/l	-0.685810
dodge diplomat	-0.519037
mercury monarch ghia	-0.416408

oldsmobile cutlass salon brougham	-0.454894
chevrolet malibu	-0.377922
ford fairmont (auto)	-0.416408
ford fairmont (man)	0.212197
plymouth volare	-0.377922
amc concord	-0.519037
buick century special	-0.365093
mercury zephyr	-0.339436
pontiac phoenix lj	-0.544694
plymouth horizon	1.379605
mercedes benz 300d	0.250683
cadillac eldorado	-0.057205
peugeot 504	0.481598
oldsmobile cutlass salon brougham	0.058253
plymouth horizon tc3	1.418091
amc spirit dl	0.507256
fiat strada custom	1.777293
buick skylark limited	0.635542
chevrolet citation	0.686857
oldsmobile omega brougham	0.430284
pontiac phoenix	1.289804
datsun 210	1.071717
dodge colt hatchback custom	1.572035
dodge st. regis	-0.672981
vw rabbit custom	1.084545
mercury zephyr 6	-0.467723
ford fairmont 4	-0.147006
amc concord dl 6	-0.416408
dodge aspen 6	-0.365093
chevrolet caprice classic	-0.826925
ford ltd landau	-0.749953
pontiac lemans v6	-0.249635
maxda glc deluxe	1.366776
buick estate wagon (sw)	-0.839754
ford country squire (sw)	-1.019355
chevrolet malibu classic (sw)	-0.544694
chrysler lebaron town @ country (sw)	-0.634495
mercury grand marquis	-0.891068
vw rabbit c (diesel)	2.675299
vw dasher (diesel)	2.559841
audi 5000s (diesel)	1.661835
mercedes-benz 240d	0.840801
honda civic 1500 gl	2.713785
datsun 280-zx	1.187175
vokswagen rabbit	0.815144
mazda rx-7 gs	0.032595

triumph tr7 coupe	1.482234
honda accord	1.148689
datsun 210	2.226296
subaru dl	1.328290
dodge colt	0.571399
mazda glc	2.970359
toyota corolla	1.123031
vw rabbit	2.316097
toyota corolla tercel	1.879922
chevrolet chevette	1.110203
chevrolet citation	0.584228
ford fairmont	0.378969
datsun 310	1.764465
dodge aspen	-0.557523
audi 4000	1.392433
toyota corona liftback	0.815144
mazda 626	1.007574
datsun 510 hatchback	1.738807
amc concord	0.109567
peugeot 505s turbo diesel	0.597056
honda prelude	1.315461
toyota corolla	1.148689
datsun 200sx	1.212832
mazda 626	1.046059
volvo diesel	0.930602
chrysler lebaron salon	-0.749953
datsun 810 maxima	0.096739
buick century	-0.134177
oldsmobile cutlass ls	0.404626
ford granada gl	-0.416408
volkswagen jetta	1.225661
toyota cressida	0.250683
ford escort 2h	0.827972
plymouth reliant	0.481598
plymouth horizon 4	1.443748
ford escort 4w	1.405262
buick skylark	0.404626
dodge aries wagon (sw)	0.301997
plymouth reliant	0.840801
toyota starlet	2.008209
chevrolet citation	0.006938
honda civic 1300	1.495063
subaru	1.135860
datsun 210 mpg	1.738807
toyota tercel	1.828608
mazda glc 4	1.366776

```
plymouth champ                          1.995380
chrysler lebaron medallion              0.327654
honda civic (auto)                      1.097374
datsun 310 gx                           1.867094
buick century limited                   0.199368
oldsmobile cutlass ciera (diesel)       1.867094
ford granada l                         -0.185492
dodge rampage                           1.097374
dodge charger 2.2                       1.610521
chevrolet camaro                        0.455941
ford mustang gl                         0.455941
vw pickup                               2.636813
honda civic                             1.867094
toyota celica gt                        1.097374
toyota corolla                          1.353947
ford ranger                             0.584228
nissan stanza xe                        1.610521
mercury lynx l                          1.610521
plymouth horizon miser                  1.867094
mazda glc custom                        0.969088
mazda glc custom l                      1.738807
volkswagen rabbit l                     1.610521
ford fairmont futura                    0.071081
pontiac phoenix                         0.455941
dodge aries se                          0.712514
pontiac j2000 se hatchback              0.969088
chevrolet cavalier 2-door               1.353947
chevrolet cavalier wagon                0.455941
chevrolet cavalier                      0.584228
honda accord                            1.610521
chevy s-10                              0.969088
Name: mpg, dtype: float64
```

```python
cols = list(Auto_os.columns)
cols.remove("mpg")
formula = " + ".join(cols)
model = smf.ols(f"mpg ~ {formula}", data=Auto_os)
results = model.fit()
results.summary()
```

Dep. Variable:	mpg	**R-squared:**	0.808
Model:	OLS	**Adj. R-squared:**	0.804
Method:	Least Squares	**F-statistic:**	201.7
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	3.05e-132
Time:	14:39:09	**Log-Likelihood:**	-232.60
No. Observations:	392	**AIC:**	483.2
Df Residuals:	383	**BIC:**	518.9
Df Model:	8		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
Intercept	-0.4186	0.041	-10.263	0.000	-0.499	-0.338
oilshock[T.1]	0.6363	0.048	13.204	0.000	0.542	0.731
cylinders	-0.1382	0.073	-1.885	0.060	-0.282	0.006
displacement	0.2845	0.107	2.659	0.008	0.074	0.495
horsepower	-0.2213	0.069	-3.192	0.002	-0.358	-0.085
weight	-0.5923	0.073	-8.085	0.000	-0.736	-0.448
acceleration	0.0053	0.036	0.146	0.884	-0.066	0.076
origin_Europe	0.3038	0.076	4.015	0.000	0.155	0.453
origin_Japan	0.3819	0.074	5.156	0.000	0.236	0.528

Omnibus:	20.039	**Durbin-Watson:**	1.331
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27.583
Skew:	0.413	**Prob(JB):**	1.02e-06
Kurtosis:	4.004	**Cond. No.**	11.9

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

i. Is there a relationship between the predictors and the response? Use the anova_lm() function from statsmodels to answer this question.

ii. Which predictors appear to have a statistically significant relationship to the response?

iii. What does the coefficient for the year variable suggest?

```
anova_lm(results)
```

	df	sum_sq	mean_sq	F	PR(>F)
oilshock	1.0	106.483141	106.483141	542.347509	2.293307e-75
cylinders	1.0	170.845795	170.845795	870.163964	1.267122e-100
displacement	1.0	11.934469	11.934469	60.785485	6.078862e-14
horsepower	1.0	3.951021	3.951021	20.123619	9.610639e-06
weight	1.0	17.796189	17.796189	90.640818	1.988543e-19
acceleration	1.0	0.009116	0.009116	0.046430	8.295108e-01
origin_Europe	1.0	0.564108	0.564108	2.873155	9.088094e-02
origin_Japan	1.0	5.218909	5.218909	26.581317	4.058867e-07

	df	sum_sq	mean_sq	F	PR(>F)
Residual	383.0	75.197253	0.196337	NaN	NaN

There seems to be a statistical relationship between all of the predictors and the response variable, mpg, except for acceleration.

Even though some of the categorical variables are insignificant, even if one of the levels is significant, it is advisable to retain them all in the model.

https://stats.stackexchange.com/questions/24298/can-i-ignore-coefficients-for-non-significant-levels-of-factors-in-a-linear-mode

Note: Year has been converted to a categorical variable oilshock to better capture the effects of the oil shock of 1973 on the mileage.

(d) Produce some of diagnostic plots of the linear regression fit as described in the lab. Comment on any problems you see with the fit. Do the residual plots suggest any unusually large outliers? Does the leverage plot identify any observations with unusually high leverage?

Before producing the diagnostic plots, let's first test for collinearity using correlation matrix and variance inflation factors.

```
Auto_os.corr(numeric_only=True)
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	origin_Europe	origin
mpg	1.000000	-0.777618	-0.805127	-0.778427	-0.832244	0.423329	0.244313	0.4514
cylinders	-0.777618	1.000000	0.950823	0.842983	0.897527	-0.504683	-0.352324	-0.404
displacement	-0.805127	0.950823	1.000000	0.897257	0.932994	-0.543800	-0.371633	-0.440
horsepower	-0.778427	0.842983	0.897257	1.000000	0.864538	-0.689196	-0.284948	-0.321
weight	-0.832244	0.897527	0.932994	0.864538	1.000000	-0.416839	-0.293841	-0.447
acceleration	0.423329	-0.504683	-0.543800	-0.689196	-0.416839	1.000000	0.208298	0.1150
origin_Europe	0.244313	-0.352324	-0.371633	-0.284948	-0.293841	0.208298	1.000000	-0.230
origin_Japan	0.451454	-0.404209	-0.440825	-0.321936	-0.447929	0.115020	-0.230157	1.0000

```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(Auto_os.columns) + " - mpg", Auto_os)
vifdf
```

	VIF
Feature	
oilshock[T.1]	1.149269
cylinders	10.737464
displacement	22.861475

Feature	VIF
horsepower	9.594564
weight	10.715246
acceleration	2.614133
origin_Europe	1.639338
origin_Japan	1.762590

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is displacement with a VIF of 22.861474853464927
Hence, we drop displacement from the model to be fitted.
```

```
('displacement', 22.861474853464927)
```

```
vifdf = calculate_VIFs(
    "mpg ~ " + " + ".join(Auto_os.columns) + " - mpg - displacement", Auto_os
)
vifdf
```

Feature	VIF
oilshock[T.1]	1.139339
cylinders	6.190903
horsepower	8.641303
weight	9.024884
acceleration	2.591157
origin_Europe	1.450726
origin_Japan	1.591434

```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

Linear Regression for mpg ~ cylinders + horsepower + weight + acceleration + oilshock + origin_Europe + origin_Japan

```
cols = list(Auto_os.columns)
cols.remove("mpg")
cols.remove("displacement")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_os)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.805
Model:                            OLS   Adj. R-squared:                  0.801
Method:                 Least Squares   F-statistic:                     225.9
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          6.41e-132
Time:                        14:39:10   Log-Likelihood:                -236.18
No. Observations:                 392   AIC:                             488.4
Df Residuals:                     384   BIC:                             520.1
Df Model:                           7
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        -0.3890      0.040     -9.837      0.000      -0.467      -0.311
oilshock[T.1]     0.6243      0.048     12.911      0.000       0.529       0.719
cylinders        -0.0113      0.056     -0.202      0.840      -0.122       0.099
horsepower       -0.1632      0.066     -2.461      0.014      -0.294      -0.033
weight           -0.5149      0.068     -7.599      0.000      -0.648      -0.382
acceleration     -0.0038      0.036     -0.103      0.918      -0.075       0.068
origin_Europe     0.2356      0.072      3.283      0.001       0.095       0.377
origin_Japan      0.3205      0.071      4.518      0.000       0.181       0.460
==============================================================================
Omnibus:                       25.646   Durbin-Watson:                   1.305
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               40.287
Skew:                           0.456   Prob(JB):                     1.79e-09
Kurtosis:                       4.278   Cond. No.                         7.67
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df      sum_sq     mean_sq           F         PR(>F)
oilshock         1.0  106.483141  106.483141  533.906720   1.149811e-74
cylinders        1.0  170.845795  170.845795  856.621225  7.985118e-100
horsepower       1.0   12.927972   12.927972   64.820882   1.039468e-14
weight           1.0   20.649905   20.649905  103.538670   1.085729e-21
acceleration     1.0    0.003626    0.003626    0.018183   8.928058e-01
origin_Europe    1.0    0.432514    0.432514    2.168627   1.416711e-01
origin_Japan     1.0    4.071523    4.071523   20.414626   8.312108e-06
Residual       384.0   76.585524    0.199441         NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x784733bb0980>
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is acceleration with a p-value of 0.917749942605
Using the backward methodology, we suggest dropping acceleration from the new model
```

Linear Regression after dropping acceleration. The model now is mpg ~ cylinders + horsepower + weight + oilshock + origin_Europe + origin_Japan

```
cols.remove("acceleration")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_os)
simple_model = results
models = []
models.append(
    {
        "name": "simple_model",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.805
Model:                            OLS   Adj. R-squared:                  0.802
Method:                 Least Squares   F-statistic:                     264.3
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          3.80e-133
Time:                        14:39:10   Log-Likelihood:                -236.19
No. Observations:                 392   AIC:                             486.4
Df Residuals:                     385   BIC:                             514.2
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        -0.3889      0.039     -9.849      0.000      -0.467      -0.311
oilshock[T.1]     0.6245      0.048     12.935      0.000       0.530       0.719
cylinders        -0.0105      0.055     -0.189      0.850      -0.120       0.099
horsepower       -0.1585      0.048     -3.285      0.001      -0.253      -0.064
weight           -0.5182      0.060     -8.704      0.000      -0.635      -0.401
origin_Europe     0.2352      0.072      3.287      0.001       0.095       0.376
origin_Japan      0.3202      0.071      4.524      0.000       0.181       0.459
==============================================================================
Omnibus:                       25.330   Durbin-Watson:                   1.305
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               39.508
Skew:                           0.454   Prob(JB):                     2.64e-09
Kurtosis:                       4.263   Cond. No.                         6.84
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
                df     sum_sq    mean_sq           F         PR(>F)
oilshock       1.0 106.483141 106.483141  535.282217   7.450502e-75
cylinders      1.0 170.845795 170.845795  858.828126  4.453195e-100
horsepower     1.0  12.927972  12.927972   64.987879   9.610211e-15
weight         1.0  20.649905  20.649905  103.805416   9.634452e-22
origin_Europe  1.0   0.434982   0.434982    2.186618   1.400323e-01
origin_Japan   1.0   4.070552   4.070552   20.462339   8.111817e-06
Residual     385.0  76.587654   0.198929         NaN            NaN
```

Linear Regression after dropping cylinders. The model now is mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan

```
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_os)
simple_model = results
models = []
models.append(
    {
        "name": "simple_model",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.805
Model:                            OLS   Adj. R-squared:                  0.802
Method:                 Least Squares   F-statistic:                     317.9
Date:                Tue, 25 Feb 2025   Prob (F-statistic):         2.06e-134
Time:                        14:39:10   Log-Likelihood:                -236.21
No. Observations:                 392   AIC:                             484.4
Df Residuals:                     386   BIC:                             508.2
Df Model:                           5
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        -0.3901      0.039    -10.030      0.000      -0.467      -0.314
oilshock[T.1]     0.6250      0.048     12.983      0.000       0.530       0.720
horsepower       -0.1613      0.046     -3.510      0.001      -0.252      -0.071
weight           -0.5245      0.050    -10.576      0.000      -0.622      -0.427
origin_Europe     0.2386      0.069      3.448      0.001       0.103       0.375
origin_Japan      0.3222      0.070      4.611      0.000       0.185       0.460
==============================================================================
```

