Multilinear Regression: CarSeats dataset

Import notebook funcs

```
from notebookfuncs import *
```

Import ISLP objects

```
from ISLP import load_data
```

Import User Funactions

```
from userfuncs import *
```

Load dataset

```
Carseats = load_data("Carseats")
Carseats.head()
```

	Sales	CompPrice	Income	Advertising	Population	Price	ShelveLoc	Age	Education	Urban	U
0	9.50	138	73	11	276	120	Bad	42	17	Yes	Y
1	11.22	111	48	16	260	83	Good	65	10	Yes	Y
2	10.06	113	35	10	269	80	Medium	59	12	Yes	Y
3	7.40	117	100	4	466	97	Medium	55	14	Yes	Y
4	4.15	141	64	3	340	128	Bad	38	13	Yes	N

```
Carseats.shape
```

```
(400, 11)
```

```
Carseats = Carseats.dropna()
Carseats.shape
```

```
(400, 11)
```

Display dataset stats

```
Carseats.describe()
```

	Sales	CompPrice	Income	Advertising	Population	Price	Age	Educati
count	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	400.0000
mean	7.496325	124.975000	68.657500	6.635000	264.840000	115.795000	53.322500	13.90000
std	2.824115	15.334512	27.986037	6.650364	147.376436	23.676664	16.200297	2.620528
min	0.000000	77.000000	21.000000	0.000000	10.000000	24.000000	25.000000	10.00000
25%	5.390000	115.000000	42.750000	0.000000	139.000000	100.000000	39.750000	12.00000
50%	7.490000	125.000000	69.000000	5.000000	272.000000	117.000000	54.500000	14.00000
75%	9.320000	135.000000	91.000000	12.000000	398.500000	131.000000	66.000000	16.00000
max	16.270000	175.000000	120.000000	29.000000	509.000000	191.000000	80.000000	18.00000

Set categorical types

```
Carseats["US"] = Carseats["US"].astype("category")
Carseats["Urban"] = Carseats["Urban"].astype("category")
```

Standardize variables

```
Carseats["Sales"] = standardize(Carseats["Sales"])
Carseats["Price"] = standardize(Carseats["Price"])
```

(a) Fit a multiple regression model to predict Sales using Price, Urban, and US.

```
formula = "Price + Urban + US"
perform_analysis("Sales", formula, Carseats)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.239
Model:                            OLS   Adj. R-squared:                  0.234
Method:                 Least Squares   F-statistic:                     41.52
Date:                Wed, 09 Oct 2024   Prob (F-statistic):           2.39e-23
Time:                        14:33:33   Log-Likelihood:                -512.88
No. Observations:                 400   AIC:                             1034.
Df Residuals:                     396   BIC:                             1050.
Df Model:                           3
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept       -0.2691      0.098     -2.734      0.007      -0.463      -0.076
Urban[T.Yes]    -0.0078      0.096     -0.081      0.936      -0.197       0.182
US[T.Yes]        0.4256      0.092      4.635      0.000       0.245       0.606
Price           -0.4566      0.044    -10.389      0.000      -0.543      -0.370
==============================================================================
Omnibus:                        0.676   Durbin-Watson:                   1.912
Prob(Omnibus):                  0.713   Jarque-Bera (JB):                0.758
Skew:                           0.093   Prob(JB):                        0.684
Kurtosis:                       2.897   Cond. No.                         4.27
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
             df      sum_sq    mean_sq           F        PR(>F)
Urban       1.0    0.095104   0.095104    0.123767  7.251713e-01
US          1.0   12.675983  12.675983   16.496407  5.877444e-05
Price       1.0   82.939070  82.939070  107.936143  1.609917e-22
Residual  396.0  304.289843   0.768409         NaN           NaN

<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x793d3f541bb0>
```

(b) Provide an interpretation of each coefficient in the model. Be careful—some of the variables in the model are qualitative!

- The coefficient of -0.0078 for Urban (True) indicates that -0.0078 of the SD of Sales can be explained by the level urban store as compared to a rural store. However, the p-value of 0.936 indicates that this difference is not significant and can be discounted or discarded.
- The coefficent of 0.4256 for US (True) indicates that 0.4256 of the typical deviation of Sales are explained by a US store as compared to a non-US store.
- The coefficient of -0.4566 for Price indicates that -0.4566 of the typical deviation of Sales is explained by one SD of change in the Price variable.
- We can also conclude that Price has the highest effect on Sales, the response variable, since the absolute value of its coefficient 0.4566 is the highest amongst all the coefficients.
- https://blogs.sas.com/content/iml/2023/07/17/standardize-reg-coeff-class.html
- https://www.statlect.com/fundamentals-of-statistics/linear-regression-with-standardized-variables

(c) Write out the model in equation form, being careful to handle the qualitative variables properly.

