Multilinear Regression: Auto dataset

Import notebook functions

```
from notebookfuncs import *
```

Import standard libraries

```
import numpy as np
import pandas as pd

pd.set_option("display.max_rows", 1000)
pd.set_option("display.max_columns", 1000)
pd.set_option("display.width", 1000)
pd.set_option("display.max.colwidth", None)
import matplotlib.pyplot as plt
from matplotlib.pyplot import subplots
import seaborn as sns
import itertools
```

New imports

```
import statsmodels.api as sm
```

Import statsmodels.objects

```
from statsmodels.stats.outliers_influence import variance_inflation_factor as
↪ VIF
from statsmodels.stats.outliers_influence import summary_table
from statsmodels.stats.anova import anova_lm
import statsmodels.formula.api as smf
```

Import ISLP objects

```
import ISLP
from ISLP import models
from ISLP import load_data
from ISLP.models import ModelSpec as MS, summarize, poly
```

Import user functions

```
from userfuncs import *
```

Set level of significance (alpha)

```
LOS_Alpha = 0.01
```

```
0.01
```

```
Auto = load_data("Auto")
Auto = Auto.sort_values(by=["year"], ascending=True)
Auto.head()
Auto.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year', 'o
```

```
Auto.shape
```

```
(392, 8)
```

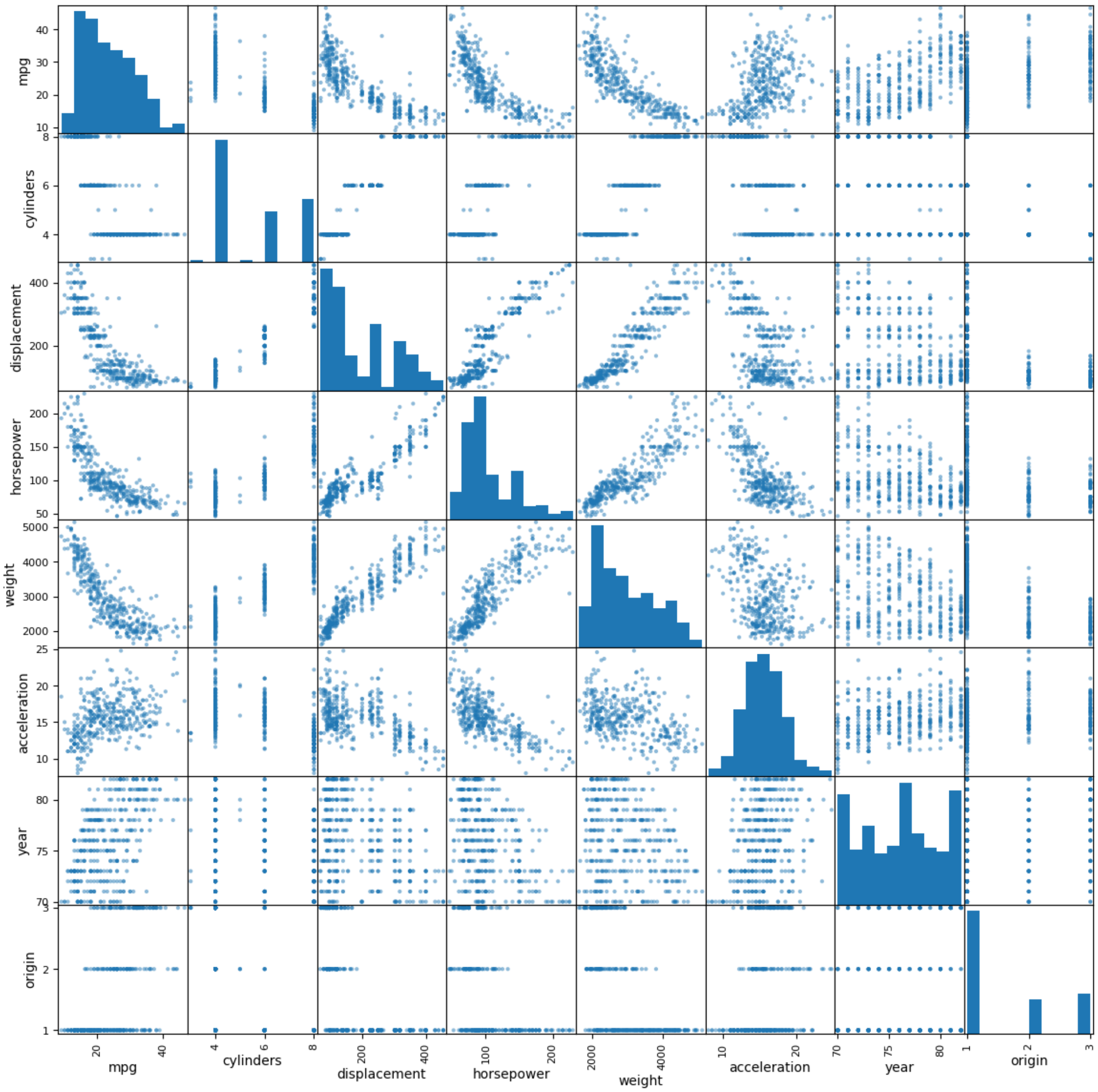
```
Auto.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origi
count	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000	392.0
mean	23.445918	5.471939	194.411990	104.469388	2977.584184	15.541327	75.979592	1.576
std	7.805007	1.705783	104.644004	38.491160	849.402560	2.758864	3.683737	0.805
min	9.000000	3.000000	68.000000	46.000000	1613.000000	8.000000	70.000000	1.000
25%	17.000000	4.000000	105.000000	75.000000	2225.250000	13.775000	73.000000	1.000
50%	22.750000	4.000000	151.000000	93.500000	2803.500000	15.500000	76.000000	1.000
75%	29.000000	8.000000	275.750000	126.000000	3614.750000	17.025000	79.000000	2.000
max	46.600000	8.000000	455.000000	230.000000	5140.000000	24.800000	82.000000	3.000

9. This question involves the use of multiple linear regression on the Auto data set.

(a) Produce a scatterplot matrix which includes all of the variables in the data set.

```
pd.plotting.scatter_matrix(Auto, figsize=(14, 14))
```



(b) Compute the matrix of correlations between the variables using the DataFrame.corr() method.

```
Auto.corr()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origi
mpg	1.000000	-0.777618	-0.805127	-0.778427	-0.832244	0.423329	0.580541	0.565
cylinders	-0.777618	1.000000	0.950823	0.842983	0.897527	-0.504683	-0.345647	-0.56
displacement	-0.805127	0.950823	1.000000	0.897257	0.932994	-0.543800	-0.369855	-0.61
horsepower	-0.778427	0.842983	0.897257	1.000000	0.864538	-0.689196	-0.416361	-0.45
weight	-0.832244	0.897527	0.932994	0.864538	1.000000	-0.416839	-0.309120	-0.58
acceleration	0.423329	-0.504683	-0.543800	-0.689196	-0.416839	1.000000	0.290316	0.21
year	0.580541	-0.345647	-0.369855	-0.416361	-0.309120	0.290316	1.000000	0.18
origin	0.565209	-0.568932	-0.614535	-0.455171	-0.585005	0.212746	0.181528	1.00

(c) Use the sm.OLS() function to perform a multiple linear regression with mpg as the response and all other variables except name as the predictors. Use the summarize() function to print the results. Comment on the output. For instance:

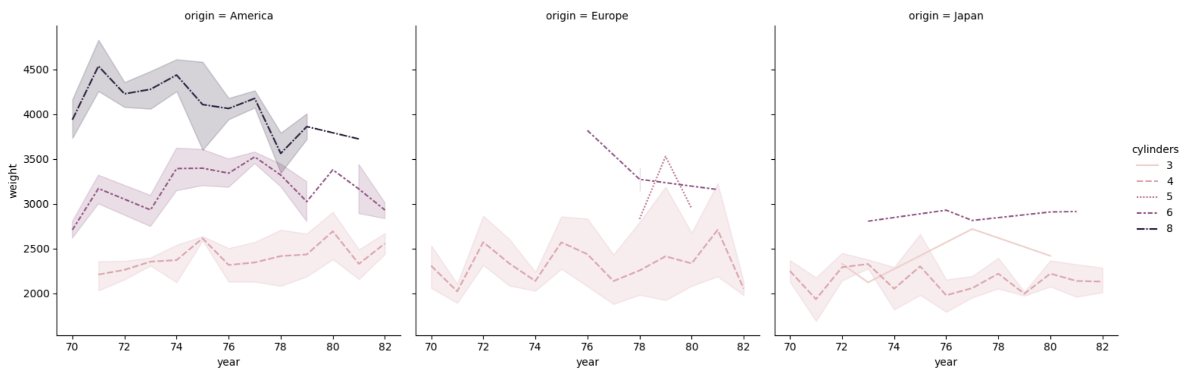
Convert year and origin columns to categorical types

```
Auto["origin"] = Auto["origin"].astype("category")
Auto["origin"] = Auto["origin"].cat.rename_categories(
    {1: "America", 2: "Europe", 3: "Japan"}
)
Auto["year"] = Auto["year"].astype("category")
Auto.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration
count	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000
mean	23.445918	5.471939	194.411990	104.469388	2977.584184	15.541327
std	7.805007	1.705783	104.644004	38.491160	849.402560	2.758864
min	9.000000	3.000000	68.000000	46.000000	1613.000000	8.000000
25%	17.000000	4.000000	105.000000	75.000000	2225.250000	13.775000
50%	22.750000	4.000000	151.000000	93.500000	2803.500000	15.500000
75%	29.000000	8.000000	275.750000	126.000000	3614.750000	17.025000
max	46.600000	8.000000	455.000000	230.000000	5140.000000	24.800000

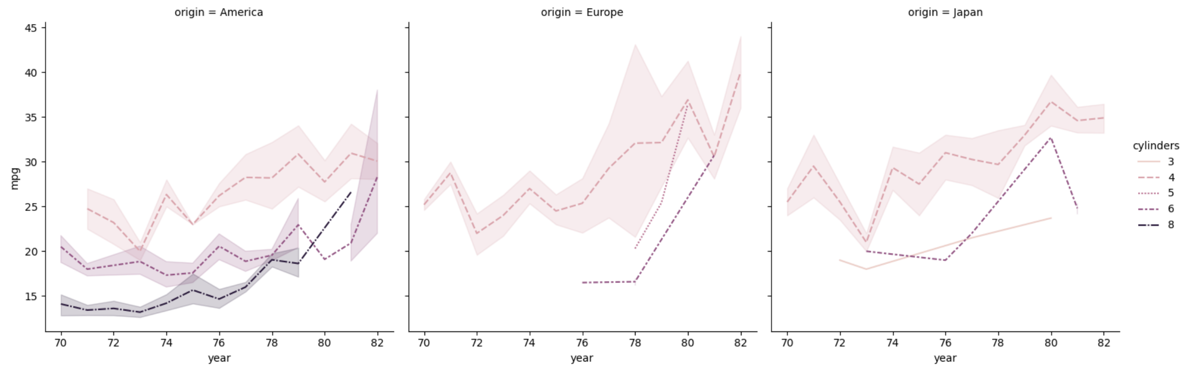
```
sns.relplot(
    Auto,
    x="year",
    y="weight",
```

