ExercisesAutoRegime

February 21, 2025

1 Auto dataset two regimes: Pre-oilshock and Post-oilshock

1.1 We can also test if there are two regimes that contribute to the heteroskedasticity by running separate regressions for pre-oilshock and post-oilshock.

1.1.1 Imports for python objects and libraries

Set up IPython libraries for customizing notebook display

[1]:
```
from notebookfuncs import *
```

Import standard libraries

[2]:
```
import numpy as np
import pandas as pd

pd.set_option("display.max_rows", 1000)
pd.set_option("display.max_columns", 1000)
pd.set_option("display.width", 1000)
pd.set_option("display.max.colwidth", None)
import matplotlib.pyplot as plt
import seaborn as sns
import itertools
```

Statsmodels imports

[3]:
```
import statsmodels.api as sm
```

Import statsmodels.objects

[4]:
```
from statsmodels.stats.outliers_influence import summary_table
```

Import ISLP objects

[5]:
```
import ISLP
from ISLP import models
from ISLP import load_data
from ISLP.models import ModelSpec as MS, summarize, poly
```

Import user functions

```
[6]: from userfuncs import display_residuals_plot
from userfuncs import identify_least_significant_feature
from userfuncs import calculate_VIFs
from userfuncs import identify_highest_VIF_feature
from userfuncs import standardize
from userfuncs import perform_analysis
```

Set level of significance (alpha)

```
[7]: LOS_Alpha = 0.01
```

1.1.2 Data Cleaning and exploratory data analysis

```
[8]: Auto = load_data("Auto")
Auto = Auto.sort_values(by=["year"], ascending=True)
Auto.head()
Auto.columns
Auto = Auto.dropna()
Auto.shape
Auto.describe()
```

```
[8]:               mpg   cylinders  displacement  horsepower       weight
acceleration        year       origin
count  392.000000  392.000000    392.000000  392.000000   392.000000
392.000000  392.000000  392.000000
mean     23.445918    5.471939    194.411990  104.469388  2977.584184
15.541327    75.979592    1.576531
std       7.805007    1.705783    104.644004   38.491160   849.402560
2.758864     3.683737     0.805518
min       9.000000    3.000000     68.000000   46.000000  1613.000000
8.000000    70.000000     1.000000
25%      17.000000    4.000000    105.000000   75.000000  2225.250000
13.775000    73.000000    1.000000
50%      22.750000    4.000000    151.000000   93.500000  2803.500000
15.500000    76.000000    1.000000
75%      29.000000    8.000000    275.750000  126.000000  3614.750000
17.025000    79.000000    2.000000
max      46.600000    8.000000    455.000000  230.000000  5140.000000
24.800000    82.000000    3.000000
```

Convert origin to categorical type

```
[9]: Auto["origin"] = Auto["origin"].astype("category")
Auto["origin"] = Auto["origin"].cat.rename_categories(
    {1: "America", 2: "Europe", 3: "Japan"}
)
Auto.describe()
```

[9]:
```
                 mpg   cylinders  displacement  horsepower       weight
acceleration        year
count  392.000000  392.000000    392.000000  392.000000   392.000000
392.000000  392.000000
mean    23.445918    5.471939    194.411990  104.469388  2977.584184
15.541327   75.979592
std      7.805007    1.705783    104.644004   38.491160   849.402560
2.758864    3.683737
min      9.000000    3.000000     68.000000   46.000000  1613.000000
8.000000   70.000000
25%     17.000000    4.000000    105.000000   75.000000  2225.250000
13.775000   73.000000
50%     22.750000    4.000000    151.000000   93.500000  2803.500000
15.500000   76.000000
75%     29.000000    8.000000    275.750000  126.000000  3614.750000
17.025000   79.000000
max     46.600000    8.000000    455.000000  230.000000  5140.000000
24.800000   82.000000
```

1.2 Create two datasets based on whether the car models have been exposed to the 1973 oil shock or not

[10]:
```python
Auto_preos = Auto[Auto["year"] <= 76]
Auto_preos.shape
Auto_preos.describe()
Auto_preos.corr(numeric_only=True)
```

[10]:
```
                   mpg  cylinders  displacement  horsepower    weight
acceleration      year
mpg           1.000000  -0.863133     -0.878385   -0.812052 -0.903557
0.494406  0.172135
cylinders    -0.863133   1.000000      0.955270    0.852144  0.906436
-0.616635 -0.157796
displacement -0.878385   0.955270      1.000000    0.900549  0.926890
-0.653019 -0.195140
horsepower   -0.812052   0.852144      0.900549    1.000000  0.861309
-0.748969 -0.294137
weight       -0.903557   0.906436      0.926890    0.861309  1.000000
-0.522137 -0.073366
acceleration  0.494406  -0.616635     -0.653019   -0.748969 -0.522137
1.000000  0.298412
year          0.172135  -0.157796     -0.195140   -0.294137 -0.073366
0.298412  1.000000
```

[11]:
```python
Auto_postos = Auto[Auto["year"] > 76]
Auto_postos.shape
Auto_postos.describe()
```

[11]:
```
               mpg   cylinders  displacement  horsepower       weight  \
acceleration        year
count  178.000000  178.000000    178.000000  178.000000   178.000000
178.000000  178.000000
mean    27.900562    4.960674    160.426966   91.410112  2726.679775
16.133146   79.455056
std      7.504963    1.466624     80.477444   27.144212   670.417952
2.504227    1.714248
min     15.000000    3.000000     70.000000   48.000000  1755.000000
11.100000   77.000000
25%     20.875000    4.000000     98.000000   70.000000  2144.250000
14.500000   78.000000
50%     28.000000    4.000000    134.500000   87.000000  2630.000000
15.800000   79.000000
75%     33.650000    6.000000    200.000000  105.000000  3208.750000
17.600000   81.000000
max     46.600000    8.000000    400.000000  190.000000  4360.000000
24.800000   82.000000
```