```
Omnibus:                       24.971   Durbin-Watson:                   1.304
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.456
Skew:                           0.453   Prob(JB):                     4.46e-09
Kurtosis:                       4.239   Cond. No.                         5.60
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                 df      sum_sq     mean_sq           F        PR(>F)
oilshock        1.0  106.483141  106.483141  536.622900  4.863116e-75
horsepower      1.0  165.048555  165.048555  831.763917  2.445119e-98
weight          1.0   39.079210   39.079210  196.940090  1.939884e-36
origin_Europe   1.0    0.574647    0.574647    2.895939  8.960825e-02
origin_Japan    1.0    4.219706    4.219706   21.265252  5.446537e-06
Residual      386.0   76.594741    0.198432         NaN           NaN
```

We can now try and plot the diagnostics for the model.

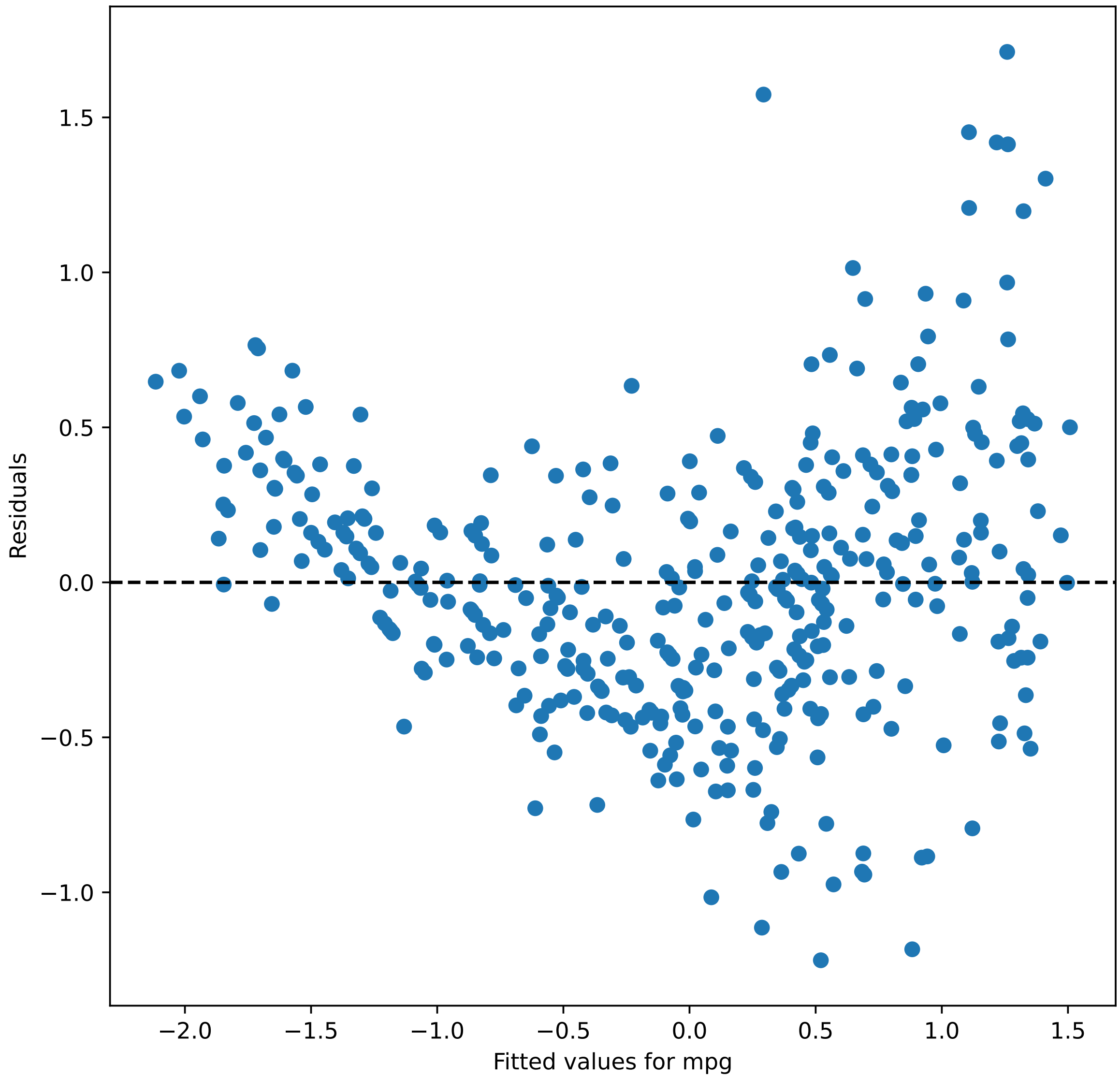
```python
TSS = np.sum((y - np.mean(y)) ** 2)
TSS
RSS = np.sum((y - results.fittedvalues) ** 2)
RSS
RSE = np.sqrt(RSS / results.df_model)
display("RSE " + str(RSE))
display("R-squared adjusted : " + str(results.rsquared_adj))
display("F-statistic : " + str(results.fvalue))
```

```
'RSE 3.9139428061794668'

'R-squared adjusted : 0.8020742313429469'

'F-statistic : 317.8976193276657'
```

```python
display_residuals_plot(results)
```



There is some evidence of non-linearity and heteroskedasticity from the residuals plot above.

(e) Fit some models with interactions as described in the lab. Do any interactions appear to be statistically significant?

```
formula = " + ".join(cols)
formula += " + " + "horsepower: weight"
results = perform_analysis("mpg", formula, Auto_os)
numeric_interactions = results
```

```python
models.append(
    {
        "name": "numeric_interactions",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.847
Model:                            OLS   Adj. R-squared:                  0.845
Method:                 Least Squares   F-statistic:                     355.8
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          1.15e-153
Time:                        14:39:10   Log-Likelihood:                -187.98
No. Observations:                 392   AIC:                             390.0
Df Residuals:                     385   BIC:                             417.8
Df Model:                           6
Covariance Type:            nonrobust
=====================================================================================
                        coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
Intercept            -0.5631      0.038    -14.715      0.000      -0.638      -0.488
oilshock[T.1]         0.6508      0.043     15.243      0.000       0.567       0.735
horsepower           -0.3723      0.045     -8.185      0.000      -0.462      -0.283
weight               -0.4926      0.044    -11.192      0.000      -0.579      -0.406
origin_Europe         0.1565      0.062      2.535      0.012       0.035       0.278
origin_Japan          0.2061      0.063      3.278      0.001       0.082       0.330
horsepower:weight     0.2300      0.022     10.364      0.000       0.186       0.274
==============================================================================
Omnibus:                       27.116   Durbin-Watson:                   1.364
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               45.242
Skew:                           0.457   Prob(JB):                     1.50e-10
Kurtosis:                       4.390   Cond. No.                         7.09
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                      df      sum_sq     mean_sq            F         PR(>F)
oilshock             1.0  106.483141  106.483141   684.553356   1.927144e-87
horsepower           1.0  165.048555  165.048555  1061.055689  1.099814e-112
weight               1.0   39.079210   39.079210   251.230425   6.522402e-44
origin_Europe        1.0    0.574647    0.574647     3.694260   5.533771e-02
origin_Japan         1.0    4.219706    4.219706    27.127429   3.109409e-07
horsepower:weight    1.0   16.707504   16.707504   107.408348   2.313061e-22
Residual           385.0   59.887237    0.155551          NaN            NaN
```

```python
formula = " + ".join(cols)
formula += " + " + "horsepower: weight"
formula += " + " + "oilshock: weight"
formula += " + " + "oilshock: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
oilshock_interactions = results
models.append(
    {
        "name": "oilshock_interactions",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.861
Model:                            OLS   Adj. R-squared:                  0.858
Method:                 Least Squares   F-statistic:                     297.5
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          3.50e-159
Time:                        14:39:11   Log-Likelihood:                -168.92
No. Observations:                 392   AIC:                             355.8
Df Residuals:                     383   BIC:                             391.6
Df Model:                           8
Covariance Type:            nonrobust
==============================================================================================
                               coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------------
Intercept                   -0.5350      0.037    -14.531      0.000      -0.607      -0.463
oilshock[T.1]                0.5913      0.042     14.071      0.000       0.509       0.674
horsepower                  -0.2801      0.051     -5.456      0.000      -0.381      -0.179
oilshock[T.1]:horsepower    -0.2276      0.088     -2.598      0.010      -0.400      -0.055
weight                      -0.4591      0.050     -9.226      0.000      -0.557      -0.361
oilshock[T.1]:weight        -0.0963      0.082     -1.181      0.238      -0.257       0.064
origin_Europe                0.1804      0.059      3.039      0.003       0.064       0.297
origin_Japan                 0.1929      0.060      3.209      0.001       0.075       0.311
horsepower:weight            0.1715      0.023      7.403      0.000       0.126       0.217
==============================================================================
Omnibus:                       23.934   Durbin-Watson:                   1.456
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               43.610
Skew:                           0.377   Prob(JB):                     3.39e-10
Kurtosis:                       4.450   Cond. No.                         11.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
                        df     sum_sq    mean_sq            F         PR(>F)
oilshock               1.0 106.483141 106.483141   750.563317   2.852400e-92
horsepower             1.0 165.048555 165.048555  1163.370934  3.985476e-118
oilshock:horsepower    1.0  19.155472  19.155472   135.020381   6.081794e-27
weight                 1.0  35.071635  35.071635   247.207986   2.471356e-43
oilshock:weight        1.0   0.468549   0.468549     3.302640   6.994992e-02
origin_Europe          1.0   0.971997   0.971997     6.851278   9.208817e-03
origin_Japan           1.0   2.687942   2.687942    18.946386   1.727234e-05
horsepower:weight      1.0   7.776131   7.776131    54.811293   8.507915e-13
Residual             383.0  54.336579   0.141871          NaN            NaN
```

```
formula = " + ".join(cols)
formula += " + " + "oilshock: horsepower"
formula += " + " + "origin_Europe: horsepower"
formula += " + " + "origin_Japan: horsepower"
formula += " + " + "origin_Europe: weight"
formula += " + " + "origin_Japan: weight"
formula += " + " + "oilshock: weight"
formula += " + " + "oilshock: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
origin_interactions = results
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.855
Model:                            OLS   Adj. R-squared:                  0.851
Method:                 Least Squares   F-statistic:                     204.1
Date:                Tue, 25 Feb 2025   Prob (F-statistic):         6.22e-152
Time:                        14:39:11   Log-Likelihood:                -177.44
No. Observations:                 392   AIC:                             378.9
Df Residuals:                     380   BIC:                             426.5
Df Model:                          11
Covariance Type:            nonrobust
=============================================================================================
                                coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------------------
Intercept                    -0.4245      0.035    -12.121      0.000      -0.493      -0.356
oilshock[T.1]                 0.5690      0.044     13.054      0.000       0.483       0.655
horsepower                   -0.0708      0.048     -1.470      0.142      -0.166       0.024
oilshock[T.1]:horsepower     -0.1615      0.096     -1.687      0.092      -0.350       0.027
weight                       -0.4713      0.054     -8.712      0.000      -0.578      -0.365
oilshock[T.1]:weight         -0.2333      0.087     -2.694      0.007      -0.404      -0.063
origin_Europe                 0.0297      0.082      0.363      0.717      -0.131       0.191
origin_Japan                 -0.0010      0.131     -0.007      0.994      -0.259       0.257
origin_Europe:horsepower     -0.5852      0.130     -4.515      0.000      -0.840      -0.330
origin_Japan:horsepower      -0.2801      0.204     -1.370      0.172      -0.682       0.122
origin_Europe:weight          0.1640      0.120      1.370      0.171      -0.071       0.399
```

```
origin_Japan:weight         -0.1326      0.245     -0.541      0.589      -0.614       0.349
==============================================================================
Omnibus:                       20.717   Durbin-Watson:                   1.585
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               32.838
Skew:                           0.373   Prob(JB):                     7.40e-08
Kurtosis:                       4.205   Cond. No.                         25.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                            df      sum_sq     mean_sq            F         PR(>F)
oilshock                   1.0  106.483141  106.483141   712.972849   3.365342e-89
horsepower                 1.0  165.048555  165.048555  1105.105820  1.586816e-114
oilshock:horsepower        1.0   19.155472   19.155472   128.258155   8.134152e-26
weight                     1.0   35.071635   35.071635   234.827067   1.302452e-41
oilshock:weight            1.0    0.468549    0.468549     3.137234   7.732495e-02
origin_Europe              1.0    0.971997    0.971997     6.508146   1.112920e-02
origin_Japan               1.0    2.687942    2.687942    17.997494   2.781984e-05
origin_Europe:horsepower   1.0    3.024522    3.024522    20.251113   9.040705e-06
origin_Japan:horsepower    1.0    1.977640    1.977640    13.241566   3.116551e-04
origin_Europe:weight       1.0    0.313437    0.313437     2.098659   1.482531e-01
origin_Japan:weight        1.0    0.043767    0.043767     0.293050   5.885897e-01
Residual                 380.0   56.753344    0.149351          NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7847302909b0>
```

- From the above analysis, we can see that there is no significant interaction between origin and weight.
- So we can omit them from the model.

```python
formula = " + ".join(cols)
formula += " + " + "oilshock: horsepower"
formula += " + " + "origin_Europe: horsepower"
formula += " + " + "origin_Japan: horsepower"
formula += " + " + "oilshock: weight"
formula += " + " + "oilshock: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
origin_interactions = results
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.854
Model:                            OLS   Adj. R-squared:                  0.851
Method:                 Least Squares   F-statistic:                     248.9
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          8.18e-154
Time:                        14:39:11   Log-Likelihood:                -178.67
No. Observations:                 392   AIC:                             377.3
Df Residuals:                     382   BIC:                             417.1
```

```
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================================
                               coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------------
Intercept                   -0.4296      0.034    -12.650      0.000      -0.496      -0.363
oilshock[T.1]                0.5693      0.043     13.280      0.000       0.485       0.654
horsepower                  -0.0797      0.047     -1.689      0.092      -0.172       0.013
oilshock[T.1]:horsepower    -0.1958      0.092     -2.133      0.034      -0.376      -0.015
weight                      -0.4571      0.051     -8.947      0.000      -0.558      -0.357
oilshock[T.1]:weight        -0.2019      0.084     -2.408      0.016      -0.367      -0.037
origin_Europe                0.0044      0.080      0.055      0.956      -0.153       0.162
origin_Japan                 0.0750      0.084      0.892      0.373      -0.090       0.240
origin_Europe:horsepower    -0.4667      0.096     -4.884      0.000      -0.655      -0.279
origin_Japan:horsepower     -0.3682      0.101     -3.637      0.000      -0.567      -0.169
==============================================================================
Omnibus:                       20.114   Durbin-Watson:                   1.577
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               31.613
Skew:                           0.366   Prob(JB):                     1.37e-07
Kurtosis:                       4.183   Cond. No.                         10.2
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                            df      sum_sq     mean_sq            F         PR(>F)
oilshock                   1.0  106.483141  106.483141   712.242504   2.550768e-89
horsepower                 1.0  165.048555  165.048555  1103.973788  9.879451e-115
oilshock:horsepower        1.0   19.155472   19.155472   128.126771   8.220841e-26
weight                     1.0   35.071635   35.071635   234.586518   1.270254e-41
oilshock:weight            1.0    0.468549    0.468549     3.134021   7.747196e-02
origin_Europe              1.0    0.971997    0.971997     6.501479   1.116822e-02
origin_Japan               1.0    2.687942    2.687942    17.979058   2.804561e-05
origin_Europe:horsepower   1.0    3.024522    3.024522    20.230368   9.121050e-06
origin_Japan:horsepower    1.0    1.977640    1.977640    13.228002   3.136339e-04
Residual                 382.0   57.110548    0.149504          NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x78473024dd00>
```

- From the above analysis, it is evident that with the interaction between origin and horsepower, the interaction between oilshock and weight and horsepower is insignificant. We can drop these from the model as well.

