Chapter 5
Image Restoration

第五章：图像复原

Notice

- 第3 版作业和 project 见课程网站

Adobe 2011年

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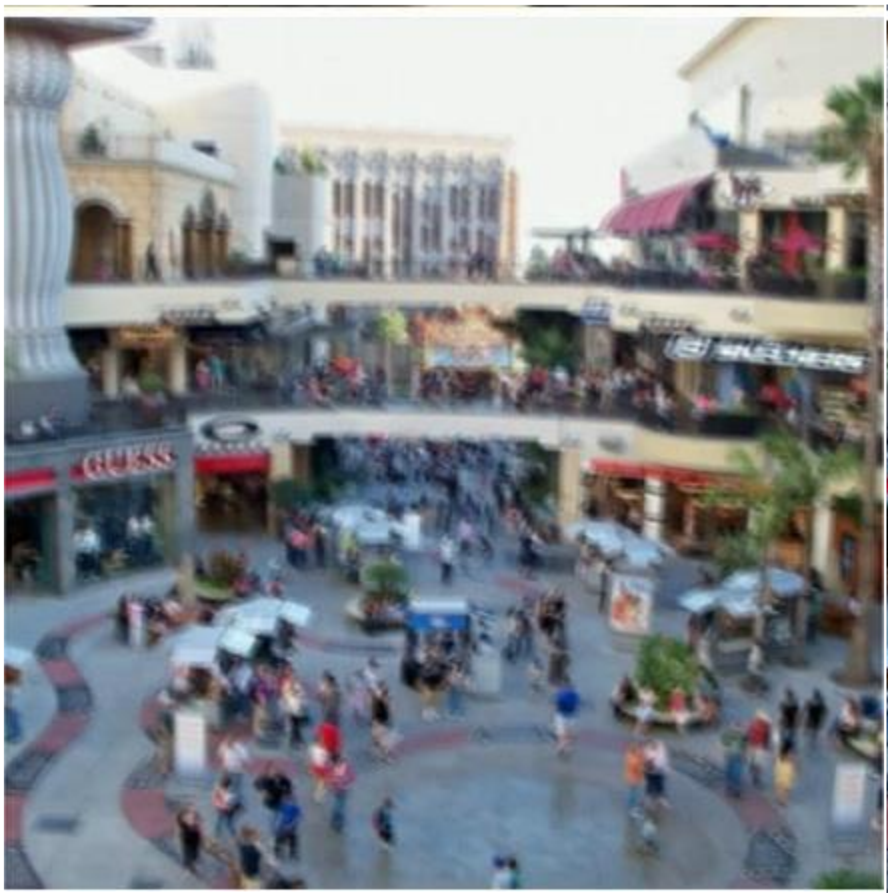