[12]:
```python
display(
    "If you look at the two datasets as displayed above, it's evident that the␣
 ↪oil shock had a major impact on the models produced since."
)
display(Auto_preos.mean(numeric_only=True), Auto_postos.mean(numeric_only=True))
display(
    "Mileage increased, number of cylinders decreased, displacement decreased,␣
 ↪horsepower decreased, weight decreased and time to acceleration increased␣
 ↪thus indicating that less powerful and less performant cars were produced in␣
 ↪the immediate period after the oil shock of 1973."
)
```

```
"If you look at the two datasets as displayed above, it's evident that the oil␣
 ↪shock had a major impact on the models produced since."
```

```
mpg               19.740654
cylinders          5.897196
displacement     222.679907
horsepower       115.331776
weight          3186.280374
acceleration      15.049065
year              73.088785
dtype: float64
```

```
mpg               27.900562
cylinders          4.960674
displacement     160.426966
horsepower        91.410112
weight          2726.679775
acceleration      16.133146
```

```
year              79.455056
dtype: float64
```

```
'Mileage increased, number of cylinders decreased, displacement decreased,
↪horsepower decreased, weight decreased and time to acceleration increased thus
↪indicating that less powerful and less performant cars were produced in the
↪immediate period after the oil shock of 1973.'
```

Standardize numeric variables in the model

[13]:
```
Auto_preos = Auto_preos.apply(standardize)
Auto_preos.describe()
```

[13]:
```
                mpg     cylinders  displacement    horsepower      weight
acceleration          year
count  2.140000e+02  2.140000e+02  2.140000e+02  2.140000e+02  214.000000
214.000000  2.140000e+02
mean  -4.150366e-17 -2.490220e-17  2.490220e-17 -1.494132e-16    0.000000
0.000000 -5.312469e-16
std    1.002345e+00  1.002345e+00  1.002345e+00  1.002345e+00    1.002345
1.002345  1.002345e+00
min   -1.829062e+00 -1.635252e+00 -1.362364e+00 -1.617309e+00   -1.705900
-2.463723 -1.552289e+00
25%   -8.073018e-01 -1.070826e+00 -9.550106e-01 -6.842252e-01   -0.944725
-0.698694 -1.049733e+00
50%   -1.261285e-01  5.802508e-02  4.685742e-02 -3.576458e-01   -0.081084
-0.017149 -4.461951e-02
75%    7.891982e-01  1.186877e+00  8.395442e-01  8.087090e-01    0.913215
0.629446  9.604936e-01
max    2.598565e+00  1.186877e+00  2.046190e+00  2.674877e+00    2.118409
2.953691  1.463050e+00
```

[14]:
```
Auto_postos = Auto_postos.apply(standardize)
Auto_postos.describe()
```

[14]:
```
                mpg     cylinders  displacement    horsepower      weight
acceleration          year
count  1.780000e+02  1.780000e+02  1.780000e+02  1.780000e+02  178.000000
1.780000e+02  1.780000e+02
mean  -3.193450e-16  2.794269e-16 -7.983626e-17 -1.796316e-16    0.000000
-1.237462e-15 -1.516889e-15
std    1.002821e+00  1.002821e+00  1.002821e+00  1.002821e+00    1.002821
1.002821e+00  1.002821e+00
min   -1.723786e+00 -1.340633e+00 -1.126801e+00 -1.603751e+00   -1.453453
-2.015529e+00 -1.436187e+00
25%   -9.387629e-01 -6.568717e-01 -7.778958e-01 -7.909792e-01   -0.871207
-6.539953e-01 -8.511958e-01
50%    1.328704e-02 -6.568717e-01 -3.230732e-01 -1.629280e-01   -0.144615
-1.334087e-01 -2.662041e-01
```

```
75%     7.682459e-01  7.106507e-01  4.931154e-01  5.020674e-01    0.721088
5.874034e-01  9.037793e-01
max     2.498638e+00  2.078173e+00  2.985294e+00  3.642323e+00    2.443144
3.470652e+00  1.488771e+00
```

Encode categorical variables as dummy variables dropping the first to remove multicollinearity.

[15]:
```python
Auto_preos = pd.get_dummies(
    Auto_preos, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_preos.columns
```

[15]:
```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight',
'acceleration', 'year', 'origin_Europe', 'origin_Japan'], dtype='object')
```

[16]:
```python
Auto_postos = pd.get_dummies(
    Auto_postos, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_postos.columns
```

[16]:
```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight',
'acceleration', 'year', 'origin_Europe', 'origin_Japan'], dtype='object')
```

1.2.1 Analysis for pre-oil shock model

Test for multicollinearity using correlation matrix and variance inflation factors

