Applied : Auto dataset - Simple Linear Regression

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- Simple Linear Regression uitlizing Auto dataset

Import notebook functions

```
from notebookfuncs import *
```

Import standard libraries

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from matplotlib.pyplot import subplots
```

New imports

```
import statsmodels.api as sm
```

Import statsmodel.objects

```
from statsmodels.stats.outliers_influence import variance_inflation_factor as VIF
from statsmodels.stats.outliers_influence import summary_table
from statsmodels.stats.anova import anova_lm
```

Import ISLP objects

```
import ISLP
from ISLP import models
from ISLP import load_data
from ISLP.models import ModelSpec as MS, summarize, poly
```

```
Auto = load_data("Auto")
Auto.columns
```

```
Index(['mpg', 'cylinders', 'displacement', 'horsepower', 'weight',
       'acceleration', 'year', 'origin'],
      dtype='object')
```

```
Auto.shape
```

```
(392, 8)
```

```
Auto.describe()
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
count	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000
mean	23.445918	5.471939	194.411990	104.469388	2977.584184	15.541327	75.979592	1.576531
std	7.805007	1.705783	104.644004	38.491160	849.402560	2.758864	3.683737	0.805518
min	9.000000	3.000000	68.000000	46.000000	1613.000000	8.000000	70.000000	1.000000
25%	17.000000	4.000000	105.000000	75.000000	2225.250000	13.775000	73.000000	1.000000
50%	22.750000	4.000000	151.000000	93.500000	2803.500000	15.500000	76.000000	1.000000
75%	29.000000	8.000000	275.750000	126.000000	3614.750000	17.025000	79.000000	2.000000
max	46.600000	8.000000	455.000000	230.000000	5140.000000	24.800000	82.000000	3.000000

Convert cylinders and origin columns to categorical types

```
Auto["cylinders"] = Auto["cylinders"].astype("category")
Auto["origin"] = Auto["origin"].astype("category")
Auto.describe()
```

	mpg	displacement	horsepower	weight	acceleration	year
count	392.000000	392.000000	392.000000	392.000000	392.000000	392.000000
mean	23.445918	194.411990	104.469388	2977.584184	15.541327	75.979592
std	7.805007	104.644004	38.491160	849.402560	2.758864	3.683737
min	9.000000	68.000000	46.000000	1613.000000	8.000000	70.000000
25%	17.000000	105.000000	75.000000	2225.250000	13.775000	73.000000
50%	22.750000	151.000000	93.500000	2803.500000	15.500000	76.000000
75%	29.000000	275.750000	126.000000	3614.750000	17.025000	79.000000
max	46.600000	455.000000	230.000000	5140.000000	24.800000	82.000000

8) This question involves the use of Simple Linear Regression on the Auto dataset

(a) Use the sm.OLS() function to perform a simple linear regression with mpg as the response and horsepower as the predictor.

Use the summarize() function to print the results.

Comment on the output. For example:

i. Is there a relationship between the predictor and the response?

ii. How strong is the relationship between the predictor and the response?

iii. Is the relationship between the predictor and the response positive or negative?

iv. What is the predicted mpg associated with a horsepower of 98? What are the associated 95 % confidence and prediction intervals?

```
y = Auto["mpg"]
y.head()
```

```
name
chevrolet chevelle malibu    18.0
buick skylark 320            15.0
plymouth satellite           18.0
amc rebel sst                16.0
ford torino                  17.0
Name: mpg, dtype: float64
```

```
design = MS(["horsepower"])
design = design.fit(Auto)
X = design.transform(Auto)
```

	intercept	horsepower
name		
chevrolet chevelle malibu	1.0	130
buick skylark 320	1.0	165
plymouth satellite	1.0	150
amc rebel sst	1.0	150
ford torino	1.0	140
...	...	...
ford mustang gl	1.0	86
vw pickup	1.0	52
dodge rampage	1.0	84
ford ranger	1.0	79
chevy s-10	1.0	82

```
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P>|t|
intercept	39.9359	0.717	55.660	0.0
horsepower	-0.1578	0.006	-24.489	0.0

- There is evidence of a linear relationship between horespower and the response mpg.

