

## Marketing Exercise

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## Import notebook functions

```
from notebookfuncs import *
```

## Import standard libraries

```
import numpy as np
import pandas as pd
from matplotlib.pyplot import subplots
```

## New imports

```
import statsmodels.api as sm
```

## Import statsmodel.objects

```
from statsmodels.stats.outliers_influence import variance_inflation_factor as VIF
from statsmodels.stats.outliers_influence import summary_table
from statsmodels.stats.anova import anova_lm
```

## Import ISLP objects

```
import ISLP
from ISLP import models
from ISLP import load_data
from ISLP.models import ModelSpec as MS, summarize, poly
```

Inspecting objects and namespaces

```
dir()
```

```
['Audio',
 'ISLP',
 'In',
 'InteractiveShell',
 'Latex',
 'MS',
 'Markdown',
 'Math',
 'Out',
 'VIF',
 '__builtin__',
 '__builtins__',
 '__name__',
 '__spec__',
 '_dh',
 '_i',
 '_i1',
 '_i2',
 '_i3',
 '_i4',
 '_i5',
 '_i6',
```

```
'_ih',
'_ii',
'_iii',
'_oh',
'allDone',
'anova_lm',
'display',
'exit',
'get_ipython',
'load_data',
'models',
'np',
'ojs_define',
'open',
'pd',
'poly',
'printlatex',
'printmd',
'quit',
'sm',
'subplots',
'summarize',
'summary_table']
```

```
Advertising = pd.read_csv("Advertising.csv")
# Drop first column
Advertising = Advertising.iloc[:, 1:]
Advertising.head()
```

	TV	Radio	Newspaper	Sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	9.3
3	151.5	41.3	58.5	18.5
4	180.8	10.8	58.4	12.9

```
Advertising.describe()
```

	TV	Radio	Newspaper	Sales
count	200.000000	200.000000	200.000000	200.000000
mean	147.042500	23.264000	30.554000	14.022500
std	85.854236	14.846809	21.778621	5.217457
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	10.375000

	TV	Radio	Newspaper	Sales
50%	149.750000	22.900000	25.750000	12.900000
75%	218.825000	36.525000	45.100000	17.400000
max	296.400000	49.600000	114.000000	27.000000

Is there a relationship between sales and advertising budget?

```
y = Advertising["Sales"]
cols = list(Advertising.columns)
cols.remove("Sales")
X = MS(cols).fit_transform(Advertising)
model = sm.OLS(y, X)
results = model.fit()
print("F-value", results.fvalue)
print("F-pvalue", results.f_pvalue)
summarize(results)
```

F-value 570.2707036590944

F-pvalue 1.575227256092416e-96

	coef	std err	t	P> t
intercept	2.9389	0.312	9.422	0.00
TV	0.0458	0.001	32.809	0.00
Radio	0.1885	0.009	21.893	0.00
Newspaper	-0.0010	0.006	-0.177	0.86

```
dir(models)
```

```
['Column',
 'Feature',
 'FeatureSelector',
 'ModelSpec',
 'Stepwise',
 'StringIO',
 '__builtins__',
 '__cached__',
 '__doc__',
 '__file__',
 '__loader__',
 '__name__',
 '__package__',
 '__path__',
```

```

'__spec__',
'bs',
'build_columns',
'columns',
'contrast',
'derived_feature',
'generic_selector',
'min_max_strategy',
'model_spec',
'np',
'ns',
'pca',
'pd',
'poly',
'sklearn_selected',
'sklearn_selection_path',
'sklearn_sm',
'sklearn_wrap',
'strategy',
'summarize']

```

- The p-value corresponding to the F-statistic is very low. Thus, clear evidence of a relationship between sales and advertising budget.

```
dir(results)
```

```

['HCO_se',
'HC1_se',
'HC2_se',
'HC3_se',
'_HCCM',
'__class__',
'__delattr__',
'__dict__',
'__dir__',
'__doc__',
'__eq__',
'__format__',
'__ge__',
'__getattribute__',
'__getstate__',
'__gt__',
'__hash__',
'__init__',
'__init_subclass__',
'__le__',
'__lt__',

```

```

'__module__',
'__ne__',
'__new__',
'__reduce__',
'__reduce_ex__',
'__repr__',
'__setattr__',
'__sizeof__',
'__str__',
'__subclasshook__',
'__weakref__',
'_abat_diagonal',
'_cache',
'_data_attr',
'_data_in_cache',
'_get_robustcov_results',
'_get_wald_nonlinear',
'_is_nested',
'_transform_predict_exog',
'_use_t',
'_wexog_singular_values',
'aic',
'bic',
'bse',
'centered_tss',
'compare_f_test',
'compare_lm_test',
'compare_lr_test',
'condition_number',
'conf_int',
'conf_int_el',
'cov_HC0',
'cov_HC1',
'cov_HC2',
'cov_HC3',
'cov_kwds',
'cov_params',
'cov_type',
'df_model',
'df_resid',
'diagn',
'eigenvals',
'el_test',
'ess',
'f_pvalue',
'f_test',