[17]:
```python
Auto_preos.corr(numeric_only=True)
```

[17]:
```
                   mpg  cylinders  displacement  horsepower    weight
acceleration      year  origin_Europe  origin_Japan
mpg           1.000000  -0.863133     -0.878385   -0.812052 -0.903557
0.494406  0.172135       0.429946      0.454576
cylinders    -0.863133   1.000000      0.955270    0.852144  0.906436
-0.616635 -0.157796      -0.507897     -0.408555
displacement -0.878385   0.955270      1.000000    0.900549  0.926890
-0.653019 -0.195140      -0.499456     -0.428045
horsepower   -0.812052   0.852144      0.900549    1.000000  0.861309
-0.748969 -0.294137      -0.373257     -0.292877
weight       -0.903557   0.906436      0.926890    0.861309  1.000000
-0.522137 -0.073366      -0.420078     -0.424328
acceleration  0.494406  -0.616635     -0.653019   -0.748969 -0.522137
1.000000  0.298412       0.215335      0.164038
year          0.172135  -0.157796     -0.195140   -0.294137 -0.073366
0.298412  1.000000       0.061819      0.030362
origin_Europe  0.429946  -0.507897     -0.499456   -0.373257 -0.420078
0.215335  0.061819       1.000000     -0.192745
origin_Japan   0.454576  -0.408555     -0.428045   -0.292877 -0.424328
```

```
0.164038  0.030362      -0.192745      1.000000
```

[18]:
```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(Auto_preos.columns) + " - mpg",␣
 ↪Auto_preos)
vifdf
```

[18]:

Feature	VIF
cylinders	12.409093
displacement	23.483690
horsepower	9.924721
weight	10.993223
acceleration	2.965117
year	1.296707
origin_Europe	2.286473
origin_Japan	2.062780

[19]:
```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is displacement with a VIF of
23.483689524756567
Hence, we drop displacement from the model to be fitted.
```

[20]:
```
vifdf = calculate_VIFs(
    "mpg ~ " + " + ".join(Auto_preos.columns) + " - mpg - displacement",␣
 ↪Auto_preos
)
vifdf
```

[20]:

Feature	VIF
cylinders	8.727646
horsepower	8.845099
weight	9.513189
acceleration	2.856231
year	1.287027
origin_Europe	1.960903
origin_Japan	1.789531

[21]:
```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

Linear Regression for mpg ~ horsepower + acceleration + weight + cylinders + year + origin_Europe + origin_Japan

[22]:
```
cols = list(Auto_preos.columns)
cols.remove("mpg")
cols.remove("displacement")
```

```python
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_preos)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.848
Model:                            OLS   Adj. R-squared:                  0.842
Method:                 Least Squares   F-statistic:                     163.8
Date:                Wed, 25 Sep 2024   Prob (F-statistic):           1.51e-80
Time:                        07:57:00   Log-Likelihood:                -102.32
No. Observations:                 214   AIC:                             220.6
Df Residuals:                     206   BIC:                             247.6
Df Model:                           7
Covariance Type:            nonrobust
================================================================================
=
                    coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
-
Intercept        -0.1025      0.040     -2.583      0.010      -0.181
-0.024
cylinders        -0.1149      0.080     -1.430      0.154      -0.273
0.043
horsepower       -0.1394      0.081     -1.724      0.086      -0.299
0.020
weight           -0.6079      0.084     -7.248      0.000      -0.773
-0.443
acceleration     -0.0653      0.046     -1.421      0.157      -0.156
0.025
year              0.0776      0.031      2.514      0.013       0.017
0.138
origin_Europe     0.2534      0.097      2.618      0.009       0.063
0.444
origin_Japan      0.3985      0.106      3.749      0.000       0.189
0.608
==============================================================================
Omnibus:                       12.372   Durbin-Watson:                   1.407
Prob(Omnibus):                  0.002   Jarque-Bera (JB):               16.578
Skew:                          -0.403   Prob(JB):                     0.000251
Kurtosis:                       4.099   Cond. No.                         9.30
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                  df      sum_sq     mean_sq            F        PR(>F)
cylinders        1.0  159.429574  159.429574  1007.436126  2.877624e-81
```

```
horsepower        1.0    4.577852    4.577852   28.927463  2.030739e-07
weight            1.0   13.283446   13.283446   83.938147  5.242535e-17
acceleration      1.0    0.533174    0.533174    3.369126  6.787066e-02
year              1.0    1.267919    1.267919    8.011985  5.107121e-03
origin_Europe     1.0    0.083174    0.083174    0.525577  4.692948e-01
origin_Japan      1.0    2.224788    2.224788   14.058446  2.302318e-04
Residual        206.0   32.600074    0.158253         NaN           NaN
```

[23]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is acceleration with a
p-value of 0.15682628665346462 and a coefficient of -0.06530735672959463
Using the backward methodology, we suggest dropping acceleration from the new
model
```

Linear Regression after dropping acceleration in pre-oil shock. The model now is mpg ~ horsepower + weight + cylinder + year + origin_Europe + origin_Japan

[24]:
```
cols.remove("acceleration")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_preos)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.846
Model:                            OLS   Adj. R-squared:                  0.842
Method:                 Least Squares   F-statistic:                     189.8
Date:                Wed, 25 Sep 2024   Prob (F-statistic):           2.86e-81
Time:                        07:57:01   Log-Likelihood:                -103.36
No. Observations:                 214   AIC:                             220.7
Df Residuals:                     207   BIC:                             244.3
Df Model:                           6
Covariance Type:            nonrobust
================================================================================
=
                    coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
-
Intercept        -0.1073      0.040     -2.705      0.007      -0.185
-0.029
cylinders        -0.0832      0.077     -1.075      0.284      -0.236
0.069
horsepower       -0.0718      0.066     -1.095      0.275      -0.201
0.057
weight           -0.6564      0.077     -8.546      0.000      -0.808
-0.505
year              0.0789      0.031      2.552      0.011       0.018
0.140
```

```
origin_Europe     0.2722      0.096      2.832      0.005      0.083
0.462
origin_Japan      0.4069      0.106      3.825      0.000      0.197
0.617
==============================================================================
Omnibus:                        9.704   Durbin-Watson:                   1.384
Prob(Omnibus):                  0.008   Jarque-Bera (JB):               10.825
Skew:                          -0.398   Prob(JB):                      0.00446
Kurtosis:                       3.763   Cond. No.                         8.52
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                  df      sum_sq     mean_sq            F        PR(>F)
cylinders        1.0  159.429574  159.429574  1002.499833  2.735220e-81
horsepower       1.0    4.577852    4.577852    28.785723  2.155884e-07
weight           1.0   13.283446   13.283446    83.526863  5.919063e-17
year             1.0    1.323199    1.323199     8.320328  4.335077e-03
origin_Europe    1.0    0.139721    0.139721     0.878569  3.496863e-01
origin_Japan     1.0    2.326581    2.326581    14.629642  1.731691e-04
Residual       207.0   32.919628    0.159032          NaN           NaN
```