```
results.summary()
```

Dep. Variable:	mpg	**R-squared:**	0.606
Model:	OLS	**Adj. R-squared:**	0.605
Method:	Least Squares	**F-statistic:**	599.7
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	7.03e-81
Time:	14:40:25	**Log-Likelihood:**	-1178.7
No. Observations:	392	**AIC:**	2361.
Df Residuals:	390	**BIC:**	2369.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
intercept	39.9359	0.717	55.660	0.000	38.525	41.347
horsepower	-0.1578	0.006	-24.489	0.000	-0.171	-0.145

Omnibus:	16.432	**Durbin-Watson:**	0.920
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.305
Skew:	0.492	**Prob(JB):**	0.000175
Kurtosis:	3.299	**Cond. No.**	322.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

- The R2 value of 60.6% indicates that the regression of horsepower on mpg explains 60.6% of the variation in the model.
- The relationship between horsepower and mpg is negative, i.e., an increase in hp of 1 unit decreases the mileage by 0.1578 miles. An increase in the car's output in power is offset by a decrease in its economy.

```
design = MS(["horsepower"])
new_df = pd.DataFrame({"horsepower": [98]})
design = design.fit(new_df)
newX = design.transform(new_df)
```

	intercept	horsepower
0	1.0	98

```
new_predictions = results.get_prediction(newX)
mileage = new_predictions.predicted_mean[0]
mileage
```

```
24.46707715251243
```

- The predicted mileage for a horsepower of 98 is 24.47 mpg.

```
new_predictions.conf_int(alpha=0.05)
```

```
array([[23.97307896, 24.96107534]])
```

- The 95% confidence interval is (23.97, 24.96)

```
new_predictions.conf_int(alpha=0.05, obs=True)
```

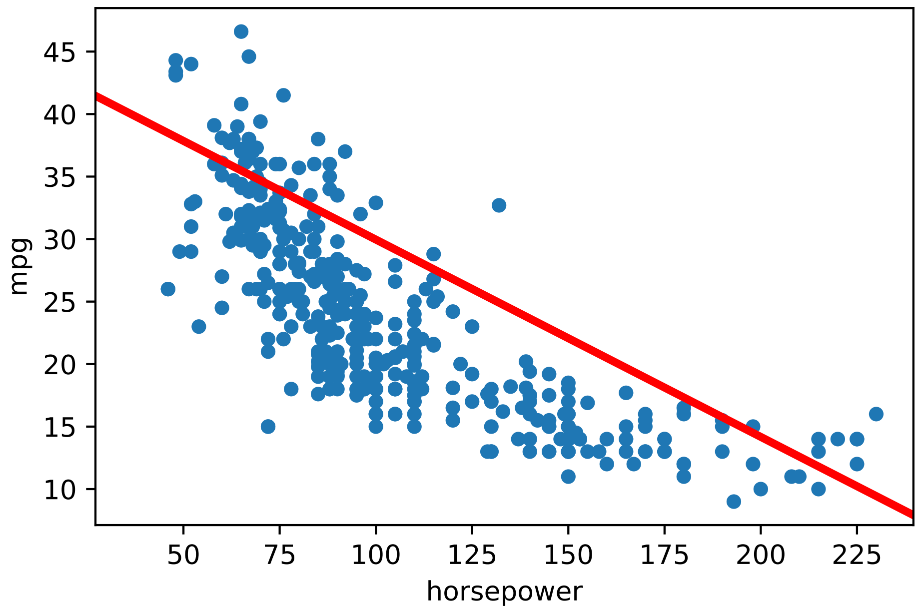
```
array([[14.80939607, 34.12475823]])
```

- The 95% prediction interval is (14.82, 34.13)

(b) Plot the response and the predictor in a new set of axes ax.

Use the ax.axline() method or the abline() function defined in the lab to display the least squares regression line.