```
    col="origin",
    hue="cylinders",
    style="cylinders",
    estimator="mean",
    kind="line",
)
```



The weight of the 8-cylinder American made models show a decline from the highs of 1972. It can also be seen that American made cars are heavier than their European and Japanese counterparts especially in the most common models with 4 cylinders.

```
sns.relplot(
    Auto,
    x="year",
    y="mpg",
    col="origin",
    hue="cylinders",
    style="cylinders",
    estimator="mean",
    kind="line",
)
```



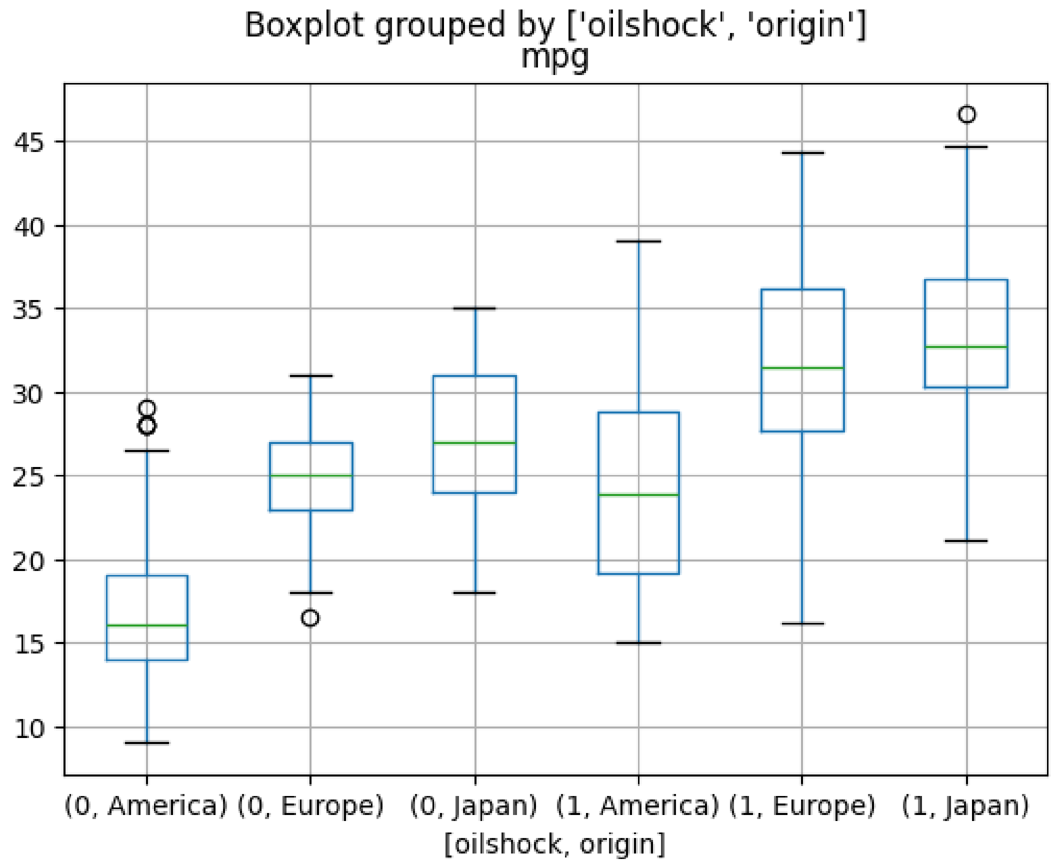
It can be seen that after the oil shock of 1973 and the regulations and actions taken by the US government, the mileage for American made cars rose across all models. This was, however, matched by the European and Japanese models which were already lighter and more fuel efficient.

Encode categorical variables as dummy variables dropping the first to remove multicollinearity.

```
def categorize_for_oil_shock(row):
    # we add 3 years because it takes approximately that long for car
     ↪  manufacturers to introduce a new model
    if row["year"] in (70, 71, 72, 73, 74, 75, 76):
        return 0
    return 1


Auto["oilshock"] = Auto.apply(categorize_for_oil_shock, axis=1)
```

```
Auto.boxplot(column="mpg", by=["oilshock", "origin"])
```




```
Auto_os = Auto.drop(["year"], axis=1)
Auto_os.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'origin',
```

```
# standardizing dataframes
Auto_os["oilshock"] = Auto_os["oilshock"].astype("category")
Auto_os = Auto_os.apply(standardize)
Auto_os.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration
count	3.920000e+02	3.920000e+02	3.920000e+02	3.920000e+02	3.920000e+02	3.920000e+02
mean	1.812609e-16	-1.087565e-16	-7.250436e-17	-1.812609e-16	-3.625218e-17	-8.519262e-16
std	1.001278e+00	1.001278e+00	1.001278e+00	1.001278e+00	1.001278e+00	1.001278e+00

	mpg	cylinders	displacement	horsepower	weight	acceleration
min	-1.853218e+00	-1.451004e+00	-1.209563e+00	-1.520975e+00	-1.608575e+00	-2.736983e+00
25%	-8.269250e-01	-8.640136e-01	-8.555316e-01	-7.665929e-01	-8.868535e-01	-6.410551e-01
50%	-8.927701e-02	-8.640136e-01	-4.153842e-01	-2.853488e-01	-2.052109e-01	-1.499869e-02
75%	7.125143e-01	1.483947e+00	7.782764e-01	5.600800e-01	7.510927e-01	5.384714e-01
max	2.970359e+00	1.483947e+00	2.493416e+00	3.265452e+00	2.549061e+00	3.360262e+00

```python
Auto_os = pd.get_dummies(
    Auto_os, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_os.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'oilshock
```

```python
y = Auto_os["mpg"]
```

```python
cols = list(Auto_os.columns)
cols.remove("mpg")
formula = " + ".join(cols)
model = smf.ols(f"mpg ~ {formula}", data=Auto_os)
results = model.fit()
results.summary()
```

Dep. Variable:	mpg	**R-squared:**	0.808
Model:	OLS	**Adj. R-squared:**	0.804
Method:	Least Squares	**F-statistic:**	201.7
Date:	Sat, 28 Sep 2024	**Prob (F-statistic):**	3.05e-132
Time:	04:09:22	**Log-Likelihood:**	-232.60
No. Observations:	392	**AIC:**	483.2
Df Residuals:	383	**BIC:**	518.9
Df Model:	8		
Covariance Type:	nonrobust		

	coef	std err	t	P> |t|	[0.025	0.975]
Intercept	-0.4186	0.041	-10.263	0.000	-0.499	-0.338
oilshock[T.1]	0.6363	0.048	13.204	0.000	0.542	0.731
cylinders	-0.1382	0.073	-1.885	0.060	-0.282	0.006
displacement	0.2845	0.107	2.659	0.008	0.074	0.495
horsepower	-0.2213	0.069	-3.192	0.002	-0.358	-0.085
weight	-0.5923	0.073	-8.085	0.000	-0.736	-0.448
acceleration	0.0053	0.036	0.146	0.884	-0.066	0.076
origin_Europe	0.3038	0.076	4.015	0.000	0.155	0.453
origin_Japan	0.3819	0.074	5.156	0.000	0.236	0.528

Omnibus:	20.039	**Durbin-Watson:**	1.331
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27.583
Skew:	0.413	**Prob(JB):**	1.02e-06
Kurtosis:	4.004	**Cond. No.**	11.9

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

i. Is there a relationship between the predictors and the response? Use the anova_lm() function from statsmodels to answer this question.

ii. Which predictors appear to have a statistically significant relationship to the response?

iii. What does the coefficient for the year variable suggest?

```
anova_lm(results)
```

	df	sum_sq	mean_sq	F	PR(>F)
oilshock	1.0	106.483141	106.483141	542.347509	2.293307e-75
cylinders	1.0	170.845795	170.845795	870.163964	1.267122e-100
displacement	1.0	11.934469	11.934469	60.785485	6.078862e-14
horsepower	1.0	3.951021	3.951021	20.123619	9.610639e-06
weight	1.0	17.796189	17.796189	90.640818	1.988543e-19
acceleration	1.0	0.009116	0.009116	0.046430	8.295108e-01
origin_Europe	1.0	0.564108	0.564108	2.873155	9.088094e-02
origin_Japan	1.0	5.218909	5.218909	26.581317	4.058867e-07
Residual	383.0	75.197253	0.196337	NaN	NaN

There seems to be a statistical relationship between all of the predictors and the response variable, mpg, except for acceleration.