```

```
'fittedvalues',  
'fvalue',  
'get_influence',  
'get_prediction',  
'get_robustcov_results',  
'info_criteria',  
'initialize',  
'k_constant',  
'llf',  
'load',  
'model',  
'mse_model',  
'mse_resid',  
'mse_total',  
'nobs',  
'normalized_cov_params',  
'outlier_test',  
'params',  
'predict',  
'pvalues',  
'remove_data',  
'resid',  
'resid_pearson',  
'rsquared',  
'rsquared_adj',  
'save',  
'scale',  
'ssr',  
'summary',  
'summary2',  
't_test',  
't_test_pairwise',  
'tvalues',  
'uncentered_tss',  
'use_t',  
'wald_test',  
'wald_test_terms',  
'wresid']
```

## How strong is the relationship?

```
results.summary()
```



<b>Dep. Variable:</b>	Sales	<b>R-squared:</b>	0.897
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.896
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	570.3
<b>Date:</b>	Tue, 25 Feb 2025	<b>Prob (F-statistic):</b>	1.58e-96
<b>Time:</b>	14:39:30	<b>Log-Likelihood:</b>	-386.18
<b>No. Observations:</b>	200	<b>AIC:</b>	780.4
<b>Df Residuals:</b>	196	<b>BIC:</b>	793.6
<b>Df Model:</b>	3		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P>  t	[0.025	0.975]
<b>intercept</b>	2.9389	0.312	9.422	0.000	2.324	3.554
<b>TV</b>	0.0458	0.001	32.809	0.000	0.043	0.049
<b>Radio</b>	0.1885	0.009	21.893	0.000	0.172	0.206
<b>Newspaper</b>	-0.0010	0.006	-0.177	0.860	-0.013	0.011

<b>Omnibus:</b>	60.414	<b>Durbin-Watson:</b>	2.084
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	151.241
<b>Skew:</b>	-1.327	<b>Prob(JB):</b>	1.44e-33
<b>Kurtosis:</b>	6.332	<b>Cond. No.</b>	454.

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
y.mean()
```

```
14.0225
```

```
results.resid.std()
```

```
1.6727572743844117
```

```
(results.resid.std() / y.mean()) * 100
```

```
11.929094486606608
```

- The residual standard error (RSE) is 1.67 and the mean value of the response is 14.023 which translates to a percentage error of roughly 11.93%

```
("R-squared", results.rsquared, "Adjusted R-squared", results.rsquared_adj)
```

```
('R-squared', 0.8972106381789522, 'Adjusted R-squared', 0.8956373316204668)
```

- The R2 explains about 90% of the variance in Sales.

## Which media are associated with Sales?

- The low p-values for Radio and TV suggest that only they are related to Sales.

## How large is the association between each medium and sales?

```
results.conf_int(alpha=0.05)
```

	0	1
intercept	2.323762	3.554016
TV	0.043014	0.048516
Radio	0.171547	0.205513
Newspaper	-0.012616	0.010541

- The confidence intervals for TV and Radio are narrow and far from zero. This provides evidence that these media are related to sales.
- The interval for Newspaper includes zero indicating that it is not statistically significant given values of TV and Radio.

```
vals = [VIF(X, i) for i in range(1, X.shape[1])]
print(vals)
```

```
[1.00461078493965, 1.1449519171055353, 1.1451873787239288]
```

- The VIF scores are 1.005, 1.145 and 1.145 respectively for TV, radio and newspaper. These suggest no evidence of collinearity as an explanation for wide standard errors for newspaper.
- In order to assess the association of each medium individually on sales, we can perform three separate linear regressions.

