Higher Order Equations

- Must make into a system of first-order equations to use ODE solvers
- Nonlinear is OK!
- Pendulum example:

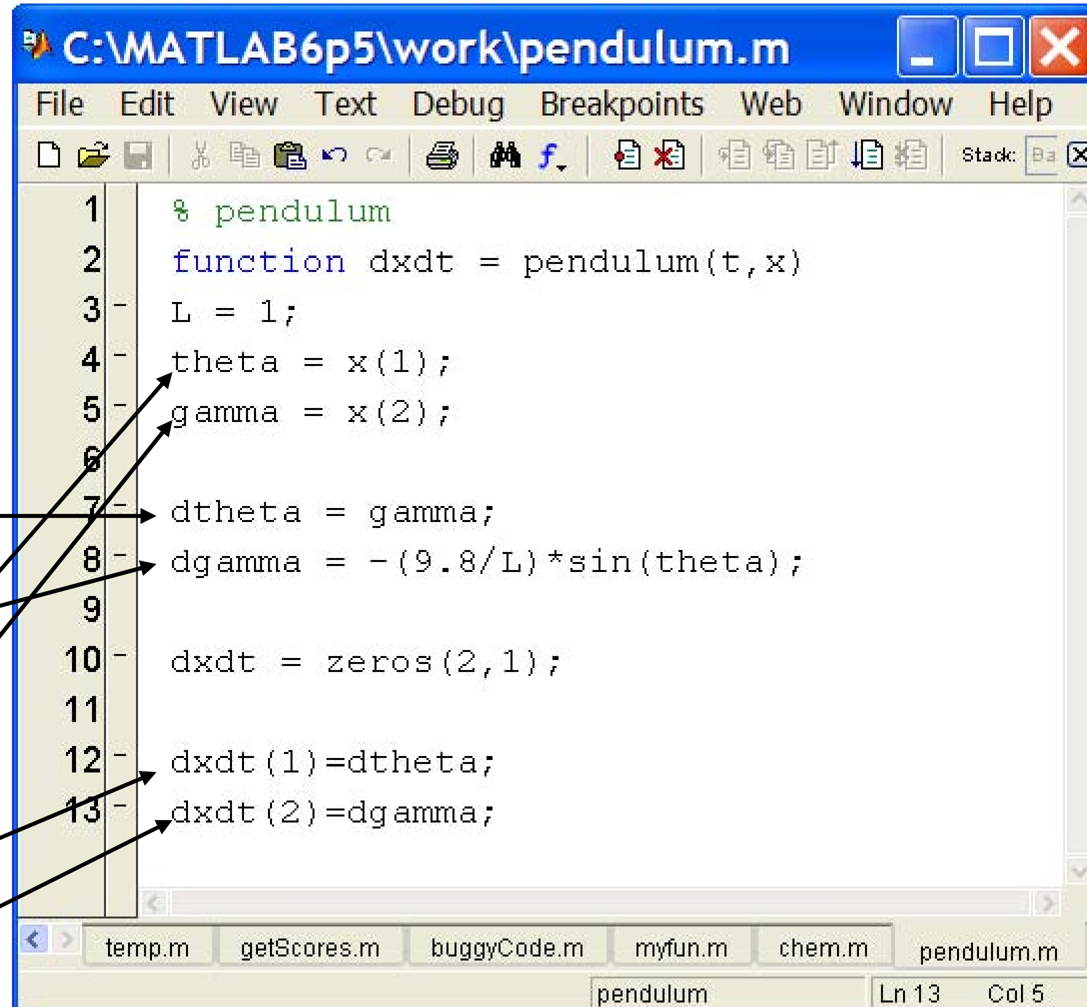
$$\ddot{\theta} + \frac{g}{L} sin(\theta) = 0$$

$$\ddot{\theta} = -\frac{g}{L} sin(\theta)$$

$$let \quad \dot{\theta} = \gamma$$

$$\dot{\gamma} = -\frac{g}{L} sin(\theta)$$

$$\boxed{\vec{x} = \begin{bmatrix} \theta \\ \gamma \end{bmatrix} \qquad \frac{d\vec{x}}{dt} = \begin{bmatrix} \dot{\theta} \\ \dot{\gamma} \end{bmatrix}}$$



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Plotting the Output

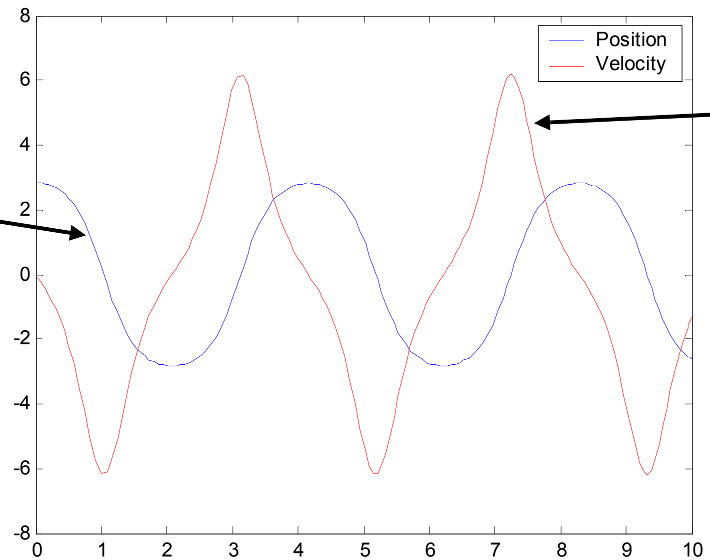
- We can solve for the position and velocity of the pendulum:

```
» [t,x]=ode45('pendulum',[0 10],[0.9*pi 0]);
```

  - ➢ assume pendulum is almost horizontal

```
» plot(t,x(:,1));
» hold on;
» plot(t,x(:,2),'r');
» legend('Position','Velocity');
```

Position in terms of angle (rad)



Velocity (m/s)

Plotting the Output

- Or we can plot in the phase plane:
  - » `plot(x(:,1),x(:,2));`
  - » `xlabel('Position');`
  - » `yLabel('Velocity');`
- The phase plane is just a plot of one variable versus the other: