Making Sense of Tristate Numbers (tnum)

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What is tnum?



Tristate

Number

Tracked



```
struct bpf_reg_state {
 enum bpf_reg_type type;
 struct tnum {
    u64 value;
    u64 mask;
  } var_off;
```



kernel/bpf/tnum.c



Number of bugs in kernel/bpf/tnum.c

since its introduction in 2017





Number of lines in kernel/bpf/tnum.c

remains unchanged since 2017



168

out of 213



75%



Good code Good API



In practice...





Backgrounds



Why tnum?



BPF Verifier

&

Safety



Address leakage?

Out-of-bound read?

BPF Verifier

Uninit stack?

Invalid return value?

&

Safety

Infinite loop?

Termination?

Is pointer aligned?



Out-of-bound read?

Invalid return

What's the values being used?

Uninit stack?

Infinite loop?

Termination?

value?

Is pointer aligned?



```
/* i is some random number */
int i = bpf_get_prandom_u32();
/* mask must be 3 */
int mask = 3;
/* i & mask can be 0, 1, 2, or 3 */
return i & mask;
```



What the verifier actually sees



```
/* random number given as the
 * return value (register r0) */
call bpf_get_prandom_u32;
/* mask stored in register r1*/
r1 = 3;
r0 &= r1; /* (i & mask) kept in r0 */
ret;
```



Out-of-bound read?

Invalid return

What's the values being used?

Uninit stack?

Infinite loop?

Termination?

value?

Is pointer aligned?



Out-of-bound read?

What's the value

within registers?

Uninit stack?

Infinite loop?

value?

Invalid return

Termination?

Is pointer aligned?



Out-of-bound read?

Uninit stack?

Invalid return value?

Value Tracking

Infinite loop?

Termination?

Is pointer aligned?



Value Tracking



```
/* Takes 2^31 GiB just to track a
 * single register */
struct values {
 char possibly_0 :1;
 char possibly_1 :1;
 char possibly_2 :1;
```



```
/* Takes 2^31 GiB just to track g
* single register */
struct values {
 char possibly 0
 char possibly_
 char possibly_2 :1;
```



```
struct values add_values(struct values *a,
                         struct values *b) {
  if (a->possibly_0 && b->possibly_0)
     ret->possibly_0 = 1;
  if (a->possibly_0 && b->possibly_1)
     ret->possibly_1 = 1;
  if (a->possibly_1 && b->possibly_0)
     ret->possibly_1 = 1;
  /* Some bit-tricks would help, but ... */
```



```
struct values add_values(struct values *a
                        struct values *
  if (a->possibly_0 && b->possibly_0)
     ret->possibly_0 = 1;
  if (a->possibly_0 && b->possibly_1)
     ret->possibly_1 = 1;
  if (a->possibly_1 && b->possibly_0)
      ret->possibly_1 = 1;
   * Some bit-tricks would help, but ...
```



Just track min & max



```
struct values {
   s64 min;
   s64 max;
};
```



```
struct values add_values(struct values *a,
                         struct values *b)
  /* Ignoring overflow for now */
  ret->min = a->min + b->min;
  ret->max = a->max + b->max;
```



```
struct bpf_reg_state {
   struct tnum var_off;
   s64 smin_value;
   s64 smax_value;
...
```



```
struct bpf_reg_state {
 struct tnum var_off;
 s64 smin_value; /* minimum possible (s64)value */
 s64 smax_value; /* maximum possible (s64)value */
 u64 umin_value; /* minimum possible (u64)value */
 u64 umax_value; /* maximum possible (u64)value */
 s32 s32_min_value; /* minimum possible (s32)value */
 s32 s32_max_value; /* maximum possible (s32)value */
 u32 u32_min_value; /* minimum possible (u32)value */
 u32 u32_max_value; /* maximum possible (u32)value */
```



What about **bitwise** operations?



```
struct values xor_values(struct values *a,
                         struct values *b)
```



Attempt #2 - Ranges

```
struct values xor_values(struct values *a,
                        struct values *b)
         ain't nobody got
         time for that<sup>1</sup>
```



Track individual bits



Attempt #3 - Bitwise Pattern

Each bit in the register can have three possible states:

- Unknown 🗆 **x**
- Known to be set

 1
- Known to be unset \square 0



Attempt #3 - Bitwise Pattern

Each bit in the register can have three possible states:

- Unknown $\square \times \square \{0, 1\}$
- Known to be set

 1
- Known to be unset \square 0

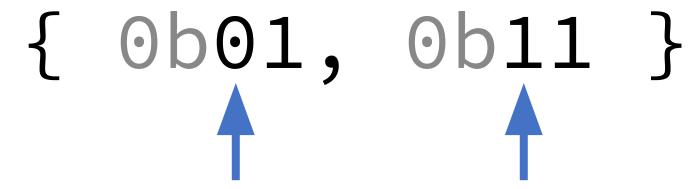


{ 1, 3 }



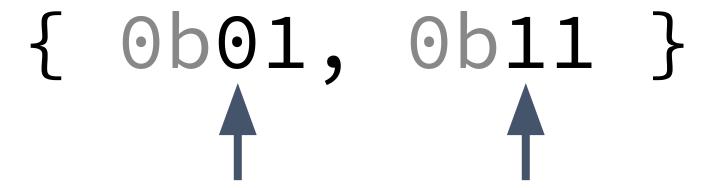
{ 0b01, 0b11 }

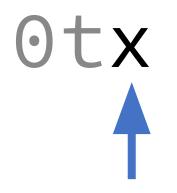




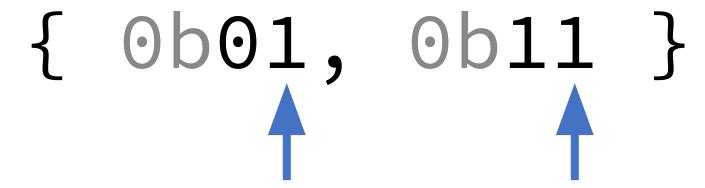






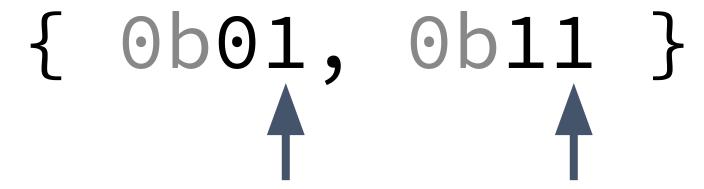


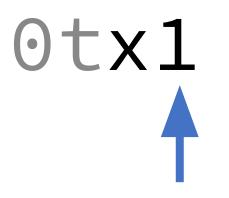














{ 0b01, 0b11 }



Abstract

{ 0b01, 0b11 }



Concrete (actual values used in register)

{ 0b01, 0b11 }

Abstract (how we represent such set of values)



Abstract

{ 0b0...01, 0b0...11 }

0t0...x1



Abstract

{ 0b01, 0b11 }



{ 1, 3 }

Abstract



{ 1, 3 }

non-consecutive values

(e.g. pointer alignment)

Abstract



Limitation



Fuzzy



{ 1, 3 }

Abstract





Abstract

{ 1, 2 }



58

Abstract

{ 0b01, 0b10 }



Abstract

{ 0b01, 0b10 } — 0t



Abstract

{ 0b01, 0b10 } — 0t



Abstract





Abstract

{ 0b01, 0b10 } — 0txx



Abstract

{ 0b01, 0b10 } — 0txx



Abstract

0txx



Abstract

{ oboo,



}



```
{ 0b00, 0b01, 0txx
```



```
{ 0b00,
 0b01,
 0b10,
}
```





```
{ 0b00,
 0b01,
 0b10,
 0b11 }
```





Concrete (ideal)