- The equation can be written out as follows:
- Sales = -0.0078 * Urban + 0.4256 * US -0.4566 * Price (Standardized) - 0.2691

(d) For which of the predictors can you reject the null hypothesis H0 : j = 0?

- The p-value for the Urban predictor is 0.936 which is much higher than our chosen level of significance 0.01. So we cannot reject the null Hypothesis in this case that its coefficient is zero.
- The p-values for US, Price and Intercept are zero. Hence, we reject the null hypothesis for them that their coefficients are zero.

(e) On the basis of your response to the previous question, fit a smaller model that only uses the predictors for which there is evidence of association with the outcome.

```
formula = "Price + US"
results = perform_analysis("Sales", formula, Carseats)
```

```
                            OLS Regression Results
==============================================================================
```

```
Dep. Variable:                  Sales   R-squared:                       0.239
Model:                            OLS   Adj. R-squared:                  0.235
Method:                 Least Squares   F-statistic:                     62.43
Date:                Wed, 09 Oct 2024   Prob (F-statistic):           2.66e-24
Time:                        14:33:33   Log-Likelihood:                -512.88
No. Observations:                 400   AIC:                             1032.
Df Residuals:                     397   BIC:                             1044.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -0.2743      0.074     -3.730      0.000      -0.419      -0.130
US[T.Yes]      0.4253      0.092      4.641      0.000       0.245       0.605
Price         -0.4567      0.044    -10.416      0.000      -0.543      -0.371
==============================================================================
Omnibus:                        0.666   Durbin-Watson:                   1.912
Prob(Omnibus):                  0.717   Jarque-Bera (JB):                0.749
Skew:                           0.092   Prob(JB):                        0.688
Kurtosis:                       2.895   Cond. No.                         3.12
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
             df      sum_sq    mean_sq           F        PR(>F)
US          1.0   12.544810  12.544810   16.366658  6.273751e-05
Price       1.0   83.160345  83.160345  108.495617  1.272157e-22
Residual  397.0  304.294845   0.766486         NaN           NaN

<statsmodels.regression.linear_model.RegressionResultsWrapper at 0x793d3f5b8c20>
```

(f) How well do the models in (a) and (e) fit the data?

- Model(a) has an explanatory value R2 adjusted value of 0.234
- Model(e) has an explanatory value R2 adjusted value of 0.235

(g) Using the model from (e), obtain 95 % confidence intervals for the coefficient(s).

