Marketing Exercise

Table of contents

Import notebook functions

```
from notebookfuncs import *
```

Import standard libraries

```
import numpy as np
import pandas as pd
from matplotlib.pyplot import subplots
```

New imports

```
import statsmodels.api as sm
```

Import statsmodel.objects

```
from statsmodels.stats.outliers_influence import variance_inflation_factor as VIF
from statsmodels.stats.outliers_influence import summary_table
from statsmodels.stats.anova import anova_lm
```

Import ISLP objects

```
import ISLP
from ISLP import models
from ISLP import load_data
from ISLP.models import ModelSpec as MS, summarize, poly
```

Inspecting objects and namespaces

```
dir()
```

```
['Audio',
 'ISLP',
 'In',
 'InteractiveShell',
 'Latex',
 'MS',
 'Markdown',
 'Math',
 'Out',
 'VIF',
 '__builtin__',
 '__builtins__',
 '__name__',
 '__spec__',
 '_dh',
 '_i',
 '_i1',
 '_i2',
 '_i3',
 '_i4',
 '_i5',
 '_i6',
```

```
 '_ih',
 '_ii',
 '_iii',
 '_oh',
 'allDone',
 'anova_lm',
 'display',
 'exit',
 'get_ipython',
 'load_data',
 'models',
 'np',
 'ojs_define',
 'open',
 'pd',
 'poly',
 'printlatex',
 'printmd',
 'quit',
 'sm',
 'subplots',
 'summarize',
 'summary_table']
```

```
Advertising = pd.read_csv("Advertising.csv")
# Drop first column
Advertising = Advertising.iloc[:, 1:]
Advertising.head()
```

	TV	Radio	Newspaper	Sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	9.3
3	151.5	41.3	58.5	18.5
4	180.8	10.8	58.4	12.9

```
Advertising.describe()
```

	TV	Radio	Newspaper	Sales
count	200.000000	200.000000	200.000000	200.000000
mean	147.042500	23.264000	30.554000	14.022500
std	85.854236	14.846809	21.778621	5.217457
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	10.375000

	TV	Radio	Newspaper	Sales
50%	149.750000	22.900000	25.750000	12.900000
75%	218.825000	36.525000	45.100000	17.400000
max	296.400000	49.600000	114.000000	27.000000

Is there a relationship between sales and advertising budget?

```
y = Advertising["Sales"]
cols = list(Advertising.columns)
cols.remove("Sales")
X = MS(cols).fit_transform(Advertising)
model = sm.OLS(y, X)
results = model.fit()
print("F-value", results.fvalue)
print("F-pvalue", results.f_pvalue)
summarize(results)
```

```
F-value 570.2707036590944
F-pvalue 1.575227256092416e-96
```

	coef	std err	t	P>|t|
intercept	2.9389	0.312	9.422	0.00
TV	0.0458	0.001	32.809	0.00
Radio	0.1885	0.009	21.893	0.00
Newspaper	-0.0010	0.006	-0.177	0.86

```
dir(models)
```

```
['Column',
 'Feature',
 'FeatureSelector',
 'ModelSpec',
 'Stepwise',
 'StringIO',
 '__builtins__',
 '__cached__',
 '__doc__',
 '__file__',
 '__loader__',
 '__name__',
 '__package__',
 '__path__',
```

```
 '__spec__',
 'bs',
 'build_columns',
 'columns',
 'contrast',
 'derived_feature',
 'generic_selector',
 'min_max_strategy',
 'model_spec',
 'np',
 'ns',
 'pca',
 'pd',
 'poly',
 'sklearn_selected',
 'sklearn_selection_path',
 'sklearn_sm',
 'sklearn_wrap',
 'strategy',
 'summarize']
```