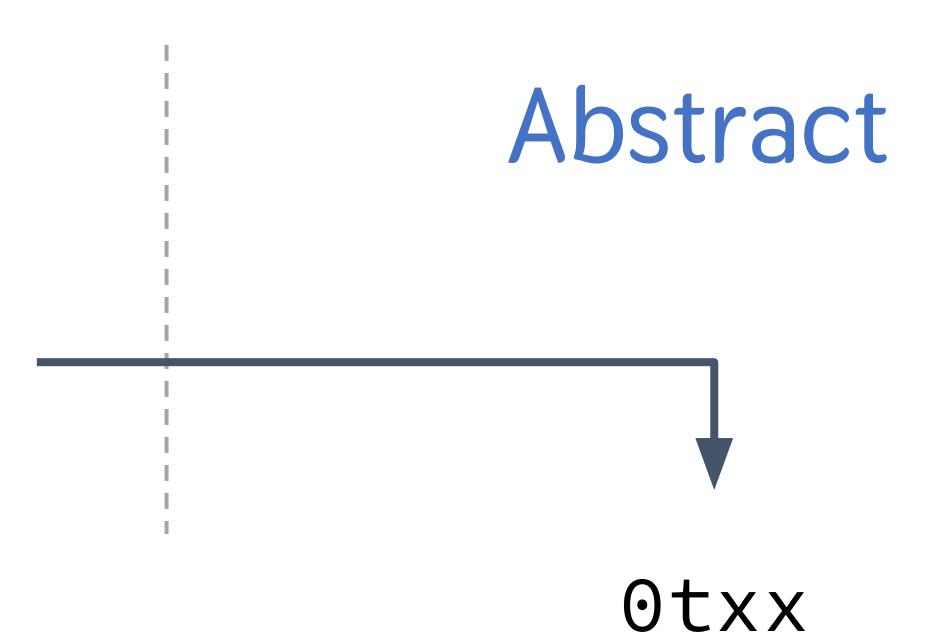
Concrete (ideal)

{ 1, 2 }



Concrete (ideal)

{ 1, 2 }



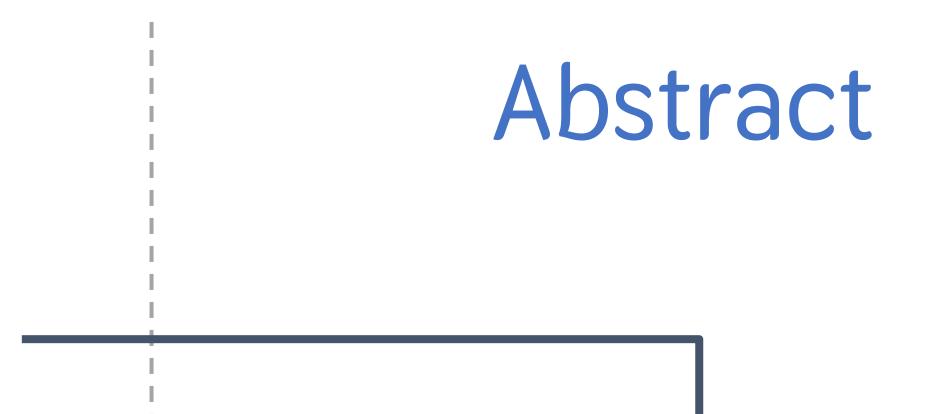


Concrete (ideal)

{ 1, 2 }

Concrete

(actual)







Concrete (ideal)

{ 1, 2 }

Abstract

0txx

Concrete (actual)

{ 1, 2, 3, 4 }





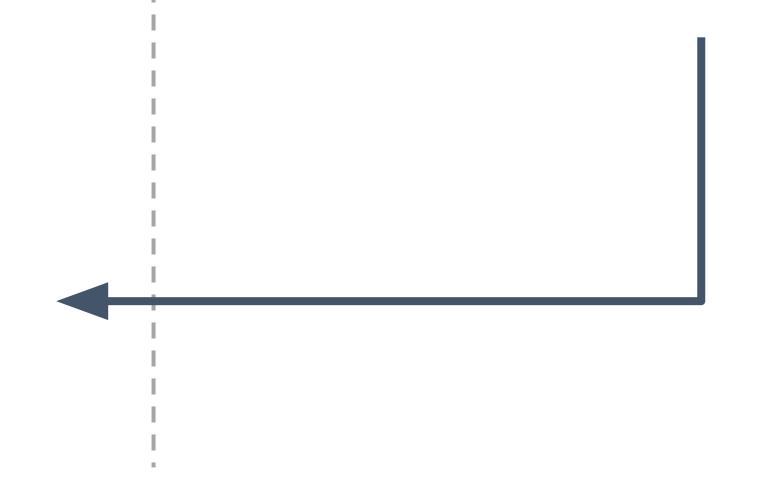
Concrete (ideal)