```
ax = Auto.plot.scatter("horsepower", "mpg")
ax.axline(
    (ax.get_xlim()[0], results.params.iloc[0]),
    slope=results.params.iloc[1],
    color="r",
    linewidth=3,
)
```



- The least squares regression line is plotted above using ax.axline(). The plot displays some evidence of non-linearity in the relationship between horsepower and mpg.

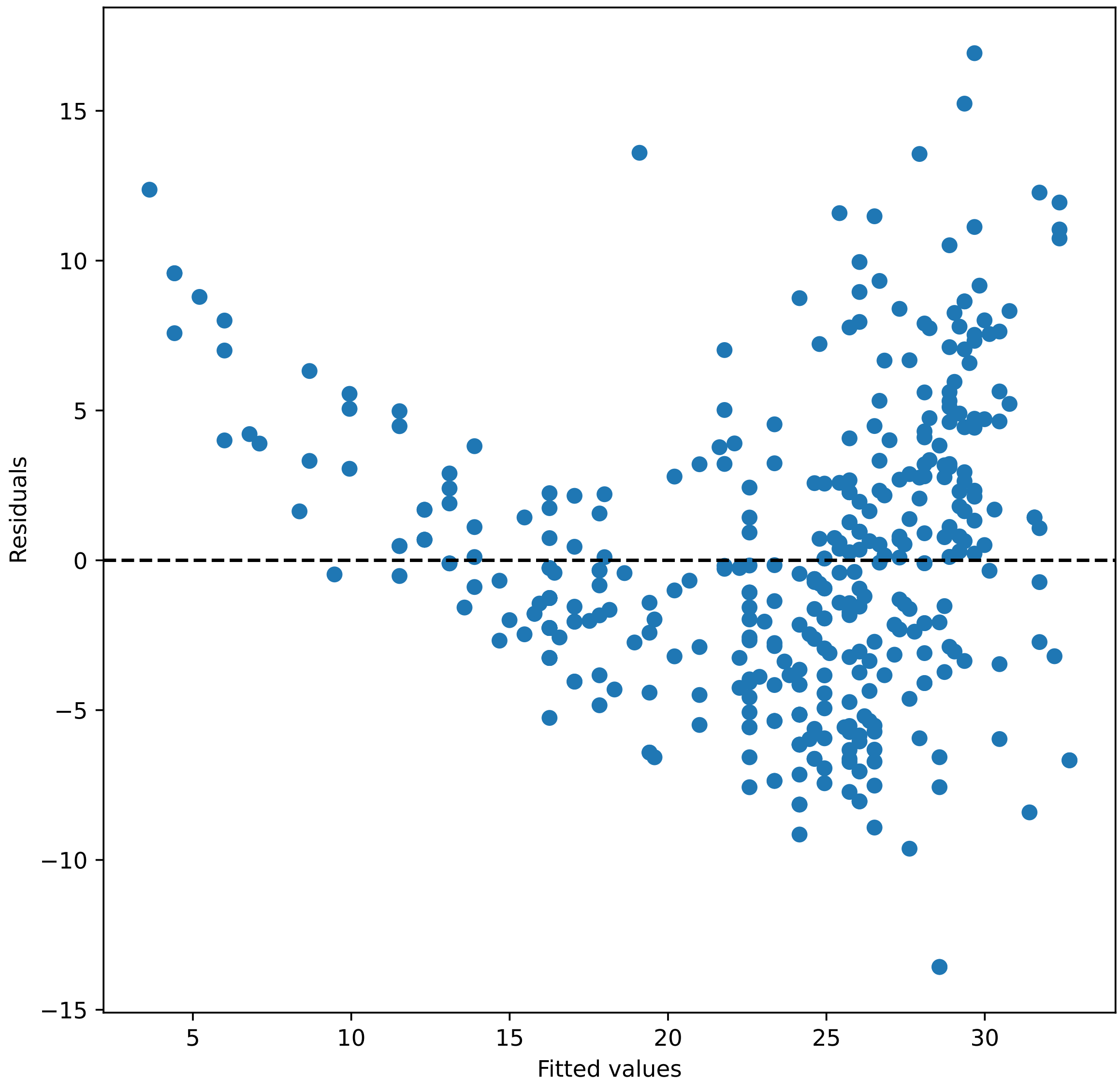
(c) Produce some of diagnostic plots of the least squares regression fit as described in the lab.

Comment on any problems you see with the fit.

Plot of fitted values versus residuals.

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(results.fittedvalues, results.resid)
ax.set_xlabel("Fitted values")
```