[25]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is cylinders with a p-value
of 0.28351001934768794 and a coefficient of -0.08318186983127318
Using the backward methodology, we suggest dropping cylinders from the new model
```

[26]:
```
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_preos)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.845
Model:                            OLS   Adj. R-squared:                  0.842
Method:                 Least Squares   F-statistic:                     227.3
Date:                Wed, 25 Sep 2024   Prob (F-statistic):           3.20e-82
Time:                        07:57:01   Log-Likelihood:                -103.95
No. Observations:                 214   AIC:                             219.9
Df Residuals:                     208   BIC:                             240.1
Df Model:                           5
Covariance Type:            nonrobust
================================================================================
=
                    coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
```

```
-
Intercept         -0.1213      0.037     -3.235      0.001      -0.195
-0.047
horsepower        -0.0964      0.061     -1.569      0.118      -0.218
0.025
weight            -0.6974      0.067    -10.455      0.000      -0.829
-0.566
year               0.0802      0.031      2.597      0.010       0.019
0.141
origin_Europe      0.3185      0.086      3.708      0.000       0.149
0.488
origin_Japan       0.4445      0.101      4.422      0.000       0.246
0.643
==============================================================================
Omnibus:                        7.861   Durbin-Watson:                   1.406
Prob(Omnibus):                  0.020   Jarque-Bera (JB):                8.096
Skew:                          -0.371   Prob(JB):                       0.0175
Kurtosis:                       3.598   Cond. No.                         6.43
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                  df      sum_sq     mean_sq           F        PR(>F)
horsepower       1.0  141.117636  141.117636  886.687803  6.026431e-77
weight           1.0   34.542884   34.542884  217.044124  4.053391e-34
year             1.0    1.552002    1.552002    9.751732  2.046623e-03
origin_Europe    1.0    0.572100    0.572100    3.594690  5.935071e-02
origin_Japan     1.0    3.111879    3.111879   19.552944  1.576086e-05
Residual       208.0   33.103499    0.159151         NaN           NaN
```

[27]:
```python
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is horsepower with a
p-value of 0.11823063227848224 and a coefficient of -0.09641477003432276
Using the backward methodology, we suggest dropping horsepower from the new
model
```

[28]:
```python
cols.remove("horsepower")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_preos)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.843
Model:                            OLS   Adj. R-squared:                  0.840
Method:                 Least Squares   F-statistic:                     281.6
Date:                Wed, 25 Sep 2024   Prob (F-statistic):           6.06e-83
Time:                        07:57:01   Log-Likelihood:                -105.21
```

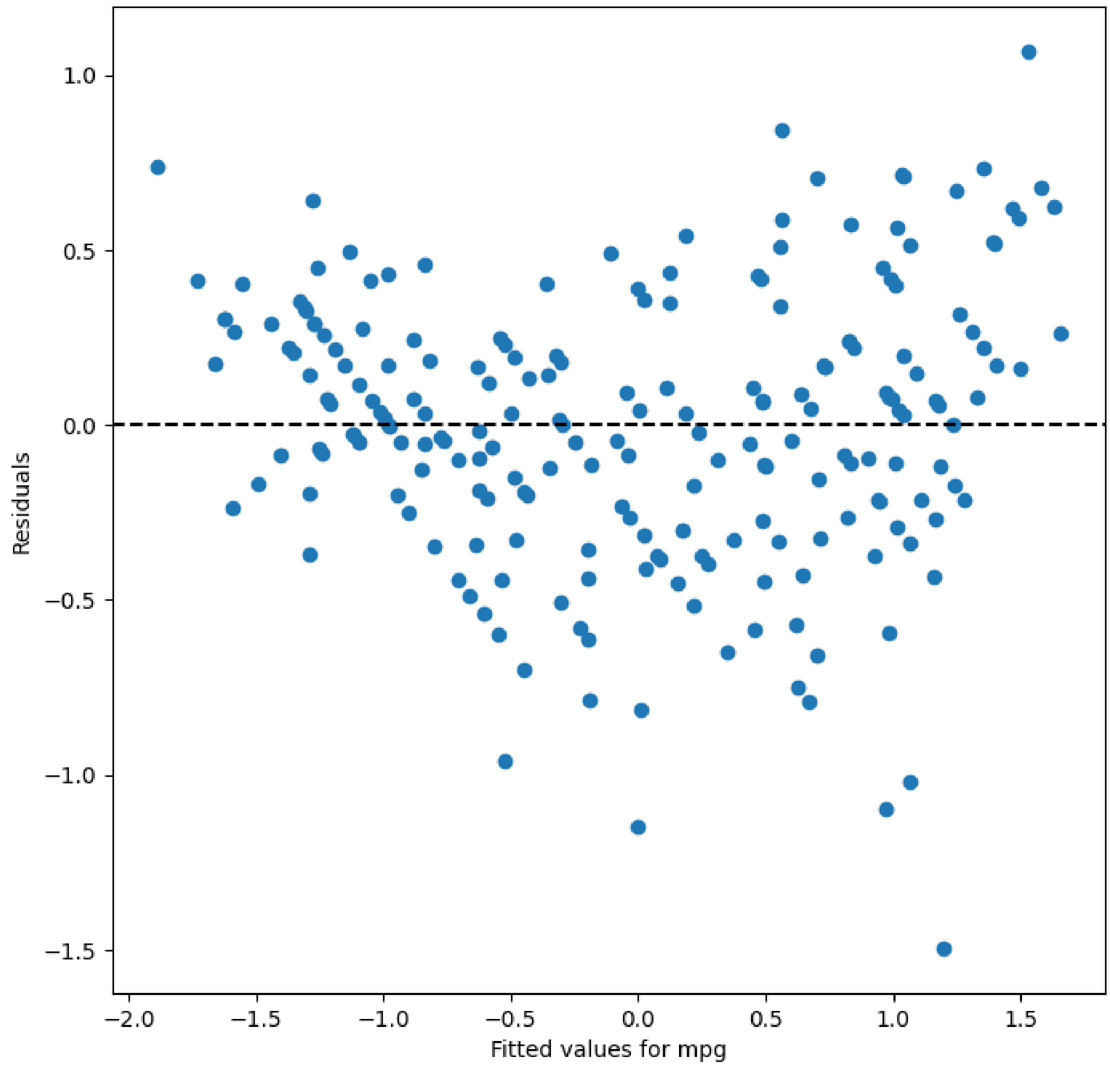
```
No. Observations:                 214   AIC:                             220.4
Df Residuals:                     209   BIC:                             237.3
Df Model:                           4
Covariance Type:            nonrobust
================================================================================
=
                   coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
-
Intercept        -0.1151      0.037     -3.077      0.002      -0.189
-0.041
weight           -0.7850      0.037    -21.422      0.000      -0.857
-0.713
year              0.1028      0.027      3.742      0.000       0.049
0.157
origin_Europe     0.3078      0.086      3.582      0.000       0.138
0.477
origin_Japan      0.4140      0.099      4.183      0.000       0.219
0.609
==============================================================================
Omnibus:                       10.672   Durbin-Watson:                   1.398
Prob(Omnibus):                  0.005   Jarque-Bera (JB):               11.650
Skew:                          -0.443   Prob(JB):                      0.00295
Kurtosis:                       3.722   Cond. No.                         4.59
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                  df      sum_sq     mean_sq            F         PR(>F)
weight           1.0  174.712905  174.712905  1090.157519  7.188986e-85
year             1.0    2.410414    2.410414    15.040281  1.409621e-04
origin_Europe    1.0    0.576721    0.576721     3.598570  5.920817e-02
origin_Japan     1.0    2.804802    2.804802    17.501148  4.226183e-05
Residual       209.0   33.495157    0.160264          NaN           NaN
```

[29]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
No variables are statistically insignificant.
The model mpg ~ weight + year + origin_Europe + origin_Japan cannot be pruned
further.
```