- The p-value corresponding to the F-statistic is very low. Thus, clear evidence of a relationship between sales and advertising budget.

```
dir(results)
```

```
['HC0_se',
 'HC1_se',
 'HC2_se',
 'HC3_se',
 '_HCCM',
 '__class__',
 '__delattr__',
 '__dict__',
 '__dir__',
 '__doc__',
 '__eq__',
 '__format__',
 '__ge__',
 '__getattribute__',
 '__getstate__',
 '__gt__',
 '__hash__',
 '__init__',
 '__init_subclass__',
 '__le__',
 '__lt__',
```

```
'__module__',
'__ne__',
'__new__',
'__reduce__',
'__reduce_ex__',
'__repr__',
'__setattr__',
'__sizeof__',
'__str__',
'__subclasshook__',
'__weakref__',
'_abat_diagonal',
'_cache',
'_data_attr',
'_data_in_cache',
'_get_robustcov_results',
'_get_wald_nonlinear',
'_is_nested',
'_transform_predict_exog',
'_use_t',
'_wexog_singular_values',
'aic',
'bic',
'bse',
'centered_tss',
'compare_f_test',
'compare_lm_test',
'compare_lr_test',
'condition_number',
'conf_int',
'conf_int_el',
'cov_HC0',
'cov_HC1',
'cov_HC2',
'cov_HC3',
'cov_kwds',
'cov_params',
'cov_type',
'df_model',
'df_resid',
'diagn',
'eigenvals',
'el_test',
'ess',
'f_pvalue',
'f_test',
```

```
 'fittedvalues',
 'fvalue',
 'get_influence',
 'get_prediction',
 'get_robustcov_results',
 'info_criteria',
 'initialize',
 'k_constant',
 'llf',
 'load',
 'model',
 'mse_model',
 'mse_resid',
 'mse_total',
 'nobs',
 'normalized_cov_params',
 'outlier_test',
 'params',
 'predict',
 'pvalues',
 'remove_data',
 'resid',
 'resid_pearson',
 'rsquared',
 'rsquared_adj',
 'save',
 'scale',
 'ssr',
 'summary',
 'summary2',
 't_test',
 't_test_pairwise',
 'tvalues',
 'uncentered_tss',
 'use_t',
 'wald_test',
 'wald_test_terms',
 'wresid']
```

How strong is the relationship?

```
results.summary()
```

Dep. Variable:	Sales	**R-squared:**	0.897
Model:	OLS	**Adj. R-squared:**	0.896
Method:	Least Squares	**F-statistic:**	570.3
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	1.58e-96
Time:	14:39:30	**Log-Likelihood:**	-386.18
No. Observations:	200	**AIC:**	780.4
Df Residuals:	196	**BIC:**	793.6
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
intercept	2.9389	0.312	9.422	0.000	2.324	3.554
TV	0.0458	0.001	32.809	0.000	0.043	0.049
Radio	0.1885	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.0010	0.006	-0.177	0.860	-0.013	0.011

Omnibus:	60.414	**Durbin-Watson:**	2.084
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	151.241
Skew:	-1.327	**Prob(JB):**	1.44e-33
Kurtosis:	6.332	**Cond. No.**	454.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
y.mean()
```

```
14.0225
```

```
results.resid.std()
```

```
1.6727572743844117
```

```
(results.resid.std() / y.mean()) * 100
```

```
11.929094486606608
```

- The residual standard error (RSE) is 1.67 and the mean value of the response is 14.023 which translates to a percentage error of roughly 11.93%

```
("R-squared", results.rsquared, "Adjusted R-squared", results.rsquared_adj)
```

```
('R-squared', 0.8972106381789522, 'Adjusted R-squared', 0.8956373316204668)
```

- The R2 explains about 90% of the variance in Sales.

Which media are associated with Sales?

- The low p-values for Radio and TV suggest that only they are related to Sales.

How large is the association between each medium and sales?

```
results.conf_int(alpha=0.05)
```

	0	1
intercept	2.323762	3.554016
TV	0.043014	0.048516
Radio	0.171547	0.205513
Newspaper	-0.012616	0.010541

- The confidence intervals for TV and Radio are narrow and far from zero. This provides evidence that these media are related to sales.
- The interval for Newspaper includes zero indicating that it is not statistically significant given values of TV and Radio.

```
vals = [VIF(X, i) for i in range(1, X.shape[1])]
print(vals)
```

```
[1.00461078493965, 1.1449519171055353, 1.1451873787239288]
```

- The VIF scores are 1.005, 1.145 and 1.145 respectively for TV, radio and newspaper. These suggest no evidence of collinearity as an explnation for wide standard errors for newspaper.
- In order to assess the association of each medium individually on sales, we can perform three separate linear regressions.