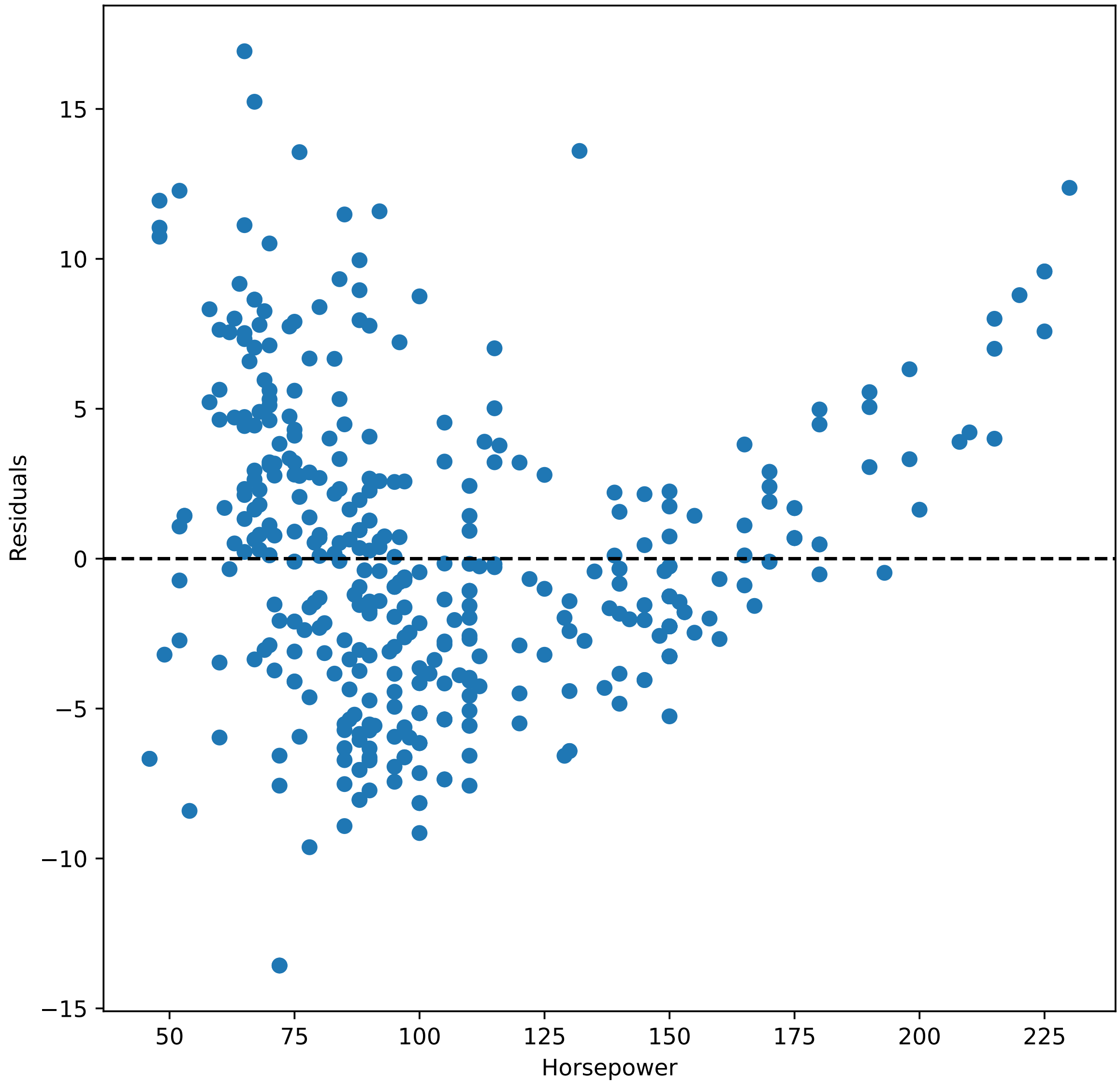
```
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



We can also plot the residuals vs predictor plot where horsepower is the predictor.

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(Auto["horsepower"], results.resid)
ax.set_xlabel("Horsepower")
```

```
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



Conclusions:

- There is evidence of non-linearity in the relationship between residuals and fitted values.
- There is evidence of heteroskedasticity i.e., non-constant variance in the residuals across the fitted values.

```
RSS = np.sum((y - results.fittedvalues) ** 2)
RSS
```

```
9385.915871932419
```

```
RSE = np.sqrt(RSS / (Auto.shape[0] - 2))
RSE
```

```
4.90575691954594
```

OLSResults.scale()

- Gives us a scale factor for the covariance matrix.
- The Default value is ssr/(n-p). Note that the square root of scale is often called the standard error of the regression.
- https://www.statsmodels.org/dev/generated/statsmodels.regression.linear_model.OLSResults.scale.html

```
np.sqrt(results.scale)
```

```
4.90575691954594
```

```
mpg_mean = Auto["mpg"].mean()
```

```
23.445918367346938
```

```
print("Percentage error in mpg estimation using model above is: ")
np.round(RSE / mpg_mean * 100, decimals=2)
```