Even though some of the categorical variables are insignificant, even if one of the levels is significant, it is advisable to retain them all in the model.

https://stats.stackexchange.com/questions/24298/can-i-ignore-coefficients-for-non-significant-levels-of-factors-in-a-linear-mode

Note: Year has been converted to a categorical variable oilshock to better capture the effects of the oil shock of 1973 on the mileage.

(d) Produce some of diagnostic plots of the linear regression fit as described in the lab. Comment on any problems you see with the fit. Do the residual plots suggest any unusually large outliers? Does the leverage plot identify any observations with unusually high leverage?

Before producing the diagnostic plots, let's first test for collinearity using correlation matrix and variance inflation factors.

```
Auto_os.corr(numeric_only=True)
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	origin_Europe
mpg	1.000000	-0.777618	-0.805127	-0.778427	-0.832244	0.423329	0.244313
cylinders	-0.777618	1.000000	0.950823	0.842983	0.897527	-0.504683	-0.352324
displacement	-0.805127	0.950823	1.000000	0.897257	0.932994	-0.543800	-0.371633
horsepower	-0.778427	0.842983	0.897257	1.000000	0.864538	-0.689196	-0.284948
weight	-0.832244	0.897527	0.932994	0.864538	1.000000	-0.416839	-0.293841
acceleration	0.423329	-0.504683	-0.543800	-0.689196	-0.416839	1.000000	0.208298
origin_Europe	0.244313	-0.352324	-0.371633	-0.284948	-0.293841	0.208298	1.000000
origin_Japan	0.451454	-0.404209	-0.440825	-0.321936	-0.447929	0.115020	-0.230157

```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(Auto_os.columns) + " - mpg",
 ↪  Auto_os)
vifdf
```

Feature	VIF
oilshock[T.1]	1.149269
cylinders	10.737464
displacement	22.861475
horsepower	9.594564
weight	10.715246
acceleration	2.614133
origin_Europe	1.639338
origin_Japan	1.762590

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is displacement with a VIF of 22.861474853464927
Hence, we drop displacement from the model to be fitted.

('displacement', 22.861474853464927)
```

```
vifdf = calculate_VIFs(
    "mpg ~ " + " + ".join(Auto_os.columns) + " - mpg - displacement", Auto_os
)
vifdf
```

Feature	VIF
oilshock[T.1]	1.139339
cylinders	6.190903
horsepower	8.641303
weight	9.024884
acceleration	2.591157
origin_Europe	1.450726
origin_Japan	1.591434

```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

Linear Regression for mpg ~ cylinders + horsepower + weight + acceleration + oilshock + origin_Europe + origin_Japan

```
cols = list(Auto_os.columns)
cols.remove("mpg")
cols.remove("displacement")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_os)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.805
Model:                            OLS   Adj. R-squared:                  0.801
Method:                 Least Squares   F-statistic:                     225.9
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          6.41e-132
Time:                        04:09:23   Log-Likelihood:                -236.18
No. Observations:                 392   AIC:                             488.4
Df Residuals:                     384   BIC:                             520.1
Df Model:                           7
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        -0.3890      0.040     -9.837      0.000      -0.467      -0.311
oilshock[T.1]     0.6243      0.048     12.911      0.000       0.529       0.719
cylinders        -0.0113      0.056     -0.202      0.840      -0.122       0.099
horsepower       -0.1632      0.066     -2.461      0.014      -0.294      -0.033
weight           -0.5149      0.068     -7.599      0.000      -0.648      -0.382
acceleration     -0.0038      0.036     -0.103      0.918      -0.075       0.068
origin_Europe     0.2356      0.072      3.283      0.001       0.095       0.377
origin_Japan      0.3205      0.071      4.518      0.000       0.181       0.460
==============================================================================
Omnibus:                       25.646   Durbin-Watson:                   1.305
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               40.287
Skew:                           0.456   Prob(JB):                     1.79e-09
Kurtosis:                       4.278   Cond. No.                         7.67
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df      sum_sq     mean_sq           F         PR(>F)
```

```
oilshock          1.0  106.483141  106.483141  533.906720   1.149811e-74
cylinders         1.0  170.845795  170.845795  856.621225  7.985118e-100
horsepower        1.0   12.927972   12.927972   64.820882   1.039468e-14
weight            1.0   20.649905   20.649905  103.538670   1.085729e-21
acceleration      1.0    0.003626    0.003626    0.018183   8.928058e-01
origin_Europe     1.0    0.432514    0.432514    2.168627   1.416711e-01
origin_Japan      1.0    4.071523    4.071523   20.414626   8.312108e-06
Residual        384.0   76.585524    0.199441         NaN            NaN
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is acceleration with a p-value of 0.9177
Using the backward methodology, we suggest dropping acceleration from the new model
```

Linear Regression after dropping acceleration. The model now is mpg ~ cylinders + horsepower + weight + oilshock + origin_Europe + origin_Japan

```
cols.remove("acceleration")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_os)
simple_model = results
models = []
models.append(
    {
        "name": "simple_model",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.805
Model:                            OLS   Adj. R-squared:                  0.802
Method:                 Least Squares   F-statistic:                     264.3
Date:                Sat, 28 Sep 2024   Prob (F-statistic):         3.80e-133
Time:                        04:09:23   Log-Likelihood:                -236.19
No. Observations:                 392   AIC:                             486.4
Df Residuals:                     385   BIC:                             514.2
Df Model:                           6
```

```
Covariance Type:            nonrobust
==================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
Intercept        -0.3889      0.039     -9.849      0.000      -0.467      -0.311
oilshock[T.1]     0.6245      0.048     12.935      0.000       0.530       0.719
cylinders        -0.0105      0.055     -0.189      0.850      -0.120       0.099
horsepower       -0.1585      0.048     -3.285      0.001      -0.253      -0.064
weight           -0.5182      0.060     -8.704      0.000      -0.635      -0.401
origin_Europe     0.2352      0.072      3.287      0.001       0.095       0.376
origin_Japan      0.3202      0.071      4.524      0.000       0.181       0.459
==============================================================================
Omnibus:                       25.330   Durbin-Watson:                   1.305
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               39.508
Skew:                           0.454   Prob(JB):                     2.64e-09
Kurtosis:                       4.263   Cond. No.                         6.84
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df      sum_sq     mean_sq           F         PR(>F)
oilshock         1.0  106.483141  106.483141  535.282217   7.450502e-75
cylinders        1.0  170.845795  170.845795  858.828126  4.453195e-100
horsepower       1.0   12.927972   12.927972   64.987879   9.610211e-15
weight           1.0   20.649905   20.649905  103.805416   9.634452e-22
origin_Europe    1.0    0.434982    0.434982    2.186618   1.400323e-01
origin_Japan     1.0    4.070552    4.070552   20.462339   8.111817e-06
Residual       385.0   76.587654    0.198929         NaN            NaN
```

Linear Regression after dropping cylinders. The model now is mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan

```
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_os)
simple_model = results
models = []
models.append(
    {
        "name": "simple_model",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
```