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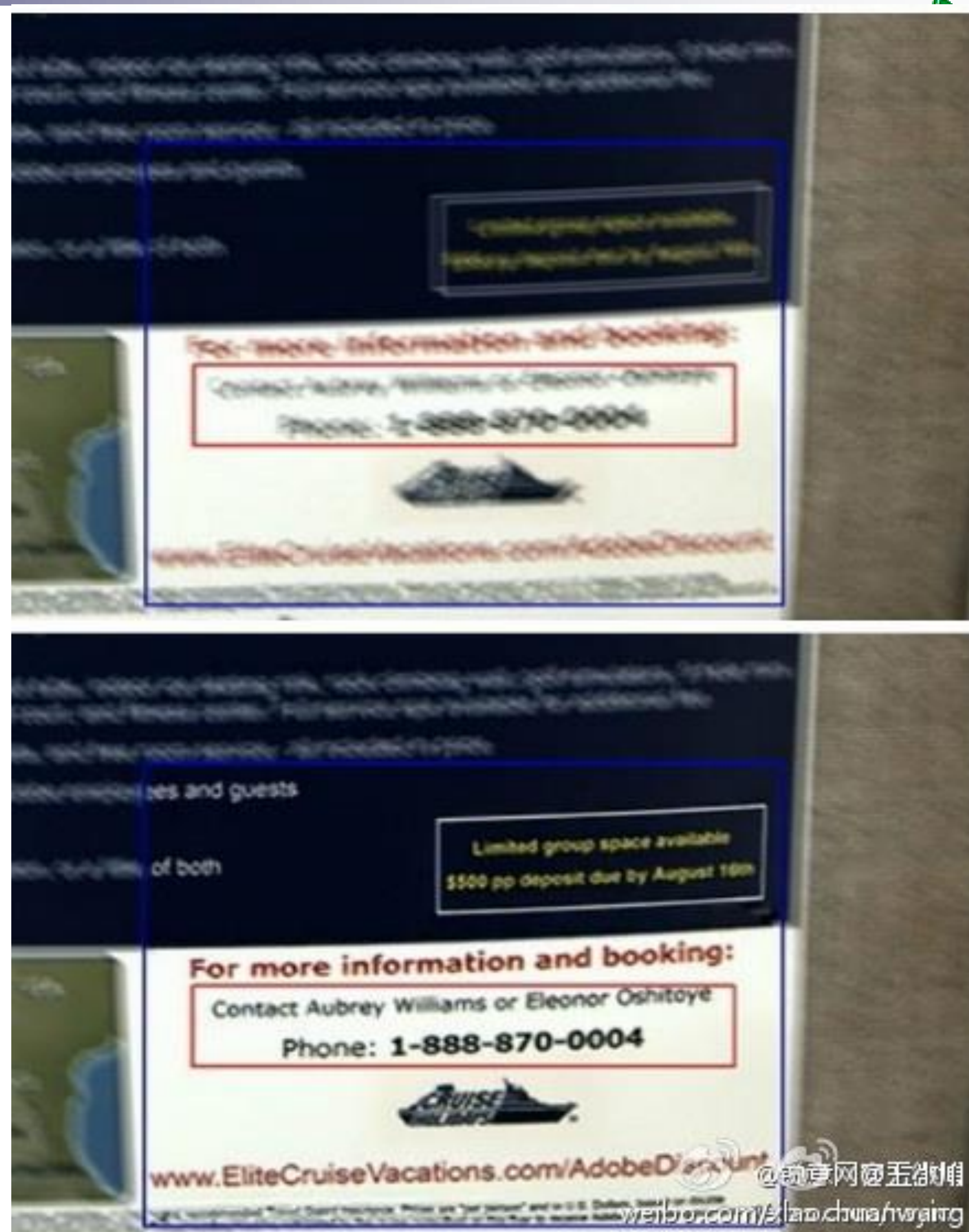

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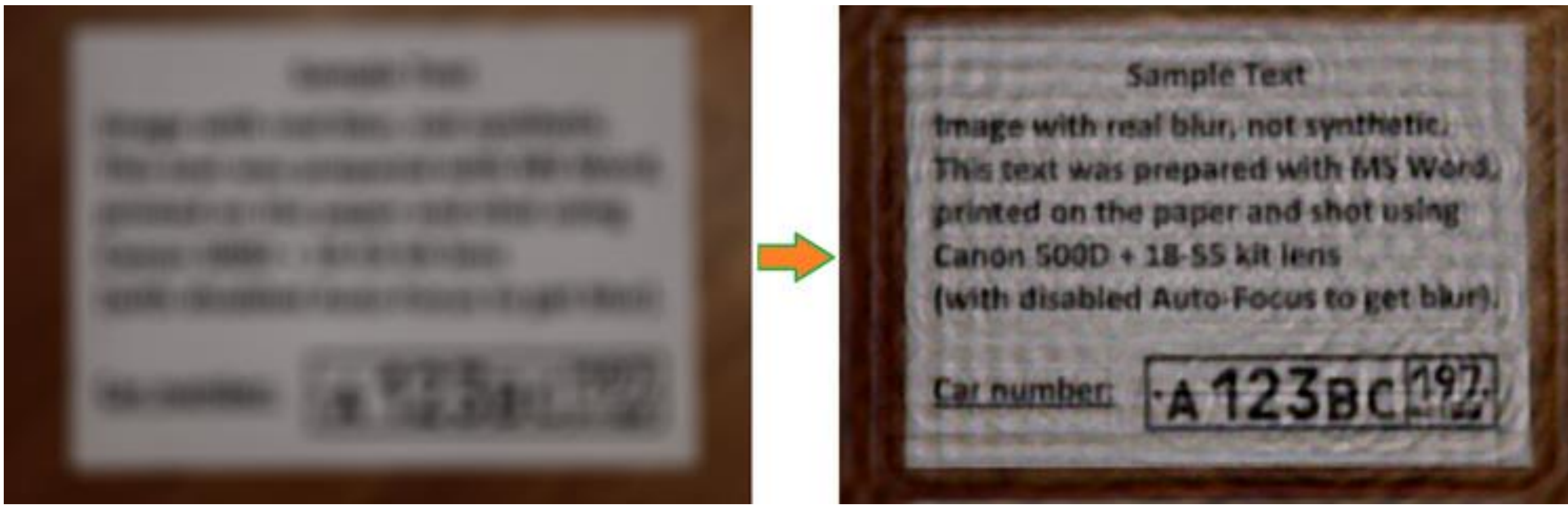