{ 1, 2 }

0txx

Concrete (actual)

{ 1, 2, 3, 4 }



Abstract



{ 1, 2 } Fine 0txx



{ 1, 2, 3, 4 }

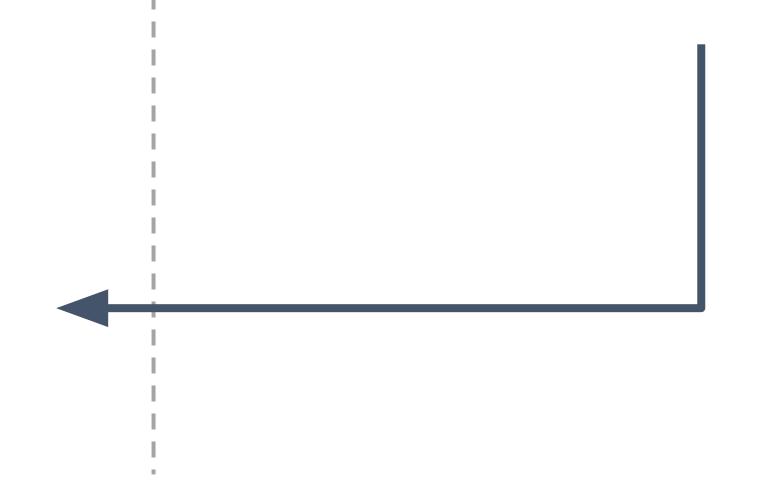
Concrete (ideal)

{ 1, 2 }

0txx

Concrete (actual)

{ 1, 2, 3, 4 }



Abstract



```
Concrete (ideal)
```

Abstract

```
{ 1, 2 }
```

Over-approximation

Concrete (Static Analysis)
(actual)

```
{ 1, 2, 3, 4 }
```



{ 1, 2 } Fine 0txx { 1, 2, 3, 4 }



Attempt #2 - Ranges

```
struct bpf_reg_state {
 struct tnum var_off;
 s64 smin_value; /* minimum possible (s64)value */
 s64 smax_value; /* maximum possible (s64)value */
 u64 umin_value; /* minimum possible (u64)value */
 u64 umax_value; /* maximum possible (u64)value */
 s32 s32_min_value; /* minimum possible (s32)value */
 s32 s32_max_value; /* maximum possible (s32)value */
 u32 u32_min_value; /* minimum possible (u32)value */
 u32 u32_max_value; /* maximum possible (u32)value */
```



Signess



crossing sign boundary



Concrete (ideal)

0tx...xx

Abstract

Concrete (actual)

$$\{0, 1, 2... 2^{64}-1\}$$



Concrete (ideal)

Abstract

Fine otx...xx

Concrete (actual)

$$\{0, 1, 2... 2^{64}-1\}$$



Attempt #2 - Ranges

```
struct bpf_reg_state {
 struct tnum var_off;
 s64 smin_value; /* minimum possible (s64)value */
 s64 smax_value; /* maximum possible (s64)value */
 u64 umin_value; /* minimum possible (u64)value */
 u64 umax_value; /* maximum possible (u64)value */
 s32 s32_min_value; /* minimum possible (s32)value */
 s32 s32_max_value; /* maximum possible (s32)value */
 u32 u32_min_value; /* minimum possible (u32)value */
 u32 u32_max_value; /* maximum possible (u32)value */
```



Can't track nothing





```
/* assume this isn't optimized out */
if (i < 0 && i > 0) {
   /* never ever */
}
```



```
/* assume this isn't optimized out */
if (i < 0 && i > 0) {
   /* IMPOSSIBLE(*) to represent i */
}
```