```python
formula = " + ".join(cols)
formula += " + " + "oilshock: horsepower"
formula += " + " + "origin_Europe: horsepower"
formula += " + " + "origin_Japan: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
```

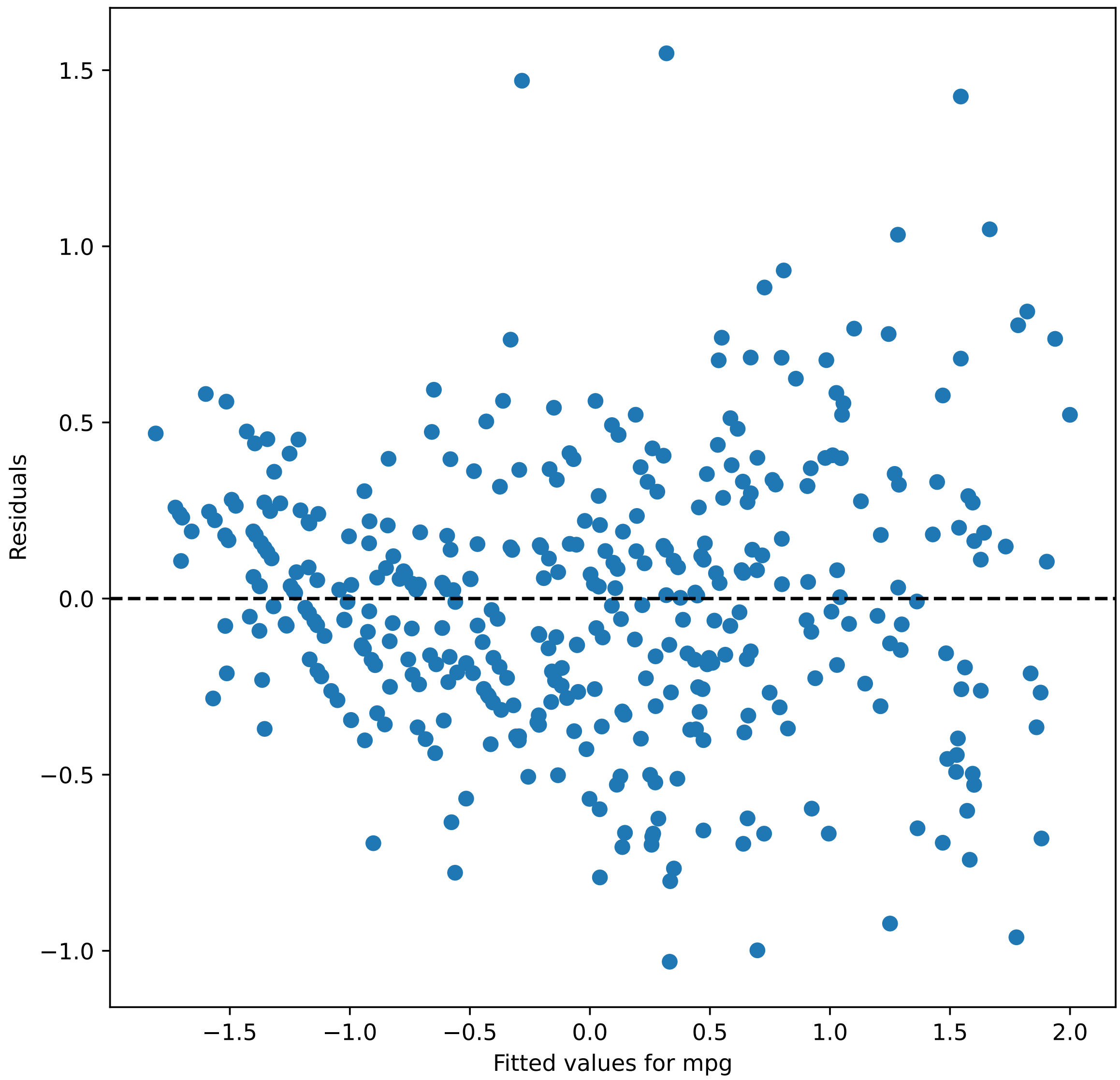
```python
origin_interactions = results
models.append(
    {
        "name": "origin_interactions",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.852
Model:                            OLS   Adj. R-squared:                  0.849
Method:                 Least Squares   F-statistic:                     275.8
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          8.41e-154
Time:                        14:39:11   Log-Likelihood:                -181.63
No. Observations:                 392   AIC:                             381.3
Df Residuals:                     383   BIC:                             417.0
Df Model:                           8
Covariance Type:            nonrobust
============================================================================================
                               coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
Intercept                   -0.4267      0.034    -12.495      0.000      -0.494      -0.360
oilshock[T.1]                0.5628      0.043     13.073      0.000       0.478       0.647
horsepower                  -0.0265      0.042     -0.633      0.527      -0.109       0.056
oilshock[T.1]:horsepower    -0.3804      0.051     -7.497      0.000      -0.480      -0.281
weight                      -0.5231      0.043    -12.051      0.000      -0.608      -0.438
origin_Europe                0.0041      0.081      0.051      0.959      -0.155       0.163
origin_Japan                 0.0877      0.084      1.039      0.299      -0.078       0.254
origin_Europe:horsepower    -0.4423      0.096     -4.626      0.000      -0.630      -0.254
origin_Japan:horsepower     -0.3589      0.102     -3.526      0.000      -0.559      -0.159
==============================================================================
Omnibus:                       19.159   Durbin-Watson:                   1.576
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               29.046
Skew:                           0.362   Prob(JB):                     4.93e-07
Kurtosis:                       4.119   Cond. No.                         9.30
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                            df      sum_sq     mean_sq            F         PR(>F)
oilshock                   1.0  106.483141  106.483141   703.425918   9.826044e-89
horsepower                 1.0  165.048555  165.048555  1090.308104  4.259700e-114
oilshock:horsepower        1.0   19.155472   19.155472   126.540737   1.468876e-25
weight                     1.0   35.071635   35.071635   231.682658   2.992243e-41
```

```
origin_Europe                    1.0    0.792887    0.792887    5.237801   2.264506e-02
origin_Japan                     1.0    2.840217    2.840217   18.762427   1.893486e-05
origin_Europe:horsepower         1.0    2.748574    2.748574   18.157033   2.563625e-05
origin_Japan:horsepower          1.0    1.881783    1.881783   12.431031   4.733982e-04
Residual                       383.0   57.977737    0.151378         NaN            NaN
```

```
display_residuals_plot(results)
```




```
anova_lm(simple_model, numeric_interactions, oilshock_interactions,
↪  origin_interactions)
```

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	386.0	76.594741	0.0	NaN	NaN	NaN
1	385.0	59.887237	1.0	16.707504	110.369506	7.218441e-23
2	383.0	54.336579	2.0	5.550658	18.333780	2.489897e-08
3	383.0	57.977737	-0.0	-3.641158	inf	NaN

```
pd.DataFrame(models)
```

	name	model
0	simple_model	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan
1	numeric_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + horsepower: wei
2	oilshock_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + horsepower: wei
3	origin_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + oilshock: horsep

(f) Try a few different transformations of the variables, such as log(X), √X, X2 . Comment on your findings.

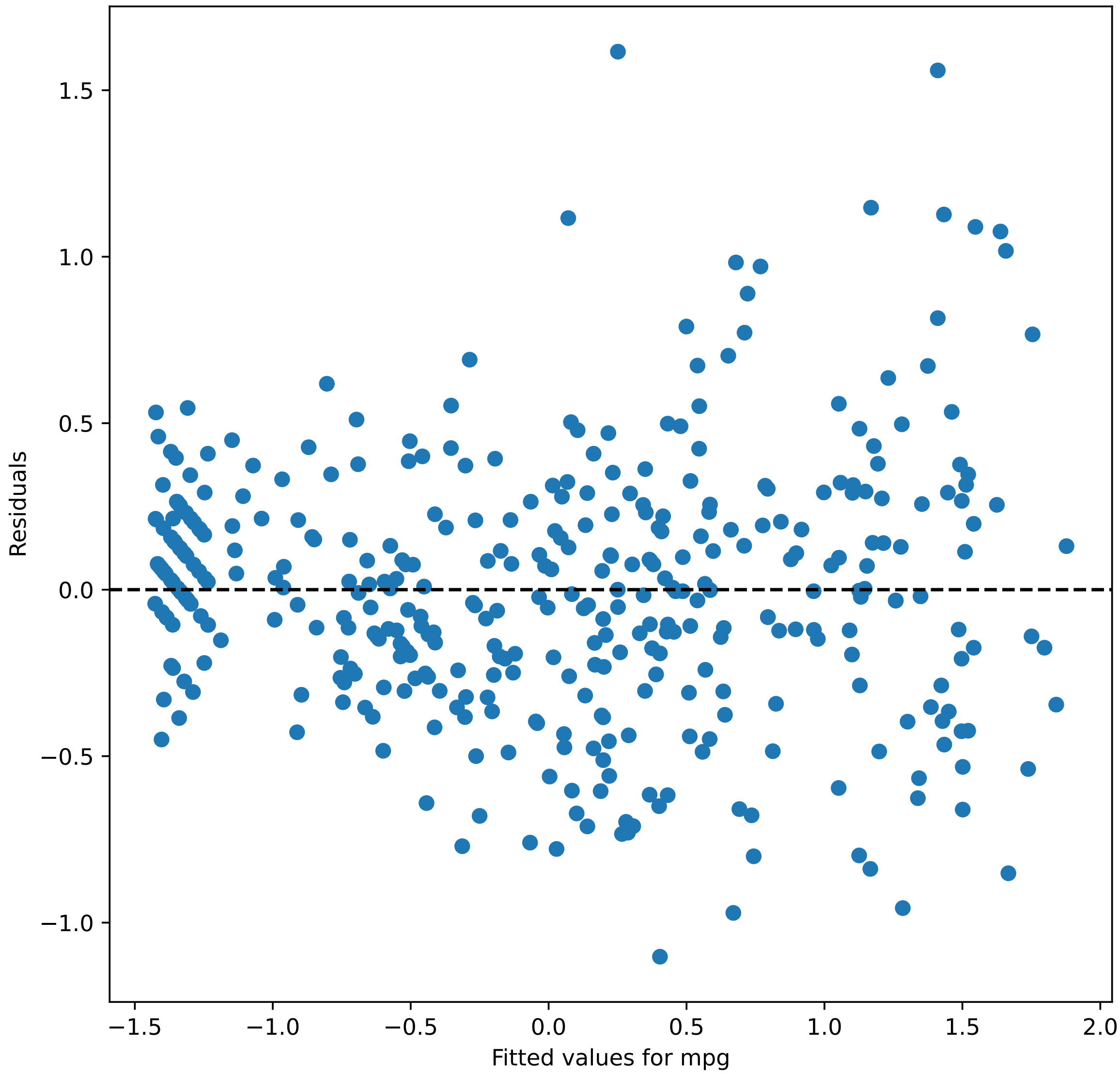
```
formula = simple_model.model.formula
formula = formula[formula.rindex("~") + 1 :]
# Add higher order transformations for horsepower and weight
formula += " + " + "I(horsepower**2)"
formula += " + " + "I(weight**2)"
results = perform_analysis("mpg", formula, Auto_os)
squared_transformations = results
models.append(
    {
        "name": "squared_transformation",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.848
Model:                            OLS   Adj. R-squared:                  0.846
Method:                 Least Squares   F-statistic:                     306.8
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          5.75e-153
Time:                        14:39:11   Log-Likelihood:                -186.58
No. Observations:                 392   AIC:                             389.2
Df Residuals:                     384   BIC:                             420.9
Df Model:                           7
Covariance Type:            nonrobust
```

```
=====================================================================================
                         coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
Intercept             -0.6020      0.040    -15.072      0.000      -0.681      -0.524
oilshock[T.1]          0.6580      0.043     15.334      0.000       0.574       0.742
horsepower            -0.3798      0.054     -7.058      0.000      -0.486      -0.274
weight                -0.5069      0.050    -10.044      0.000      -0.606      -0.408
origin_Europe          0.1436      0.062      2.324      0.021       0.022       0.265
origin_Japan           0.1959      0.064      3.047      0.002       0.069       0.322
I(horsepower ** 2)     0.0976      0.021      4.667      0.000       0.056       0.139
I(weight ** 2)         0.1413      0.026      5.382      0.000       0.090       0.193
==============================================================================
Omnibus:                       26.672   Durbin-Watson:                   1.394
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               46.435
Skew:                           0.436   Prob(JB):                     8.26e-11
Kurtosis:                       4.443   Cond. No.                         10.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                       df      sum_sq     mean_sq            F         PR(>F)
oilshock              1.0  106.483141  106.483141   687.677177   1.332036e-87
horsepower            1.0  165.048555  165.048555  1065.897602  7.775452e-113
weight                1.0   39.079210   39.079210   252.376864   4.864770e-44
origin_Europe         1.0    0.574647    0.574647     3.711118   5.478894e-02
origin_Japan          1.0    4.219706    4.219706    27.251220   2.932534e-07
I(horsepower ** 2)    1.0   12.648520   12.648520    81.685220   7.946432e-18
I(weight ** 2)        1.0    4.485871    4.485871    28.970134   1.281966e-07
Residual            384.0   59.460351    0.154845          NaN            NaN
```

```
display_residuals_plot(results)
```




```
anova_lm(simple_model, squared_transformations)
```

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	386.0	76.594741	0.0	NaN	NaN	NaN
1	384.0	59.460351	2.0	17.134391	55.327677	7.681995e-22

```
pd.DataFrame(models)
```

	name	model
0	simple__model	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan
1	numeric__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + horsepower:
2	oilshock__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + horsepower:
3	origin__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + oilshock: hor
4	squared__transformation	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + I(horsepower

- Since we've standardized the variables, we cannot run log or square root transformations on the negative valued columns.
- We can reload the data and run the log and sqrt transformations on the original un-standardized data.