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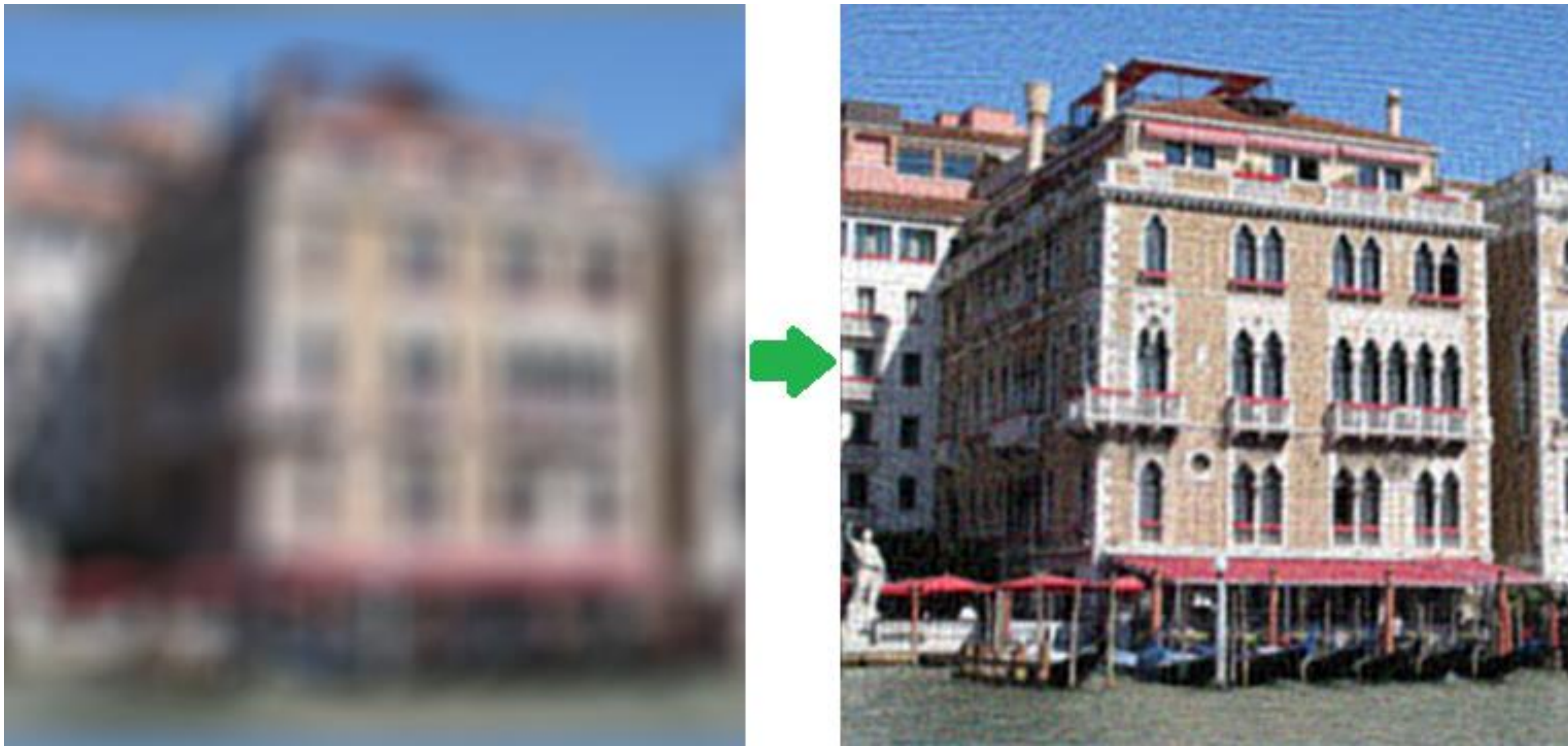

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Things which we see are not by themselves what we see…… It remains completely unknown to us what the objects may be by themselves and apart from the receptivity of our senses. We know nothing but our manner of perceiving them.