Residual plot for model for pre-oil shock

[30]:
```
display_residuals_plot(results)
```



[31]:
```
preoilshock_model = results
```

1.2.2 Analysis for post Oil Shock

Test for multicollinearity using correlation matrix and variance inflation factors

[32]:
```
Auto_postos.corr(numeric_only=True)
```

[32]:
```
                   mpg  cylinders  displacement  horsepower    weight  \
acceleration       year  origin_Europe  origin_Japan
mpg           1.000000  -0.710232     -0.771039   -0.796617 -0.837333
0.278650  0.460200         0.212795      0.405159
cylinders    -0.710232   1.000000      0.936943    0.796697  0.860088
-0.247767 -0.338905        -0.181385     -0.359263
displacement -0.771039   0.936943      1.000000    0.854454  0.929346
```

```
-0.264374 -0.319411       -0.240143     -0.436964
horsepower     -0.796617   0.796697      0.854454    1.000000   0.837067
-0.535033 -0.353954       -0.214702     -0.317954
weight         -0.837333   0.860088      0.929346    0.837067   1.000000
-0.130152 -0.319783       -0.144152     -0.459869
acceleration    0.278650  -0.247767     -0.264374   -0.535033 -0.130152
1.000000   0.157159        0.235217      0.000714
year            0.460200  -0.338905     -0.319411   -0.353954 -0.319783
0.157159   1.000000       -0.057596      0.155368
origin_Europe   0.212795  -0.181385     -0.240143   -0.214702 -0.144152
0.235217 -0.057596        1.000000     -0.264286
origin_Japan    0.405159  -0.359263     -0.436964   -0.317954 -0.459869
0.000714   0.155368       -0.264286      1.000000
```

[33]:
```python
vifdf = calculate_VIFs(
    "mpg ~ " + " + ".join(Auto_postos.columns) + " - mpg", Auto_postos
)
vifdf
```

[33]:
```
                     VIF
Feature
cylinders       9.017020
displacement   20.423355
horsepower      9.245687
weight         12.693737
acceleration    2.788052
year            1.185236
origin_Europe   1.452328
origin_Japan    1.651675
```

[34]:
```python
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is displacement with a VIF of
20.423354692792778
Hence, we drop displacement from the model to be fitted.
```

