

MECH 547 Mechanics of Biological Materials

Homework 1

(6 problems, 30 points)

The material property charts you need for this assignment are in the appendix

Problem 1

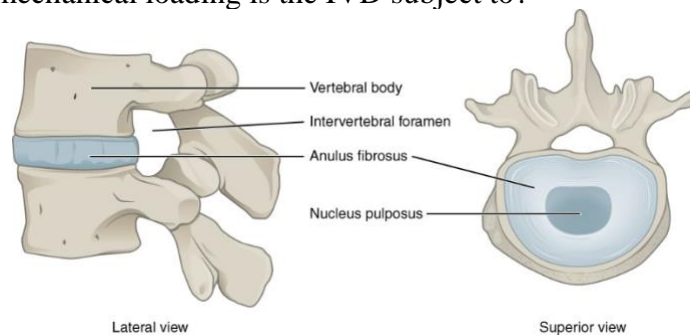
This Gastropod shell is subjected to a point load as showed. What types of stresses would you expect in the shell?



Problem 2

An intervertebral disc (IVD) is a soft tissue bonded between adjacent vertebrae in the vertebral column. As shown below, the IVD contains two substructures: an inner core of nucleus pulposus and an outer rim of anulus fibrosus. It is critical for the function of the spine.

- 1) What are the mechanical properties of the IVD? Which property is critical to their mechanical function?
- 2) Regarding the critically important mechanical properties of the IVD, which mechanical tests will you perform? Please elaborate the rationale and potential challenges during testing.
- 3) What kind of mechanical loading is the IVD subject to?



Problem 3

Derive the stress-stretch relations from the strain energy function for a compressible Neo-Hookean material under unidirectional tension (in terms of true stress). The strain energy function takes the following form.

$$W = C_1 (I_1 - 3 - 2 \ln J) + D_1 (J - 1)^2 ; \quad J = \det(\mathbf{F}) = \lambda_1 \lambda_2 \lambda_3$$

The true stress can be obtained from the strain energy function via $\sigma_i = \frac{\lambda_i}{J} \frac{\partial W}{\partial \lambda_i}$; $i = 1, 2, 3$.

Problem 4

Compared to the necking of metals, explain why rubbers do not neck under tension. Assume the rubber material follows the Neo-Hookean model.

Problem 5

Examine the Modulus-Strength charts for engineering materials and natural materials.

- How do natural polymers compare with engineering polymers?
- How do natural elastomers compare with engineering elastomers?
- How do natural ceramic composites compare with engineering ceramics?

Problem 6

- Show that for a beam of circular cross section with a required flexural strength the material performance index is $\frac{\sigma_f^{2/3}}{\rho}$.
- Name some of the best natural material for this and compare with the actual function of these materials.

Materials Property Charts