Immanuel Kant (伊曼努尔·康德)

目的和内容

掌握图像复原的基本理论和方法:

- 建立退化和干扰模型
- 学习某些分析问题建立模型的思想
- 设计去除这些因素的算法

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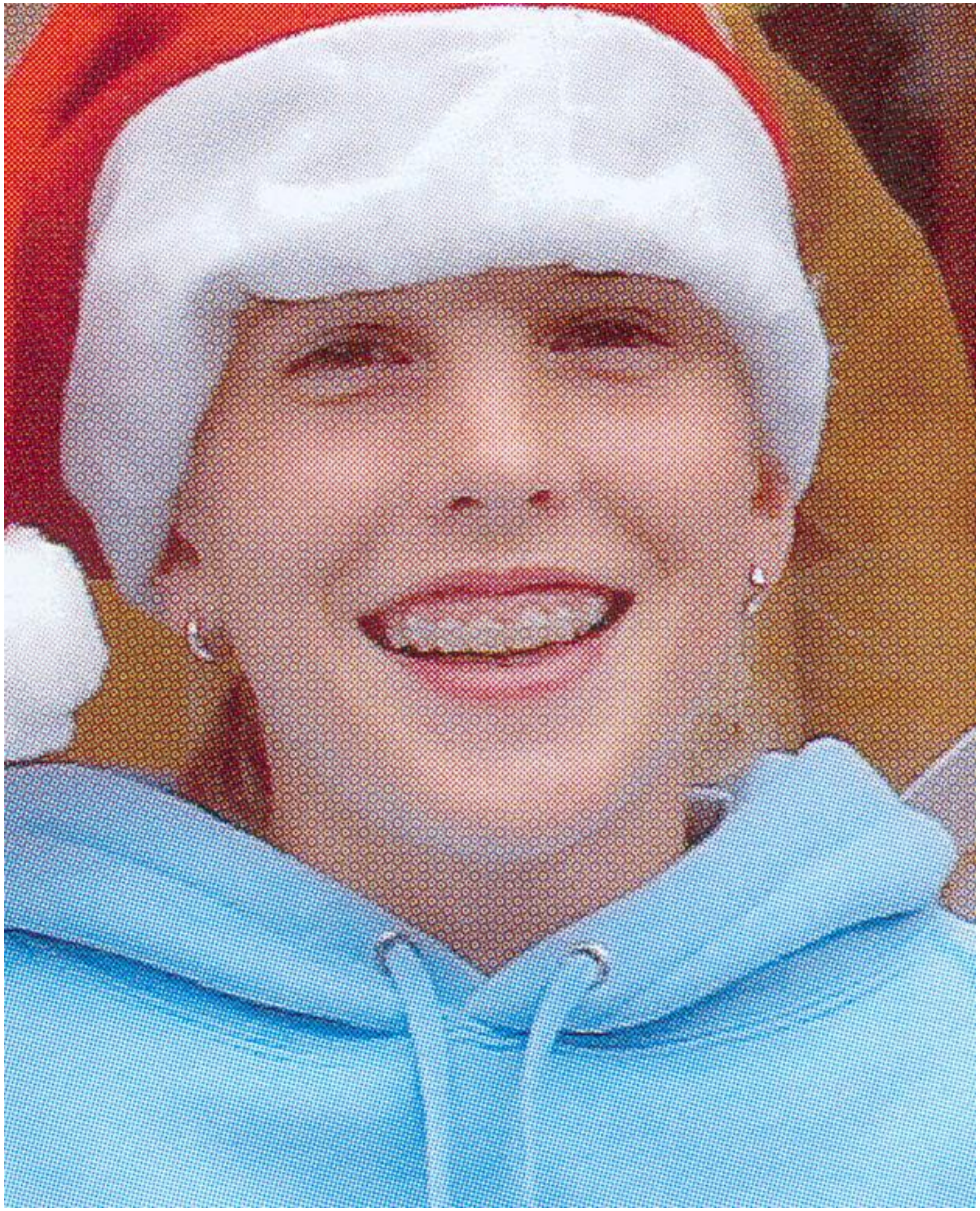
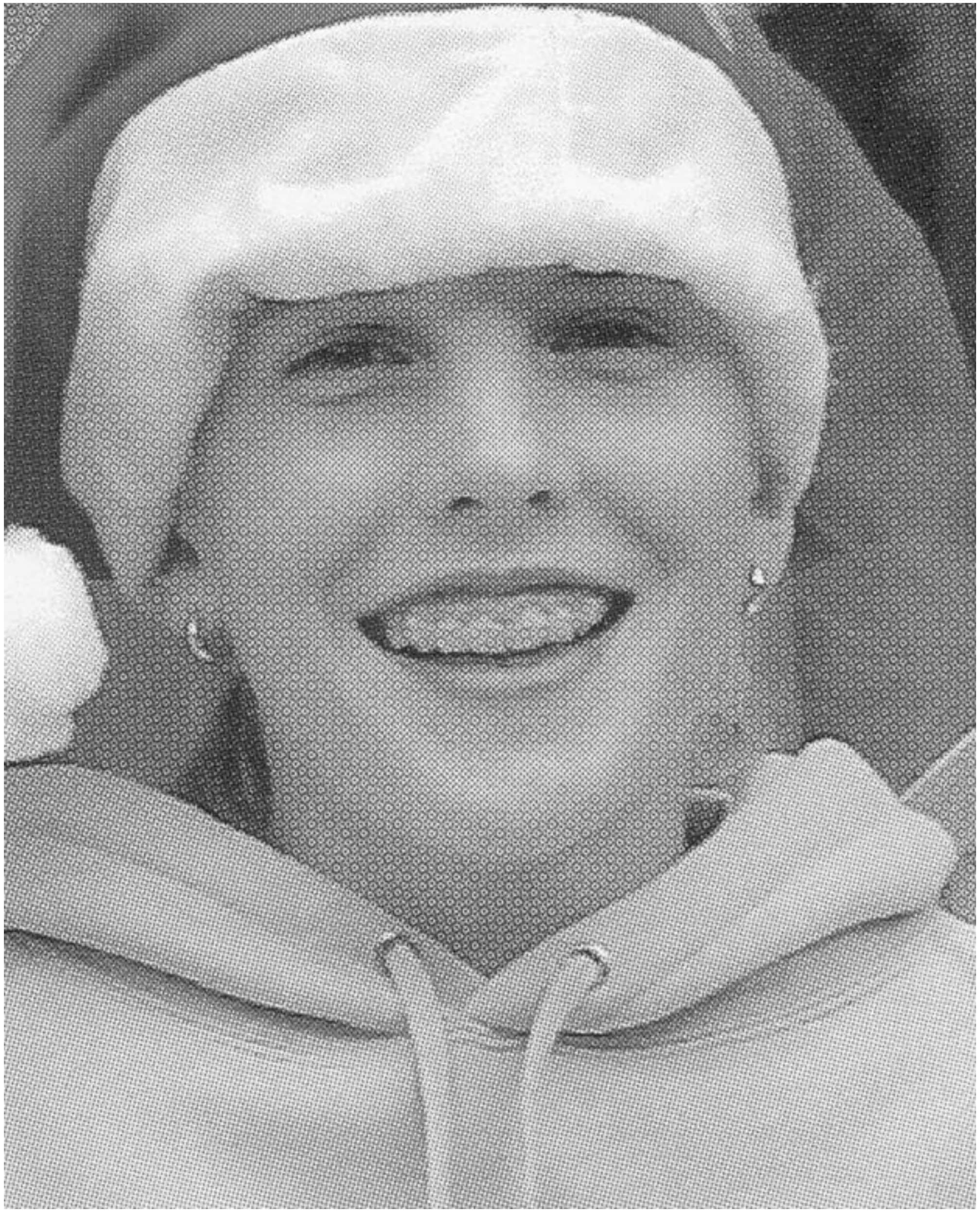
图像复原: 利用退化现象和噪声干扰的某些先验知识来重建或恢复被干扰和退化的图像, 尽可能的恢复图像的原貌.

与图像增强的异同:

- 图像增强侧重主观的探索性过程，主要是为了人类视觉系统的生理感知特点而设计的。例如：增强对比度、使图像变柔和等;
- 图像复原大部分是一个客观过程, 为现实服务的. 需要利用某些先验知识从退化和干扰的图像中去除模糊因素和噪声、尽可能恢复图像的本来面貌。复原技术一般是先把退化和干扰模型化 (key), 然后采用相反的过程进行处理，复原图像;
- 两者的目的都是为了改善图像，在去除噪声方面所使用的方法有很多交叉相同的部分。





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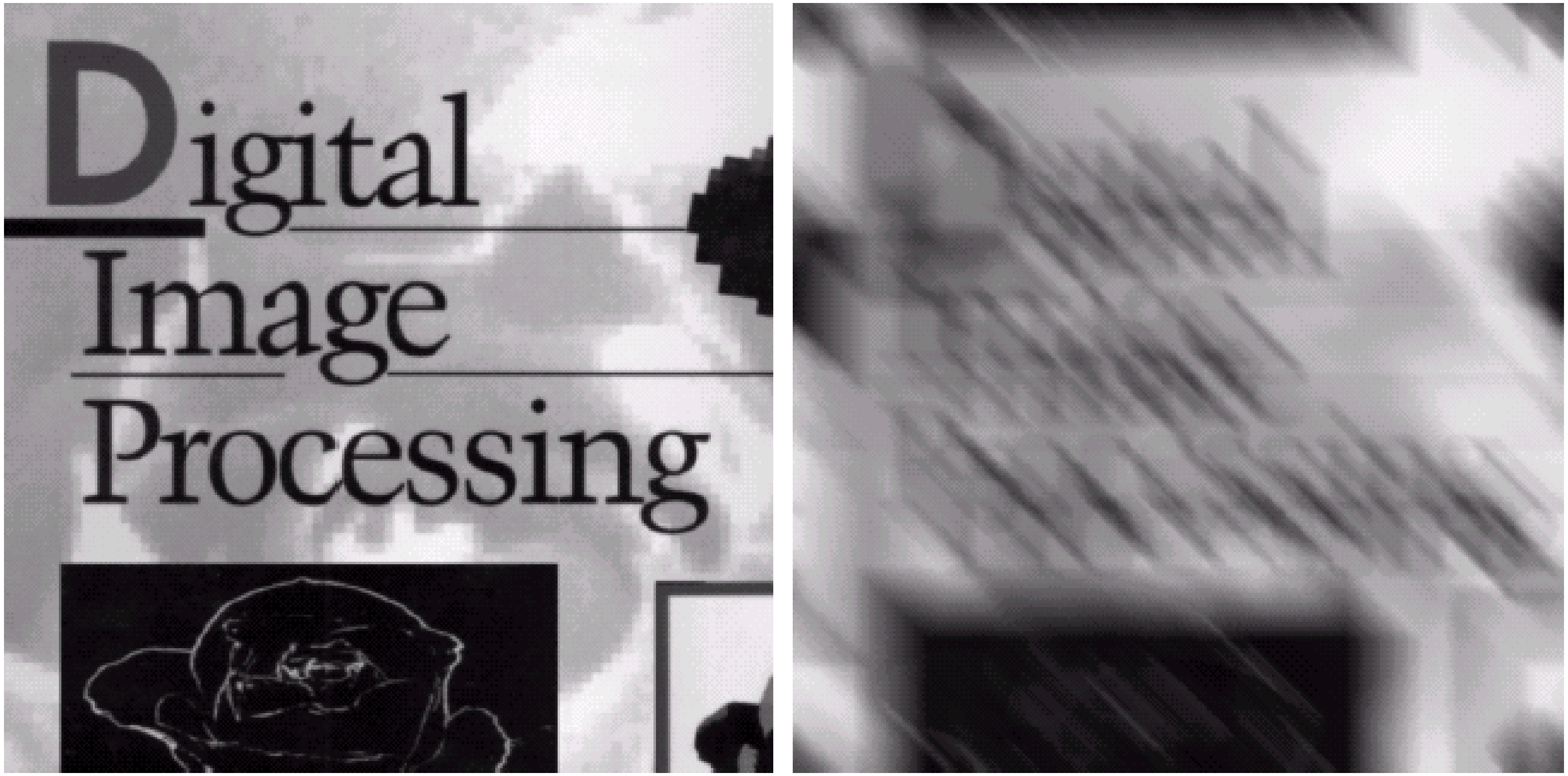