```
TV = MS(["TV"]).fit_transform(Advertising)
model = sm.OLS(y, TV)
results = model.fit()
print(summarize(results))
Radio = MS(["Radio"]).fit_transform(Advertising)
model = sm.OLS(y, Radio)
results = model.fit()
print(summarize(results))
Newspaper = MS(["Newspaper"]).fit_transform(Advertising)
model = sm.OLS(y, Newspaper)
results = model.fit()
print(summarize(results))
```

	coef	std err	t	P> t
intercept	7.0326	0.458	15.360	0.0
TV	0.0475	0.003	17.668	0.0

	coef	std err	t	P> t
intercept	9.3116	0.563	16.542	0.0
Radio	0.2025	0.020	9.921	0.0

	coef	std err	t	P> t
--	------	---------	---	------

```

intercept  12.3514    0.621  19.876  0.000
Newspaper   0.0547    0.017   3.300  0.001

```

Looking at the p-values, there is evidence of a strong association b/w TV and sales and radio and sales. There is evidence of a mild association between Newspaper and sales when TV and radio are ignored.

## How accurately can we predict future sales?

- Given that \$100,000 is spent on TV advertising, and \$20,000 is spent on Radio advertising, we need to compute the 95% Confidence intervals for each city (i.e., the mean) and the prediction interval for a particular city (also at 95% confidence intervals).

## Fit the regression dropping the Newspaper column as insignificant

```

y = Advertising["Sales"]
cols = list(Advertising.columns)
cols.remove("Sales")
cols.remove("Newspaper")
X = MS(cols).fit_transform(Advertising)
model = sm.OLS(y, X)
results = model.fit()
print("F-value", results.fvalue)
print("F-pvalue", results.f_pvalue)
summarize(results)

```

F-value 859.6177183058211

F-pvalue 4.8273618513354486e-98

	coef	std err	t	P> t
intercept	2.9211	0.294	9.919	0.0
TV	0.0458	0.001	32.909	0.0
Radio	0.1880	0.008	23.382	0.0

```
results.summary()
```

<b>Dep. Variable:</b>	Sales	<b>R-squared:</b>	0.897
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.896
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	859.6
<b>Date:</b>	Tue, 25 Feb 2025	<b>Prob (F-statistic):</b>	4.83e-98
<b>Time:</b>	14:39:30	<b>Log-Likelihood:</b>	-386.20
<b>No. Observations:</b>	200	<b>AIC:</b>	778.4
<b>Df Residuals:</b>	197	<b>BIC:</b>	788.3
<b>Df Model:</b>	2		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
<b>intercept</b>	2.9211	0.294	9.919	0.000	2.340	3.502
<b>TV</b>	0.0458	0.001	32.909	0.000	0.043	0.048
<b>Radio</b>	0.1880	0.008	23.382	0.000	0.172	0.204
<b>Omnibus:</b>	60.022		<b>Durbin-Watson:</b>		2.081	
<b>Prob(Omnibus):</b>	0.000		<b>Jarque-Bera (JB):</b>		148.679	
<b>Skew:</b>	-1.323		<b>Prob(JB):</b>		5.19e-33	
<b>Kurtosis:</b>	6.292		<b>Cond. No.</b>		425.	

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
design = MS(["TV", "Radio"])
new_df = pd.DataFrame({"TV": [100], "Radio": [20]})
print(new_df)
new_X = design.fit_transform(new_df)
new_predictions = results.get_prediction(new_X)
new_predictions.predicted_mean
```

```
      TV  Radio
0  100    20
array([11.25646595])
```

We predict the confidence intervals at 95% as follows:

```
new_predictions.conf_int(alpha=0.05)

array([[10.98525445, 11.52767746]])
```