```
Auto = load_data("Auto")
Auto = Auto.sort_values(by=["year"], ascending=True)
Auto.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year', 'origin'],
```

```
print("Minimums:")
print(Auto.min())
print("Maximums:")
print(Auto.max())
```

```
Minimums:
mpg                9.0
cylinders          3.0
displacement      68.0
horsepower        46.0
weight          1613.0
acceleration       8.0
year              70.0
origin             1.0
dtype: float64
Maximums:
mpg               46.6
cylinders          8.0
displacement     455.0
horsepower       230.0
weight          5140.0
acceleration      24.8
year              82.0
origin             3.0
dtype: float64
```

- From the above, we can see that the values for displacement, horsepower and weight are quite large.

- Hence, we log or square root transform only these variables.

Now let's categorize the variables

```
Auto["origin"] = Auto["origin"].astype("category")
Auto["origin"] = Auto["origin"].cat.rename_categories(
    {1: "America", 2: "Europe", 3: "Japan"}
)
Auto["year"] = Auto["year"].astype("category")
Auto["oilshock"] = Auto.apply(categorize_for_oil_shock, axis=1)
```

Log Transformed Model

```
Auto_log = Auto.copy(deep=True)
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
chevrolet chevelle malibu	18.0	8	307.0	130	3504	12.0	70	A
hi 1200d	9.0	8	304.0	193	4732	18.5	70	A
dodge d200	11.0	8	318.0	210	4382	13.5	70	A
chevy c20	10.0	8	307.0	200	4376	15.0	70	A
ford f250	10.0	8	360.0	215	4615	14.0	70	A
amc gremlin	21.0	6	199.0	90	2648	15.0	70	A
bmw 2002	26.0	4	121.0	113	2234	12.5	70	E
saab 99e	25.0	4	104.0	95	2375	17.5	70	E
audi 100 ls	24.0	4	107.0	90	2430	14.5	70	E
volkswagen 1131 deluxe sedan	26.0	4	97.0	46	1835	20.5	70	E
datsun pl510	27.0	4	97.0	88	2130	14.5	70	Ja
ford maverick	21.0	6	200.0	85	2587	16.0	70	A
amc hornet	18.0	6	199.0	97	2774	15.5	70	A
plymouth duster	22.0	6	198.0	95	2833	15.5	70	A
peugeot 504	25.0	4	110.0	87	2672	17.5	70	E
buick estate wagon (sw)	14.0	8	455.0	225	3086	10.0	70	A
toyota corona mark ii	24.0	4	113.0	95	2372	15.0	70	Ja
plymouth satellite	18.0	8	318.0	150	3436	11.0	70	A
amc rebel sst	16.0	8	304.0	150	3433	12.0	70	A
ford torino	17.0	8	302.0	140	3449	10.5	70	A
ford galaxie 500	15.0	8	429.0	198	4341	10.0	70	A
chevrolet impala	14.0	8	454.0	220	4354	9.0	70	A
buick skylark 320	15.0	8	350.0	165	3693	11.5	70	A
pontiac catalina	14.0	8	455.0	225	4425	10.0	70	A
amc ambassador dpl	15.0	8	390.0	190	3850	8.5	70	A
dodge challenger se	15.0	8	383.0	170	3563	10.0	70	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
plymouth 'cuda 340	14.0	8	340.0	160	3609	8.0	70	A
chevrolet monte carlo	15.0	8	400.0	150	3761	9.5	70	A
plymouth fury iii	14.0	8	440.0	215	4312	8.5	70	A
amc hornet sportabout (sw)	18.0	6	258.0	110	2962	13.5	71	A
chevrolet vega (sw)	22.0	4	140.0	72	2408	19.0	71	A
pontiac firebird	19.0	6	250.0	100	3282	15.0	71	A
ford mustang	18.0	6	250.0	88	3139	14.5	71	A
mercury capri 2000	23.0	4	122.0	86	2220	14.0	71	A
toyota corolla 1200	31.0	4	71.0	65	1773	19.0	71	Ja
peugeot 304	30.0	4	79.0	70	2074	19.5	71	E
datsun 1200	35.0	4	72.0	69	1613	18.0	71	Ja
volkswagen model 111	27.0	4	97.0	60	1834	19.0	71	E
plymouth cricket	26.0	4	91.0	70	1955	20.5	71	A
pontiac safari (sw)	13.0	8	400.0	175	5140	12.0	71	A
opel 1900	28.0	4	116.0	90	2123	14.0	71	E
ford country squire (sw)	13.0	8	400.0	170	4746	12.0	71	A
fiat 124b	30.0	4	88.0	76	2065	14.5	71	E
plymouth fury iii	14.0	8	318.0	150	4096	13.0	71	A
dodge monaco (sw)	12.0	8	383.0	180	4955	11.5	71	A
chevrolet vega 2300	28.0	4	140.0	90	2264	15.5	71	A
toyota corona	25.0	4	113.0	95	2228	14.0	71	Ja
amc gremlin	19.0	6	232.0	100	2634	13.0	71	A
plymouth satellite custom	16.0	6	225.0	105	3439	15.5	71	A
datsun pl510	27.0	4	97.0	88	2130	14.5	71	Ja
ford torino 500	19.0	6	250.0	88	3302	15.5	71	A
amc matador	18.0	6	232.0	100	3288	15.5	71	A
chevrolet impala	14.0	8	350.0	165	4209	12.0	71	A
pontiac catalina brougham	14.0	8	400.0	175	4464	11.5	71	A
ford galaxie 500	14.0	8	351.0	153	4154	13.5	71	A
chevrolet chevelle malibu	17.0	6	250.0	100	3329	15.5	71	A
chevrolet chevelle concours (sw)	13.0	8	307.0	130	4098	14.0	72	A
plymouth satellite custom (sw)	14.0	8	318.0	150	4077	14.0	72	A
volvo 145e (sw)	18.0	4	121.0	112	2933	14.5	72	E
volkswagen 411 (sw)	22.0	4	121.0	76	2511	18.0	72	E
peugeot 504 (sw)	21.0	4	120.0	87	2979	19.5	72	E
ford pinto (sw)	22.0	4	122.0	86	2395	16.0	72	A
datsun 510 (sw)	28.0	4	97.0	92	2288	17.0	72	Ja
toyouta corona mark ii (sw)	23.0	4	120.0	97	2506	14.5	72	Ja
dodge colt (sw)	28.0	4	98.0	80	2164	15.0	72	A
amc matador (sw)	15.0	8	304.0	150	3892	12.5	72	A
toyota corolla 1600 (sw)	27.0	4	97.0	88	2100	16.5	72	Ja
renault 12 (sw)	26.0	4	96.0	69	2189	18.0	72	E
mazda rx2 coupe	19.0	3	70.0	97	2330	13.5	72	Ja

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
ford gran torino (sw)	13.0	8	302.0	140	4294	16.0	72	A
oldsmobile delta 88 royale	12.0	8	350.0	160	4456	13.5	72	A
chrysler newport royal	13.0	8	400.0	190	4422	12.5	72	A
toyota corona hardtop	24.0	4	113.0	95	2278	15.5	72	Ja
volkswagen type 3	23.0	4	97.0	54	2254	23.5	72	E
chevrolet vega	20.0	4	140.0	90	2408	19.5	72	A
ford pinto runabout	21.0	4	122.0	86	2226	16.5	72	A
chevrolet impala	13.0	8	350.0	165	4274	12.0	72	A
dodge colt hardtop	25.0	4	97.5	80	2126	17.0	72	A
plymouth fury iii	15.0	8	318.0	150	4135	13.5	72	A
ford galaxie 500	14.0	8	351.0	153	4129	13.0	72	A
amc ambassador sst	17.0	8	304.0	150	3672	11.5	72	A
mercury marquis	11.0	8	429.0	208	4633	11.0	72	A
buick lesabre custom	13.0	8	350.0	155	4502	13.5	72	A
pontiac catalina	14.0	8	400.0	175	4385	12.0	72	A
fiat 124 sport coupe	26.0	4	98.0	90	2265	15.5	73	E
amc gremlin	18.0	6	232.0	100	2789	15.0	73	A
toyota carina	20.0	4	97.0	88	2279	19.0	73	Ja
chevrolet vega	21.0	4	140.0	72	2401	19.5	73	A
datsun 610	22.0	4	108.0	94	2379	16.5	73	Ja
maxda rx3	18.0	3	70.0	90	2124	13.5	73	Ja
ford pinto	19.0	4	122.0	85	2310	18.5	73	A
mercury capri v6	21.0	6	155.0	107	2472	14.0	73	A
chevrolet monte carlo s	15.0	8	350.0	145	4082	13.0	73	A
saab 99le	24.0	4	121.0	110	2660	14.0	73	E
fiat 128	29.0	4	68.0	49	1867	19.5	73	E
opel manta	24.0	4	116.0	75	2158	15.5	73	E
audi 100ls	20.0	4	114.0	91	2582	14.0	73	E
volvo 144ea	19.0	4	121.0	112	2868	15.5	73	E
dodge dart custom	15.0	8	318.0	150	3399	11.0	73	A
toyota mark ii	20.0	6	156.0	122	2807	13.5	73	Ja
oldsmobile omega	11.0	8	350.0	180	3664	11.0	73	A
oldsmobile vista cruiser	12.0	8	350.0	180	4499	12.5	73	A
pontiac grand prix	16.0	8	400.0	230	4278	9.5	73	A
plymouth custom suburb	13.0	8	360.0	170	4654	13.0	73	A
amc hornet	18.0	6	232.0	100	2945	16.0	73	A
chevrolet impala	11.0	8	400.0	150	4997	14.0	73	A
buick century 350	13.0	8	350.0	175	4100	13.0	73	A
amc matador	14.0	8	304.0	150	3672	11.5	73	A
chevrolet malibu	13.0	8	350.0	145	3988	13.0	73	A
dodge coronet custom	15.0	8	318.0	150	3777	12.5	73	A
mercury marquis brougham	12.0	8	429.0	198	4952	11.5	73	A
chevrolet caprice classic	13.0	8	400.0	150	4464	12.0	73	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
ford ltd	13.0	8	351.0	158	4363	13.0	73	A
plymouth fury gran sedan	14.0	8	318.0	150	4237	14.5	73	A
ford gran torino	14.0	8	302.0	137	4042	14.5	73	A
buick electra 225 custom	12.0	8	455.0	225	4951	11.0	73	A
amc ambassador brougham	13.0	8	360.0	175	3821	11.0	73	A
plymouth valiant	18.0	6	225.0	105	3121	16.5	73	A
chevrolet nova custom	16.0	6	250.0	100	3278	18.0	73	A
ford country	12.0	8	400.0	167	4906	12.5	73	A
ford maverick	18.0	6	250.0	88	3021	16.5	73	A
plymouth duster	23.0	6	198.0	95	2904	16.0	73	A
volkswagen super beetle	26.0	4	97.0	46	1950	21.0	73	E
chrysler new yorker brougham	13.0	8	440.0	215	4735	11.0	73	A
audi fox	29.0	4	98.0	83	2219	16.5	74	E
volkswagen dasher	26.0	4	79.0	67	1963	15.5	74	E
opel manta	26.0	4	97.0	78	2300	14.5	74	E
toyota corona	31.0	4	76.0	52	1649	16.5	74	Ja
datsun 710	32.0	4	83.0	61	2003	19.0	74	Ja
subaru	26.0	4	108.0	93	2391	15.5	74	Ja
fiat 128	24.0	4	90.0	75	2108	15.5	74	E
fiat 124 tc	26.0	4	116.0	75	2246	14.0	74	E
honda civic	24.0	4	120.0	97	2489	15.0	74	Ja
fiat x1.9	31.0	4	79.0	67	2000	16.0	74	E
amc matador (sw)	14.0	8	304.0	150	4257	15.5	74	A
dodge colt	28.0	4	90.0	75	2125	14.5	74	A
ford gran torino (sw)	14.0	8	302.0	140	4638	16.0	74	A
ford gran torino	16.0	8	302.0	140	4141	14.0	74	A
buick century luxus (sw)	13.0	8	350.0	150	4699	14.5	74	A
dodge coronet custom (sw)	14.0	8	318.0	150	4457	13.5	74	A
plymouth duster	20.0	6	198.0	95	3102	16.5	74	A
amc hornet	19.0	6	232.0	100	2901	16.0	74	A
chevrolet nova	15.0	6	250.0	100	3336	17.0	74	A
ford pinto	26.0	4	122.0	80	2451	16.5	74	A
datsun b210	31.0	4	79.0	67	1950	19.0	74	Ja
chevrolet vega	25.0	4	140.0	75	2542	17.0	74	A
chevrolet chevelle malibu classic	16.0	6	250.0	100	3781	17.0	74	A
amc matador	16.0	6	258.0	110	3632	18.0	74	A
plymouth satellite sebring	18.0	6	225.0	105	3613	16.5	74	A
toyota corolla 1200	32.0	4	71.0	65	1836	21.0	74	Ja
datsun 710	24.0	4	119.0	97	2545	17.0	75	Ja
pontiac astro	23.0	4	140.0	78	2592	18.5	75	A
amc gremlin	20.0	6	232.0	100	2914	16.0	75	A
toyota corona	24.0	4	134.0	96	2702	13.5	75	Ja
volkswagen dasher	25.0	4	90.0	71	2223	16.5	75	E

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
ford pinto	18.0	6	171.0	97	2984	14.5	75	A
saab 99le	25.0	4	121.0	115	2671	13.5	75	E
amc pacer	19.0	6	232.0	90	3211	17.0	75	A
audi 100ls	23.0	4	115.0	95	2694	15.0	75	E
peugeot 504	23.0	4	120.0	88	2957	17.0	75	E
volvo 244dl	22.0	4	121.0	98	2945	14.5	75	E
honda civic cvcc	33.0	4	91.0	53	1795	17.5	75	Ja
ford pinto	23.0	4	140.0	83	2639	17.0	75	A
volkswagen rabbit	29.0	4	90.0	70	1937	14.0	75	E
toyota corolla	29.0	4	97.0	75	2171	16.0	75	Ja
plymouth valiant custom	19.0	6	225.0	95	3264	16.0	75	A
chevrolet monza 2+2	20.0	8	262.0	110	3221	13.5	75	A
ford mustang ii	13.0	8	302.0	129	3169	12.0	75	A
chevrolet nova	18.0	6	250.0	105	3459	16.0	75	A
mercury monarch	15.0	6	250.0	72	3432	21.0	75	A
pontiac catalina	16.0	8	400.0	170	4668	11.5	75	A
chevrolet bel air	15.0	8	350.0	145	4440	14.0	75	A
plymouth grand fury	16.0	8	318.0	150	4498	14.5	75	A
ford maverick	15.0	6	250.0	72	3158	19.5	75	A
buick century	17.0	6	231.0	110	3907	21.0	75	A
chevroelt chevelle malibu	16.0	6	250.0	105	3897	18.5	75	A
amc matador	15.0	6	258.0	110	3730	19.0	75	A
plymouth fury	18.0	6	225.0	95	3785	19.0	75	A
buick skyhawk	21.0	6	231.0	110	3039	15.0	75	A
ford ltd	14.0	8	351.0	148	4657	13.5	75	A
ford pinto	26.5	4	140.0	72	2565	13.6	76	A
pontiac ventura sj	18.5	6	250.0	110	3645	16.2	76	A
amc pacer d/l	17.5	6	258.0	95	3193	17.8	76	A
volkswagen rabbit	29.5	4	97.0	71	1825	12.2	76	E
datsun b-210	32.0	4	85.0	70	1990	17.0	76	Ja
toyota corolla	28.0	4	97.0	75	2155	16.4	76	Ja
volvo 245	20.0	4	130.0	102	3150	15.7	76	E
ford f108	13.0	8	302.0	130	3870	15.0	76	A
peugeot 504	19.0	4	120.0	88	3270	21.9	76	E
toyota mark ii	19.0	6	156.0	108	2930	15.5	76	Ja
mercedes-benz 280s	16.5	6	168.0	120	3820	16.7	76	E
cadillac seville	16.5	8	350.0	180	4380	12.1	76	A
chevy c10	13.0	8	350.0	145	4055	12.0	76	A
dodge d100	13.0	8	318.0	150	3755	14.0	76	A
ford granada ghia	18.0	6	250.0	78	3574	21.0	76	A
plymouth volare premier v8	13.0	8	318.0	150	3940	13.2	76	A
dodge aspen se	20.0	6	225.0	100	3651	17.7	76	A
vw rabbit	29.0	4	90.0	70	1937	14.2	76	E

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
opel 1900	25.0	4	116.0	81	2220	16.9	76	E
honda civic	33.0	4	91.0	53	1795	17.4	76	Ja
fiat 131	28.0	4	107.0	86	2464	15.5	76	E
capri ii	25.0	4	140.0	92	2572	14.9	76	A
dodge colt	26.0	4	98.0	79	2255	17.7	76	A
renault 12tl	27.0	4	101.0	83	2202	15.3	76	E
dodge coronet brougham	16.0	8	318.0	150	4190	13.0	76	A
amc matador	15.5	8	304.0	120	3962	13.9	76	A
chevrolet chevelle malibu classic	17.5	8	305.0	140	4215	13.0	76	A
plymouth valiant	22.0	6	225.0	100	3233	15.4	76	A
chevrolet nova	22.0	6	250.0	105	3353	14.5	76	A
ford maverick	24.0	6	200.0	81	3012	17.6	76	A
amc hornet	22.5	6	232.0	90	3085	17.6	76	A
chevrolet chevette	29.0	4	85.0	52	2035	22.2	76	A
chevrolet woody	24.5	4	98.0	60	2164	22.1	76	A
ford gran torino	14.5	8	351.0	152	4215	12.8	76	A
ford mustang ii 2+2	25.5	4	140.0	89	2755	15.8	77	A
volkswagen rabbit custom	29.0	4	97.0	78	1940	14.5	77	E
pontiac sunbird coupe	24.5	4	151.0	88	2740	16.0	77	A
toyota corolla liftback	26.0	4	97.0	75	2265	18.2	77	Ja
chevrolet chevette	30.5	4	98.0	63	2051	17.0	77	A
bmw 320i	21.5	4	121.0	110	2600	12.8	77	E
subaru dl	30.0	4	97.0	67	1985	16.4	77	Ja
volkswagen dasher	30.5	4	97.0	78	2190	14.1	77	E
datsun 810	22.0	6	146.0	97	2815	14.5	77	Ja
mazda rx-4	21.5	3	80.0	110	2720	13.5	77	Ja
ford thunderbird	16.0	8	351.0	149	4335	14.5	77	A
dodge colt m/m	33.5	4	98.0	83	2075	15.9	77	A
chrysler cordoba	15.5	8	400.0	190	4325	12.2	77	A
chevrolet monte carlo landau	15.5	8	350.0	170	4165	11.4	77	A
plymouth arrow gs	25.5	4	122.0	96	2300	15.5	77	A
buick opel isuzu deluxe	30.0	4	111.0	80	2155	14.8	77	A
renault 5 gtl	36.0	4	79.0	58	1825	18.6	77	E
datsun f-10 hatchback	33.5	4	85.0	70	1945	16.8	77	Ja
pontiac grand prix lj	16.0	8	400.0	180	4220	11.1	77	A
oldsmobile cutlass supreme	17.0	8	260.0	110	4060	19.0	77	A
chevrolet caprice classic	17.5	8	305.0	145	3880	12.5	77	A
mercury cougar brougham	15.0	8	302.0	130	4295	14.9	77	A
chevrolet concours	17.5	6	250.0	110	3520	16.4	77	A
buick skylark	20.5	6	231.0	105	3425	16.9	77	A
plymouth volare custom	19.0	6	225.0	100	3630	17.7	77	A
ford granada	18.5	6	250.0	98	3525	19.0	77	A
dodge monaco brougham	15.5	8	318.0	145	4140	13.7	77	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
honda accord cvcc	31.5	4	98.0	68	2045	18.5	77	Ja
datsun 510	27.2	4	119.0	97	2300	14.7	78	Ja
toyota corona	27.5	4	134.0	95	2560	14.2	78	Ja
chevrolet chevette	30.0	4	98.0	68	2155	16.5	78	A
buick regal sport coupe (turbo)	17.7	6	231.0	165	3445	13.4	78	A
ford futura	18.1	8	302.0	139	3205	11.2	78	A
dodge omni	30.9	4	105.0	75	2230	14.5	78	A
dodge magnum xe	17.5	8	318.0	140	4080	13.7	78	A
toyota celica gt liftback	21.1	4	134.0	95	2515	14.8	78	Ja
peugeot 604sl	16.2	6	163.0	133	3410	15.8	78	E
oldsmobile starfire sx	23.8	4	151.0	85	2855	17.6	78	A
datsun 200-sx	23.9	4	119.0	97	2405	14.9	78	Ja
audi 5000	20.3	5	131.0	103	2830	15.9	78	E
volvo 264gl	17.0	6	163.0	125	3140	13.6	78	E
saab 99gle	21.6	4	121.0	115	2795	15.7	78	E
volkswagen scirocco	31.5	4	89.0	71	1990	14.9	78	E
honda accord lx	29.5	4	98.0	68	2135	16.6	78	Ja
plymouth sapporo	23.2	4	156.0	105	2745	16.7	78	A
chevrolet monte carlo landau	19.2	8	305.0	145	3425	13.2	78	A
mazda glc deluxe	32.8	4	78.0	52	1985	19.4	78	Ja
dodge aspen	18.6	6	225.0	110	3620	18.7	78	A
volkswagen rabbit custom diesel	43.1	4	90.0	48	1985	21.5	78	E
ford fiesta	36.1	4	98.0	66	1800	14.4	78	A
datsun b210 gx	39.4	4	85.0	70	2070	18.6	78	Ja
honda civic cvcc	36.1	4	91.0	60	1800	16.4	78	Ja
amc concord d/l	18.1	6	258.0	120	3410	15.1	78	A
dodge diplomat	19.4	8	318.0	140	3735	13.2	78	A
mercury monarch ghia	20.2	8	302.0	139	3570	12.8	78	A
oldsmobile cutlass salon brougham	19.9	8	260.0	110	3365	15.5	78	A
chevrolet malibu	20.5	6	200.0	95	3155	18.2	78	A
ford fairmont (auto)	20.2	6	200.0	85	2965	15.8	78	A
ford fairmont (man)	25.1	4	140.0	88	2720	15.4	78	A
plymouth volare	20.5	6	225.0	100	3430	17.2	78	A
amc concord	19.4	6	232.0	90	3210	17.2	78	A
buick century special	20.6	6	231.0	105	3380	15.8	78	A
mercury zephyr	20.8	6	200.0	85	3070	16.7	78	A
pontiac phoenix lj	19.2	6	231.0	105	3535	19.2	78	A
plymouth horizon	34.2	4	105.0	70	2200	13.2	79	A
mercedes benz 300d	25.4	5	183.0	77	3530	20.1	79	E
cadillac eldorado	23.0	8	350.0	125	3900	17.4	79	A
peugeot 504	27.2	4	141.0	71	3190	24.8	79	E
oldsmobile cutlass salon brougham	23.9	8	260.0	90	3420	22.2	79	A
plymouth horizon tc3	34.5	4	105.0	70	2150	14.9	79	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
amc spirit dl	27.4	4	121.0	80	2670	15.0	79	A
fiat strada custom	37.3	4	91.0	69	2130	14.7	79	E
buick skylark limited	28.4	4	151.0	90	2670	16.0	79	A
chevrolet citation	28.8	6	173.0	115	2595	11.3	79	A
oldsmobile omega brougham	26.8	6	173.0	115	2700	12.9	79	A
pontiac phoenix	33.5	4	151.0	90	2556	13.2	79	A
datsun 210	31.8	4	85.0	65	2020	19.2	79	Ja
dodge colt hatchback custom	35.7	4	98.0	80	1915	14.4	79	A
dodge st. regis	18.2	8	318.0	135	3830	15.2	79	A
vw rabbit custom	31.9	4	89.0	71	1925	14.0	79	E
mercury zephyr 6	19.8	6	200.0	85	2990	18.2	79	A
ford fairmont 4	22.3	4	140.0	88	2890	17.3	79	A
amc concord dl 6	20.2	6	232.0	90	3265	18.2	79	A
dodge aspen 6	20.6	6	225.0	110	3360	16.6	79	A
chevrolet caprice classic	17.0	8	305.0	130	3840	15.4	79	A
ford ltd landau	17.6	8	302.0	129	3725	13.4	79	A
pontiac lemans v6	21.5	6	231.0	115	3245	15.4	79	A
maxda glc deluxe	34.1	4	86.0	65	1975	15.2	79	Ja
buick estate wagon (sw)	16.9	8	350.0	155	4360	14.9	79	A
ford country squire (sw)	15.5	8	351.0	142	4054	14.3	79	A
chevrolet malibu classic (sw)	19.2	8	267.0	125	3605	15.0	79	A
chrysler lebaron town @ country (sw)	18.5	8	360.0	150	3940	13.0	79	A
mercury grand marquis	16.5	8	351.0	138	3955	13.2	79	A
vw rabbit c (diesel)	44.3	4	90.0	48	2085	21.7	80	E
vw dasher (diesel)	43.4	4	90.0	48	2335	23.7	80	E
audi 5000s (diesel)	36.4	5	121.0	67	2950	19.9	80	E
mercedes-benz 240d	30.0	4	146.0	67	3250	21.8	80	E
honda civic 1500 gl	44.6	4	91.0	67	1850	13.8	80	Ja
datsun 280-zx	32.7	6	168.0	132	2910	11.4	80	Ja
vokswagen rabbit	29.8	4	89.0	62	1845	15.3	80	E
mazda rx-7 gs	23.7	3	70.0	100	2420	12.5	80	Ja
triumph tr7 coupe	35.0	4	122.0	88	2500	15.1	80	E
honda accord	32.4	4	107.0	72	2290	17.0	80	Ja
datsun 210	40.8	4	85.0	65	2110	19.2	80	Ja
subaru dl	33.8	4	97.0	67	2145	18.0	80	Ja
dodge colt	27.9	4	156.0	105	2800	14.4	80	A
mazda glc	46.6	4	86.0	65	2110	17.9	80	Ja
toyota corolla	32.2	4	108.0	75	2265	15.2	80	Ja
vw rabbit	41.5	4	98.0	76	2144	14.7	80	E
toyota corolla tercel	38.1	4	89.0	60	1968	18.8	80	Ja
chevrolet chevette	32.1	4	98.0	70	2120	15.5	80	A
chevrolet citation	28.0	4	151.0	90	2678	16.5	80	A
ford fairmont	26.4	4	140.0	88	2870	18.1	80	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
datsun 310	37.2	4	86.0	65	2019	16.4	80	Ja
dodge aspen	19.1	6	225.0	90	3381	18.7	80	A
audi 4000	34.3	4	97.0	78	2188	15.8	80	E
toyota corona liftback	29.8	4	134.0	90	2711	15.5	80	Ja
mazda 626	31.3	4	120.0	75	2542	17.5	80	Ja
datsun 510 hatchback	37.0	4	119.0	92	2434	15.0	80	Ja
amc concord	24.3	4	151.0	90	3003	20.1	80	A
peugeot 505s turbo diesel	28.1	4	141.0	80	3230	20.4	81	E
honda prelude	33.7	4	107.0	75	2210	14.4	81	Ja
toyota corolla	32.4	4	108.0	75	2350	16.8	81	Ja
datsun 200sx	32.9	4	119.0	100	2615	14.8	81	Ja
mazda 626	31.6	4	120.0	74	2635	18.3	81	Ja
volvo diesel	30.7	6	145.0	76	3160	19.6	81	E
chrysler lebaron salon	17.6	6	225.0	85	3465	16.6	81	A
datsun 810 maxima	24.2	6	146.0	120	2930	13.8	81	Ja
buick century	22.4	6	231.0	110	3415	15.8	81	A
oldsmobile cutlass ls	26.6	8	350.0	105	3725	19.0	81	A
ford granada gl	20.2	6	200.0	88	3060	17.1	81	A
volkswagen jetta	33.0	4	105.0	74	2190	14.2	81	E
toyota cressida	25.4	6	168.0	116	2900	12.6	81	Ja
ford escort 2h	29.9	4	98.0	65	2380	20.7	81	A
plymouth reliant	27.2	4	135.0	84	2490	15.7	81	A
plymouth horizon 4	34.7	4	105.0	63	2215	14.9	81	A
ford escort 4w	34.4	4	98.0	65	2045	16.2	81	A
buick skylark	26.6	4	151.0	84	2635	16.4	81	A
dodge aries wagon (sw)	25.8	4	156.0	92	2620	14.4	81	A
plymouth reliant	30.0	4	135.0	84	2385	12.9	81	A
toyota starlet	39.1	4	79.0	58	1755	16.9	81	Ja
chevrolet citation	23.5	6	173.0	110	2725	12.6	81	A
honda civic 1300	35.1	4	81.0	60	1760	16.1	81	Ja
subaru	32.3	4	97.0	67	2065	17.8	81	Ja
datsun 210 mpg	37.0	4	85.0	65	1975	19.4	81	Ja
toyota tercel	37.7	4	89.0	62	2050	17.3	81	Ja
mazda glc 4	34.1	4	91.0	68	1985	16.0	81	Ja
plymouth champ	39.0	4	86.0	64	1875	16.4	81	A
chrysler lebaron medallion	26.0	4	156.0	92	2585	14.5	82	A
honda civic (auto)	32.0	4	91.0	67	1965	15.7	82	Ja
datsun 310 gx	38.0	4	91.0	67	1995	16.2	82	Ja
buick century limited	25.0	6	181.0	110	2945	16.4	82	A
oldsmobile cutlass ciera (diesel)	38.0	6	262.0	85	3015	17.0	82	A
ford granada l	22.0	6	232.0	112	2835	14.7	82	A
dodge rampage	32.0	4	135.0	84	2295	11.6	82	A
dodge charger 2.2	36.0	4	135.0	84	2370	13.0	82	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
chevrolet camaro	27.0	4	151.0	90	2950	17.3	82	A
ford mustang gl	27.0	4	140.0	86	2790	15.6	82	A
vw pickup	44.0	4	97.0	52	2130	24.6	82	E
honda civic	38.0	4	91.0	67	1965	15.0	82	Ja
toyota celica gt	32.0	4	144.0	96	2665	13.9	82	Ja
toyota corolla	34.0	4	108.0	70	2245	16.9	82	Ja
ford ranger	28.0	4	120.0	79	2625	18.6	82	A
nissan stanza xe	36.0	4	120.0	88	2160	14.5	82	Ja
mercury lynx l	36.0	4	98.0	70	2125	17.3	82	A
plymouth horizon miser	38.0	4	105.0	63	2125	14.7	82	A
mazda glc custom	31.0	4	91.0	68	1970	17.6	82	Ja
mazda glc custom l	37.0	4	91.0	68	2025	18.2	82	Ja
volkswagen rabbit l	36.0	4	105.0	74	1980	15.3	82	E
ford fairmont futura	24.0	4	140.0	92	2865	16.4	82	A
pontiac phoenix	27.0	4	151.0	90	2735	18.0	82	A
dodge aries se	29.0	4	135.0	84	2525	16.0	82	A
pontiac j2000 se hatchback	31.0	4	112.0	85	2575	16.2	82	A
chevrolet cavalier 2-door	34.0	4	112.0	88	2395	18.0	82	A
chevrolet cavalier wagon	27.0	4	112.0	88	2640	18.6	82	A
chevrolet cavalier	28.0	4	112.0	88	2605	19.6	82	A
honda accord	36.0	4	107.0	75	2205	14.5	82	Ja
chevy s-10	31.0	4	119.0	82	2720	19.4	82	A

```
Auto_log["log_displacement"] = np.log(Auto_log["displacement"])
Auto_log["log_horsepower"] = np.log(Auto_log["horsepower"])
Auto_log["log_weight"] = np.log(Auto_log["weight"])
Auto_log = Auto_log.drop(
    columns=[
        "displacement",
        "weight",
        "horsepower",
        "year",
    ]
)
Auto_log.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'origin', 'oilshock', 'log_displacement', 'log_horsepower'
```

```
Auto_log.corr(numeric_only=True)
```

	mpg	cylinders	acceleration	oilshock	log_displacement	log_horsepower	log_weight
mpg	1.000000	-0.777618	0.423329	0.521192	-0.828453	-0.817517	-0.844194

	mpg	cylinders	acceleration	oilshock	log_displacement	log_horsepower	log_weight
cylinders	-0.777618	1.000000	-0.504683	-0.273703	0.942814	0.843204	0.884303
acceleration	0.423329	-0.504683	1.000000	0.195892	-0.497107	-0.698328	-0.401563
oilshock	0.521192	-0.273703	0.195892	1.000000	-0.268161	-0.299037	-0.250520
log_displacement	-0.828453	0.942814	-0.497107	-0.268161	1.000000	0.872149	0.942850
log_horsepower	-0.817517	0.843204	-0.698328	-0.299037	0.872149	1.000000	0.873956
log_weight	-0.844194	0.884303	-0.401563	-0.250520	0.942850	0.873956	1.000000

```
Auto_log = pd.get_dummies(
    Auto_log, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_log.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'oilshock', 'log_displacement', 'log_horsepower', 'log_wei
```

```
cols = list(Auto_log.columns)
cols.remove("mpg")
```

```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_log)
vifdf
```