```
TV = MS(["TV"]).fit_transform(Advertising)
model = sm.OLS(y, TV)
results = model.fit()
print(summarize(results))
Radio = MS(["Radio"]).fit_transform(Advertising)
model = sm.OLS(y, Radio)
results = model.fit()
print(summarize(results))
Newspaper = MS(["Newspaper"]).fit_transform(Advertising)
model = sm.OLS(y, Newspaper)
results = model.fit()
print(summarize(results))
```

```
             coef  std err       t  P>|t|
intercept  7.0326    0.458  15.360    0.0
TV         0.0475    0.003  17.668    0.0
             coef  std err       t  P>|t|
intercept  9.3116    0.563  16.542    0.0
Radio      0.2025    0.020   9.921    0.0
              coef  std err       t  P>|t|
```

```
intercept  12.3514    0.621  19.876  0.000
Newspaper   0.0547    0.017   3.300  0.001
```

Looking at the p-values, there is evidence of a strong association b/w TV and sales and radio and sales. There is evidence of a mild association between Newspaper and sales when TV and radio are ignored.

How accurately can we predict future sales?

- Given that \$100,000 is spent on TV advertising, and \$20,000 is spent on Radio advertising, we need to compute the 95% Confidence intervals for each city (i.e., the mean) and the prediction interval for a particular city (also at 95% confidence intervals).

Fit the regression dropping the Newspaper column as insignificant

```
y = Advertising["Sales"]
cols = list(Advertising.columns)
cols.remove("Sales")
cols.remove("Newspaper")
X = MS(cols).fit_transform(Advertising)
model = sm.OLS(y, X)
results = model.fit()
print("F-value", results.fvalue)
print("F-pvalue", results.f_pvalue)
summarize(results)
```

```
F-value 859.6177183058211
F-pvalue 4.8273618513354486e-98
```

	coef	std err	t	P>|t|
intercept	2.9211	0.294	9.919	0.0
TV	0.0458	0.001	32.909	0.0
Radio	0.1880	0.008	23.382	0.0

```
results.summary()
```

Dep. Variable:	Sales	**R-squared:**	0.897
Model:	OLS	**Adj. R-squared:**	0.896
Method:	Least Squares	**F-statistic:**	859.6
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	4.83e-98
Time:	14:39:30	**Log-Likelihood:**	-386.20
No. Observations:	200	**AIC:**	778.4
Df Residuals:	197	**BIC:**	788.3
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P> |t|	[0.025	0.975]
intercept	2.9211	0.294	9.919	0.000	2.340	3.502
TV	0.0458	0.001	32.909	0.000	0.043	0.048
Radio	0.1880	0.008	23.382	0.000	0.172	0.204

Omnibus:	60.022	**Durbin-Watson:**	2.081
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	148.679
Skew:	-1.323	**Prob(JB):**	5.19e-33
Kurtosis:	6.292	**Cond. No.**	425.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
design = MS(["TV", "Radio"])
new_df = pd.DataFrame({"TV": [100], "Radio": [20]})
print(new_df)
new_X = design.fit_transform(new_df)
new_predictions = results.get_prediction(new_X)
new_predictions.predicted_mean
```

```
    TV  Radio
0  100     20
```

```
array([11.25646595])
```

We predict the confidence intervals at 95% as follows:

```
new_predictions.conf_int(alpha=0.05)
```

```
array([[10.98525445, 11.52767746]])
```

We predict the prediction interval for a particular city as follows:

```
new_predictions.conf_int(alpha=0.05, obs=True)
```