We predict the prediction interval for a particular city as follows:

```
new_predictions.conf_int(alpha=0.05, obs=True)

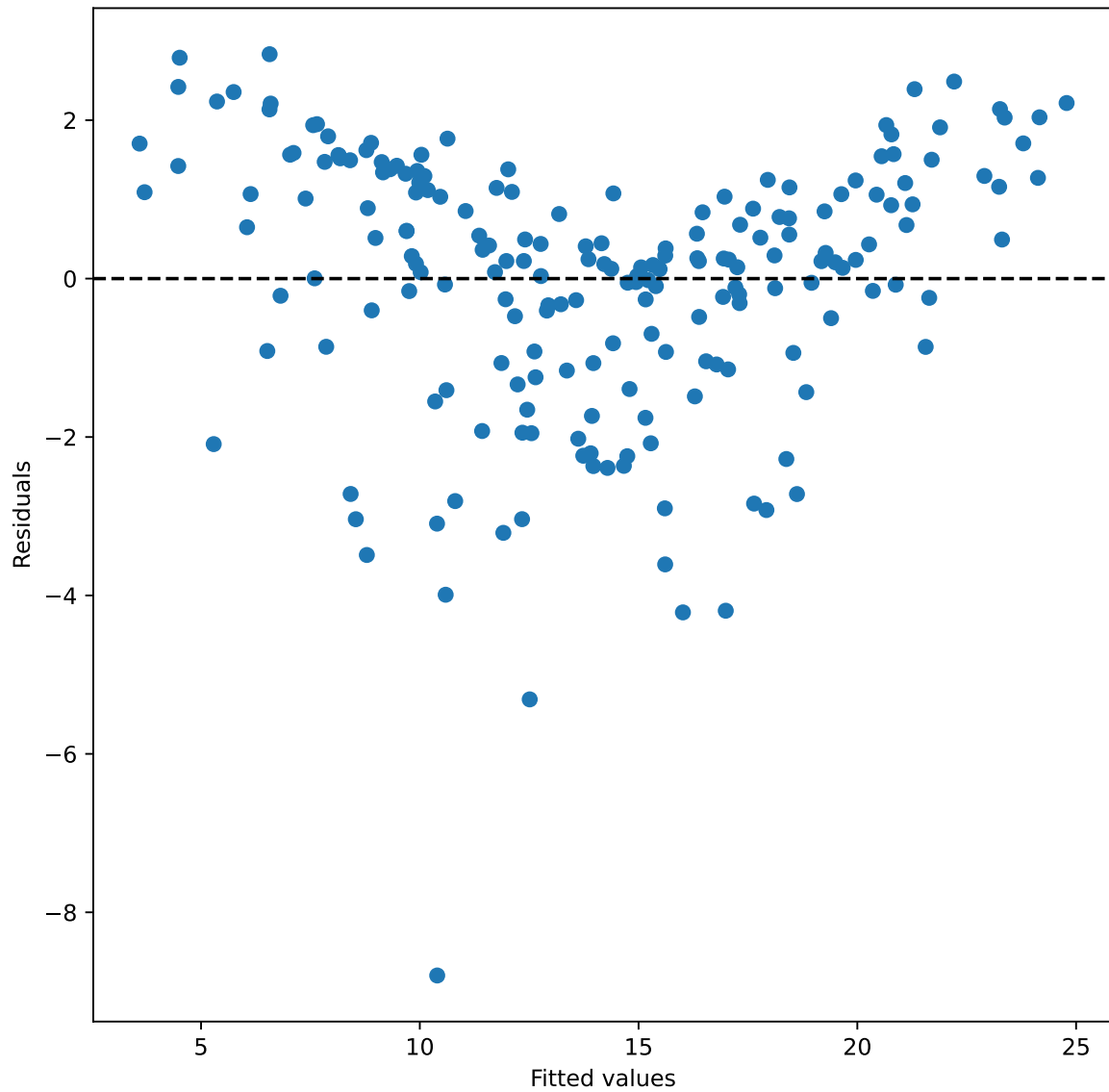
array([[ 7.92961607, 14.58331584]])
```

- Both intervals are centered at 11,256 but the prediction intervals are wider reflecting the additional uncertainty around sales for a particular city as against the average sales for many locations.

