A. type check

A.1 Syntax

$e,\ \tau$	$::=$		Expressions
	$\mid$	x	Variable
	$\mid$	$\star$	Type of Types
	$\mid$	$e_1\ e_2$	Application
	$\mid$	$\lambda x.\ e$	Abstraction
	$\mid$	$\lambda x : \sigma\ .\ e$	Abstraction with annotation
	$\mid$	$\mathsf{cast}^{\uparrow}\ e$	Cast Up
	$\mid$	$\mathsf{cast}_{\downarrow}\ e$	Cast Down
	$\mid$	$\Pi\,(x : \tau).\ \tau_2$	Explicit Pi type
	$\mid$	$\mathbf{let}\ x = e_1\ \mathbf{in}\ e_2$	Let binding
	$\mid$	$e :: \sigma$	Annotation
σ	$::=$		Polytype
	$\mid$	$\forall\, x : \tau.\ \rho$	Implicit Pi Type
ρ	$::=$		Rhotype
	$\mid$	τ	Mono
	$\mid$	$\Pi\,(x : \sigma_1).\sigma_2$	FunPoly

A.2 non syntax-directed Typing

$\boxed{\Gamma \vdash e : \sigma}$

$$\Gamma \vdash \star : \star \ \textsc{Ax}$$

$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma}\ \textsc{Var}$$

$$\frac{\Gamma \vdash e_1 : (\Pi x : \sigma_1.\sigma_2) \qquad \Gamma \vdash e_2 : \sigma_1}{\Gamma \vdash e_1\, e_2 : \sigma_2[x \mapsto e_2]}\ \textsc{App}$$

$$\frac{\Gamma, x : \tau \vdash e : \sigma}{\Gamma \vdash (\lambda x.\, e) : (\Pi x : \tau.\sigma)}\ \textsc{Lam}$$

$$\frac{\Gamma, x : \sigma \vdash e : \sigma_2}{\Gamma \vdash (\lambda x : \sigma.\, e) : (\Pi x : \sigma.\sigma_2)}\ \textsc{LamAnn}$$

$$\frac{\Gamma \vdash \tau_1 : \star \qquad \Gamma, x : \tau_1 \vdash \tau_2 : \star}{\Gamma \vdash (\Pi x : \tau_1.\tau_2) : \star}\ \textsc{ExplicitPi}$$

$$\frac{\Gamma \vdash e : \tau_2 \qquad \Gamma \vdash \tau_1 : \star \qquad \tau_1 \longrightarrow \tau_2}{\Gamma \vdash (\mathsf{cast}^{\uparrow}\ e) : \tau_1}\ \textsc{CastUp}$$

$$\frac{\Gamma \vdash e : \tau_1 \qquad \Gamma \vdash \tau_2 : \star \qquad \tau_1 \longrightarrow \tau_2}{\Gamma \vdash (\mathsf{cast}_{\downarrow}\ e) : \tau_2}\ \textsc{CastDown}$$

$$\frac{\Gamma \vdash e_1 : \sigma \qquad \Gamma, x : \sigma \vdash e_2 : \tau}{\Gamma \vdash (let\ x = e_1\ in\ e_2) : \tau}\ \textsc{Let}$$

$$\frac{\Gamma \vdash e : \forall(\alpha_i : \tau_i).\sigma \qquad \tau_{\beta i} : \tau_i}{\Gamma \vdash e : \sigma[\alpha_i \mapsto \tau_{\beta i}]}\ \textsc{Inst}$$

$$\frac{\Gamma \vdash e : \sigma \qquad \alpha_i \notin free(\Gamma) \qquad \tau_i : \star \qquad \alpha_i : \tau_i \vdash \sigma : \star}{\Gamma \vdash e : \forall(\alpha_i : \tau_i).\sigma}\ \textsc{Gen}$$

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash (e :: \sigma) : \sigma}\ \textsc{Ann}$$

$\boxed{\Gamma \vdash \sigma : \star}$

$$\frac{\Gamma \vdash \tau : \star \qquad \Gamma, x : \tau \vdash \rho : \star}{\Gamma \vdash (\forall x : \tau.\rho) : \star}\ \textsc{ImplicitPi}$$

$\boxed{\Gamma \vdash \rho : \star}$

$$\frac{\Gamma \vdash \sigma_1 : \star \qquad \Gamma, x : \sigma_1 \vdash \sigma_2 : \star}{\Gamma \vdash (\Pi x : \sigma_1.\sigma_2) : \star}\ \textsc{FunPoly}$$