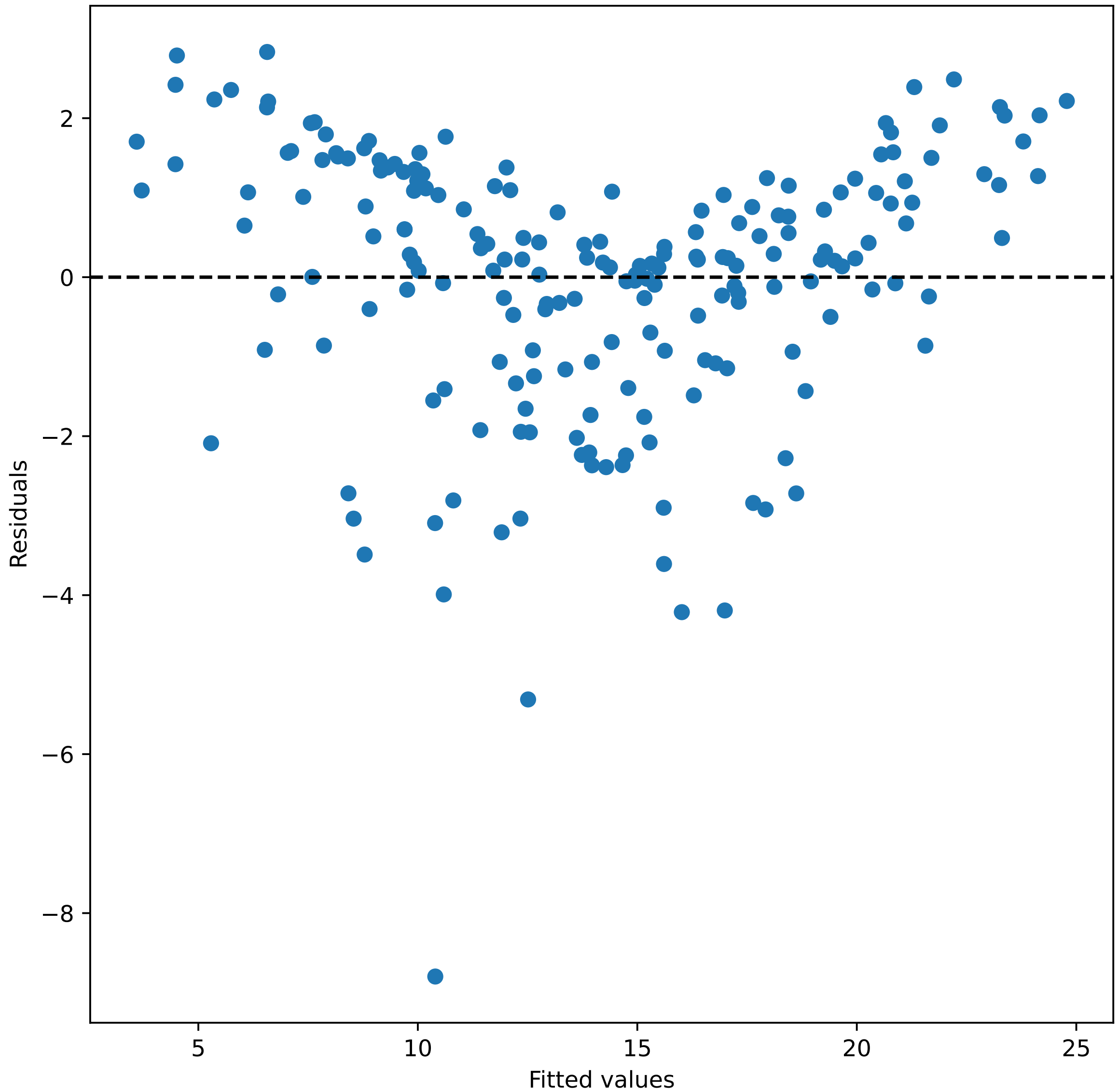
```
array([[ 7.92961607, 14.58331584]])
```

- Both intervals are centered at 11,256 but the prediction intervals are wider reflecting the additional uncertainty around sales for a particular city as against the average sales for many locations.