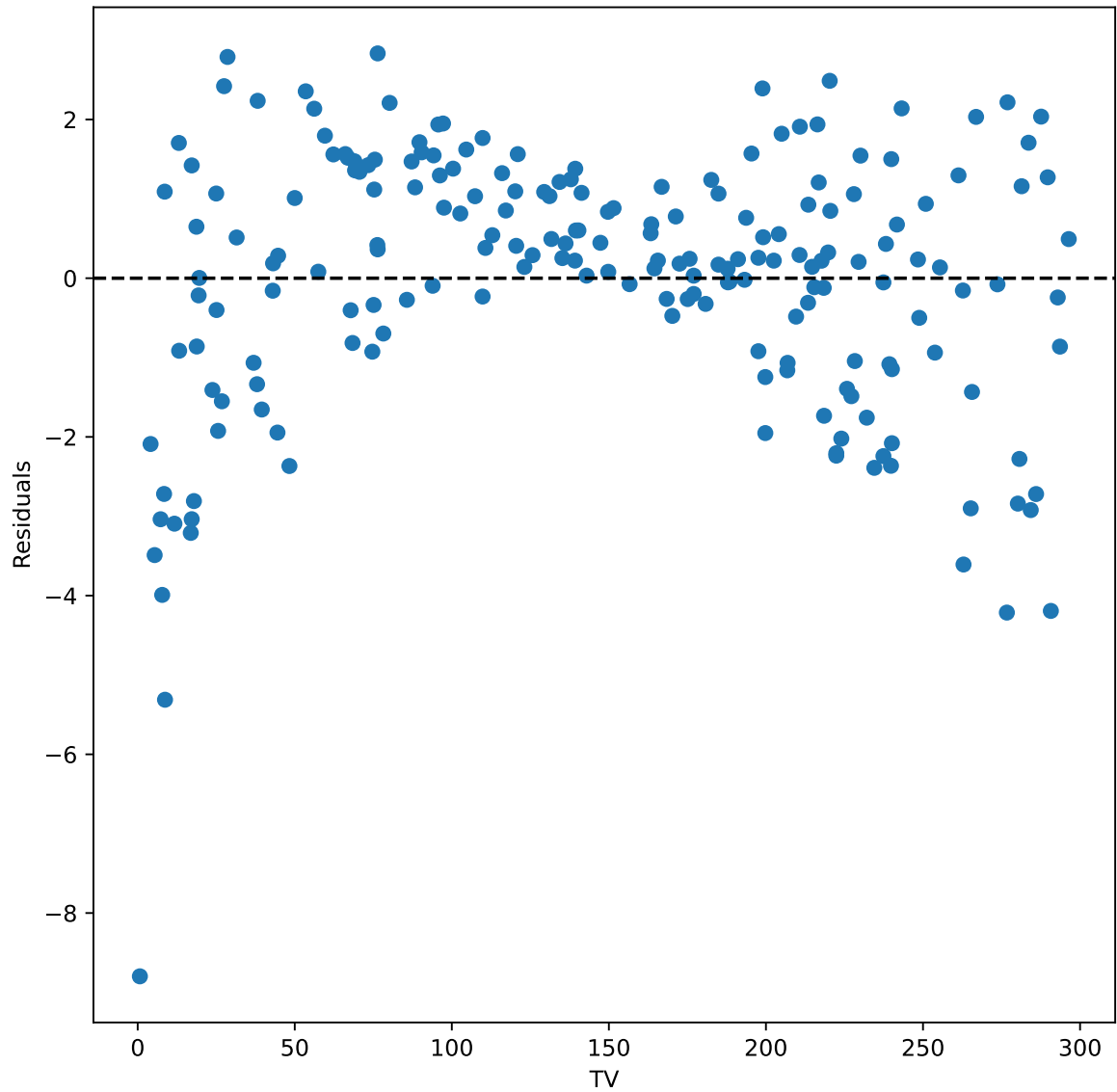
Is the relationship linear?

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(results.fittedvalues, results.resid)
ax.set_xlabel("Fitted values")
```

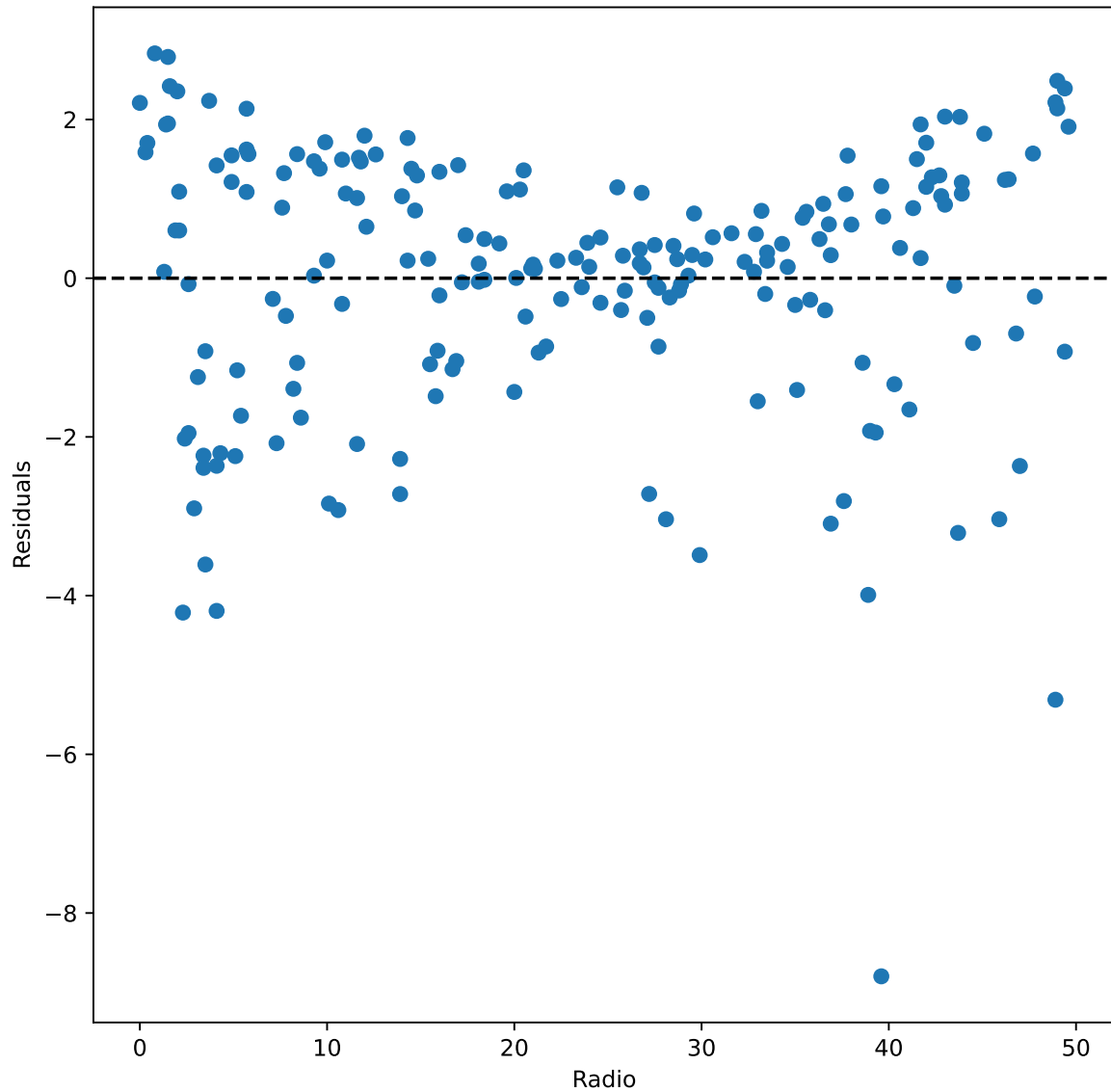
```
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



```
_, ax = subplots(figsize=(8, 8))
ax.scatter(Advertising["TV"], results.resid)
ax.set_xlabel("TV")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