Feature	VIF
cylinders	9.828626
acceleration	3.304749
oilshock	1.147770
log_displacement	25.969595
log_horsepower	11.414709
log_weight	16.146573
origin_Europe	1.876698
origin_Japan	2.097688

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is log_displacement with a VIF of 25.96959512578754
Hence, we drop log_displacement from the model to be fitted.
```

```
('log_displacement', 25.96959512578754)
```

```
cols.remove("log_displacement")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_log)
vifdf
```

Feature	VIF
cylinders	5.535070
acceleration	3.179336
oilshock	1.142791
log_horsepower	11.411764
log_weight	10.608718
origin_Europe	1.451961
origin_Japan	1.652749

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is log_horsepower with a VIF of 11.411764499222897
Hence, we drop log_horsepower from the model to be fitted.
```

```
('log_horsepower', 11.411764499222897)
```

```
cols.remove("log_horsepower")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_log)
vifdf
```

Feature	VIF
cylinders	5.517868
acceleration	1.377517
oilshock	1.118666
log_weight	5.014899
origin_Europe	1.451265
origin_Japan	1.608682

```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

```
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_log)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.823
Model:                            OLS   Adj. R-squared:                  0.821
Method:                 Least Squares   F-statistic:                     299.3
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          1.36e-141
Time:                        14:39:13   Log-Likelihood:                -1021.3
```

```
No. Observations:                 392   AIC:                             2057.
Df Residuals:                     385   BIC:                             2084.
Df Model:                           6
Covariance Type:            nonrobust
==================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
Intercept       168.2416      9.696     17.351      0.000     149.177     187.306
cylinders         0.0129      0.230      0.056      0.955      -0.440       0.465
acceleration      0.1805      0.071      2.538      0.012       0.041       0.320
oilshock          5.1312      0.355     14.470      0.000       4.434       5.828
log_weight      -18.9156      1.331    -14.211      0.000     -21.533     -16.299
origin_Europe     1.3692      0.531      2.578      0.010       0.325       2.413
origin_Japan      1.5602      0.528      2.956      0.003       0.522       2.598
==============================================================================
Omnibus:                       30.158   Durbin-Watson:                   1.253
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               51.811
Skew:                           0.493   Prob(JB):                     5.62e-12
Kurtosis:                       4.484   Cond. No.                     1.08e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.08e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
                  df        sum_sq       mean_sq            F         PR(>F)
cylinders        1.0  14403.083079  14403.083079  1318.559127  2.133814e-126
acceleration     1.0     30.471304     30.471304     2.789557   9.569322e-02
oilshock         1.0   2422.051542   2422.051542   221.731566   6.308582e-40
log_weight       1.0   2639.878573   2639.878573   241.672978   1.215817e-42
origin_Europe    1.0     22.569818     22.569818     2.066199   1.514086e-01
origin_Japan     1.0     95.449335     95.449335     8.738101   3.308129e-03
Residual       385.0   4205.489819     10.923350          NaN            NaN

<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7847303e30e0>
```

```python
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is cylinders with a p-value of 0.955447999790515
Using the backward methodology, we suggest dropping cylinders from the new model
```

```python
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_log)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.823
```

```
Model:                            OLS   Adj. R-squared:                  0.821
Method:                 Least Squares   F-statistic:                     360.0
Date:                Tue, 25 Feb 2025   Prob (F-statistic):         6.83e-143
Time:                        14:39:13   Log-Likelihood:                -1021.3
No. Observations:                 392   AIC:                             2055.
Df Residuals:                     386   BIC:                             2078.
Df Model:                           5
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept       167.8689      7.032     23.870      0.000     154.042     181.696
acceleration      0.1792      0.067      2.673      0.008       0.047       0.311
oilshock          5.1289      0.352     14.574      0.000       4.437       5.821
log_weight      -18.8571      0.820    -22.992      0.000     -20.470     -17.245
origin_Europe     1.3628      0.518      2.631      0.009       0.344       2.381
origin_Japan      1.5576      0.525      2.967      0.003       0.525       2.590
==============================================================================
Omnibus:                       30.308   Durbin-Watson:                   1.253
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               52.282
Skew:                           0.493   Prob(JB):                     4.44e-12
Kurtosis:                       4.492   Cond. No.                         751.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df        sum_sq       mean_sq           F         PR(>F)
acceleration     1.0   4268.531557   4268.531557  391.783092   1.076057e-60
oilshock         1.0   4757.627552   4757.627552  436.674301   2.076230e-65
log_weight       1.0  10466.602734  10466.602734  960.667136  8.818191e-107
origin_Europe    1.0     24.823504     24.823504    2.278402   1.320051e-01
origin_Japan     1.0     95.884166     95.884166    8.800637   3.198611e-03
Residual       386.0   4205.523957     10.895140         NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7847302e0c20>
```

```python
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
No variables are statistically insignificant.
The model mpg ~ acceleration + oilshock + log_weight + origin_Europe + origin_Japan cannot be pruned
```

```python
models.append(
    {
        "name": "log_transformation",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
```

```
)
```

```
pd.DataFrame(models)
```

	name	model
0	simple__model	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan
1	numeric__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + horsepower:
2	oilshock__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + horsepower:
3	origin__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + oilshock: hor
4	squared__transformation	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + I(horsepower
5	log__transformation	mpg ~ acceleration + oilshock + log__weight + origin__Europe + origin__Japan

Square Root Transformed Model

```
Auto_sqrt = Auto.copy(deep=True)
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
chevrolet chevelle malibu	18.0	8	307.0	130	3504	12.0	70	A
hi 1200d	9.0	8	304.0	193	4732	18.5	70	A
dodge d200	11.0	8	318.0	210	4382	13.5	70	A
chevy c20	10.0	8	307.0	200	4376	15.0	70	A
ford f250	10.0	8	360.0	215	4615	14.0	70	A
amc gremlin	21.0	6	199.0	90	2648	15.0	70	A
bmw 2002	26.0	4	121.0	113	2234	12.5	70	E
saab 99e	25.0	4	104.0	95	2375	17.5	70	E
audi 100 ls	24.0	4	107.0	90	2430	14.5	70	E
volkswagen 1131 deluxe sedan	26.0	4	97.0	46	1835	20.5	70	E
datsun pl510	27.0	4	97.0	88	2130	14.5	70	Ja
ford maverick	21.0	6	200.0	85	2587	16.0	70	A
amc hornet	18.0	6	199.0	97	2774	15.5	70	A
plymouth duster	22.0	6	198.0	95	2833	15.5	70	A
peugeot 504	25.0	4	110.0	87	2672	17.5	70	E
buick estate wagon (sw)	14.0	8	455.0	225	3086	10.0	70	A
toyota corona mark ii	24.0	4	113.0	95	2372	15.0	70	Ja
plymouth satellite	18.0	8	318.0	150	3436	11.0	70	A
amc rebel sst	16.0	8	304.0	150	3433	12.0	70	A
ford torino	17.0	8	302.0	140	3449	10.5	70	A
ford galaxie 500	15.0	8	429.0	198	4341	10.0	70	A
chevrolet impala	14.0	8	454.0	220	4354	9.0	70	A
buick skylark 320	15.0	8	350.0	165	3693	11.5	70	A
pontiac catalina	14.0	8	455.0	225	4425	10.0	70	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
amc ambassador dpl	15.0	8	390.0	190	3850	8.5	70	A
dodge challenger se	15.0	8	383.0	170	3563	10.0	70	A
plymouth 'cuda 340	14.0	8	340.0	160	3609	8.0	70	A
chevrolet monte carlo	15.0	8	400.0	150	3761	9.5	70	A
plymouth fury iii	14.0	8	440.0	215	4312	8.5	70	A
amc hornet sportabout (sw)	18.0	6	258.0	110	2962	13.5	71	A
chevrolet vega (sw)	22.0	4	140.0	72	2408	19.0	71	A
pontiac firebird	19.0	6	250.0	100	3282	15.0	71	A
ford mustang	18.0	6	250.0	88	3139	14.5	71	A
mercury capri 2000	23.0	4	122.0	86	2220	14.0	71	A
toyota corolla 1200	31.0	4	71.0	65	1773	19.0	71	Ja
peugeot 304	30.0	4	79.0	70	2074	19.5	71	E
datsun 1200	35.0	4	72.0	69	1613	18.0	71	Ja
volkswagen model 111	27.0	4	97.0	60	1834	19.0	71	E
plymouth cricket	26.0	4	91.0	70	1955	20.5	71	A
pontiac safari (sw)	13.0	8	400.0	175	5140	12.0	71	A
opel 1900	28.0	4	116.0	90	2123	14.0	71	E
ford country squire (sw)	13.0	8	400.0	170	4746	12.0	71	A
fiat 124b	30.0	4	88.0	76	2065	14.5	71	E
plymouth fury iii	14.0	8	318.0	150	4096	13.0	71	A
dodge monaco (sw)	12.0	8	383.0	180	4955	11.5	71	A
chevrolet vega 2300	28.0	4	140.0	90	2264	15.5	71	A
toyota corona	25.0	4	113.0	95	2228	14.0	71	Ja
amc gremlin	19.0	6	232.0	100	2634	13.0	71	A
plymouth satellite custom	16.0	6	225.0	105	3439	15.5	71	A
datsun pl510	27.0	4	97.0	88	2130	14.5	71	Ja
ford torino 500	19.0	6	250.0	88	3302	15.5	71	A
amc matador	18.0	6	232.0	100	3288	15.5	71	A
chevrolet impala	14.0	8	350.0	165	4209	12.0	71	A
pontiac catalina brougham	14.0	8	400.0	175	4464	11.5	71	A
ford galaxie 500	14.0	8	351.0	153	4154	13.5	71	A
chevrolet chevelle malibu	17.0	6	250.0	100	3329	15.5	71	A
chevrolet chevelle concours (sw)	13.0	8	307.0	130	4098	14.0	72	A
plymouth satellite custom (sw)	14.0	8	318.0	150	4077	14.0	72	A
volvo 145e (sw)	18.0	4	121.0	112	2933	14.5	72	E
volkswagen 411 (sw)	22.0	4	121.0	76	2511	18.0	72	E
peugeot 504 (sw)	21.0	4	120.0	87	2979	19.5	72	E
ford pinto (sw)	22.0	4	122.0	86	2395	16.0	72	A
datsun 510 (sw)	28.0	4	97.0	92	2288	17.0	72	Ja
toyouta corona mark ii (sw)	23.0	4	120.0	97	2506	14.5	72	Ja
dodge colt (sw)	28.0	4	98.0	80	2164	15.0	72	A
amc matador (sw)	15.0	8	304.0	150	3892	12.5	72	A
toyota corolla 1600 (sw)	27.0	4	97.0	88	2100	16.5	72	Ja

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
renault 12 (sw)	26.0	4	96.0	69	2189	18.0	72	E
mazda rx2 coupe	19.0	3	70.0	97	2330	13.5	72	Ja
ford gran torino (sw)	13.0	8	302.0	140	4294	16.0	72	A
oldsmobile delta 88 royale	12.0	8	350.0	160	4456	13.5	72	A
chrysler newport royal	13.0	8	400.0	190	4422	12.5	72	A
toyota corona hardtop	24.0	4	113.0	95	2278	15.5	72	Ja
volkswagen type 3	23.0	4	97.0	54	2254	23.5	72	E
chevrolet vega	20.0	4	140.0	90	2408	19.5	72	A
ford pinto runabout	21.0	4	122.0	86	2226	16.5	72	A
chevrolet impala	13.0	8	350.0	165	4274	12.0	72	A
dodge colt hardtop	25.0	4	97.5	80	2126	17.0	72	A
plymouth fury iii	15.0	8	318.0	150	4135	13.5	72	A
ford galaxie 500	14.0	8	351.0	153	4129	13.0	72	A
amc ambassador sst	17.0	8	304.0	150	3672	11.5	72	A
mercury marquis	11.0	8	429.0	208	4633	11.0	72	A
buick lesabre custom	13.0	8	350.0	155	4502	13.5	72	A
pontiac catalina	14.0	8	400.0	175	4385	12.0	72	A
fiat 124 sport coupe	26.0	4	98.0	90	2265	15.5	73	E
amc gremlin	18.0	6	232.0	100	2789	15.0	73	A
toyota carina	20.0	4	97.0	88	2279	19.0	73	Ja
chevrolet vega	21.0	4	140.0	72	2401	19.5	73	A
datsun 610	22.0	4	108.0	94	2379	16.5	73	Ja
maxda rx3	18.0	3	70.0	90	2124	13.5	73	Ja
ford pinto	19.0	4	122.0	85	2310	18.5	73	A
mercury capri v6	21.0	6	155.0	107	2472	14.0	73	A
chevrolet monte carlo s	15.0	8	350.0	145	4082	13.0	73	A
saab 99le	24.0	4	121.0	110	2660	14.0	73	E
fiat 128	29.0	4	68.0	49	1867	19.5	73	E
opel manta	24.0	4	116.0	75	2158	15.5	73	E
audi 100ls	20.0	4	114.0	91	2582	14.0	73	E
volvo 144ea	19.0	4	121.0	112	2868	15.5	73	E
dodge dart custom	15.0	8	318.0	150	3399	11.0	73	A
toyota mark ii	20.0	6	156.0	122	2807	13.5	73	Ja
oldsmobile omega	11.0	8	350.0	180	3664	11.0	73	A
oldsmobile vista cruiser	12.0	8	350.0	180	4499	12.5	73	A
pontiac grand prix	16.0	8	400.0	230	4278	9.5	73	A
plymouth custom suburb	13.0	8	360.0	170	4654	13.0	73	A
amc hornet	18.0	6	232.0	100	2945	16.0	73	A
chevrolet impala	11.0	8	400.0	150	4997	14.0	73	A
buick century 350	13.0	8	350.0	175	4100	13.0	73	A
amc matador	14.0	8	304.0	150	3672	11.5	73	A
chevrolet malibu	13.0	8	350.0	145	3988	13.0	73	A
dodge coronet custom	15.0	8	318.0	150	3777	12.5	73	A