```
Percentage error in mpg estimation using model above is:
```

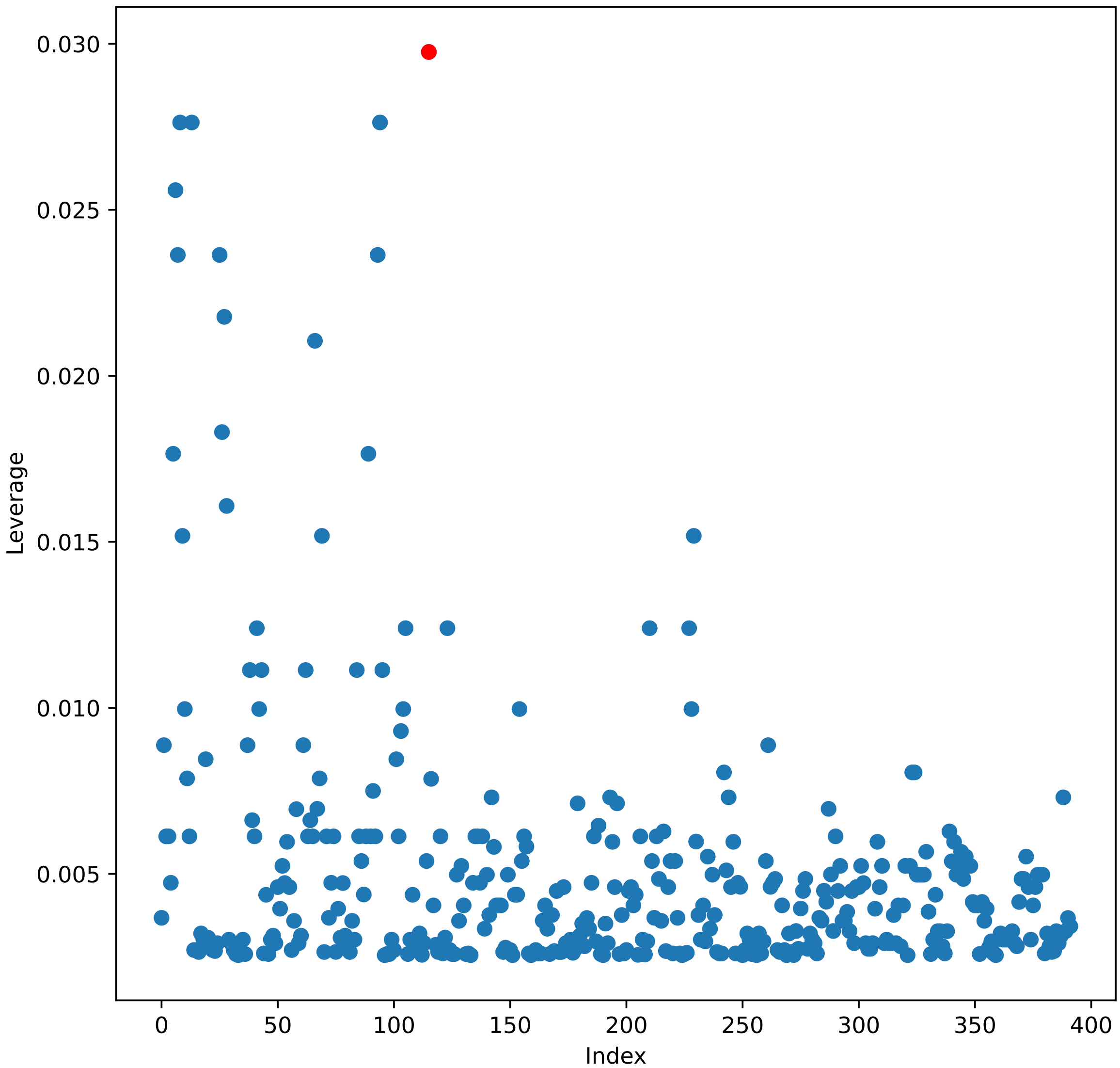
```
20.92
```

Leverage statistics

```
infl = results.get_influence()
_, ax = subplots(figsize=(8, 8))
ax.scatter(np.arange(X.shape[0]), infl.hat_matrix_diag)
ax.set_xlabel("Index")
ax.set_ylabel("Leverage")
high_leverage = np.argmax(infl.hat_matrix_diag)
max_leverage = np.max(infl.hat_matrix_diag)
print("Max leverage point:")
print(high_leverage, np.round(max_leverage, decimals=2))
ax.plot(high_leverage, max_leverage, "ro")
```