Just don't follow such branch



```
/* compute branch direction of the expression "if
 * (<reg1> opcode <reg2>) goto target;" and return:
 * 1 - branch will be taken
 * 0 - branch will not be taken
 * -1 - unknown. Example: "if (reg1 < 5)" is unknown
        when register value range [0,10]
 */
static int is_branch_taken(struct bpf_reg_state *reg1,
                           struct bpf_reg_state *reg2,
                           u8 opcode, bool is_jmp32);
```



```
static int is_scalar_branch_taken(...) {
 switch (opcode) {
 case BPF_JEQ:
  if (tnum_is_const(t1) && tnum_is_const(t2))
   return t1.value == t2.value;
  return -1;
```



Can't track relation



```
int j = i - 1; /* int i is unknown */
if (i < 1 | | i > 3)
  return;
/* From here on 1 \le i \le 3
/* with j == i - 1 we know 0 \le j \le 2
 */
if (j == 4)
  /* never ever */
```



Track relationship separately



```
struct bpf_reg_state {
 /* Upper bit of ID is used to remember relationship
  * between "linked" registers, e.g.:
  * r1 = r2; both will have r1->id == r2->id == N
  * r1 += 10; r1->id == N | BPF_ADD_CONST and
          r1->off == 10
  *
  */
#define BPF_ADD_CONST (1U << 31)</pre>
 u32 id;
```



Implementation

Data Structure

Concrete

{ 0b01, 0b11 }

Abstract

0tx1



Concrete

Abstract

Conceptual

Implementation

{ 0b01, 0b11 }

0tx1



```
struct tnum {
u64 value; /* whether bits are
             * set/unset, if known
 u64 mask; /* which bits are
            * unknown */
```



- Unknown 🗆 **x**
- Known to be set

 1
- Known to be unset \square 0



- Unknown 🗆 x
- Known to be set \Box 1 (mask[] = 0, value[] = 1)
- Known to be unset 🗆 0



- Unknown 🗆 x
- Known to be set

 1
- Known to be unset \square 0 (mask[] = 0, value[] = 0)



- Unknown $\square x (mask[] = 1)$
- Known to be set

 1
- Known to be unset

 O



- Unknown $\square x$ (mask[] = 1, value[] = 0)
- Known to be set

 1
- Known to be unset

 O



- Unknown 🗆 x
- Known to be set

 1
- Known to be unset

 O
- Invalid \square (mask[] = 1, value[] = 1)



Concrete

Abstract

Conceptual

Implementation

{ 0b01, 0b11 }

$$.mask = 0b$$

$$.$$
value = $0b$

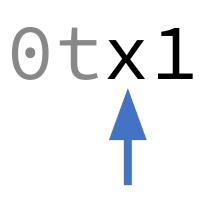


Concrete

Abstract

Conceptual

Implementation



$$.mask = 0b$$

$$.$$
value = $0b$

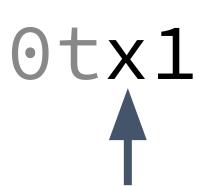


{ 0b01, 0b11 }

Abstract

Conceptual

Implementation



.mask =
$$0b1$$

.value = $0b$



Abstract

Conceptual

Implementation



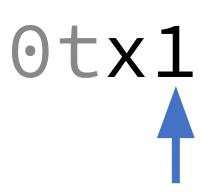


Abstract

Conceptual

Implementation

{ 0b01, 0b11 }



.mask =
$$0b1$$

.value = $0b0$

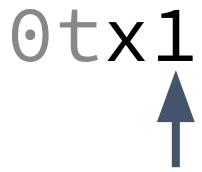


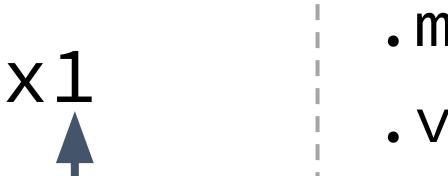
{ 0b01, 0b11 }

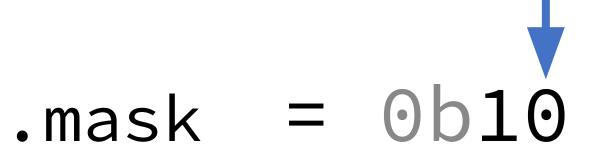
Abstract

Conceptual

Implementation







.value = 0b0



Abstract

Conceptual

Implementation





Abstract

Conceptual

Implementation

{ 0b01, 0b11 }

0tx1

.mask = 0b10.value = 0b01



Abstract

Conceptual

Implementation

{ 1, 3 }

0tx1

.mask = 0b10

.value = 0b01



Implementation

Helper



u64 tnum_umin(struct tnum a)

minimum possible unsigned value in a tnum



a.value



u64 tnum_umax(struct tnum a)

maximum possible unsigned value in a tnum



a.value a.mask



u64 tnum_and(struct tnum a, struct tnum b)

bitwise-and of two tnums



Crafting



How well do you need to know tnum?