```
_, ax = subplots(figsize=(8, 8))
ax.scatter(Advertising["Radio"], results.resid)
ax.set_xlabel("Radio")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



- There is evidence of non-linearity in the model from the residuals plotted against the fitted values. Looking at the residuals versus predictors plots, it appears that TV is a better candidate for quadratification.

```
X = MS([poly("TV", degree=2, raw=True), "Radio"]).fit_transform(Advertising)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P> t
intercept	1.2876	0.359000	3.588	0.0
poly(TV, degree=2, raw=True)[0]	0.0784	0.005000	15.736	0.0
poly(TV, degree=2, raw=True)[1]	-0.0001	0.000017	-6.775	0.0
Radio	0.1930	0.007000	26.465	0.0

```
results.summary()
```

<b>Dep. Variable:</b>	Sales	<b>R-squared:</b>	0.917
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.915
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	719.0
<b>Date:</b>	Tue, 25 Feb 2025	<b>Prob (F-statistic):</b>	1.80e-105
<b>Time:</b>	14:39:31	<b>Log-Likelihood:</b>	-365.16
<b>No. Observations:</b>	200	<b>AIC:</b>	738.3
<b>Df Residuals:</b>	196	<b>BIC:</b>	751.5
<b>Df Model:</b>	3		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P>  t	[0.025	0.975]
intercept	1.2876	0.359	3.588	0.000	0.580	1.995
poly(TV, degree=2, raw=True)[0]	0.0784	0.005	15.736	0.000	0.069	0.088
poly(TV, degree=2, raw=True)[1]	-0.0001	1.68e-05	-6.775	0.000	-0.000	-8.05e-05
Radio	0.1930	0.007	26.465	0.000	0.179	0.207

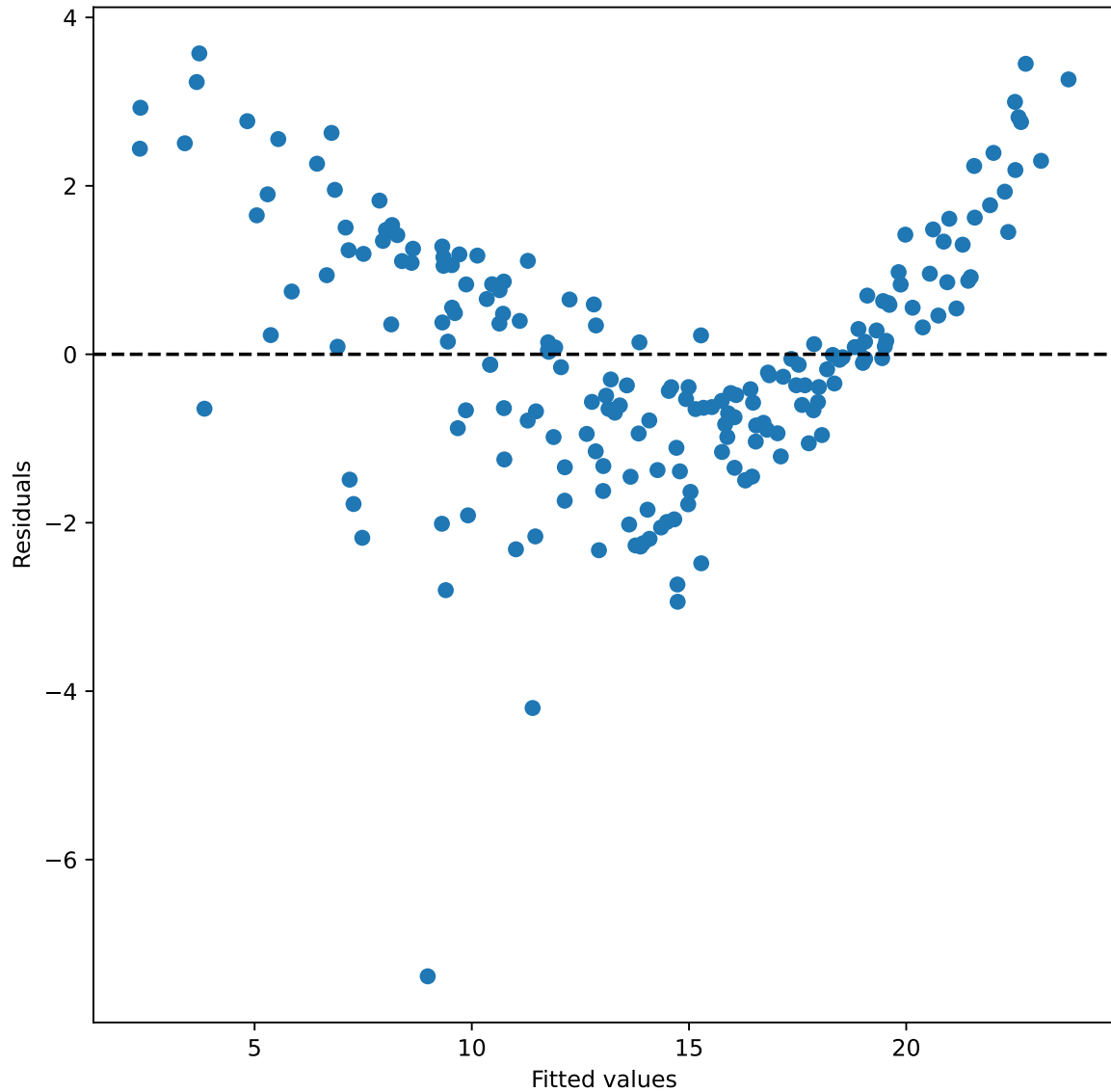
<b>Omnibus:</b>	19.524	<b>Durbin-Watson:</b>	2.136
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	44.712
<b>Skew:</b>	-0.413	<b>Prob(JB):</b>	1.95e-10
<b>Kurtosis:</b>	5.164	<b>Cond. No.</b>	1.29e+05

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.  
[2] The condition number is large, 1.29e+05. This might indicate that there are strong multicollinearity or other numerical problems.