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Chapter 5

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5.1 A Model of the Image Degradation/Restoration Process

An image is usually degraded by noise, vibration and electro-mechanical interactions, hence it needs restoration by various means.

The main Model of degraded image discussed here is:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$f(x, y)$ — "Original" Image , $h(x, y)$ — Degradation Function

$\eta(x, y)$ — Additive Noise, $g(x, y)$ — Degraded Image

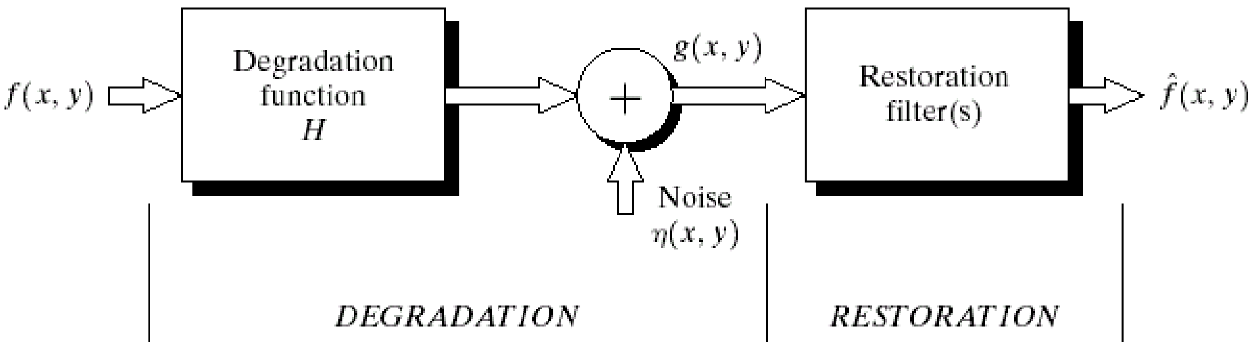

FIGURE 5.1 A model of the image degradation/ restoration process.

5.1 A Model of the Image Degradation/Restoration Process

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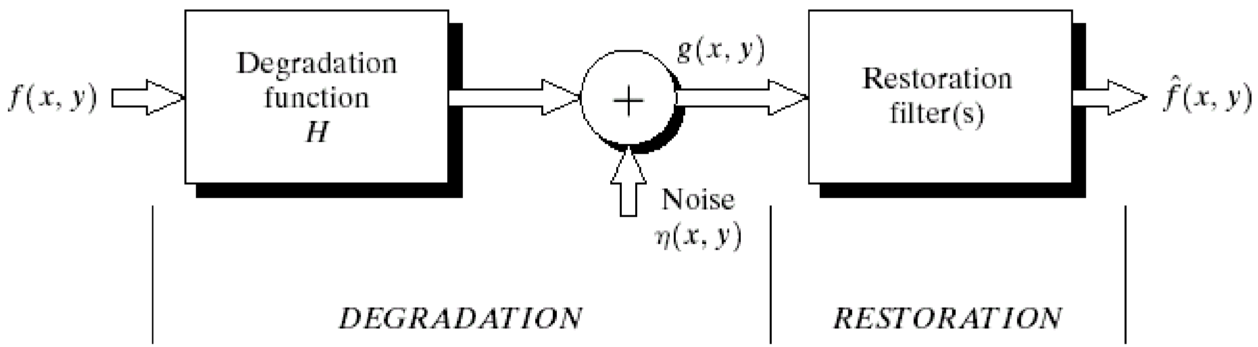


FIGURE 5.1 A model of the image degradation/restoration process.