to craft an operator



Very little



u64 tnum_and(struct tnum a, struct tnum b)

bitwise-and of two tnums



		a
&	0	1
0		
1		



&	0			1		
0	0	&	0	0	&	1
1	1	&	0	1	&	1



&	0	1
0	0	0
1	0	1



&	0	1	X
0	0	0	
1	0	1	
X			



&	0	1	X
0	0	0	
1	0	1	
X	?		



&	0	1	X
0	•	0	
1	0	1	
X			



&	0	1	X
0	0	0	
1	0	1	
X	•		



&	0	1	X
0	0	0	
1	0	1	
X	0		



&	0	1	X
0	0	0	
1	0	1	
X	0	{0, 1}	



&	0	1	X
0	0	0	
1	0	1	
X	0	X	



&	0	1	X
0	0	0	•
1	0	1	
X	0	X	



&	0	1	X
0	0	0	0
1	0	1	X
X	0	X	



&	0	1	X
0	0	0	0
1	0	1	X
X	0	X	?



&	0	1	X
0	0	0	0
1	0	1	X
X	0	X	



&	0	1	X
0	0	0	0
1	0	1	X
X	0	X	{0, 1}



&	0	1	X
0	0	0	0
1	0	1	X
X	0	X	X



&	0	1	X
0	0	0	0
1	0	1	X
X	0	X	X



&	0	1	X
0	0	•	O
1	•	1	X
X	0	X	X



&	m=0 v=0	1	X
m=0	m=0	m=0	m=0
v=0	V=0	V=0	V=0
1	m=0	1	X
	V=0		
X	m=0	X	X
	V=0		



&	m=0 v=0	1	X
m=0	m=0	m=0	m=0
V=0	V=0	V=0	V=0
1	m=0	1	X
	V=0		
X	m=0	X	X
	V=0		



&	m=0 v=0	m=0 v=1	X
m=0	m=0	m=0	m=0
v=0	V=0	V=0	V=0
m=0	m=0	m=0	X
v=1	V=0	v=1	
X	m=0 v=0	X	X



&	m=0 v=0	m=0 v=1	X
m=0	m=0	m=0	m=0
v=0	V=0	V=0	V=0
m=0	m=0	m=0	X
v=1	V=0	v=1	
X	m=0 v=0	X	X



&	m=0 v=0	m=0 v=1	m=1 v=0
m=0	m=0	m=0	m=0
v=0	V=0	V=0	V=0
m=0	m=0	m=0	m=1
v=1	V=0	v=1	V=0
m=1	m=0	m=1	m=1
V=0	V=0	V=0	V=0



&	m=0 v=0	m=0 v=1	m=1 v=0	&	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	V=0	V=0	V=0	m=0 v=0	m=0	m=0	m=0
m=0 v=1	V=0	v=1	V=0	m=0 v=1	m=0	m=0	m=1
m=1 v=0	V=0	V=0	V=0	m=1 v=0	m=0	m=1	m=1



&. v	m=0 v=0	m=0 v=1	m=1 v=0	& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	•	0	•	m=0 v=0	0	0	0
m=0 v=1	0	1	0	m=0 v=1	0	0	1
m=1 v=0	•	0	0	m=1 v=0	0	1	1

&. v	m=0 v=0	m=0 v=1	m=1 v=0	& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	0	m=0 v=0	0	0	0
m=0 v=1	0	1	•	m=0 v=1	0	0	1
m=1 v=0	0	0	0	m=1 v=0	0	1	1