```
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.805
Model:                            OLS   Adj. R-squared:                  0.802
Method:                 Least Squares   F-statistic:                     317.9
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          2.06e-134
Time:                        04:09:23   Log-Likelihood:                -236.21
No. Observations:                 392   AIC:                             484.4
Df Residuals:                     386   BIC:                             508.2
Df Model:                           5
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        -0.3901      0.039    -10.030      0.000      -0.467      -0.314
oilshock[T.1]     0.6250      0.048     12.983      0.000       0.530       0.720
horsepower       -0.1613      0.046     -3.510      0.001      -0.252      -0.071
weight           -0.5245      0.050    -10.576      0.000      -0.622      -0.427
origin_Europe     0.2386      0.069      3.448      0.001       0.103       0.375
origin_Japan      0.3222      0.070      4.611      0.000       0.185       0.460
==============================================================================
Omnibus:                       24.971   Durbin-Watson:                   1.304
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.456
Skew:                           0.453   Prob(JB):                     4.46e-09
Kurtosis:                       4.239   Cond. No.                         5.60
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df      sum_sq     mean_sq           F        PR(>F)
oilshock         1.0  106.483141  106.483141  536.622900  4.863116e-75
horsepower       1.0  165.048555  165.048555  831.763917  2.445119e-98
weight           1.0   39.079210   39.079210  196.940090  1.939884e-36
origin_Europe    1.0    0.574647    0.574647    2.895939  8.960825e-02
origin_Japan     1.0    4.219706    4.219706   21.265252  5.446537e-06
Residual       386.0   76.594741    0.198432         NaN           NaN
```

We can now try and plot the diagnostics for the model.

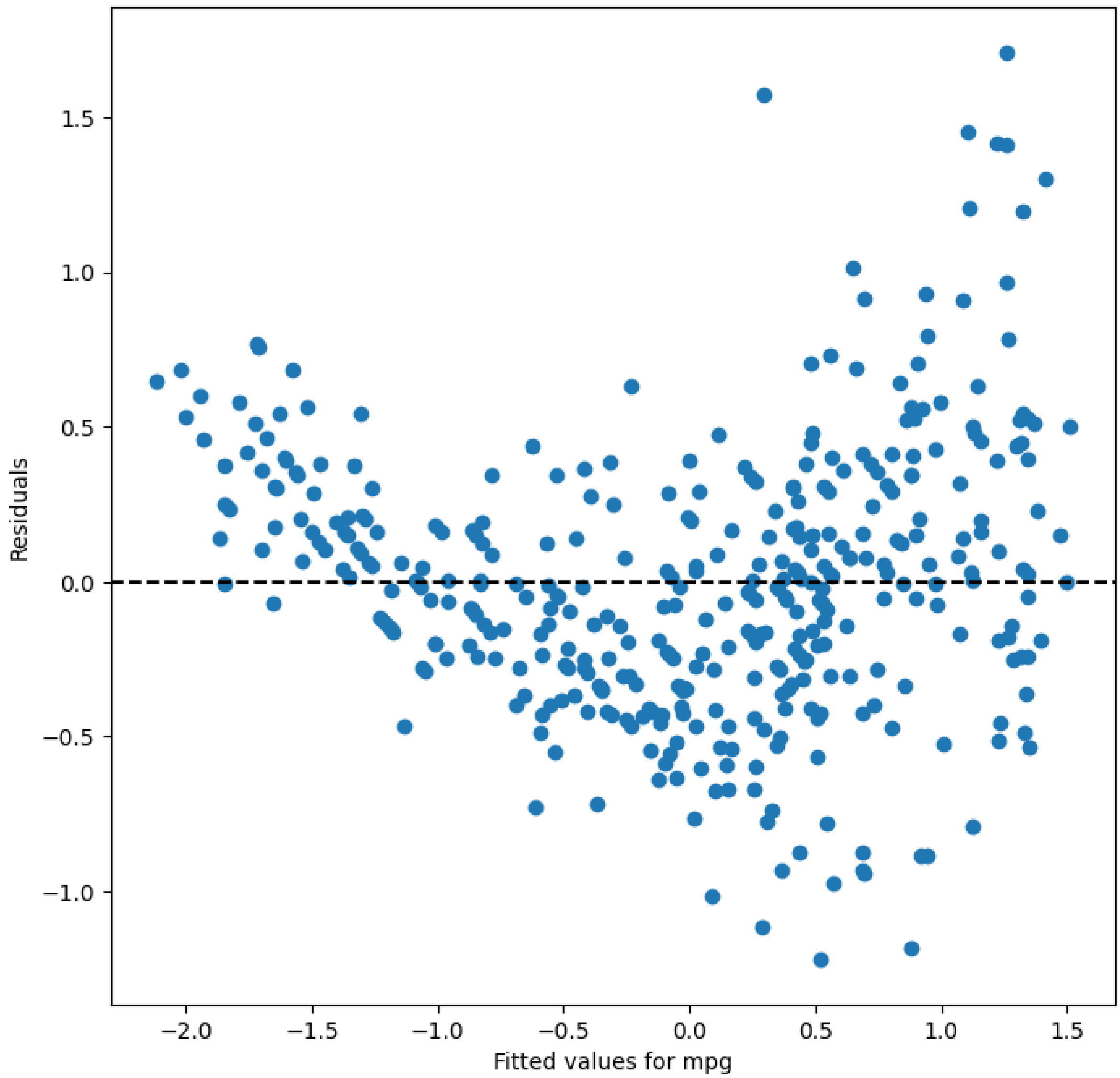
```python
TSS = np.sum((y - np.mean(y)) ** 2)
TSS
RSS = np.sum((y - results.fittedvalues) ** 2)
RSS
RSE = np.sqrt(RSS / results.df_model)
display("RSE " + str(RSE))
display("R-squared adjusted : " + str(results.rsquared_adj))
display("F-statistic : " + str(results.fvalue))
```

```
'RSE 3.9139428061794668'
```

```
'R-squared adjusted : 0.8020742313429469'
```

```
'F-statistic : 317.8976193276657'
```

```python
display_residuals_plot(results)
```



There is some evidence of non-linearity and heteroskedasticity from the residuals plot above.

(e) Fit some models with interactions as described in the lab. Do any interactions appear to be statistically significant?

```
formula = " + ".join(cols)
formula += " + " + "horsepower: weight"
results = perform_analysis("mpg", formula, Auto_os)
```

```
numeric_interactions = results
models.append(
    {
        "name": "numeric_interactions",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.847
Model:                            OLS   Adj. R-squared:                  0.845
Method:                 Least Squares   F-statistic:                     355.8
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          1.15e-153
Time:                        04:09:24   Log-Likelihood:                -187.98
No. Observations:                 392   AIC:                             390.0
Df Residuals:                     385   BIC:                             417.8
Df Model:                           6
Covariance Type:            nonrobust
=====================================================================================
                        coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
Intercept            -0.5631      0.038    -14.715      0.000      -0.638      -0.488
oilshock[T.1]         0.6508      0.043     15.243      0.000       0.567       0.735
horsepower           -0.3723      0.045     -8.185      0.000      -0.462      -0.283
weight               -0.4926      0.044    -11.192      0.000      -0.579      -0.406
origin_Europe         0.1565      0.062      2.535      0.012       0.035       0.278
origin_Japan          0.2061      0.063      3.278      0.001       0.082       0.330
horsepower:weight     0.2300      0.022     10.364      0.000       0.186       0.274
==============================================================================
Omnibus:                       27.116   Durbin-Watson:                   1.364
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               45.242
Skew:                           0.457   Prob(JB):                     1.50e-10
Kurtosis:                       4.390   Cond. No.                         7.09
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                     df      sum_sq     mean_sq           F         PR(>F)
oilshock            1.0  106.483141  106.483141  684.553356   1.927144e-87
```

```
horsepower           1.0  165.048555  165.048555  1061.055689  1.099814e-112
weight               1.0   39.079210   39.079210   251.230425   6.522402e-44
origin_Europe        1.0    0.574647    0.574647     3.694260   5.533771e-02
origin_Japan         1.0    4.219706    4.219706    27.127429   3.109409e-07
horsepower:weight    1.0   16.707504   16.707504   107.408348   2.313061e-22
Residual           385.0   59.887237    0.155551          NaN            NaN
```

```
formula = " + ".join(cols)
formula += " + " + "horsepower: weight"
formula += " + " + "oilshock: weight"
formula += " + " + "oilshock: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
oilshock_interactions = results
models.append(
    {
        "name": "oilshock_interactions",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.861
Model:                            OLS   Adj. R-squared:                  0.858
Method:                 Least Squares   F-statistic:                     297.5
Date:                Sat, 28 Sep 2024   Prob (F-statistic):         3.50e-159
Time:                        04:09:24   Log-Likelihood:                -168.92
No. Observations:                 392   AIC:                             355.8
Df Residuals:                     383   BIC:                             391.6
Df Model:                           8
Covariance Type:            nonrobust
=================================================================================================
                                coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------------------
Intercept                    -0.5350      0.037    -14.531      0.000      -0.607      -0.463
oilshock[T.1]                 0.5913      0.042     14.071      0.000       0.509       0.674
horsepower                   -0.2801      0.051     -5.456      0.000      -0.381      -0.179
oilshock[T.1]:horsepower     -0.2276      0.088     -2.598      0.010      -0.400      -0.055
weight                       -0.4591      0.050     -9.226      0.000      -0.557      -0.361
oilshock[T.1]:weight         -0.0963      0.082     -1.181      0.238      -0.257       0.064
origin_Europe                 0.1804      0.059      3.039      0.003       0.064       0.297
```

```
origin_Japan                 0.1929      0.060      3.209      0.001       0.075       0.311
horsepower:weight            0.1715      0.023      7.403      0.000       0.126       0.217
==============================================================================
Omnibus:                       23.934   Durbin-Watson:                   1.456
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               43.610
Skew:                           0.377   Prob(JB):                     3.39e-10
Kurtosis:                       4.450   Cond. No.                         11.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                          df      sum_sq     mean_sq            F         PR(>F)
oilshock                 1.0  106.483141  106.483141   750.563317   2.852400e-92
horsepower               1.0  165.048555  165.048555  1163.370934  3.985476e-118
oilshock:horsepower      1.0   19.155472   19.155472   135.020381   6.081794e-27
weight                   1.0   35.071635   35.071635   247.207986   2.471356e-43
oilshock:weight          1.0    0.468549    0.468549     3.302640   6.994992e-02
origin_Europe            1.0    0.971997    0.971997     6.851278   9.208817e-03
origin_Japan             1.0    2.687942    2.687942    18.946386   1.727234e-05
horsepower:weight        1.0    7.776131    7.776131    54.811293   8.507915e-13
Residual               383.0   54.336579    0.141871          NaN            NaN
```