[35]:
```python
vifdf = calculate_VIFs(
    "mpg ~ " + " + ".join(Auto_postos.columns) + " - mpg - displacement",␣
 ↪Auto_postos
)
vifdf
```

[35]:
```
                    VIF
Feature
cylinders      4.251590
horsepower     9.104343
weight         9.540921
acceleration   2.770794
```

```
year            1.182561
origin_Europe   1.278261
origin_Japan    1.512852
```

[36]:
```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

Linear Regression Analysis for post oil shock dropping feature displacement

[37]:
```
cols = list(Auto_postos.columns)
cols.remove("mpg")
cols.remove("displacement")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_postos)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.788
Model:                            OLS   Adj. R-squared:                  0.779
Method:                 Least Squares   F-statistic:                     90.11
Date:                Wed, 25 Sep 2024   Prob (F-statistic):           7.20e-54
Time:                        07:57:02   Log-Likelihood:                -114.64
No. Observations:                 178   AIC:                             245.3
Df Residuals:                     170   BIC:                             270.7
Df Model:                           7
Covariance Type:            nonrobust
================================================================================
=
                    coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
-
Intercept        -0.1072      0.051     -2.096      0.038      -0.208
-0.006
cylinders         0.1988      0.073      2.728      0.007       0.055
0.343
horsepower       -0.1879      0.107     -1.762      0.080      -0.398
0.023
weight           -0.7149      0.109     -6.550      0.000      -0.930
-0.499
acceleration      0.0713      0.059      1.212      0.227      -0.045
0.187
year              0.2148      0.038      5.589      0.000       0.139
0.291
origin_Europe     0.3461      0.111      3.108      0.002       0.126
0.566
origin_Japan      0.1946      0.097      2.012      0.046       0.004
0.385
```

```
==============================================================================
Omnibus:                        6.408   Durbin-Watson:                   1.583
Prob(Omnibus):                  0.041   Jarque-Bera (JB):                6.069
Skew:                           0.398   Prob(JB):                       0.0481
Kurtosis:                       3.431   Cond. No.                         7.71
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                   df     sum_sq    mean_sq           F        PR(>F)
cylinders         1.0  89.788355  89.788355  403.941459  8.824859e-47
horsepower        1.0  25.953062  25.953062  116.758098  4.752402e-21
weight            1.0  15.387223  15.387223   69.224316  2.748274e-14
acceleration      1.0   0.660414   0.660414    2.971082  8.658318e-02
year              1.0   6.087213   6.087213   27.385264  4.863030e-07
origin_Europe     1.0   1.436421   1.436421    6.462195  1.191261e-02
origin_Japan      1.0   0.899608   0.899608    4.047172  4.582475e-02
Residual        170.0  37.787704   0.222281         NaN           NaN
```

[38]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is acceleration with a
p-value of 0.22719512270297804 and a coefficient of 0.07129263347256862
Using the backward methodology, we suggest dropping acceleration from the new
model
```

[39]:
```
cols.remove("acceleration")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_postos)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.786
Model:                            OLS   Adj. R-squared:                  0.778
Method:                 Least Squares   F-statistic:                     104.6
Date:                Wed, 25 Sep 2024   Prob (F-statistic):           1.39e-54
Time:                        07:57:03   Log-Likelihood:                -115.40
No. Observations:                 178   AIC:                             244.8
Df Residuals:                     171   BIC:                             267.1
Df Model:                           6
Covariance Type:            nonrobust
================================================================================
=
                    coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
-
Intercept        -0.1148      0.051     -2.261      0.025      -0.215
```

```
-0.015
cylinders          0.1915      0.073      2.633      0.009       0.048
0.335
horsepower        -0.2864      0.069     -4.148      0.000      -0.423
-0.150
weight            -0.6311      0.085     -7.462      0.000      -0.798
-0.464
year               0.2149      0.038      5.584      0.000       0.139
0.291
origin_Europe      0.3689      0.110      3.355      0.001       0.152
0.586
origin_Japan       0.2096      0.096      2.183      0.030       0.020
0.399
==============================================================================
Omnibus:                        6.875   Durbin-Watson:                   1.555
Prob(Omnibus):                  0.032   Jarque-Bera (JB):                6.653
Skew:                           0.400   Prob(JB):                       0.0359
Kurtosis:                       3.507   Cond. No.                         6.16
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                  df     sum_sq    mean_sq           F        PR(>F)
cylinders        1.0  89.788355  89.788355  402.836748  8.041561e-47
horsepower       1.0  25.953062  25.953062  116.438785  4.931248e-21
weight           1.0  15.387223  15.387223   69.034999  2.864764e-14
year             1.0   6.001042   6.001042   26.923762  5.941221e-07
origin_Europe    1.0   1.693907   1.693907    7.599738  6.471569e-03
origin_Japan     1.0   1.062190   1.062190    4.765532  3.039795e-02
Residual       171.0  38.114221   0.222890         NaN           NaN
```

[40]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is origin_Japan with a
p-value of 0.030397952937226073 and a coefficient of 0.20964740213594493
Using the backward methodology, we suggest dropping origin_Japan from the new
model
```

- However, origin_Japan is one of three levels with origin_Europe significant. So we do not drop it from the model.
- We can check what will happen with dropping the Intercept with it also insignificant especially since we have standardized the variables.
- https://stats.stackexchange.com/questions/197923/difference-between-centered-and-uncentere

[41]:
```
postoilshock_model_intercept = results
formula = " + ".join(cols)
formula += " - 1"
```