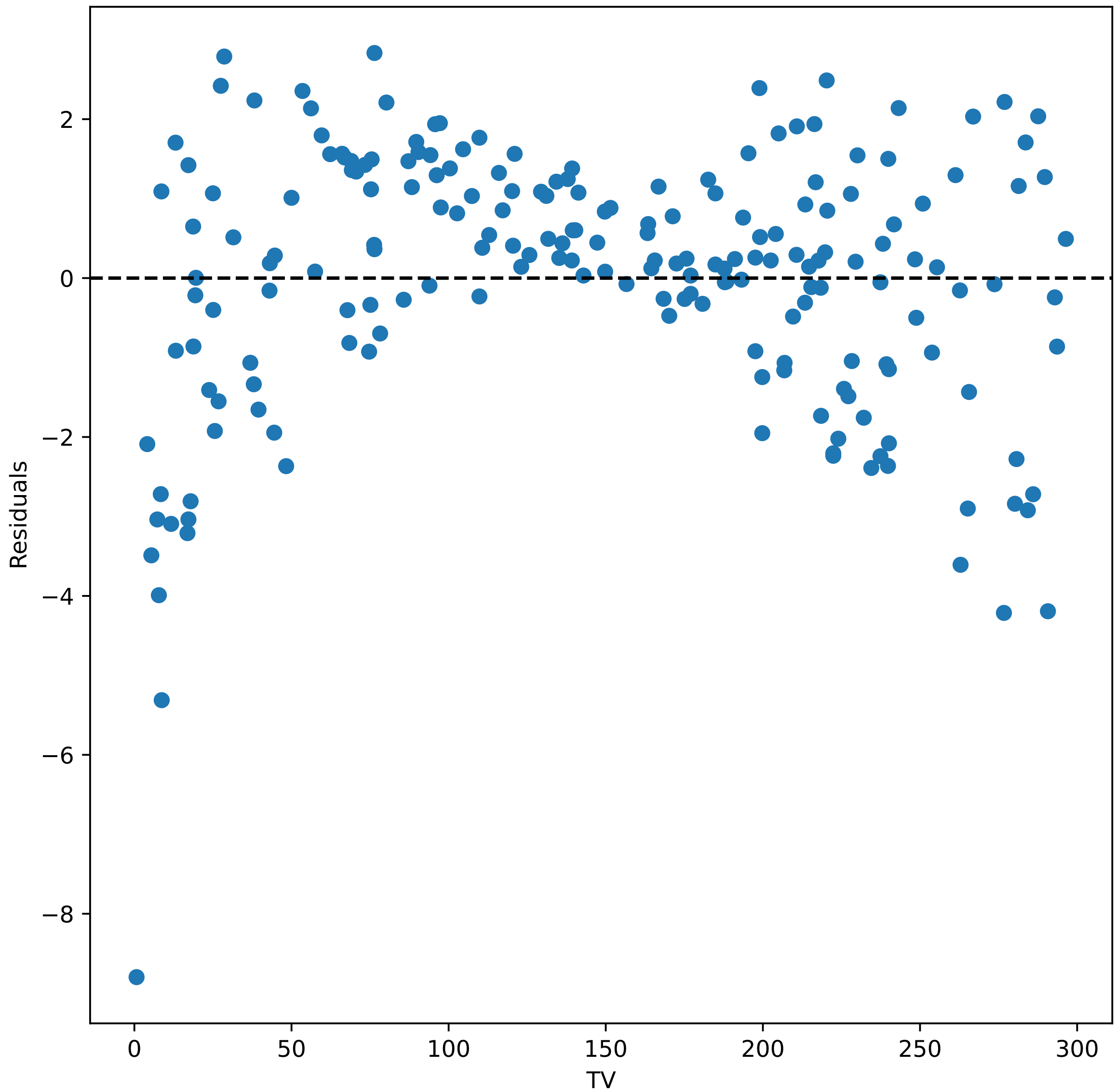
Is the relationship linear?

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(results.fittedvalues, results.resid)
ax.set_xlabel("Fitted values")
```

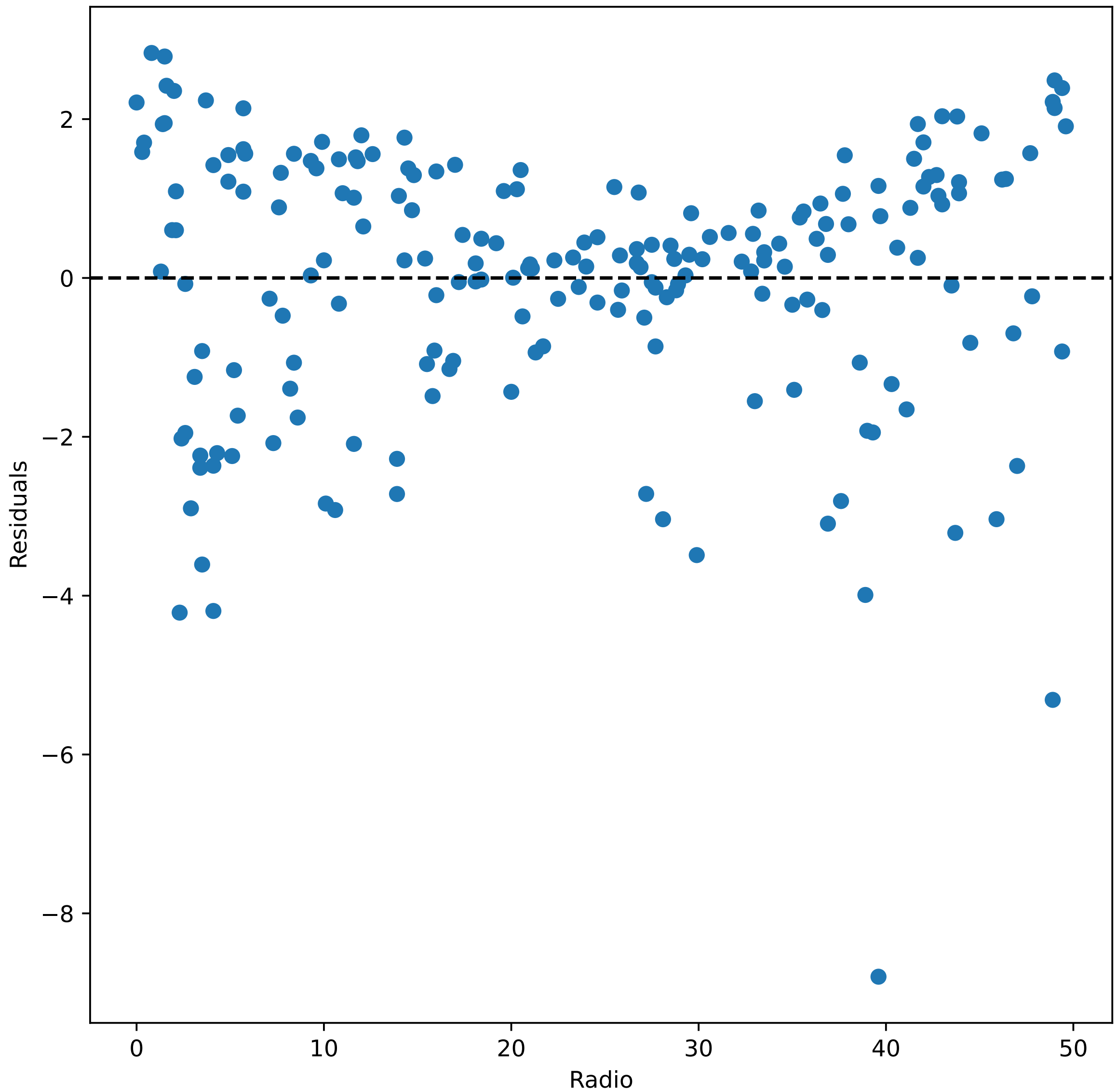
```
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```




```
_, ax = subplots(figsize=(8, 8))
ax.scatter(Advertising["TV"], results.resid)
ax.set_xlabel("TV")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```