```
Max leverage point:
```

```
115 0.03
```



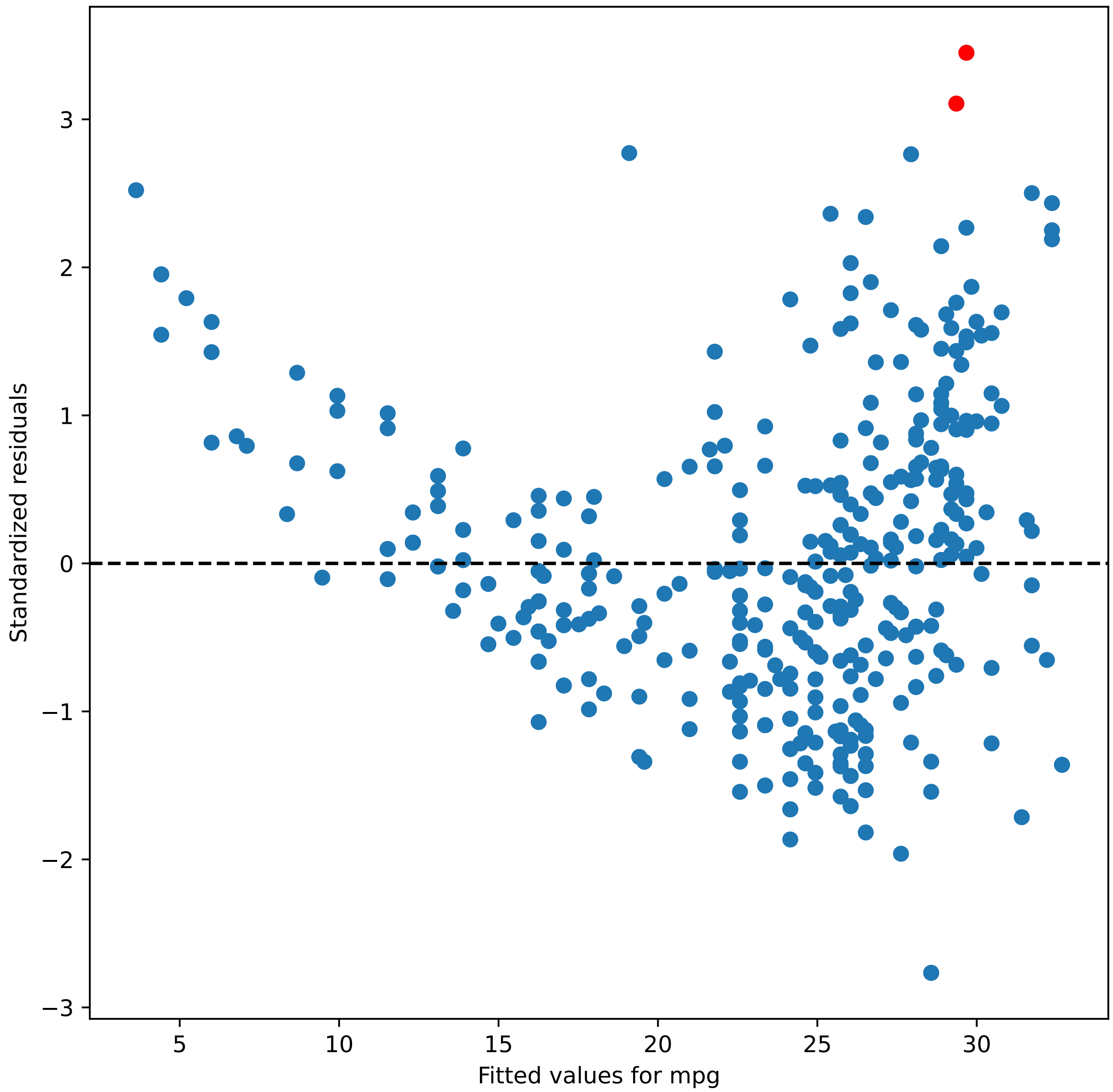
Outlier identification using Standardized Residuals versus Fitted Values plot

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(results.fittedvalues, results.resid_pearson)
ax.set_xlabel("Fitted values for mpg")
ax.set_ylabel("Standardized residuals")
ax.axhline(0, c="k", ls="--")
outliers_indexes = np.where(
```

```
    (results.resid_pearson > 3.0) | (results.resid_pearson < -3.0)
)[0]
for idx in range(len(outliers_indexes)):
    ax.plot(
        results.fittedvalues.iloc[outliers_indexes[idx]],
        results.resid_pearson[outliers_indexes[idx]],
        "ro",
    )
print("Outlier rows: ")
print(Auto.iloc[outliers_indexes])
```

```
Outlier rows:
                      mpg  cylinders  displacement  horsepower  weight  \
name
mazda glc            46.6          4          86.0          65    2110
honda civic 1500 gl  44.6          4          91.0          67    1850

                     acceleration  year  origin
name
mazda glc                    17.9    80       3
honda civic 1500 gl          13.8    80       3
```