```python
formula = " + ".join(cols)
formula += " + " + "oilshock: horsepower"
formula += " + " + "origin_Europe: horsepower"
formula += " + " + "origin_Japan: horsepower"
formula += " + " + "origin_Europe: weight"
formula += " + " + "origin_Japan: weight"
formula += " + " + "oilshock: weight"
formula += " + " + "oilshock: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
origin_interactions = results
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.855
Model:                            OLS   Adj. R-squared:                  0.851
Method:                 Least Squares   F-statistic:                     204.1
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          6.22e-152
Time:                        04:09:24   Log-Likelihood:                -177.44
No. Observations:                 392   AIC:                             378.9
Df Residuals:                     380   BIC:                             426.5
```

```
Df Model:                          11
Covariance Type:            nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
Intercept                 -0.4245      0.035    -12.121      0.000      -0.493      -0.356
oilshock[T.1]              0.5690      0.044     13.054      0.000       0.483       0.655
horsepower                -0.0708      0.048     -1.470      0.142      -0.166       0.024
oilshock[T.1]:horsepower  -0.1615      0.096     -1.687      0.092      -0.350       0.027
weight                    -0.4713      0.054     -8.712      0.000      -0.578      -0.365
oilshock[T.1]:weight      -0.2333      0.087     -2.694      0.007      -0.404      -0.063
origin_Europe              0.0297      0.082      0.363      0.717      -0.131       0.191
origin_Japan              -0.0010      0.131     -0.007      0.994      -0.259       0.257
origin_Europe:horsepower  -0.5852      0.130     -4.515      0.000      -0.840      -0.330
origin_Japan:horsepower   -0.2801      0.204     -1.370      0.172      -0.682       0.122
origin_Europe:weight       0.1640      0.120      1.370      0.171      -0.071       0.399
origin_Japan:weight       -0.1326      0.245     -0.541      0.589      -0.614       0.349
==============================================================================
Omnibus:                       20.717   Durbin-Watson:                   1.585
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               32.838
Skew:                           0.373   Prob(JB):                     7.40e-08
Kurtosis:                       4.205   Cond. No.                         25.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                             df      sum_sq     mean_sq            F         PR(>F)
oilshock                    1.0  106.483141  106.483141   712.972849   3.365342e-89
horsepower                  1.0  165.048555  165.048555  1105.105820  1.586816e-114
oilshock:horsepower         1.0   19.155472   19.155472   128.258155   8.134152e-26
weight                      1.0   35.071635   35.071635   234.827067   1.302452e-41
oilshock:weight             1.0    0.468549    0.468549     3.137234   7.732495e-02
origin_Europe               1.0    0.971997    0.971997     6.508146   1.112920e-02
origin_Japan                1.0    2.687942    2.687942    17.997494   2.781984e-05
origin_Europe:horsepower    1.0    3.024522    3.024522    20.251113   9.040705e-06
origin_Japan:horsepower     1.0    1.977640    1.977640    13.241566   3.116551e-04
origin_Europe:weight        1.0    0.313437    0.313437     2.098659   1.482531e-01
origin_Japan:weight         1.0    0.043767    0.043767     0.293050   5.885897e-01
Residual                  380.0   56.753344    0.149351          NaN            NaN
```

- From the above analysis, we can see that there is no significant interaction between origin and weight.
- So we can omit them from the model.

```python
formula = " + ".join(cols)
formula += " + " + "oilshock: horsepower"
formula += " + " + "origin_Europe: horsepower"
formula += " + " + "origin_Japan: horsepower"
formula += " + " + "oilshock: weight"
formula += " + " + "oilshock: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
origin_interactions = results
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.854
Model:                            OLS   Adj. R-squared:                  0.851
Method:                 Least Squares   F-statistic:                     248.9
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          8.18e-154
Time:                        04:09:24   Log-Likelihood:                -178.67
No. Observations:                 392   AIC:                             377.3
Df Residuals:                     382   BIC:                             417.1
Df Model:                           9
Covariance Type:            nonrobust
=============================================================================================
                               coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------------------
Intercept                   -0.4296      0.034    -12.650      0.000      -0.496      -0.363
oilshock[T.1]                0.5693      0.043     13.280      0.000       0.485       0.654
horsepower                  -0.0797      0.047     -1.689      0.092      -0.172       0.013
oilshock[T.1]:horsepower    -0.1958      0.092     -2.133      0.034      -0.376      -0.015
weight                      -0.4571      0.051     -8.947      0.000      -0.558      -0.357
oilshock[T.1]:weight        -0.2019      0.084     -2.408      0.016      -0.367      -0.037
origin_Europe                0.0044      0.080      0.055      0.956      -0.153       0.162
origin_Japan                 0.0750      0.084      0.892      0.373      -0.090       0.240
origin_Europe:horsepower    -0.4667      0.096     -4.884      0.000      -0.655      -0.279
origin_Japan:horsepower     -0.3682      0.101     -3.637      0.000      -0.567      -0.169
==============================================================================
Omnibus:                       20.114   Durbin-Watson:                   1.577
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               31.613
Skew:                           0.366   Prob(JB):                     1.37e-07
Kurtosis:                       4.183   Cond. No.                         10.2
==============================================================================

Notes:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                            df      sum_sq    mean_sq            F         PR(>F)
oilshock                    1.0  106.483141  106.483141   712.242504   2.550768e-89
horsepower                  1.0  165.048555  165.048555  1103.973788  9.879451e-115
oilshock:horsepower         1.0   19.155472   19.155472   128.126771   8.220841e-26
weight                      1.0   35.071635   35.071635   234.586518   1.270254e-41
oilshock:weight             1.0    0.468549    0.468549     3.134021   7.747196e-02
origin_Europe               1.0    0.971997    0.971997     6.501479   1.116822e-02
origin_Japan                1.0    2.687942    2.687942    17.979058   2.804561e-05
origin_Europe:horsepower    1.0    3.024522    3.024522    20.230368   9.121050e-06
origin_Japan:horsepower     1.0    1.977640    1.977640    13.228002   3.136339e-04
Residual                  382.0   57.110548    0.149504          NaN            NaN
```

- From the above analysis, it is evident that with the interaction between origin and horsepower, the interaction between oilshock and weight and horsepower is insignificant. We can drop these from the model as well.

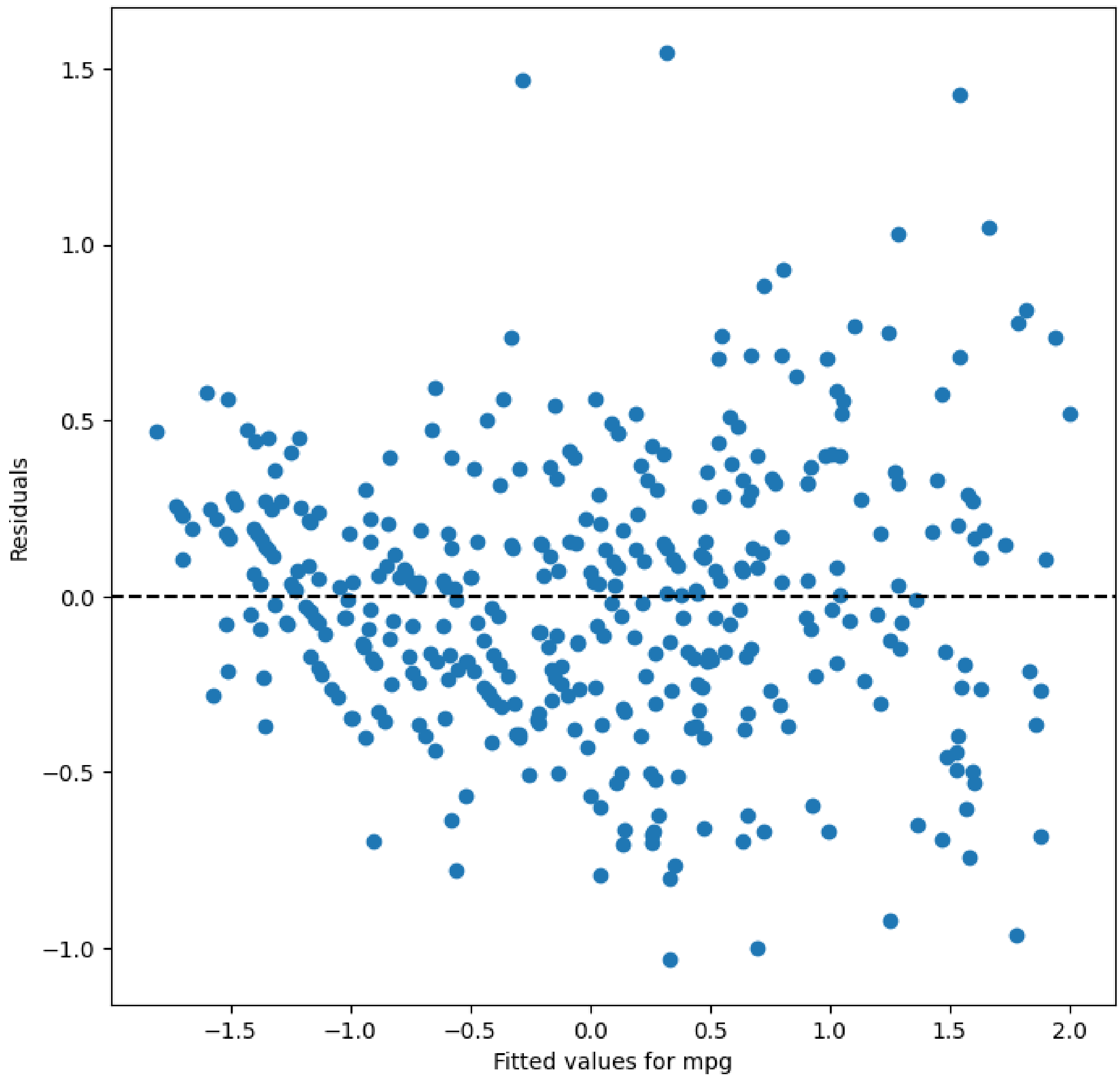
```
formula = " + ".join(cols)
formula += " + " + "oilshock: horsepower"
formula += " + " + "origin_Europe: horsepower"
formula += " + " + "origin_Japan: horsepower"
results = perform_analysis("mpg", formula, Auto_os)
origin_interactions = results
models.append(
    {
        "name": "origin_interactions",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.852
Model:                            OLS   Adj. R-squared:                  0.849
Method:                 Least Squares   F-statistic:                     275.8
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          8.41e-154
Time:                        04:09:24   Log-Likelihood:                -181.63
No. Observations:                 392   AIC:                             381.3
Df Residuals:                     383   BIC:                             417.0
Df Model:                           8
Covariance Type:            nonrobust
```