&. v		m=0 v=1	m=1 v=0	&.m	m=0 v=0		
m=0 v=0	0	0	0	m=0 v=0	0	0	0
m=0 v=1	0	1	0	m=0 v=1	0	•	1
m=1 v=0	0	0	0	m=1 v=0	0	1	1

& . m	m=0	m=0	m=1
	V=0	v=1	V=0
m=0	0	0	0
V=0			
m=0	0	0	1
v=1			
m=1	0	1	1
V=0			



&. V	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	•	0	0
m=0 v=1	O	1	O
m=1 v=0	0	•	•



&. v	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	•	0	0
m=0 v=1	•	1	0
m=1 v=0	0	0	•



&. V	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	0
m=0 v=1	0	1	0
m=1 v=0	0	0	•

value =
a.value & b.value

&. V	m=0	m=0	m=1
	v=0	v=1	V=0
m=0	0	0	0
V=0			
m=0	0	1	0
v=1			
m=1	0	0	0
V=0			

	m=0 v=0	m=0 v=1		& . m		m=0 v=1	m=1 v=0
= 0 = 0	•	0	0	m=0 v=0	0	0	0
=0 =1	0	1	0	m=0 v=1	0	•	1
=1 =0	0	0	0	m=1 v=0	0	1	1



& . m	m=0 v=0	m=0 v=1	m=1 v=0	
m=0 v=0		•	0	
m=0 v=1		0	1	
m=1 v=0		1	1	•



& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	•	O
m=0 v=1	•	•	1
m=1 v=0	•	1	1



& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	0
m=0 v=1	•	•	1
m=1 v=0	•	1	1



	& . m	m=0 v=0		m=1 v=0
	m=0 v=0	0	•	•
	m=0 v=1	0	•	1
	m=1 v=0	0	1	1



mask =
 (a.value | a.mask)
 &
 (b.value | b.mask)

& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	0
m=0 v=1	0	•	1
m=1 v=0	0	1	1



& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	•
m=0 v=1	0	•	1
m=1 v=0	0	1	1



& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	•	•
m=0 v=1	0	•	1
m=1 v=0	0	1	1



```
mask =
  (a.value | a.mask)
    &
  (b.value | b.mask)
    &
```

& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	•	0
m=0 v=1	0	•	1
m=1 v=0	0	1	1



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
 beta = b.value | b.mask;
 v = a.value & b.value;
 return TNUM(v, alpha & beta & ~v);
```



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
 beta = b.value | b.mask;
 v = a.value & b.value;
 return TNUM(v, alpha & beta & ~v);
```



&. v	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	•	0	0
m=0 v=1	•	1	•
m=1 v=0	•	•	•

value =



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
 beta = b.value | b.mask;
 v = a.value & b.value;
 return TNUM(v, alpha & beta & ~v);
```



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
 beta = b.value | b.mask;
 v = a.value & b.value;
 return TNUM(v, alpha & beta & ~v);
```



```
mask =
  (a.value | a.mask)
    &
  (b.value | b.mask)
    &
```

& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	0
m=0 v=1	0	0	1
m=1 v=0	0	1	1



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
 beta = b.value b.mask;
 v = a.value & b.value;
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```
mask =
  (a.value | a.mask)
    &
  (b.value | b.mask)
    &
```

& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	0	0
m=0 v=1	•	0	1
m=1 v=0	0	1	1



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
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 return TNUM(v, alpha & beta & ~v);
```



& . m	m=0 v=0	m=0 v=1	m=1 v=0
m=0 v=0	0	•	0
m=0 v=1	0	•	1
m=1 v=0	0	1	1



```
struct tnum tnum_and(struct tnum a, struct tnum b)
 u64 alpha, beta, v;
 alpha = a.value | a.mask;
 beta = b.value | b.mask;
 v = a.value & b.value;
 return TNUM(v, alpha & beta & ~v);
```



u64 tnum_and(struct tnum a, struct tnum b)

bitwise-and of two tnums



```
/* Return @a with lowest @size bytes
 * retained, and all other bits set
 * to equal the sign bit (which might
 * be unknown).
struct tnum tnum_scast(struct tnum a,
                       u8 size)
```