```python
results = perform_analysis("mpg", formula, Auto_postos)
```

```
                                 OLS Regression Results
================================================================================
=======
Dep. Variable:                    mpg   R-squared (uncentered):
0.779
Model:                            OLS   Adj. R-squared (uncentered):
0.772
Method:                 Least Squares   F-statistic:
101.3
Date:                Wed, 25 Sep 2024   Prob (F-statistic):
8.07e-54
Time:                        07:57:03   Log-Likelihood:
-118.03
No. Observations:                 178   AIC:
248.1
Df Residuals:                     172   BIC:
267.1
Df Model:                           6
Covariance Type:            nonrobust
================================================================================
=
                    coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
-
cylinders         0.1892      0.074      2.572      0.011       0.044
0.334
horsepower       -0.2877      0.070     -4.117      0.000      -0.426
-0.150
weight           -0.6656      0.084     -7.905      0.000      -0.832
-0.499
year              0.2098      0.039      5.398      0.000       0.133
0.287
origin_Europe     0.2400      0.095      2.523      0.013       0.052
0.428
origin_Japan      0.0688      0.074      0.930      0.353      -0.077
0.215
==============================================================================
Omnibus:                        9.950   Durbin-Watson:                   1.526
Prob(Omnibus):                  0.007   Jarque-Bera (JB):               10.241
Skew:                           0.498   Prob(JB):                      0.00597
Kurtosis:                       3.622   Cond. No.                         5.06
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not
```

```
contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                  df     sum_sq    mean_sq           F        PR(>F)
cylinders        1.0  89.788355  89.788355  393.430947  2.582371e-46
horsepower       1.0  25.953062  25.953062  113.720066  1.057893e-20
weight           1.0  15.387223  15.387223   67.423107  5.008832e-14
year             1.0   6.001042   6.001042   26.295121  7.831453e-07
origin_Europe    1.0   1.419140   1.419140    6.218329  1.358813e-02
origin_Japan     1.0   0.197537   0.197537    0.865561  3.534910e-01
Residual       172.0  39.253641   0.228219         NaN           NaN
```

[42]:
```python
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is origin_Japan with a
p-value of 0.35349096552578385 and a coefficient of 0.06881297255714136
Using the backward methodology, we suggest dropping origin_Japan from the new
model
```

- We drop both origin_Europe and origin_Japan from the model.

[43]:
```python
cols.remove("origin_Europe")
cols.remove("origin_Japan")
formula = " + ".join(cols)
formula += " - 1"
results = perform_analysis("mpg", formula, Auto_postos)
```

```
                                 OLS Regression Results
================================================================================
=======
Dep. Variable:                    mpg   R-squared (uncentered):
0.770
Model:                            OLS   Adj. R-squared (uncentered):
0.765
Method:                 Least Squares   F-statistic:
146.0
Date:                Wed, 25 Sep 2024   Prob (F-statistic):
1.73e-54
Time:                        07:57:03   Log-Likelihood:
-121.62
No. Observations:                 178   AIC:
251.2
Df Residuals:                     174   BIC:
264.0
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
```

```
cylinders      0.1776      0.074      2.388      0.018      0.031      0.324
horsepower    -0.3084      0.070     -4.424      0.000     -0.446     -0.171
weight        -0.6688      0.082     -8.173      0.000     -0.830     -0.507
year           0.1974      0.039      5.055      0.000      0.120      0.274
==============================================================================
Omnibus:                       13.678   Durbin-Watson:                   1.582
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               14.628
Skew:                           0.630   Prob(JB):                     0.000666
Kurtosis:                       3.619   Cond. No.                         4.67
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not
contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
               df     sum_sq    mean_sq           F        PR(>F)
cylinders     1.0  89.788355  89.788355  382.262105  8.909529e-46
horsepower    1.0  25.953062  25.953062  110.491745  2.548773e-20
weight        1.0  15.387223  15.387223   65.509078  9.616073e-14
year          1.0   6.001042   6.001042   25.548647  1.085130e-06
Residual    174.0  40.870318   0.234887         NaN           NaN
```

[44]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is cylinders with a p-value
of 0.018006259822639592 and a coefficient of 0.17760733234778334
Using the backward methodology, we suggest dropping cylinders from the new model
```

[45]:
```
cols.remove("cylinders")
formula = " + ".join(cols)
formula += " - 1"
results = perform_analysis("mpg", formula, Auto_postos)
```

```
                                 OLS Regression Results
==============================================================================
=======
Dep. Variable:                    mpg   R-squared (uncentered):
0.763
Model:                            OLS   Adj. R-squared (uncentered):
0.759
Method:                 Least Squares   F-statistic:
187.7
Date:                Wed, 25 Sep 2024   Prob (F-statistic):
1.90e-54
Time:                        07:57:04   Log-Likelihood:
-124.49
No. Observations:                 178   AIC:
255.0
```

```
Df Residuals:                     175   BIC:
264.5
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
horsepower    -0.2653      0.068     -3.888      0.000      -0.400      -0.131
weight        -0.5548      0.067     -8.238      0.000      -0.688      -0.422
year           0.1889      0.039      4.793      0.000       0.111       0.267
==============================================================================
Omnibus:                       15.435   Durbin-Watson:                   1.592
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               16.821
Skew:                           0.690   Prob(JB):                     0.000223
Kurtosis:                       3.601   Cond. No.                         3.56
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not
contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
               df      sum_sq     mean_sq           F        PR(>F)
horsepower    1.0  112.958534  112.958534  468.320143  2.378397e-51
weight        1.0   17.289976   17.289976   71.683331  9.907388e-15
year          1.0    5.541596    5.541596   22.975165  3.490133e-06
Residual    175.0   42.209894    0.241199         NaN           NaN
```