```
==============================================================================================
                                 coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------------
Intercept                     -0.4267      0.034    -12.495      0.000      -0.494      -0.360
oilshock[T.1]                  0.5628      0.043     13.073      0.000       0.478       0.647
horsepower                    -0.0265      0.042     -0.633      0.527      -0.109       0.056
oilshock[T.1]:horsepower      -0.3804      0.051     -7.497      0.000      -0.480      -0.281
weight                        -0.5231      0.043    -12.051      0.000      -0.608      -0.438
origin_Europe                  0.0041      0.081      0.051      0.959      -0.155       0.163
origin_Japan                   0.0877      0.084      1.039      0.299      -0.078       0.254
origin_Europe:horsepower      -0.4423      0.096     -4.626      0.000      -0.630      -0.254
origin_Japan:horsepower       -0.3589      0.102     -3.526      0.000      -0.559      -0.159
==============================================================================
Omnibus:                       19.159   Durbin-Watson:                   1.576
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               29.046
Skew:                           0.362   Prob(JB):                     4.93e-07
Kurtosis:                       4.119   Cond. No.                         9.30
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                             df     sum_sq    mean_sq            F         PR(>F)
oilshock                    1.0 106.483141 106.483141   703.425918   9.826044e-89
horsepower                  1.0 165.048555 165.048555  1090.308104  4.259700e-114
oilshock:horsepower         1.0  19.155472  19.155472   126.540737   1.468876e-25
weight                      1.0  35.071635  35.071635   231.682658   2.992243e-41
origin_Europe               1.0   0.792887   0.792887     5.237801   2.264506e-02
origin_Japan                1.0   2.840217   2.840217    18.762427   1.893486e-05
origin_Europe:horsepower    1.0   2.748574   2.748574    18.157033   2.563625e-05
origin_Japan:horsepower     1.0   1.881783   1.881783    12.431031   4.733982e-04
Residual                  383.0  57.977737   0.151378          NaN            NaN
```

```
display_residuals_plot(results)
```




```
anova_lm(simple_model, numeric_interactions, oilshock_interactions,
↪   origin_interactions)
```

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	386.0	76.594741	0.0	NaN	NaN	NaN
1	385.0	59.887237	1.0	16.707504	110.369506	7.218441e-23
2	383.0	54.336579	2.0	5.550658	18.333780	2.489897e-08
3	383.0	57.977737	-0.0	-3.641158	inf	NaN

```
pd.DataFrame(models)
```

	name	model
0	simple_model	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan
1	numeric_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + horsep
2	oilshock_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + horsep
3	origin_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + oilshoc

(f) Try a few different transformations of the variables, such as log(X), √X, X2 . Comment on your findings.

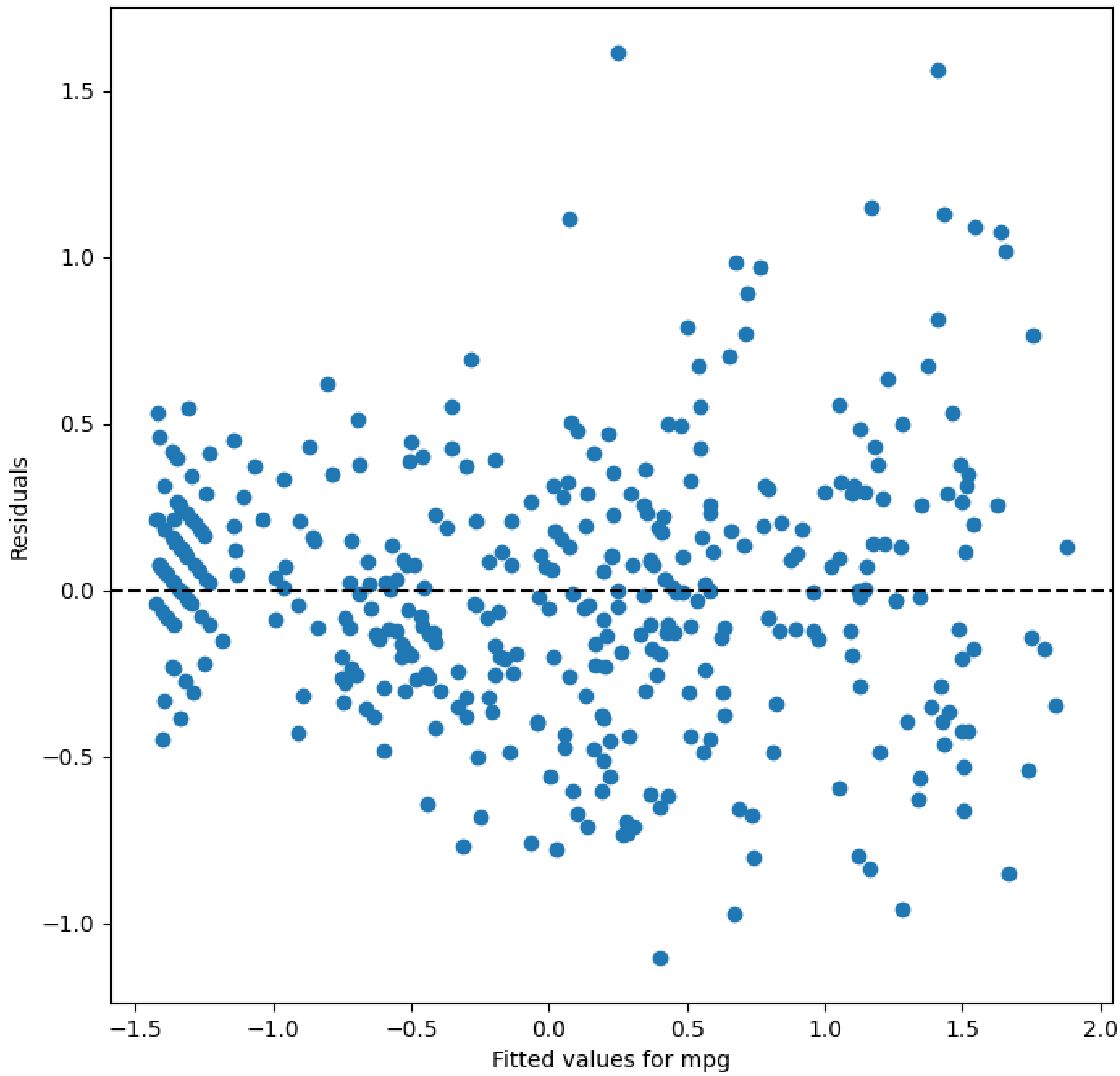
```
formula = simple_model.model.formula
formula = formula[formula.rindex("~") + 1 :]
# Add higher order transformations for horsepower and weight
formula += " + " + "I(horsepower**2)"
formula += " + " + "I(weight**2)"
results = perform_analysis("mpg", formula, Auto_os)
squared_transformations = results
models.append(
    {
        "name": "squared_transformation",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.848
Model:                            OLS   Adj. R-squared:                  0.846
Method:                 Least Squares   F-statistic:                     306.8
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          5.75e-153
Time:                        04:09:25   Log-Likelihood:                -186.58
No. Observations:                 392   AIC:                             389.2
Df Residuals:                     384   BIC:                             420.9
Df Model:                           7
Covariance Type:            nonrobust
==================================================================================
```

```
                        coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------
Intercept            -0.6020      0.040    -15.072      0.000      -0.681      -0.524
oilshock[T.1]         0.6580      0.043     15.334      0.000       0.574       0.742
horsepower           -0.3798      0.054     -7.058      0.000      -0.486      -0.274
weight               -0.5069      0.050    -10.044      0.000      -0.606      -0.408
origin_Europe         0.1436      0.062      2.324      0.021       0.022       0.265
origin_Japan          0.1959      0.064      3.047      0.002       0.069       0.322
I(horsepower ** 2)    0.0976      0.021      4.667      0.000       0.056       0.139
I(weight ** 2)        0.1413      0.026      5.382      0.000       0.090       0.193
==============================================================================
Omnibus:                       26.672   Durbin-Watson:                   1.394
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               46.435
Skew:                           0.436   Prob(JB):                     8.26e-11
Kurtosis:                       4.443   Cond. No.                         10.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                       df      sum_sq     mean_sq            F         PR(>F)
oilshock              1.0  106.483141  106.483141   687.677177   1.332036e-87
horsepower            1.0  165.048555  165.048555  1065.897602  7.775452e-113
weight                1.0   39.079210   39.079210   252.376864   4.864770e-44
origin_Europe         1.0    0.574647    0.574647     3.711118   5.478894e-02
origin_Japan          1.0    4.219706    4.219706    27.251220   2.932534e-07
I(horsepower ** 2)    1.0   12.648520   12.648520    81.685220   7.946432e-18
I(weight ** 2)        1.0    4.485871    4.485871    28.970134   1.281966e-07
Residual            384.0   59.460351    0.154845          NaN            NaN
```

```
display_residuals_plot(results)
```




```
anova_lm(simple_model, squared_transformations)
```

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	386.0	76.594741	0.0	NaN	NaN	NaN
1	384.0	59.460351	2.0	17.134391	55.327677	7.681995e-22

```
pd.DataFrame(models)
```

	name	model
0	simple_model	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan
1	numeric_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + hor
2	oilshock_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + hor
3	origin_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + oils
4	squared_transformation	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + I(h

- Since we've standardized the variables, we cannot run log or square root transformations on the negative valued columns.
- We can reload the data and run the log and sqrt transformations on the original unstandardized data.