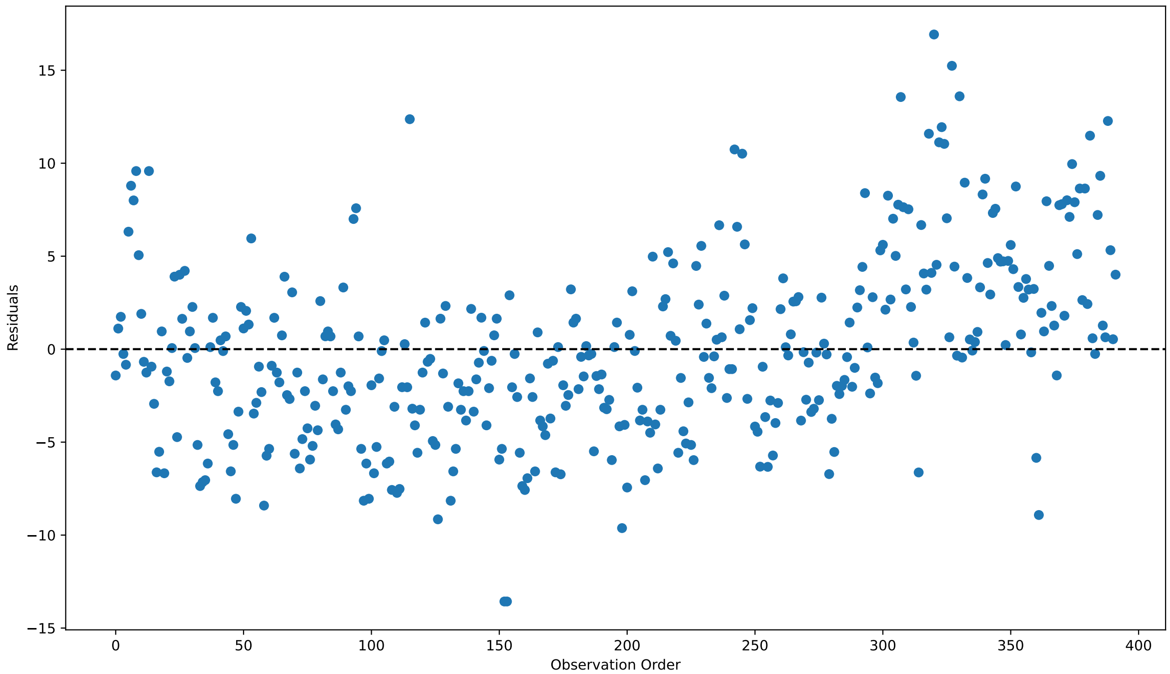


Conclusions: + From the standardized residuals versus fitted values, there are two outliers present in the data. + These points can be investigated further whether to retain them in the dataset.

Note: - We could drop the outliers from the data and regress the model without these points. That is an exercise for you!

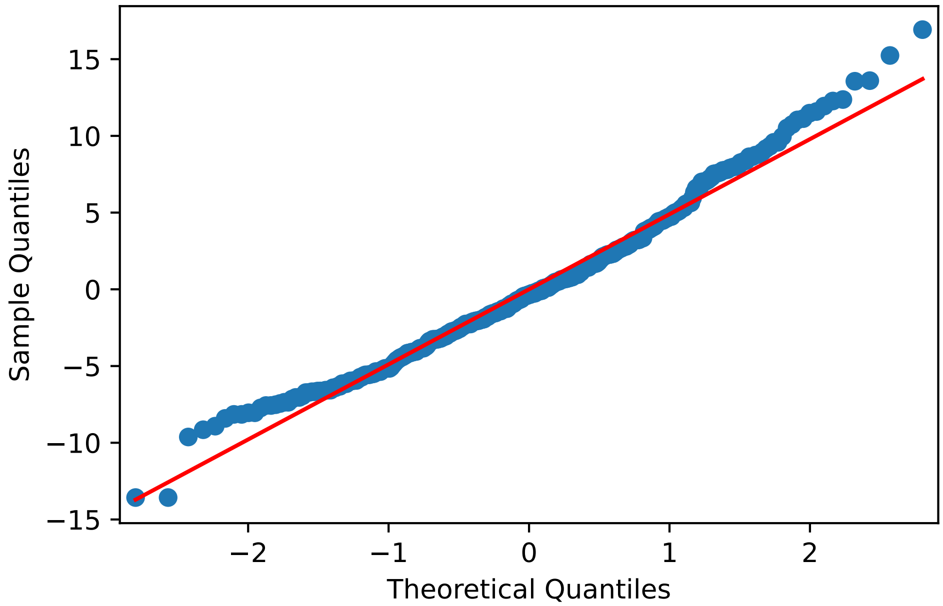
We can also plot residuals versus order.