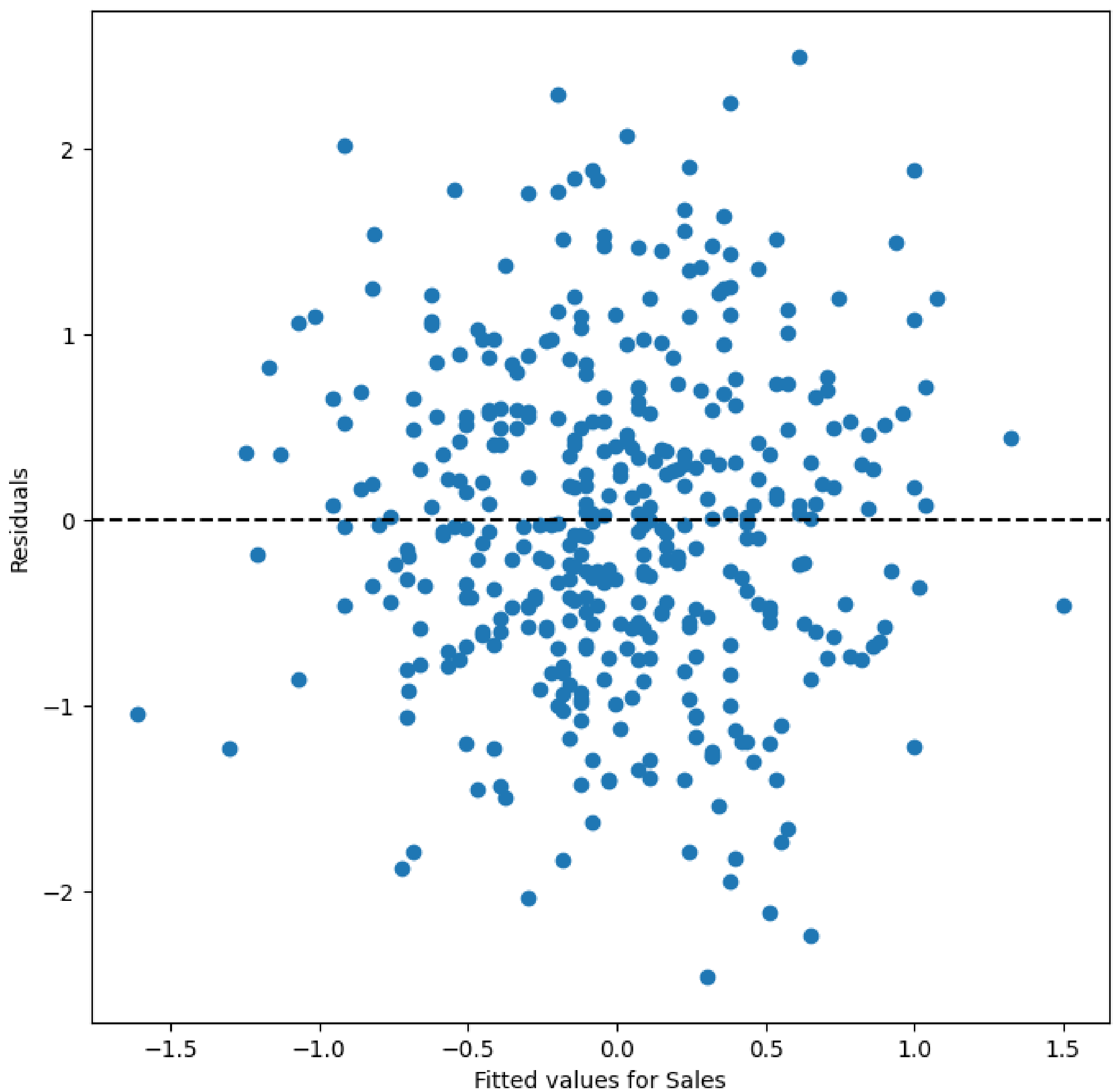
From the summary analysis, it can be seen that the 95% confidence limits for the three terms are as follows: + Intercept (-0.419, -0.130) + US[T.Yes] (0.245, 0.605) + Price (-0.543, -0.371)

+ None of them include zero in their range unlike that for Urban[T.Yes] in Model(a) which is another indicator that the coefficient is not significant.

(h) Is there evidence of outliers or high leverage observations in the model from (e)?

We can check for presence of outliers by plotting the residuals plot and seeing if there are any outliers.