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(results.fittedvalues, results.resid)
ax.set_xlabel("Fitted values")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```





While the fit has improved as seen from the  $R^2$  increasing by 2 percentage points, there is still some non-linearity visible in the residuals plot against fitted values.

References:

[https://www.kellogg.northwestern.edu/faculty/weber/emp/\\_session\\_3/nonlinearities.htm](https://www.kellogg.northwestern.edu/faculty/weber/emp/_session_3/nonlinearities.htm)

<https://online.stat.psu.edu/stat462/node/120/>

## Is there synergy among the advertising media?

Synergy implies an interaction effect. That's what we test out now.

```
X = MS([poly("TV", raw=True, degree=2), "Radio", ("TV", "Radio"))].fit_transform(
    Advertising
)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P> t
intercept	5.1371	0.193000	26.663	0.0
poly(TV, degree=2, raw=True)[0]	0.0509	0.002000	22.810	0.0
poly(TV, degree=2, raw=True)[1]	-0.0001	0.000007	-15.920	0.0
Radio	0.0352	0.006000	5.959	0.0
TV:Radio	0.0011	0.000035	31.061	0.0

```
results.summary()
```

<b>Dep. Variable:</b>	Sales	<b>R-squared:</b>	0.986
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.986
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	3432.
<b>Date:</b>	Tue, 25 Feb 2025	<b>Prob (F-statistic):</b>	1.79e-179
<b>Time:</b>	14:39:32	<b>Log-Likelihood:</b>	-186.86
<b>No. Observations:</b>	200	<b>AIC:</b>	383.7
<b>Df Residuals:</b>	195	<b>BIC:</b>	400.2
<b>Df Model:</b>	4		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
intercept	5.1371	0.193	26.663	0.000	4.757	5.517
poly(TV, degree=2, raw=True)[0]	0.0509	0.002	22.810	0.000	0.047	0.055
poly(TV, degree=2, raw=True)[1]	-0.0001	6.89e-06	-15.920	0.000	-0.000	-9.61e-05
Radio	0.0352	0.006	5.959	0.000	0.024	0.047
TV:Radio	0.0011	3.47e-05	31.061	0.000	0.001	0.001