```
Auto = load_data("Auto")
Auto = Auto.sort_values(by=["year"], ascending=True)
Auto.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year', 'c
```

```
print("Minimums:")
print(Auto.min())
print("Maximums:")
print(Auto.max())
```

```
Minimums:
mpg              9.0
cylinders        3.0
displacement    68.0
horsepower      46.0
weight        1613.0
acceleration     8.0
year            70.0
origin           1.0
dtype: float64
Maximums:
mpg             46.6
cylinders        8.0
```

```
displacement     455.0
horsepower       230.0
weight          5140.0
acceleration      24.8
year              82.0
origin             3.0
dtype: float64
```

- From the above, we can see that the values for displacement, horsepower and weight are quite large.
- Hence, we log or square root transform only these variables.

Now let's categorize the variables

```
Auto["origin"] = Auto["origin"].astype("category")
Auto["origin"] = Auto["origin"].cat.rename_categories(
    {1: "America", 2: "Europe", 3: "Japan"}
)
Auto["year"] = Auto["year"].astype("category")
Auto["oilshock"] = Auto.apply(categorize_for_oil_shock, axis=1)
```

Log Transformed Model

```
Auto_log = Auto.copy(deep=True)
```

```
Auto_log["log_displacement"] = np.log(Auto_log["displacement"])
Auto_log["log_horsepower"] = np.log(Auto_log["horsepower"])
Auto_log["log_weight"] = np.log(Auto_log["weight"])
Auto_log = Auto_log.drop(
    columns=[
        "displacement",
        "weight",
        "horsepower",
        "year",
    ]
)
Auto_log.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'origin', 'oilshock', 'log_displacement', 'log_ho
```

```
Auto_log.corr(numeric_only=True)
```

	mpg	cylinders	acceleration	oilshock	log_displacement	log_horsepower	log
mpg	1.000000	-0.777618	0.423329	0.521192	-0.828453	-0.817517	-0.
cylinders	-0.777618	1.000000	-0.504683	-0.273703	0.942814	0.843204	0.8
acceleration	0.423329	-0.504683	1.000000	0.195892	-0.497107	-0.698328	-0.
oilshock	0.521192	-0.273703	0.195892	1.000000	-0.268161	-0.299037	-0.
log_displacement	-0.828453	0.942814	-0.497107	-0.268161	1.000000	0.872149	0.9
log_horsepower	-0.817517	0.843204	-0.698328	-0.299037	0.872149	1.000000	0.8
log_weight	-0.844194	0.884303	-0.401563	-0.250520	0.942850	0.873956	1.0

```
Auto_log = pd.get_dummies(
    Auto_log, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_log.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'oilshock', 'log_displacement', 'log_horsepower',
```

```
cols = list(Auto_log.columns)
cols.remove("mpg")
```

```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_log)
vifdf
```

	VIF
Feature	
cylinders	9.828626
acceleration	3.304749
oilshock	1.147770
log_displacement	25.969595
log_horsepower	11.414709
log_weight	16.146573
origin_Europe	1.876698
origin_Japan	2.097688

```python
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is log_displacement with a VIF of 25.96959512578754
Hence, we drop log_displacement from the model to be fitted.
```

```
('log_displacement', 25.96959512578754)
```

```python
cols.remove("log_displacement")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_log)
vifdf
```

Feature	VIF
cylinders	5.535070
acceleration	3.179336
oilshock	1.142791
log_horsepower	11.411764
log_weight	10.608718
origin_Europe	1.451961
origin_Japan	1.652749

```python
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is log_horsepower with a VIF of 11.411764499222897
Hence, we drop log_horsepower from the model to be fitted.
```

```
('log_horsepower', 11.411764499222897)
```

```python
cols.remove("log_horsepower")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_log)
vifdf
```

Feature	VIF
cylinders	5.517868

Feature	VIF
acceleration	1.377517
oilshock	1.118666
log_weight	5.014899
origin_Europe	1.451265
origin_Japan	1.608682

```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

```
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_log)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.823
Model:                            OLS   Adj. R-squared:                  0.821
Method:                 Least Squares   F-statistic:                     299.3
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          1.36e-141
Time:                        04:09:28   Log-Likelihood:                -1021.3
No. Observations:                 392   AIC:                             2057.
Df Residuals:                     385   BIC:                             2084.
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept       168.2416      9.696     17.351      0.000     149.177     187.306
cylinders         0.0129      0.230      0.056      0.955      -0.440       0.465
acceleration      0.1805      0.071      2.538      0.012       0.041       0.320
oilshock          5.1312      0.355     14.470      0.000       4.434       5.828
log_weight      -18.9156      1.331    -14.211      0.000     -21.533     -16.299
origin_Europe     1.3692      0.531      2.578      0.010       0.325       2.413
origin_Japan      1.5602      0.528      2.956      0.003       0.522       2.598
==============================================================================
Omnibus:                       30.158   Durbin-Watson:                   1.253
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               51.811
```

```
Skew:                           0.493   Prob(JB):                     5.62e-12
Kurtosis:                       4.484   Cond. No.                     1.08e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.08e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
                 df        sum_sq       mean_sq            F         PR(>F)
cylinders       1.0  14403.083079  14403.083079  1318.559127  2.133814e-126
acceleration    1.0     30.471304     30.471304     2.789557   9.569322e-02
oilshock        1.0   2422.051542   2422.051542   221.731566   6.308582e-40
log_weight      1.0   2639.878573   2639.878573   241.672978   1.215817e-42
origin_Europe   1.0     22.569818     22.569818     2.066199   1.514086e-01
origin_Japan    1.0     95.449335     95.449335     8.738101   3.308129e-03
Residual      385.0   4205.489819     10.923350          NaN            NaN
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is cylinders with a p-value of 0.9554479
Using the backward methodology, we suggest dropping cylinders from the new model
```

```
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_log)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.823
Model:                            OLS   Adj. R-squared:                  0.821
Method:                 Least Squares   F-statistic:                     360.0
Date:                Sat, 28 Sep 2024   Prob (F-statistic):          6.83e-143
Time:                        04:09:28   Log-Likelihood:                -1021.3
No. Observations:                 392   AIC:                             2055.
Df Residuals:                     386   BIC:                             2078.
Df Model:                           5
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
```

```
Intercept       167.8689      7.032     23.870      0.000     154.042     181.696
acceleration      0.1792      0.067      2.673      0.008       0.047       0.311
oilshock          5.1289      0.352     14.574      0.000       4.437       5.821
log_weight      -18.8571      0.820    -22.992      0.000     -20.470     -17.245
origin_Europe     1.3628      0.518      2.631      0.009       0.344       2.381
origin_Japan      1.5576      0.525      2.967      0.003       0.525       2.590
==============================================================================
Omnibus:                       30.308   Durbin-Watson:                   1.253
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               52.282
Skew:                           0.493   Prob(JB):                     4.44e-12
Kurtosis:                       4.492   Cond. No.                         751.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                  df         sum_sq        mean_sq           F         PR(>F)
acceleration     1.0    4268.531557    4268.531557  391.783092   1.076057e-60
oilshock         1.0    4757.627552    4757.627552  436.674301   2.076230e-65
log_weight       1.0   10466.602734   10466.602734  960.667136  8.818191e-107
origin_Europe    1.0      24.823504      24.823504    2.278402   1.320051e-01
origin_Japan     1.0      95.884166      95.884166    8.800637   3.198611e-03
Residual       386.0    4205.523957      10.895140         NaN            NaN
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
No variables are statistically insignificant.
The model mpg ~ acceleration + oilshock + log_weight + origin_Europe + origin_Japan cannot be
```

```
models.append(
    {
        "name": "log_transformation",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```
pd.DataFrame(models)
```

	name	model
0	simple_model	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan
1	numeric_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + hor
2	oilshock_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + hor
3	origin_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + oils
4	squared_transformation	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + I(h
5	log_transformation	mpg ~ acceleration + oilshock + log_weight + origin_Europe + origin_Japan