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
mercury marquis brougham	12.0	8	429.0	198	4952	11.5	73	A
chevrolet caprice classic	13.0	8	400.0	150	4464	12.0	73	A
ford ltd	13.0	8	351.0	158	4363	13.0	73	A
plymouth fury gran sedan	14.0	8	318.0	150	4237	14.5	73	A
ford gran torino	14.0	8	302.0	137	4042	14.5	73	A
buick electra 225 custom	12.0	8	455.0	225	4951	11.0	73	A
amc ambassador brougham	13.0	8	360.0	175	3821	11.0	73	A
plymouth valiant	18.0	6	225.0	105	3121	16.5	73	A
chevrolet nova custom	16.0	6	250.0	100	3278	18.0	73	A
ford country	12.0	8	400.0	167	4906	12.5	73	A
ford maverick	18.0	6	250.0	88	3021	16.5	73	A
plymouth duster	23.0	6	198.0	95	2904	16.0	73	A
volkswagen super beetle	26.0	4	97.0	46	1950	21.0	73	E
chrysler new yorker brougham	13.0	8	440.0	215	4735	11.0	73	A
audi fox	29.0	4	98.0	83	2219	16.5	74	E
volkswagen dasher	26.0	4	79.0	67	1963	15.5	74	E
opel manta	26.0	4	97.0	78	2300	14.5	74	E
toyota corona	31.0	4	76.0	52	1649	16.5	74	Ja
datsun 710	32.0	4	83.0	61	2003	19.0	74	Ja
subaru	26.0	4	108.0	93	2391	15.5	74	Ja
fiat 128	24.0	4	90.0	75	2108	15.5	74	E
fiat 124 tc	26.0	4	116.0	75	2246	14.0	74	E
honda civic	24.0	4	120.0	97	2489	15.0	74	Ja
fiat x1.9	31.0	4	79.0	67	2000	16.0	74	E
amc matador (sw)	14.0	8	304.0	150	4257	15.5	74	A
dodge colt	28.0	4	90.0	75	2125	14.5	74	A
ford gran torino (sw)	14.0	8	302.0	140	4638	16.0	74	A
ford gran torino	16.0	8	302.0	140	4141	14.0	74	A
buick century luxus (sw)	13.0	8	350.0	150	4699	14.5	74	A
dodge coronet custom (sw)	14.0	8	318.0	150	4457	13.5	74	A
plymouth duster	20.0	6	198.0	95	3102	16.5	74	A
amc hornet	19.0	6	232.0	100	2901	16.0	74	A
chevrolet nova	15.0	6	250.0	100	3336	17.0	74	A
ford pinto	26.0	4	122.0	80	2451	16.5	74	A
datsun b210	31.0	4	79.0	67	1950	19.0	74	Ja
chevrolet vega	25.0	4	140.0	75	2542	17.0	74	A
chevrolet chevelle malibu classic	16.0	6	250.0	100	3781	17.0	74	A
amc matador	16.0	6	258.0	110	3632	18.0	74	A
plymouth satellite sebring	18.0	6	225.0	105	3613	16.5	74	A
toyota corolla 1200	32.0	4	71.0	65	1836	21.0	74	Ja
datsun 710	24.0	4	119.0	97	2545	17.0	75	Ja
pontiac astro	23.0	4	140.0	78	2592	18.5	75	A
amc gremlin	20.0	6	232.0	100	2914	16.0	75	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
toyota corona	24.0	4	134.0	96	2702	13.5	75	Ja
volkswagen dasher	25.0	4	90.0	71	2223	16.5	75	E
ford pinto	18.0	6	171.0	97	2984	14.5	75	A
saab 99le	25.0	4	121.0	115	2671	13.5	75	E
amc pacer	19.0	6	232.0	90	3211	17.0	75	A
audi 100ls	23.0	4	115.0	95	2694	15.0	75	E
peugeot 504	23.0	4	120.0	88	2957	17.0	75	E
volvo 244dl	22.0	4	121.0	98	2945	14.5	75	E
honda civic cvcc	33.0	4	91.0	53	1795	17.5	75	Ja
ford pinto	23.0	4	140.0	83	2639	17.0	75	A
volkswagen rabbit	29.0	4	90.0	70	1937	14.0	75	E
toyota corolla	29.0	4	97.0	75	2171	16.0	75	Ja
plymouth valiant custom	19.0	6	225.0	95	3264	16.0	75	A
chevrolet monza 2+2	20.0	8	262.0	110	3221	13.5	75	A
ford mustang ii	13.0	8	302.0	129	3169	12.0	75	A
chevrolet nova	18.0	6	250.0	105	3459	16.0	75	A
mercury monarch	15.0	6	250.0	72	3432	21.0	75	A
pontiac catalina	16.0	8	400.0	170	4668	11.5	75	A
chevrolet bel air	15.0	8	350.0	145	4440	14.0	75	A
plymouth grand fury	16.0	8	318.0	150	4498	14.5	75	A
ford maverick	15.0	6	250.0	72	3158	19.5	75	A
buick century	17.0	6	231.0	110	3907	21.0	75	A
chevroelt chevelle malibu	16.0	6	250.0	105	3897	18.5	75	A
amc matador	15.0	6	258.0	110	3730	19.0	75	A
plymouth fury	18.0	6	225.0	95	3785	19.0	75	A
buick skyhawk	21.0	6	231.0	110	3039	15.0	75	A
ford ltd	14.0	8	351.0	148	4657	13.5	75	A
ford pinto	26.5	4	140.0	72	2565	13.6	76	A
pontiac ventura sj	18.5	6	250.0	110	3645	16.2	76	A
amc pacer d/l	17.5	6	258.0	95	3193	17.8	76	A
volkswagen rabbit	29.5	4	97.0	71	1825	12.2	76	E
datsun b-210	32.0	4	85.0	70	1990	17.0	76	Ja
toyota corolla	28.0	4	97.0	75	2155	16.4	76	Ja
volvo 245	20.0	4	130.0	102	3150	15.7	76	E
ford f108	13.0	8	302.0	130	3870	15.0	76	A
peugeot 504	19.0	4	120.0	88	3270	21.9	76	E
toyota mark ii	19.0	6	156.0	108	2930	15.5	76	Ja
mercedes-benz 280s	16.5	6	168.0	120	3820	16.7	76	E
cadillac seville	16.5	8	350.0	180	4380	12.1	76	A
chevy c10	13.0	8	350.0	145	4055	12.0	76	A
dodge d100	13.0	8	318.0	150	3755	14.0	76	A
ford granada ghia	18.0	6	250.0	78	3574	21.0	76	A
plymouth volare premier v8	13.0	8	318.0	150	3940	13.2	76	A

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
dodge aspen se	20.0	6	225.0	100	3651	17.7	76	A
vw rabbit	29.0	4	90.0	70	1937	14.2	76	E
opel 1900	25.0	4	116.0	81	2220	16.9	76	E
honda civic	33.0	4	91.0	53	1795	17.4	76	Ja
fiat 131	28.0	4	107.0	86	2464	15.5	76	E
capri ii	25.0	4	140.0	92	2572	14.9	76	A
dodge colt	26.0	4	98.0	79	2255	17.7	76	A
renault 12tl	27.0	4	101.0	83	2202	15.3	76	E
dodge coronet brougham	16.0	8	318.0	150	4190	13.0	76	A
amc matador	15.5	8	304.0	120	3962	13.9	76	A
chevrolet chevelle malibu classic	17.5	8	305.0	140	4215	13.0	76	A
plymouth valiant	22.0	6	225.0	100	3233	15.4	76	A
chevrolet nova	22.0	6	250.0	105	3353	14.5	76	A
ford maverick	24.0	6	200.0	81	3012	17.6	76	A
amc hornet	22.5	6	232.0	90	3085	17.6	76	A
chevrolet chevette	29.0	4	85.0	52	2035	22.2	76	A
chevrolet woody	24.5	4	98.0	60	2164	22.1	76	A
ford gran torino	14.5	8	351.0	152	4215	12.8	76	A
ford mustang ii 2+2	25.5	4	140.0	89	2755	15.8	77	A
volkswagen rabbit custom	29.0	4	97.0	78	1940	14.5	77	E
pontiac sunbird coupe	24.5	4	151.0	88	2740	16.0	77	A
toyota corolla liftback	26.0	4	97.0	75	2265	18.2	77	Ja
chevrolet chevette	30.5	4	98.0	63	2051	17.0	77	A
bmw 320i	21.5	4	121.0	110	2600	12.8	77	E
subaru dl	30.0	4	97.0	67	1985	16.4	77	Ja
volkswagen dasher	30.5	4	97.0	78	2190	14.1	77	E
datsun 810	22.0	6	146.0	97	2815	14.5	77	Ja
mazda rx-4	21.5	3	80.0	110	2720	13.5	77	Ja
ford thunderbird	16.0	8	351.0	149	4335	14.5	77	A
dodge colt m/m	33.5	4	98.0	83	2075	15.9	77	A
chrysler cordoba	15.5	8	400.0	190	4325	12.2	77	A
chevrolet monte carlo landau	15.5	8	350.0	170	4165	11.4	77	A
plymouth arrow gs	25.5	4	122.0	96	2300	15.5	77	A
buick opel isuzu deluxe	30.0	4	111.0	80	2155	14.8	77	A
renault 5 gtl	36.0	4	79.0	58	1825	18.6	77	E
datsun f-10 hatchback	33.5	4	85.0	70	1945	16.8	77	Ja
pontiac grand prix lj	16.0	8	400.0	180	4220	11.1	77	A
oldsmobile cutlass supreme	17.0	8	260.0	110	4060	19.0	77	A
chevrolet caprice classic	17.5	8	305.0	145	3880	12.5	77	A
mercury cougar brougham	15.0	8	302.0	130	4295	14.9	77	A
chevrolet concours	17.5	6	250.0	110	3520	16.4	77	A
buick skylark	20.5	6	231.0	105	3425	16.9	77	A
plymouth volare custom	19.0	6	225.0	100	3630	17.7	77	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
ford granada	18.5	6	250.0	98	3525	19.0	77	A
dodge monaco brougham	15.5	8	318.0	145	4140	13.7	77	A
honda accord cvcc	31.5	4	98.0	68	2045	18.5	77	Ja
datsun 510	27.2	4	119.0	97	2300	14.7	78	Ja
toyota corona	27.5	4	134.0	95	2560	14.2	78	Ja
chevrolet chevette	30.0	4	98.0	68	2155	16.5	78	A
buick regal sport coupe (turbo)	17.7	6	231.0	165	3445	13.4	78	A
ford futura	18.1	8	302.0	139	3205	11.2	78	A
dodge omni	30.9	4	105.0	75	2230	14.5	78	A
dodge magnum xe	17.5	8	318.0	140	4080	13.7	78	A
toyota celica gt liftback	21.1	4	134.0	95	2515	14.8	78	Ja
peugeot 604sl	16.2	6	163.0	133	3410	15.8	78	E
oldsmobile starfire sx	23.8	4	151.0	85	2855	17.6	78	A
datsun 200-sx	23.9	4	119.0	97	2405	14.9	78	Ja
audi 5000	20.3	5	131.0	103	2830	15.9	78	E
volvo 264gl	17.0	6	163.0	125	3140	13.6	78	E
saab 99gle	21.6	4	121.0	115	2795	15.7	78	E
volkswagen scirocco	31.5	4	89.0	71	1990	14.9	78	E
honda accord lx	29.5	4	98.0	68	2135	16.6	78	Ja
plymouth sapporo	23.2	4	156.0	105	2745	16.7	78	A
chevrolet monte carlo landau	19.2	8	305.0	145	3425	13.2	78	A
mazda glc deluxe	32.8	4	78.0	52	1985	19.4	78	Ja
dodge aspen	18.6	6	225.0	110	3620	18.7	78	A
volkswagen rabbit custom diesel	43.1	4	90.0	48	1985	21.5	78	E
ford fiesta	36.1	4	98.0	66	1800	14.4	78	A
datsun b210 gx	39.4	4	85.0	70	2070	18.6	78	Ja
honda civic cvcc	36.1	4	91.0	60	1800	16.4	78	Ja
amc concord d/l	18.1	6	258.0	120	3410	15.1	78	A
dodge diplomat	19.4	8	318.0	140	3735	13.2	78	A
mercury monarch ghia	20.2	8	302.0	139	3570	12.8	78	A
oldsmobile cutlass salon brougham	19.9	8	260.0	110	3365	15.5	78	A
chevrolet malibu	20.5	6	200.0	95	3155	18.2	78	A
ford fairmont (auto)	20.2	6	200.0	85	2965	15.8	78	A
ford fairmont (man)	25.1	4	140.0	88	2720	15.4	78	A
plymouth volare	20.5	6	225.0	100	3430	17.2	78	A
amc concord	19.4	6	232.0	90	3210	17.2	78	A
buick century special	20.6	6	231.0	105	3380	15.8	78	A
mercury zephyr	20.8	6	200.0	85	3070	16.7	78	A
pontiac phoenix lj	19.2	6	231.0	105	3535	19.2	78	A
plymouth horizon	34.2	4	105.0	70	2200	13.2	79	A
mercedes benz 300d	25.4	5	183.0	77	3530	20.1	79	E
cadillac eldorado	23.0	8	350.0	125	3900	17.4	79	A
peugeot 504	27.2	4	141.0	71	3190	24.8	79	E