<b>Omnibus:</b>	169.759	<b>Durbin-Watson:</b>	2.204
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	4031.167
<b>Skew:</b>	-2.988	<b>Prob(JB):</b>	0.00
<b>Kurtosis:</b>	24.166	<b>Cond. No.</b>	1.70e+05

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.7e+05. This might indicate that there are strong multicollinearity or other numerical problems.

- Finally, when we add an interaction term TV \* Radio to the model, we can see that the residual fit exhibits no pattern. And the R2 is 98.6%.

## Compute VIFs and List Comprehension

```
vals = [VIF(X, i) for i in range(1, X.shape[1])]
print(vals)
```

```
[18.787830609925035, 15.885268501061871, 3.9253174186837008, 6.940088238444382]
```

```
vif = pd.DataFrame({"vif": vals}, index=X.columns[1:])
print(vif)
("VIF Range:", np.min(vif), np.max(vif))
```

```

                                vif
poly(TV, degree=2, raw=True)[0]  18.787831
poly(TV, degree=2, raw=True)[1]  15.885269
Radio                          3.925317
TV:Radio                       6.940088
```

```
('VIF Range:', 3.9253174186837008, 18.787830609925035)
```

- The VIF ranges are high. These can be reduced by transforming variables to mean 0.
- <https://stats.stackexchange.com/questions/23538/quadratic-term-and-variance-inflation-factor-in-ols-estimation>

```
Advertising["TV"] = Advertising["TV"] - Advertising["TV"].mean()
Advertising["Radio"] = Advertising["Radio"] - Advertising["Radio"].mean()
```

```
X = MS([poly("TV", raw=True, degree=2), "Radio", ("TV", "Radio")]).fit_transform(
    Advertising
)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P> t
intercept	14.7525	0.067000	219.634	0.0
poly(TV, degree=2, raw=True)[0]	0.0437	0.001000	84.111	0.0
poly(TV, degree=2, raw=True)[1]	-0.0001	0.000007	-15.920	0.0
Radio	0.1935	0.003000	64.526	0.0
TV:Radio	0.0011	0.000035	31.061	0.0

```
results.summary()
```

<b>Dep. Variable:</b>	Sales	<b>R-squared:</b>	0.986
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.986
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	3432.
<b>Date:</b>	Tue, 25 Feb 2025	<b>Prob (F-statistic):</b>	1.79e-179
<b>Time:</b>	14:39:32	<b>Log-Likelihood:</b>	-186.86
<b>No. Observations:</b>	200	<b>AIC:</b>	383.7
<b>Df Residuals:</b>	195	<b>BIC:</b>	400.2
<b>Df Model:</b>	4		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P>  t	[0.025	0.975]
intercept	14.7525	0.067	219.634	0.000	14.620	14.885
poly(TV, degree=2, raw=True)[0]	0.0437	0.001	84.111	0.000	0.043	0.045
poly(TV, degree=2, raw=True)[1]	-0.0001	6.89e-06	-15.920	0.000	-0.000	-9.61e-05
Radio	0.1935	0.003	64.526	0.000	0.188	0.199
TV:Radio	0.0011	3.47e-05	31.061	0.000	0.001	0.001

<b>Omnibus:</b>	169.759	<b>Durbin-Watson:</b>	2.204
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	4031.167
<b>Skew:</b>	-2.988	<b>Prob(JB):</b>	0.00
<b>Kurtosis:</b>	24.166	<b>Cond. No.</b>	1.49e+04

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.49e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```
vals = [VIF(X, i) for i in range(1, X.shape[1])]
print(vals)
```

```
[1.0172717815970211, 1.017084612216564, 1.013513326764562, 1.0075840215785734]
```

```
vif = pd.DataFrame({"vif": vals}, index=X.columns[1:])
print(vif)
("VIF Range:", np.min(vif), np.max(vif))
```

```

              vif
poly(TV, degree=2, raw=True)[0]  1.017272
poly(TV, degree=2, raw=True)[1]  1.017085
Radio                          1.013513
TV:Radio                       1.007584
```

```
('VIF Range:', 1.0075840215785734, 1.0172717815970211)
```

```
allDone()
```

```
<IPython.lib.display.Audio object>
```