Usage



Bound-syncing



```
static void reg_bounds_sync(struct bpf_reg_state *reg)
 /* tnum -> u64, s64, u32, s32 */
 __update_reg_bounds(reg);
 /* u64 -> u32, s32; s64 -> u32, s32
  * u64 -> s64; s64 -> u64
  * u32 -> u64, s64; s32 -> u64, s64 */
 __reg_deduce_bounds(reg);
 __reg_deduce_bounds(reg); /* 2nd time */
 /* u64 -> tnum; u32 -> tnum */
 __reg_bound_offset(reg);
 /* tnum -> u64, s64, u32, s32 */
 __update_reg_bounds(reg);
```



```
static void __update_reg64_bounds(struct bpf_reg_state *reg)
 /* min signed is max(sign bit) | min(other bits) */
 reg->smin_value = max_t(s64, reg->smin_value,
      reg->var_off.value | (reg->var_off.mask & S64_MIN));
 /* max signed is min(sign bit) | max(other bits) */
 reg->smax_value = min_t(s64, reg->smax_value,
      reg->var_off.value | (reg->var_off.mask & S64_MAX));
 reg->umin_value = max(reg->umin_value, reg->var_off.value);
 reg->umax_value = min(reg->umax_value,
           reg->var_off.value | reg->var_off.mask);
```



Testing



Does it work?



Is it correct?



Would it allow unsafe program to pass?



BPF selftests



Agni



Z3Py



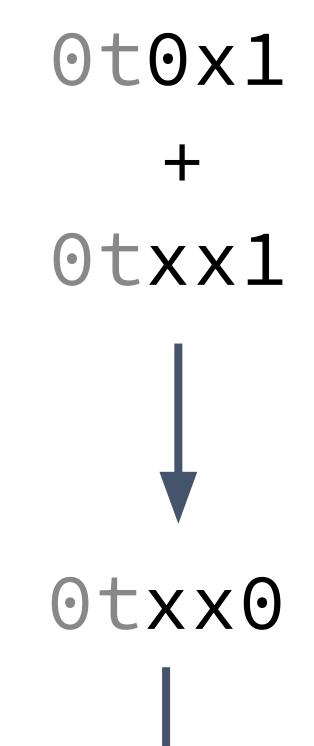
Sound, Precise, and Fast Abstract Interpretation with Tristate Numbers

Harishankar Vishwanathan, Matan Shachnai, Srinivas Narayana, and Santosh Nagarakatte



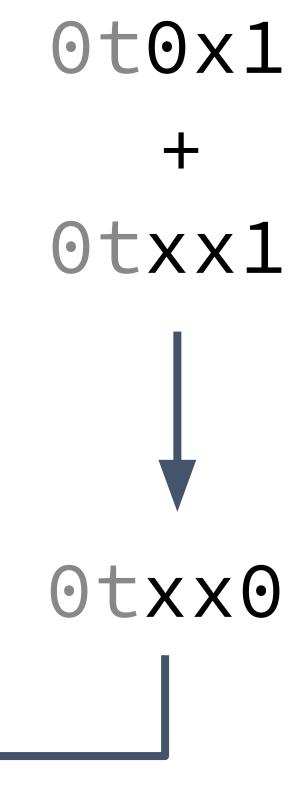
Concrete

```
{ 1, 3 }
   { 3, 5 }
 { 4, 6, 8 }
{ 2, 4, 6, 8 }
```



Concrete

```
{ 1, 3 }
   { 3, 5 }
 { 4, 6, 8 }
{ 2, 4, 6, 8 }
```





Would it allow unsafe program to pass?





Would it reject safe program (too often)?



BPF selftests



Agni



Conclusion



Tracks bit pattern

- Simple (maybe not intuitively easy to understand)
- Can't track min/max/sign-crossing precisely

Correct operation should

- Not left any possible values out (i.e. sound)
- Tries to exclude as much impossible values (i.e. precise)
 - without introducing unnecessary complexity



Resources



- Sound, Precise, and Fast Abstract Interpretation with Tristate Numbers
- Peeking into the BPF verifier
- More than you want to know about BPF verifier
- Value Tracking in BPF verifier
- Model Checking (a very small part) of BPF Verifer