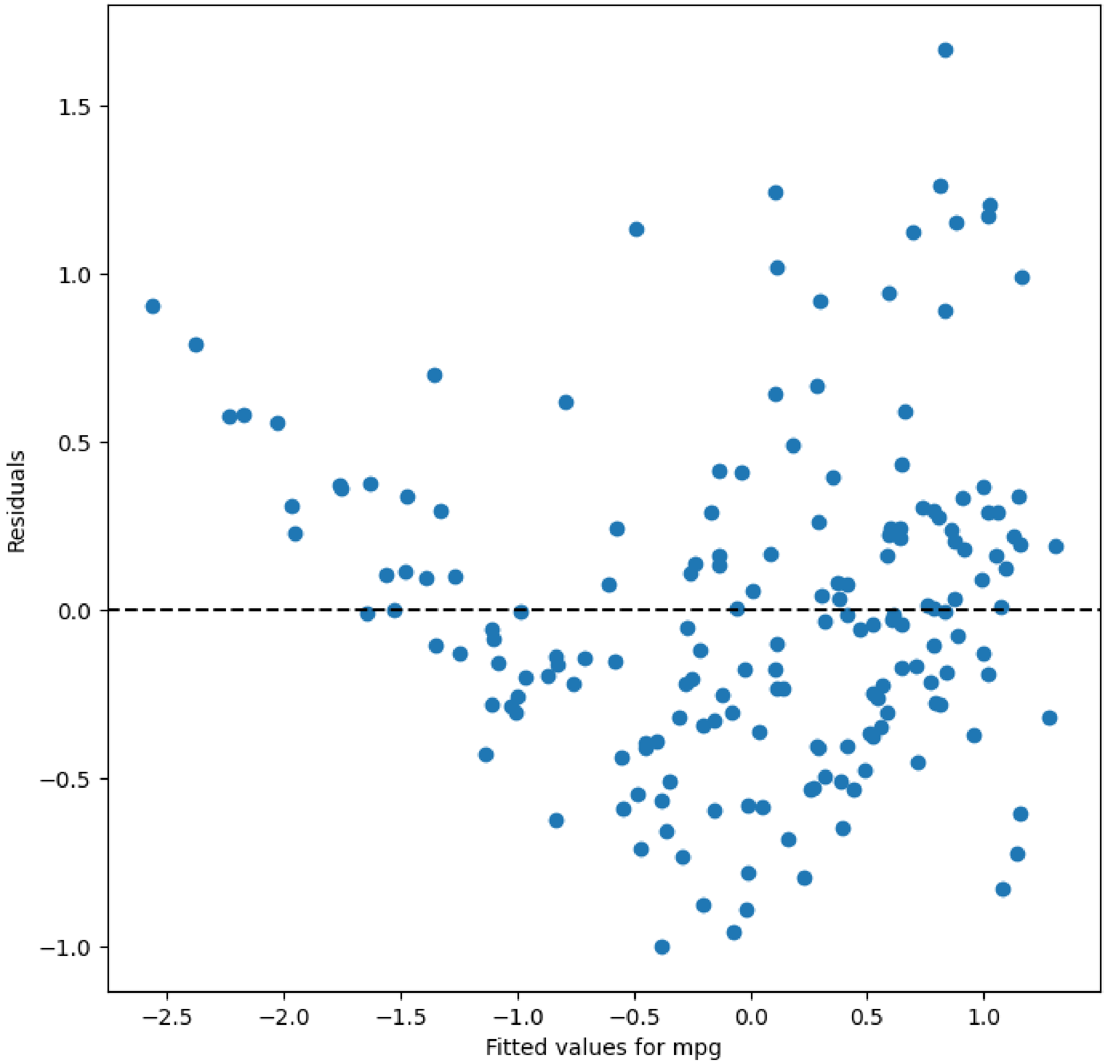
[46]:
```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
No variables are statistically insignificant.
The model mpg ~ horsepower + weight + year - 1 cannot be pruned further.
```

[47]:
```
postoilshock_model = results
```

Residual plot for model for post-oil shock

[48]:
```
display_residuals_plot(results)
```



1.2.3 Pre-oilshock model

[49]:
```
preoilshock_model.model.formula
```

[49]: 'mpg ~ weight + year + origin_Europe + origin_Japan'

1.2.4 Explanatory power of preoilshock model

[50]:
```
preoilshock_model.rsquared_adj
```

[50]: 0.8404849876892488

Post-oil shock model without intercept

[51]:
```
postoilshock_model.model.formula
```

[51]: 'mpg ~ horsepower + weight + year - 1'

1.2.5 Explanatory power of postoilshock model

[52]:
```
postoilshock_model.rsquared_adj
```

[52]: 0.7588006068263029

- Thus, we can conclude that prior to the oil shock of 1973, mileage was determined mostly by weight, year and origin.
- Post the oil shock of 1973, mileage was determined by horsepower, weight and year. Origin no longer played an important role as before.

1.2.6 Post oil shock model with intercept (Corollary)

[53]:
```
postoilshock_model_intercept.model.formula
```

[53]: 'mpg ~ cylinders + horsepower + weight + year + origin_Europe + origin_Japan'

1.2.7 Explanatory power of postoilshock model with intercept

[54]:
```
postoilshock_model_intercept.rsquared_adj
```

[54]: 0.7783620129852484

1.3 Finished

[55]:
```
allDone()
```

```
<IPython.lib.display.Audio object>
```