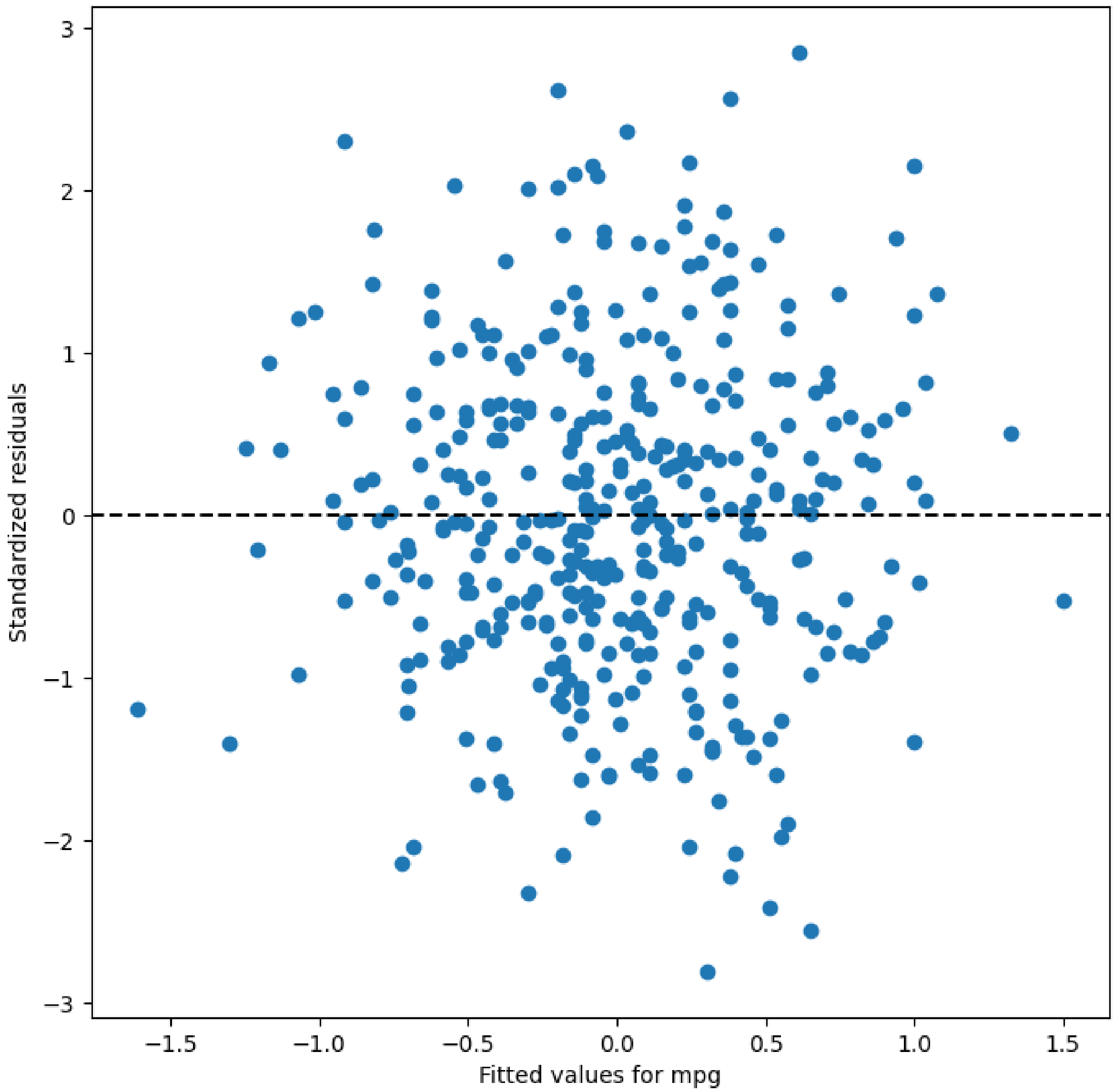
```
display_residuals_plot(results)
```



- From the plot above, there doesn't appear to be any obvious outliers.

We can plot studentized residuals to see whether there are any visible there.

```
display_studentized_residuals(results)
```



- From the above plot, no observation lies outside the (-3,3) range. Hence, we can safely conclude that there are no evident outliers in the dataset.

```
display_hat_leverage_cutoffs(results)
```

- We can see from the above graph that we have a few leverage points that exceed the cutoff of 3 * average leverage value. These are plotted in red.
- The ones in yellow exceed the less conservative estimate of 2 * average leverage value
- We could also use more conservative estimates for the cutoff of either 4 * average leverage value or 5 * average cutoff value

References: - https://online.stat.psu.edu/stat501/lesson/11/11.2

```
display_cooks_distance_plot(results)
```

```
display_DFFITS_plot(results)
```

```
display_hat_leverage_plot(results)
```

- Looking at all three studentized plots for leverage, it can be concluded that even if there are a few outliers, none wield a significant influence on the regression since the points with high leverage values have low studentized residual values.

```
inf_df, _ = get_influence_points(results)
inf_df
print(_)
```

For a more conservative cutoff values for hat_diag, we have the following infuence point(s):

```
inf_df[inf_df["hat_diag"] > (3 * np.mean(inf_df["hat_diag"]))]
```

Using DFFITS cutoff, we have the following influential points

```
inf_df[inf_df["dffits"] > 2 * np.sqrt(len(results.params) / results.nobs)]
```

Using Cooks Distance, we have the following influential points

```python
inf_df[inf_df["cooks_d"] > 1.0]
```

Using Cooks Distance p-values, we have the following influential points

```python
inf_df[inf_df["cooks_d_pvalue"] < 0.05]
```

Using DFBeta for intercept, we have the following influential points

```python
inf_df[inf_df["dfb_Intercept"] > (3 / np.sqrt(results.nobs))]
```

Using DFBeta for US, we have the following influential points

```python
inf_df[inf_df["dfb_US[T.Yes]"] > (3 / np.sqrt(results.nobs))]
```

Using DFBeta for Price, we have the following influential points

```python
inf_df[inf_df["dfb_Price"] > (3 / np.sqrt(results.nobs))]
```

```python
allDone()
```