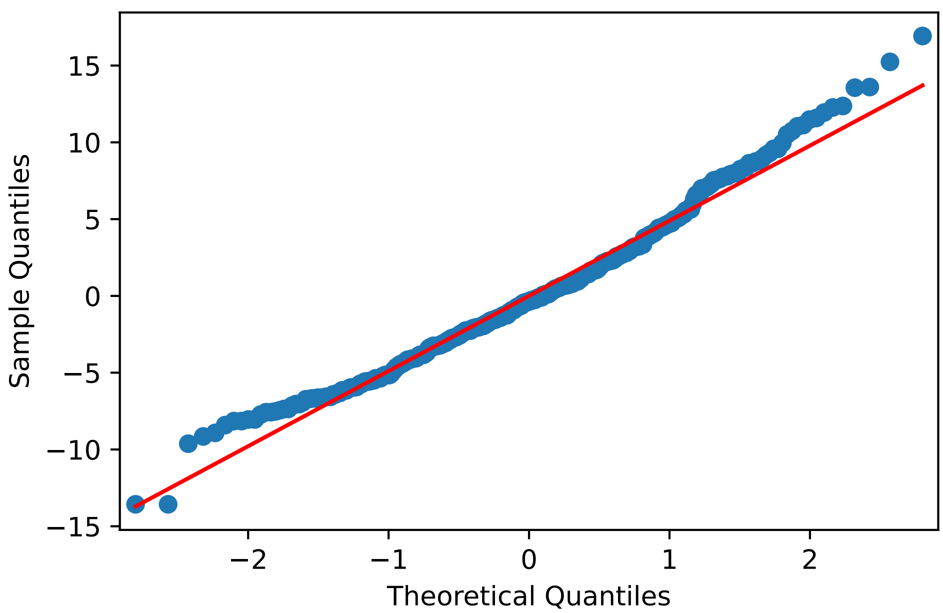
```python
_, ax = subplots(figsize=(14, 8))
ax.scatter(np.arange(X.shape[0]), results.resid)
ax.set_xlabel("Observation Order")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



Conclusions: - While there seems to be little evidence of negative or positive correlation over time, there is evidence of underestimation from observations 300 onwards. There also seems to be a time trend in the data from observation 300 or so where the expectation of the model is that mpg will be lower, but the actual values are much higher. This indicates that fuel mileage improved much more than expected in the later models from observation 300 onwards. This indicates that column year should be added to the model.

```python
sm.qqplot(results.resid, line="s")
```

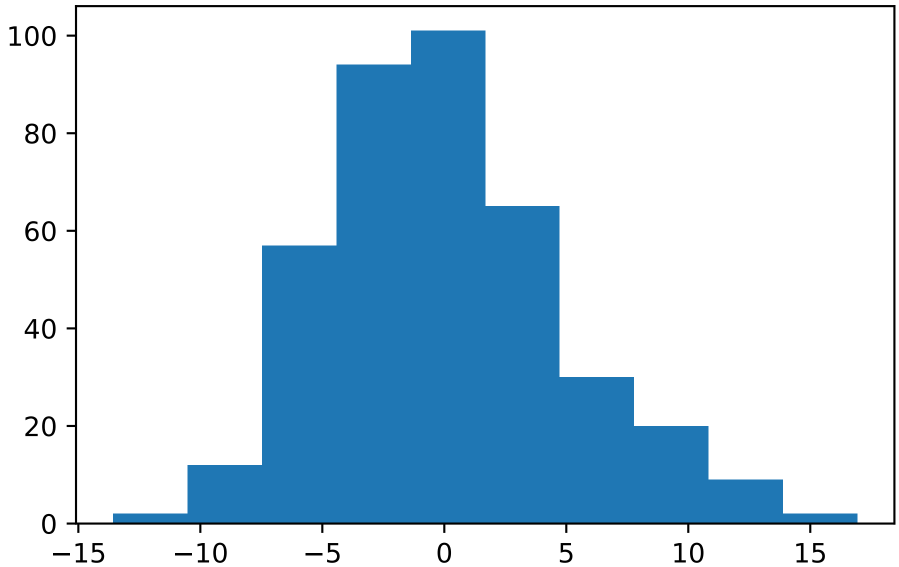





```
# Plot histogram of residuals
plt.hist(results.resid, bins=10)
```

```
(array([  2.,  12.,  57.,  94., 101.,  65.,  30.,  20.,   9.,   2.]),
 array([-13.57104022, -10.52153153,  -7.47202285,  -4.42251416,
```

```
        -1.37300547,   1.67650321,   4.7260119 ,   7.77552059,
        10.82502927,  13.87453796,  16.92404665]),
 <BarContainer object of 10 artists>)
```



Conclusions: - From the above two plots for qq and histograms for residuals, we can deduce that the residuals are approximately normal.

References: https://github.com/linusjf/LearnR/tree/development/Stats462

```
allDone()
```

```
<IPython.lib.display.Audio object>
```