```
_, ax = subplots(figsize=(8, 8))
ax.scatter(Advertising["Radio"], results.resid)
ax.set_xlabel("Radio")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



- There is evidence of non-linearity in the model from the residuals plotted against the fitted values. Looking at the residuals versus predictors plots, it appears that TV is a better candidate for quadratification.

```
X = MS([poly("TV", degree=2, raw=True), "Radio"]).fit_transform(Advertising)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P>|t|
intercept	1.2876	0.359000	3.588	0.0
poly(TV, degree=2, raw=True)[0]	0.0784	0.005000	15.736	0.0
poly(TV, degree=2, raw=True)[1]	-0.0001	0.000017	-6.775	0.0
Radio	0.1930	0.007000	26.465	0.0

```
results.summary()
```

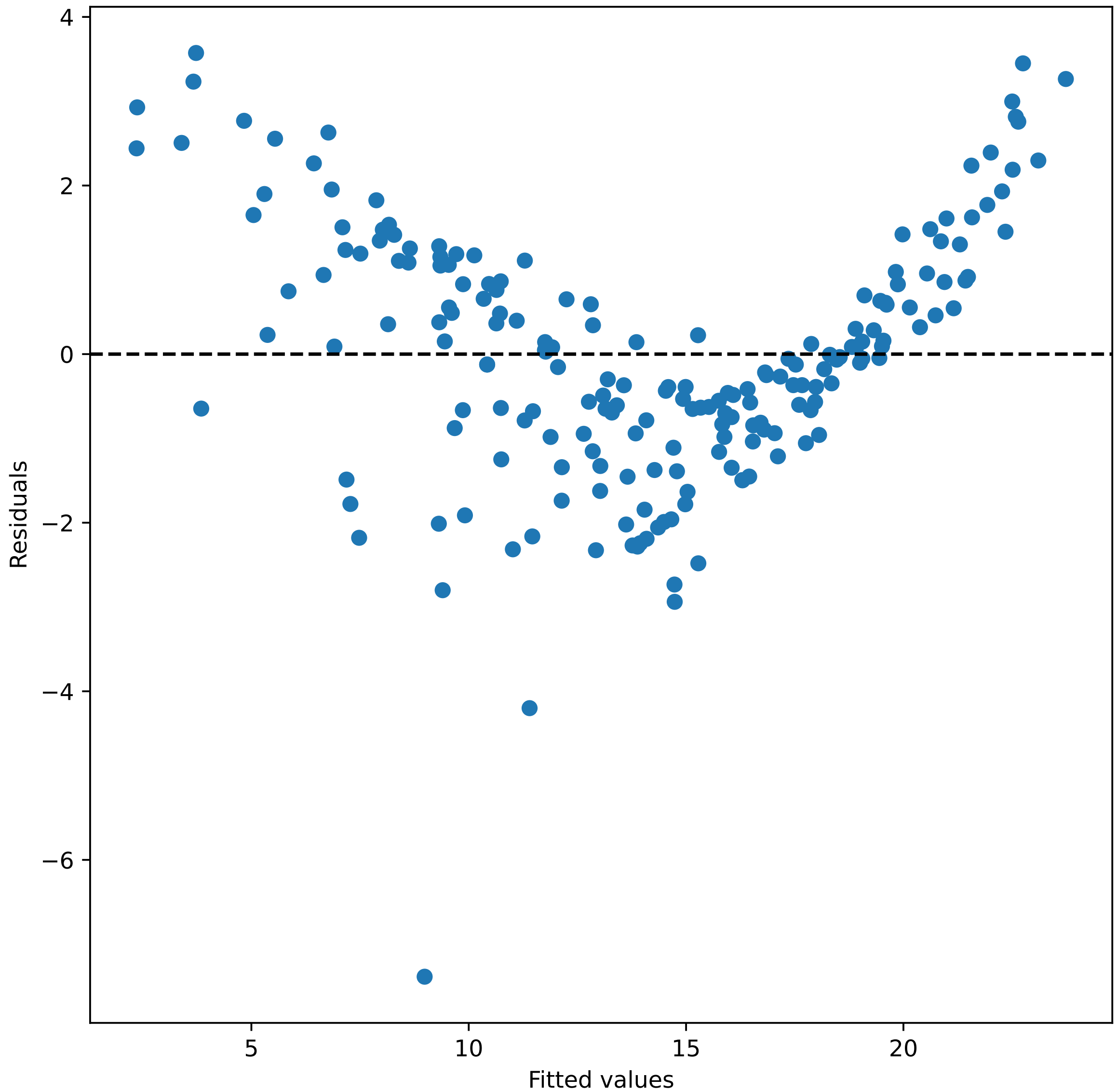
Dep. Variable:	Sales	**R-squared:**	0.917
Model:	OLS	**Adj. R-squared:**	0.915
Method:	Least Squares	**F-statistic:**	719.0
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	1.80e-105
Time:	14:39:31	**Log-Likelihood:**	-365.16
No. Observations:	200	**AIC:**	738.3
Df Residuals:	196	**BIC:**	751.5
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
intercept	1.2876	0.359	3.588	0.000	0.580	1.995
poly(TV, degree=2, raw=True)[0]	0.0784	0.005	15.736	0.000	0.069	0.088
poly(TV, degree=2, raw=True)[1]	-0.0001	1.68e-05	-6.775	0.000	-0.000	-8.05e-05
Radio	0.1930	0.007	26.465	0.000	0.179	0.207

Omnibus:	19.524	**Durbin-Watson:**	2.136
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	44.712
Skew:	-0.413	**Prob(JB):**	1.95e-10
Kurtosis:	5.164	**Cond. No.**	1.29e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.29e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

```
_, ax = subplots(figsize=(8, 8))
ax.scatter(results.fittedvalues, results.resid)
ax.set_xlabel("Fitted values")
ax.set_ylabel("Residuals")
ax.axhline(0, c="k", ls="--")
```



While the fit has improved as seen from the R2 increasing by 2 percentage points, there is still some non-linearity visible in the residuals plot against fitted values.

References:

https://www.kellogg.northwestern.edu/faculty/weber/emp/_session_3/nonlinearities.htm

https://online.stat.psu.edu/stat462/node/120/

Is there synergy among the advertising media?

Synergy implies an interaction effect. That's what we test out now.

```
X = MS([poly("TV", raw=True, degree=2), "Radio", ("TV", "Radio")]).fit_transform(
    Advertising
)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P>|t|
intercept	5.1371	0.193000	26.663	0.0
poly(TV, degree=2, raw=True)[0]	0.0509	0.002000	22.810	0.0
poly(TV, degree=2, raw=True)[1]	-0.0001	0.000007	-15.920	0.0
Radio	0.0352	0.006000	5.959	0.0
TV:Radio	0.0011	0.000035	31.061	0.0

```
results.summary()
```