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
name								
oldsmobile cutlass salon brougham	23.9	8	260.0	90	3420	22.2	79	A
plymouth horizon tc3	34.5	4	105.0	70	2150	14.9	79	A
amc spirit dl	27.4	4	121.0	80	2670	15.0	79	A
fiat strada custom	37.3	4	91.0	69	2130	14.7	79	E
buick skylark limited	28.4	4	151.0	90	2670	16.0	79	A
chevrolet citation	28.8	6	173.0	115	2595	11.3	79	A
oldsmobile omega brougham	26.8	6	173.0	115	2700	12.9	79	A
pontiac phoenix	33.5	4	151.0	90	2556	13.2	79	A
datsun 210	31.8	4	85.0	65	2020	19.2	79	Ja
dodge colt hatchback custom	35.7	4	98.0	80	1915	14.4	79	A
dodge st. regis	18.2	8	318.0	135	3830	15.2	79	A
vw rabbit custom	31.9	4	89.0	71	1925	14.0	79	E
mercury zephyr 6	19.8	6	200.0	85	2990	18.2	79	A
ford fairmont 4	22.3	4	140.0	88	2890	17.3	79	A
amc concord dl 6	20.2	6	232.0	90	3265	18.2	79	A
dodge aspen 6	20.6	6	225.0	110	3360	16.6	79	A
chevrolet caprice classic	17.0	8	305.0	130	3840	15.4	79	A
ford ltd landau	17.6	8	302.0	129	3725	13.4	79	A
pontiac lemans v6	21.5	6	231.0	115	3245	15.4	79	A
maxda glc deluxe	34.1	4	86.0	65	1975	15.2	79	Ja
buick estate wagon (sw)	16.9	8	350.0	155	4360	14.9	79	A
ford country squire (sw)	15.5	8	351.0	142	4054	14.3	79	A
chevrolet malibu classic (sw)	19.2	8	267.0	125	3605	15.0	79	A
chrysler lebaron town @ country (sw)	18.5	8	360.0	150	3940	13.0	79	A
mercury grand marquis	16.5	8	351.0	138	3955	13.2	79	A
vw rabbit c (diesel)	44.3	4	90.0	48	2085	21.7	80	E
vw dasher (diesel)	43.4	4	90.0	48	2335	23.7	80	E
audi 5000s (diesel)	36.4	5	121.0	67	2950	19.9	80	E
mercedes-benz 240d	30.0	4	146.0	67	3250	21.8	80	E
honda civic 1500 gl	44.6	4	91.0	67	1850	13.8	80	Ja
datsun 280-zx	32.7	6	168.0	132	2910	11.4	80	Ja
vokswagen rabbit	29.8	4	89.0	62	1845	15.3	80	E
mazda rx-7 gs	23.7	3	70.0	100	2420	12.5	80	Ja
triumph tr7 coupe	35.0	4	122.0	88	2500	15.1	80	E
honda accord	32.4	4	107.0	72	2290	17.0	80	Ja
datsun 210	40.8	4	85.0	65	2110	19.2	80	Ja
subaru dl	33.8	4	97.0	67	2145	18.0	80	Ja
dodge colt	27.9	4	156.0	105	2800	14.4	80	A
mazda glc	46.6	4	86.0	65	2110	17.9	80	Ja
toyota corolla	32.2	4	108.0	75	2265	15.2	80	Ja
vw rabbit	41.5	4	98.0	76	2144	14.7	80	E
toyota corolla tercel	38.1	4	89.0	60	1968	18.8	80	Ja
chevrolet chevette	32.1	4	98.0	70	2120	15.5	80	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
chevrolet citation	28.0	4	151.0	90	2678	16.5	80	A
ford fairmont	26.4	4	140.0	88	2870	18.1	80	A
datsun 310	37.2	4	86.0	65	2019	16.4	80	Ja
dodge aspen	19.1	6	225.0	90	3381	18.7	80	A
audi 4000	34.3	4	97.0	78	2188	15.8	80	E
toyota corona liftback	29.8	4	134.0	90	2711	15.5	80	Ja
mazda 626	31.3	4	120.0	75	2542	17.5	80	Ja
datsun 510 hatchback	37.0	4	119.0	92	2434	15.0	80	Ja
amc concord	24.3	4	151.0	90	3003	20.1	80	A
peugeot 505s turbo diesel	28.1	4	141.0	80	3230	20.4	81	E
honda prelude	33.7	4	107.0	75	2210	14.4	81	Ja
toyota corolla	32.4	4	108.0	75	2350	16.8	81	Ja
datsun 200sx	32.9	4	119.0	100	2615	14.8	81	Ja
mazda 626	31.6	4	120.0	74	2635	18.3	81	Ja
volvo diesel	30.7	6	145.0	76	3160	19.6	81	E
chrysler lebaron salon	17.6	6	225.0	85	3465	16.6	81	A
datsun 810 maxima	24.2	6	146.0	120	2930	13.8	81	Ja
buick century	22.4	6	231.0	110	3415	15.8	81	A
oldsmobile cutlass ls	26.6	8	350.0	105	3725	19.0	81	A
ford granada gl	20.2	6	200.0	88	3060	17.1	81	A
volkswagen jetta	33.0	4	105.0	74	2190	14.2	81	E
toyota cressida	25.4	6	168.0	116	2900	12.6	81	Ja
ford escort 2h	29.9	4	98.0	65	2380	20.7	81	A
plymouth reliant	27.2	4	135.0	84	2490	15.7	81	A
plymouth horizon 4	34.7	4	105.0	63	2215	14.9	81	A
ford escort 4w	34.4	4	98.0	65	2045	16.2	81	A
buick skylark	26.6	4	151.0	84	2635	16.4	81	A
dodge aries wagon (sw)	25.8	4	156.0	92	2620	14.4	81	A
plymouth reliant	30.0	4	135.0	84	2385	12.9	81	A
toyota starlet	39.1	4	79.0	58	1755	16.9	81	Ja
chevrolet citation	23.5	6	173.0	110	2725	12.6	81	A
honda civic 1300	35.1	4	81.0	60	1760	16.1	81	Ja
subaru	32.3	4	97.0	67	2065	17.8	81	Ja
datsun 210 mpg	37.0	4	85.0	65	1975	19.4	81	Ja
toyota tercel	37.7	4	89.0	62	2050	17.3	81	Ja
mazda glc 4	34.1	4	91.0	68	1985	16.0	81	Ja
plymouth champ	39.0	4	86.0	64	1875	16.4	81	A
chrysler lebaron medallion	26.0	4	156.0	92	2585	14.5	82	A
honda civic (auto)	32.0	4	91.0	67	1965	15.7	82	Ja
datsun 310 gx	38.0	4	91.0	67	1995	16.2	82	Ja
buick century limited	25.0	6	181.0	110	2945	16.4	82	A
oldsmobile cutlass ciera (diesel)	38.0	6	262.0	85	3015	17.0	82	A
ford granada l	22.0	6	232.0	112	2835	14.7	82	A

name	mpg	cylinders	displacement	horsepower	weight	acceleration	year	or
dodge rampage	32.0	4	135.0	84	2295	11.6	82	A
dodge charger 2.2	36.0	4	135.0	84	2370	13.0	82	A
chevrolet camaro	27.0	4	151.0	90	2950	17.3	82	A
ford mustang gl	27.0	4	140.0	86	2790	15.6	82	A
vw pickup	44.0	4	97.0	52	2130	24.6	82	E
honda civic	38.0	4	91.0	67	1965	15.0	82	Ja
toyota celica gt	32.0	4	144.0	96	2665	13.9	82	Ja
toyota corolla	34.0	4	108.0	70	2245	16.9	82	Ja
ford ranger	28.0	4	120.0	79	2625	18.6	82	A
nissan stanza xe	36.0	4	120.0	88	2160	14.5	82	Ja
mercury lynx l	36.0	4	98.0	70	2125	17.3	82	A
plymouth horizon miser	38.0	4	105.0	63	2125	14.7	82	A
mazda glc custom	31.0	4	91.0	68	1970	17.6	82	Ja
mazda glc custom l	37.0	4	91.0	68	2025	18.2	82	Ja
volkswagen rabbit l	36.0	4	105.0	74	1980	15.3	82	E
ford fairmont futura	24.0	4	140.0	92	2865	16.4	82	A
pontiac phoenix	27.0	4	151.0	90	2735	18.0	82	A
dodge aries se	29.0	4	135.0	84	2525	16.0	82	A
pontiac j2000 se hatchback	31.0	4	112.0	85	2575	16.2	82	A
chevrolet cavalier 2-door	34.0	4	112.0	88	2395	18.0	82	A
chevrolet cavalier wagon	27.0	4	112.0	88	2640	18.6	82	A
chevrolet cavalier	28.0	4	112.0	88	2605	19.6	82	A
honda accord	36.0	4	107.0	75	2205	14.5	82	Ja
chevy s-10	31.0	4	119.0	82	2720	19.4	82	A

```python
Auto_sqrt["sqrt_displacement"] = np.sqrt(Auto_sqrt["displacement"])
Auto_sqrt["sqrt_horsepower"] = np.sqrt(Auto_sqrt["horsepower"])
Auto_sqrt["sqrt_weight"] = np.sqrt(Auto_sqrt["weight"])
Auto_sqrt = Auto_sqrt.drop(
    columns=[
        "displacement",
        "weight",
        "horsepower",
        "year",
    ]
)
Auto_sqrt.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'origin', 'oilshock', 'sqrt_displacement', 'sqrt_horsepowe
```

```python
Auto_sqrt.corr(numeric_only=True)
```

	mpg	cylinders	acceleration	oilshock	sqrt_displacement	sqrt_horsepower	sqrt_we
mpg	1.000000	-0.777618	0.423329	0.521192	-0.821331	-0.802311	-0.84009
cylinders	-0.777618	1.000000	-0.504683	-0.273703	0.953208	0.849266	0.893465
acceleration	0.423329	-0.504683	1.000000	0.195892	-0.521812	-0.696702	-0.40982
oilshock	0.521192	-0.273703	0.195892	1.000000	-0.284587	-0.306247	-0.26066
sqrt_displacement	-0.821331	0.953208	-0.521812	-0.284587	1.000000	0.886470	0.939868
sqrt_horsepower	-0.802311	0.849266	-0.696702	-0.306247	0.886470	1.000000	0.872045
sqrt_weight	-0.840095	0.893465	-0.409829	-0.260664	0.939868	0.872045	1.00000

```
Auto_sqrt = pd.get_dummies(
    Auto_sqrt, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_sqrt.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'oilshock', 'sqrt_displacement', 'sqrt_horsepower', 'sqrt_
```

```
cols = list(Auto_sqrt.columns)
cols.remove("mpg")
```

```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_sqrt)
vifdf
```

Feature	VIF
cylinders	11.465746
acceleration	3.010771
oilshock	1.151324
sqrt_displacement	27.042946
sqrt_horsepower	10.615281
sqrt_weight	13.450552
origin_Europe	1.774827
origin_Japan	1.944729

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is sqrt_displacement with a VIF of 27.042946454149405
Hence, we drop sqrt_displacement from the model to be fitted.
```

```
('sqrt_displacement', 27.042946454149405)
```

```
cols.remove("sqrt_displacement")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_sqrt)
vifdf
```

Feature	VIF
cylinders	5.974510
acceleration	2.934605
oilshock	1.141428
sqrt_horsepower	10.446261
sqrt_weight	9.963350
origin_Europe	1.450840
origin_Japan	1.623907

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is sqrt_horsepower with a VIF of 10.446261176837464
Hence, we drop sqrt_horsepower from the model to be fitted.
```

```
('sqrt_horsepower', 10.446261176837464)
```

```
cols.remove("sqrt_horsepower")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_sqrt)
vifdf
```

Feature	VIF
cylinders	5.907717
acceleration	1.377206
oilshock	1.119726
sqrt_weight	5.331435
origin_Europe	1.446456
origin_Japan	1.581460

```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

```
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_sqrt)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.814
Model:                            OLS   Adj. R-squared:                  0.811
Method:                 Least Squares   F-statistic:                     281.0
Date:                Tue, 25 Feb 2025   Prob (F-statistic):         2.76e-137
Time:                        14:39:15   Log-Likelihood:                -1031.4
```

```
No. Observations:                 392   AIC:                             2077.
Df Residuals:                     385   BIC:                             2105.
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        54.3622      2.396     22.687      0.000      49.651      59.073
cylinders         0.0148      0.244      0.061      0.952      -0.466       0.495
acceleration      0.1748      0.073      2.395      0.017       0.031       0.318
oilshock          5.0506      0.364     13.873      0.000       4.335       5.766
sqrt_weight      -0.6785      0.052    -13.130      0.000      -0.780      -0.577
origin_Europe     1.5511      0.544      2.851      0.005       0.481       2.621
origin_Japan      1.9033      0.537      3.544      0.000       0.847       2.959
==============================================================================
Omnibus:                       25.773   Durbin-Watson:                   1.269
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               41.514
Skew:                           0.449   Prob(JB):                     9.67e-10
Kurtosis:                       4.317   Cond. No.                         803.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                 df        sum_sq       mean_sq            F         PR(>F)
cylinders       1.0  14403.083079  14403.083079  1252.209748  4.496819e-123
acceleration    1.0     30.471304     30.471304     2.649187   1.044209e-01
oilshock        1.0   2422.051542   2422.051542   210.574120   2.284075e-38
sqrt_weight     1.0   2365.801178   2365.801178   205.683691   1.125346e-37
origin_Europe   1.0     24.780703     24.780703     2.154444   1.429743e-01
origin_Japan    1.0    144.484455    144.484455    12.561536   4.421781e-04
Residual      385.0   4428.321210     11.502133          NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x78473024f7a0>
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is cylinders with a p-value of 0.951625687031507
Using the backward methodology, we suggest dropping cylinders from the new model
```

```
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_sqrt)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.814
Model:                            OLS   Adj. R-squared:                  0.812
Method:                 Least Squares   F-statistic:                     338.0
```

```
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          1.43e-138
Time:                        14:39:15   Log-Likelihood:                -1031.4
No. Observations:                 392   AIC:                             2075.
Df Residuals:                     386   BIC:                             2099.
Df Model:                           5
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        54.3324      2.342     23.198      0.000      49.728      58.937
acceleration      0.1733      0.069      2.512      0.012       0.038       0.309
oilshock          5.0486      0.362     13.946      0.000       4.337       5.760
sqrt_weight      -0.6760      0.031    -21.968      0.000      -0.737      -0.616
origin_Europe     1.5438      0.530      2.913      0.004       0.502       2.586
origin_Japan      1.8999      0.534      3.561      0.000       0.851       2.949
==============================================================================
Omnibus:                       25.925   Durbin-Watson:                   1.269
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               41.952
Skew:                           0.450   Prob(JB):                     7.77e-10
Kurtosis:                       4.326   Cond. No.                         783.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df        sum_sq       mean_sq           F         PR(>F)
acceleration     1.0   4268.531557   4268.531557  372.068179   1.546709e-58
oilshock         1.0   4757.627552   4757.627552  414.700418   3.906846e-63
sqrt_weight      1.0  10191.062423  10191.062423  888.307839  3.798699e-102
origin_Europe    1.0     27.910362     27.910362    2.432817   1.196386e-01
origin_Japan     1.0    145.497978    145.497978   12.682387   4.152294e-04
Residual       386.0   4428.363596     11.472445         NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x78472e585e80>
```

```python
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is acceleration with a p-value of 0.012396804420
Using the backward methodology, we suggest dropping acceleration from the new model
```

```python
cols.remove("acceleration")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_sqrt)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.811
Model:                            OLS   Adj. R-squared:                  0.809
Method:                 Least Squares   F-statistic:                     415.3
```

```
Date:                Tue, 25 Feb 2025   Prob (F-statistic):          1.50e-138
Time:                        14:39:15   Log-Likelihood:                -1034.6
No. Observations:                 392   AIC:                             2079.
Df Residuals:                     387   BIC:                             2099.
Df Model:                           4
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        58.2668      1.753     33.230      0.000      54.819      61.714
oilshock          5.1556      0.362     14.244      0.000       4.444       5.867
sqrt_weight      -0.6999      0.029    -23.759      0.000      -0.758      -0.642
origin_Europe     1.6530      0.532      3.108      0.002       0.607       2.699
origin_Japan      1.8263      0.536      3.405      0.001       0.772       2.881
==============================================================================
Omnibus:                       31.883   Durbin-Watson:                   1.241
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               60.472
Skew:                           0.483   Prob(JB):                     7.39e-14
Kurtosis:                       4.664   Cond. No.                         574.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                 df        sum_sq       mean_sq            F         PR(>F)
oilshock        1.0   6470.207217   6470.207217   556.341155   7.004177e-77
sqrt_weight     1.0  12674.256790  12674.256790  1089.796728  1.354903e-114
origin_Europe   1.0     38.907051     38.907051     3.345425   6.816142e-02
origin_Japan    1.0    134.840521    134.840521    11.594270   7.307851e-04
Residual      387.0   4500.781890     11.629927          NaN            NaN
```

```
<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x7847302bb710>
```

```
models.append(
    {
        "name": "sqrt_transformation",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
pd.DataFrame(models)
```

	name	model
0	simple_model	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan
1	numeric_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + horsepower:
2	oilshock_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + horsepower:

	name	model
3	origin__interactions	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + oilshock: hor
4	squared__transformation	mpg ~ horsepower + weight + oilshock + origin__Europe + origin__Japan + I(horsepower
5	log__transformation	mpg ~ acceleration + oilshock + log__weight + origin__Europe + origin__Japan
6	sqrt__transformation	mpg ~ oilshock + sqrt__weight + origin__Europe + origin__Japan

```python
allDone()
```

```
<IPython.lib.display.Audio object>
```