Square Root Transformed Model

```
Auto_sqrt = Auto.copy(deep=True)
```

```
Auto_sqrt["sqrt_displacement"] = np.sqrt(Auto_sqrt["displacement"])
Auto_sqrt["sqrt_horsepower"] = np.sqrt(Auto_sqrt["horsepower"])
Auto_sqrt["sqrt_weight"] = np.sqrt(Auto_sqrt["weight"])
Auto_sqrt = Auto_sqrt.drop(
    columns=[
        "displacement",
        "weight",
        "horsepower",
        "year",
    ]
)
Auto_sqrt.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'origin', 'oilshock', 'sqrt_displacement', 'sqrt_
```

```
Auto_sqrt.corr(numeric_only=True)
```

	mpg	cylinders	acceleration	oilshock	sqrt_displacement	sqrt_horsepower
mpg	1.000000	-0.777618	0.423329	0.521192	-0.821331	-0.802311
cylinders	-0.777618	1.000000	-0.504683	-0.273703	0.953208	0.849266
acceleration	0.423329	-0.504683	1.000000	0.195892	-0.521812	-0.696702
oilshock	0.521192	-0.273703	0.195892	1.000000	-0.284587	-0.306247
sqrt_displacement	-0.821331	0.953208	-0.521812	-0.284587	1.000000	0.886470
sqrt_horsepower	-0.802311	0.849266	-0.696702	-0.306247	0.886470	1.000000

	mpg	cylinders	acceleration	oilshock	sqrt_displacement	sqrt_horsepower
sqrt_weight	-0.840095	0.893465	-0.409829	-0.260664	0.939868	0.872045

```
Auto_sqrt = pd.get_dummies(
    Auto_sqrt, columns=list(["origin"]), drop_first=True, dtype=np.uint8
)
Auto_sqrt.columns
```

```
Index(['mpg', 'cylinders', 'acceleration', 'oilshock', 'sqrt_displacement', 'sqrt_horsepower
```

```
cols = list(Auto_sqrt.columns)
cols.remove("mpg")
```

```
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_sqrt)
vifdf
```

	VIF
Feature	
cylinders	11.465746
acceleration	3.010771
oilshock	1.151324
sqrt_displacement	27.042946
sqrt_horsepower	10.615281
sqrt_weight	13.450552
origin_Europe	1.774827
origin_Japan	1.944729

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is sqrt_displacement with a VIF of 27.042946454149405
Hence, we drop sqrt_displacement from the model to be fitted.
```

```
('sqrt_displacement', 27.042946454149405)
```

```
cols.remove("sqrt_displacement")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_sqrt)
vifdf
```

Feature	VIF
cylinders	5.974510
acceleration	2.934605
oilshock	1.141428
sqrt_horsepower	10.446261
sqrt_weight	9.963350
origin_Europe	1.450840
origin_Japan	1.623907

```
identify_highest_VIF_feature(vifdf)
```

```
We find the highest VIF in this model is sqrt_horsepower with a VIF of 10.446261176837464
Hence, we drop sqrt_horsepower from the model to be fitted.

('sqrt_horsepower', 10.446261176837464)
```

```
cols.remove("sqrt_horsepower")
vifdf = calculate_VIFs("mpg ~ " + " + ".join(cols), Auto_sqrt)
vifdf
```

Feature	VIF
cylinders	5.907717
acceleration	1.377206
oilshock	1.119726
sqrt_weight	5.331435
origin_Europe	1.446456
origin_Japan	1.581460

```
identify_highest_VIF_feature(vifdf)
```

```
No variables are significantly collinear.
```

```python
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_sqrt)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.814
Model:                            OLS   Adj. R-squared:                  0.811
Method:                 Least Squares   F-statistic:                     281.0
Date:                Sat, 28 Sep 2024   Prob (F-statistic):         2.76e-137
Time:                        04:09:30   Log-Likelihood:                -1031.4
No. Observations:                 392   AIC:                             2077.
Df Residuals:                     385   BIC:                             2105.
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        54.3622      2.396     22.687      0.000      49.651      59.073
cylinders         0.0148      0.244      0.061      0.952      -0.466       0.495
acceleration      0.1748      0.073      2.395      0.017       0.031       0.318
oilshock          5.0506      0.364     13.873      0.000       4.335       5.766
sqrt_weight      -0.6785      0.052    -13.130      0.000      -0.780      -0.577
origin_Europe     1.5511      0.544      2.851      0.005       0.481       2.621
origin_Japan      1.9033      0.537      3.544      0.000       0.847       2.959
==============================================================================
Omnibus:                       25.773   Durbin-Watson:                   1.269
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               41.514
Skew:                           0.449   Prob(JB):                     9.67e-10
Kurtosis:                       4.317   Cond. No.                         803.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                df        sum_sq       mean_sq            F         PR(>F)
cylinders      1.0  14403.083079  14403.083079  1252.209748  4.496819e-123
acceleration   1.0     30.471304     30.471304     2.649187   1.044209e-01
oilshock       1.0   2422.051542   2422.051542   210.574120   2.284075e-38
sqrt_weight    1.0   2365.801178   2365.801178   205.683691   1.125346e-37
origin_Europe  1.0     24.780703     24.780703     2.154444   1.429743e-01
origin_Japan   1.0    144.484455    144.484455    12.561536   4.421781e-04
```

```
Residual       385.0   4428.321210     11.502133          NaN            NaN
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is cylinders with a p-value of 0.951625
Using the backward methodology, we suggest dropping cylinders from the new model
```

```
cols.remove("cylinders")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_sqrt)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.814
Model:                            OLS   Adj. R-squared:                  0.812
Method:                 Least Squares   F-statistic:                     338.0
Date:                Sat, 28 Sep 2024   Prob (F-statistic):         1.43e-138
Time:                        04:09:30   Log-Likelihood:                -1031.4
No. Observations:                 392   AIC:                             2075.
Df Residuals:                     386   BIC:                             2099.
Df Model:                           5
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        54.3324      2.342     23.198      0.000      49.728      58.937
acceleration      0.1733      0.069      2.512      0.012       0.038       0.309
oilshock          5.0486      0.362     13.946      0.000       4.337       5.760
sqrt_weight      -0.6760      0.031    -21.968      0.000      -0.737      -0.616
origin_Europe     1.5438      0.530      2.913      0.004       0.502       2.586
origin_Japan      1.8999      0.534      3.561      0.000       0.851       2.949
==============================================================================
Omnibus:                       25.925   Durbin-Watson:                   1.269
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               41.952
Skew:                           0.450   Prob(JB):                     7.77e-10
Kurtosis:                       4.326   Cond. No.                         783.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
                  df        sum_sq       mean_sq           F         PR(>F)
acceleration     1.0   4268.531557   4268.531557  372.068179   1.546709e-58
oilshock         1.0   4757.627552   4757.627552  414.700418   3.906846e-63
sqrt_weight      1.0  10191.062423  10191.062423  888.307839  3.798699e-102
origin_Europe    1.0     27.910362     27.910362    2.432817   1.196386e-01
origin_Japan     1.0    145.497978    145.497978   12.682387   4.152294e-04
Residual       386.0   4428.363596     11.472445         NaN            NaN
```

```
identify_least_significant_feature(results, alpha=LOS_Alpha)
```

```
We find the least significant variable in this model is acceleration with a p-value of 0.0123
Using the backward methodology, we suggest dropping acceleration from the new model
```

```
cols.remove("acceleration")
formula = " + ".join(cols)
results = perform_analysis("mpg", formula, Auto_sqrt)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.811
Model:                            OLS   Adj. R-squared:                  0.809
Method:                 Least Squares   F-statistic:                     415.3
Date:                Sat, 28 Sep 2024   Prob (F-statistic):         1.50e-138
Time:                        04:09:31   Log-Likelihood:                -1034.6
No. Observations:                 392   AIC:                             2079.
Df Residuals:                     387   BIC:                             2099.
Df Model:                           4
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept        58.2668      1.753     33.230      0.000      54.819      61.714
oilshock          5.1556      0.362     14.244      0.000       4.444       5.867
sqrt_weight      -0.6999      0.029    -23.759      0.000      -0.758      -0.642
origin_Europe     1.6530      0.532      3.108      0.002       0.607       2.699
origin_Japan      1.8263      0.536      3.405      0.001       0.772       2.881
==============================================================================
Omnibus:                       31.883   Durbin-Watson:                   1.241
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               60.472
Skew:                           0.483   Prob(JB):                     7.39e-14
```

```
Kurtosis:                       4.664   Cond. No.                         574.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
                 df        sum_sq       mean_sq            F         PR(>F)
oilshock        1.0   6470.207217   6470.207217   556.341155   7.004177e-77
sqrt_weight     1.0  12674.256790  12674.256790  1089.796728  1.354903e-114
origin_Europe   1.0     38.907051     38.907051     3.345425   6.816142e-02
origin_Japan    1.0    134.840521    134.840521    11.594270   7.307851e-04
Residual      387.0   4500.781890     11.629927          NaN            NaN
```

```python
models.append(
    {
        "name": "sqrt_transformation",
        "model": results.model.formula,
        "R-squared adjusted": results.rsquared_adj,
    }
)
```

```python
pd.DataFrame(models)
```

	name	model
0	simple_model	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan
1	numeric_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + hor
2	oilshock_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + hor
3	origin_interactions	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + oils
4	squared_transformation	mpg ~ horsepower + weight + oilshock + origin_Europe + origin_Japan + I(h
5	log_transformation	mpg ~ acceleration + oilshock + log_weight + origin_Europe + origin_Japan
6	sqrt_transformation	mpg ~ oilshock + sqrt_weight + origin_Europe + origin_Japan

```python
allDone()
```

<IPython.lib.display.Audio object>