Dep. Variable:	Sales	**R-squared:**	0.986
Model:	OLS	**Adj. R-squared:**	0.986
Method:	Least Squares	**F-statistic:**	3432.
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	1.79e-179
Time:	14:39:32	**Log-Likelihood:**	-186.86
No. Observations:	200	**AIC:**	383.7
Df Residuals:	195	**BIC:**	400.2
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
intercept	5.1371	0.193	26.663	0.000	4.757	5.517
poly(TV, degree=2, raw=True)[0]	0.0509	0.002	22.810	0.000	0.047	0.055
poly(TV, degree=2, raw=True)[1]	-0.0001	6.89e-06	-15.920	0.000	-0.000	-9.61e-05
Radio	0.0352	0.006	5.959	0.000	0.024	0.047
TV:Radio	0.0011	3.47e-05	31.061	0.000	0.001	0.001

Omnibus:	169.759	**Durbin-Watson:**	2.204
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4031.167
Skew:	-2.988	**Prob(JB):**	0.00
Kurtosis:	24.166	**Cond. No.**	1.70e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.7e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

- Finally, when we add an interaction term TV * Radio to the model, we can see that the residual fit exhibits no pattern. And the R2 is 98.6%.

Compute VIFs and List Comprehension

```
vals = [VIF(X, i) for i in range(1, X.shape[1])]
print(vals)
```

```
[18.787830609925035, 15.885268501061871, 3.9253174186837008, 6.940088238444382]
```

```
vif = pd.DataFrame({"vif": vals}, index=X.columns[1:])
print(vif)
("VIF Range:", np.min(vif), np.max(vif))
```

```
                                    vif
poly(TV, degree=2, raw=True)[0]  18.787831
poly(TV, degree=2, raw=True)[1]  15.885269
Radio                             3.925317
TV:Radio                          6.940088
```

```
('VIF Range:', 3.9253174186837008, 18.787830609925035)
```

- The VIF ranges are high. These can be reduced by transforming variables to mean 0.
- https://stats.stackexchange.com/questions/23538/quadratic-term-and-variance-inflation-factor-in-ols-estimation

```
Advertising["TV"] = Advertising["TV"] - Advertising["TV"].mean()
Advertising["Radio"] = Advertising["Radio"] - Advertising["Radio"].mean()
```

```
X = MS([poly("TV", raw=True, degree=2), "Radio", ("TV", "Radio")]).fit_transform(
    Advertising
)
model = sm.OLS(y, X)
results = model.fit()
summarize(results)
```

	coef	std err	t	P>|t|
intercept	14.7525	0.067000	219.634	0.0
poly(TV, degree=2, raw=True)[0]	0.0437	0.001000	84.111	0.0
poly(TV, degree=2, raw=True)[1]	-0.0001	0.000007	-15.920	0.0
Radio	0.1935	0.003000	64.526	0.0
TV:Radio	0.0011	0.000035	31.061	0.0

```
results.summary()
```

Dep. Variable:	Sales	**R-squared:**	0.986
Model:	OLS	**Adj. R-squared:**	0.986
Method:	Least Squares	**F-statistic:**	3432.
Date:	Tue, 25 Feb 2025	**Prob (F-statistic):**	1.79e-179
Time:	14:39:32	**Log-Likelihood:**	-186.86
No. Observations:	200	**AIC:**	383.7
Df Residuals:	195	**BIC:**	400.2
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
intercept	14.7525	0.067	219.634	0.000	14.620	14.885
poly(TV, degree=2, raw=True)[0]	0.0437	0.001	84.111	0.000	0.043	0.045
poly(TV, degree=2, raw=True)[1]	-0.0001	6.89e-06	-15.920	0.000	-0.000	-9.61e-05
Radio	0.1935	0.003	64.526	0.000	0.188	0.199
TV:Radio	0.0011	3.47e-05	31.061	0.000	0.001	0.001

Omnibus:	169.759	**Durbin-Watson:**	2.204
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4031.167
Skew:	-2.988	**Prob(JB):**	0.00
Kurtosis:	24.166	**Cond. No.**	1.49e+04

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.49e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```
vals = [VIF(X, i) for i in range(1, X.shape[1])]
print(vals)
```

```
[1.0172717815970211, 1.017084612216564, 1.013513326764562, 1.0075840215785734]
```

```
vif = pd.DataFrame({"vif": vals}, index=X.columns[1:])
print(vif)
("VIF Range:", np.min(vif), np.max(vif))
```

```
                                      vif
poly(TV, degree=2, raw=True)[0]  1.017272
poly(TV, degree=2, raw=True)[1]  1.017085
Radio                            1.013513
TV:Radio                         1.007584
```

```
('VIF Range:', 1.0075840215785734, 1.0172717815970211)
```

```
allDone()
```

```
<IPython.lib.display.Audio object>
```