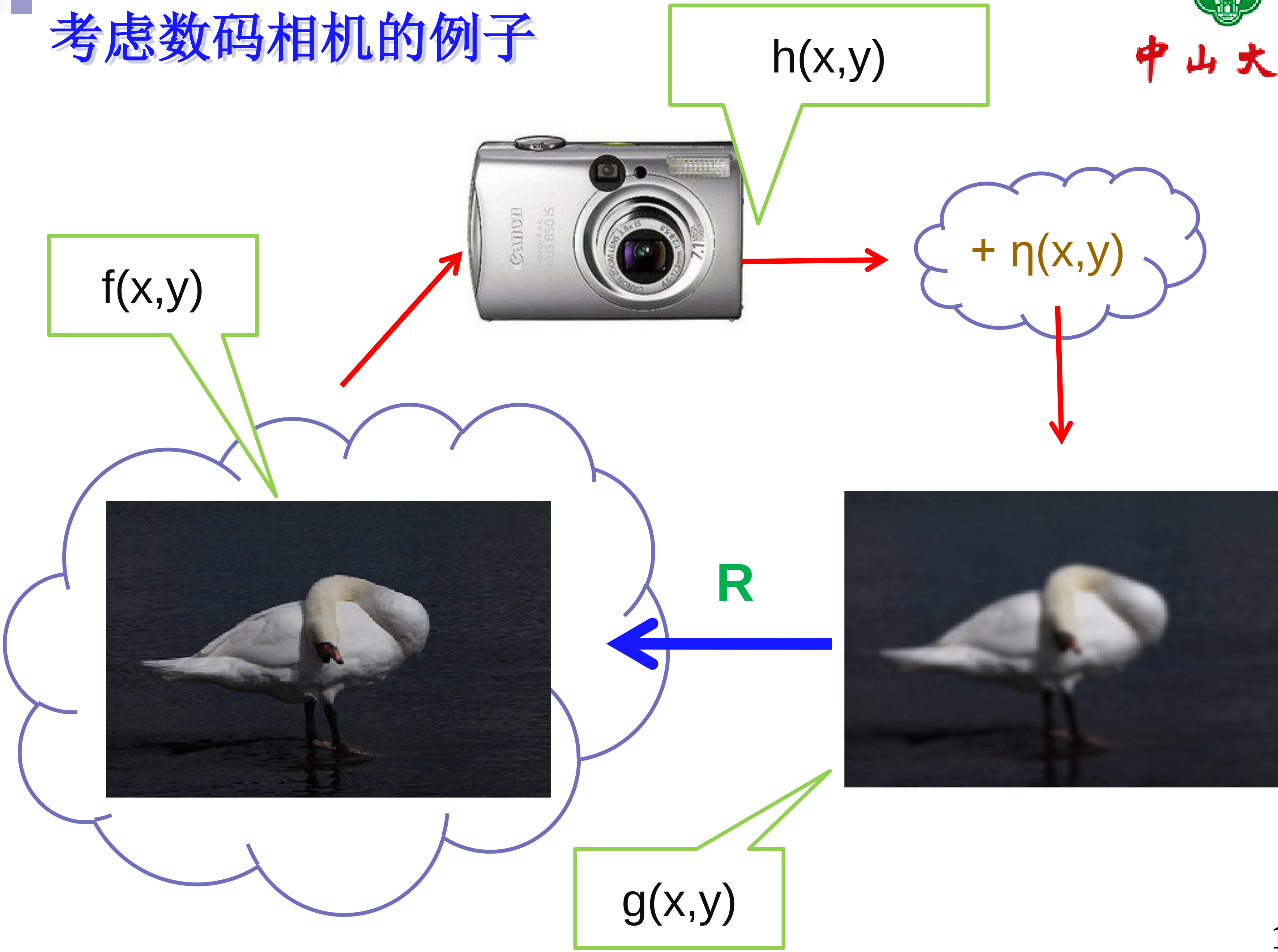
According to Fourier Transform and the Convolution Theory, we have equivalent forms:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$\longrightarrow \quad G(u, v) = H(u, v) F(u, v) + N(u, v)$$

按上述模型, 本章的图像复原分为两部分: 噪声(加性污染)处理和退化(乘性污染)处理.

考虑数码相机的例子



5.1 A Model of the Image Degradation/Restoration Process

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$f(x, y)$ — "Original" Image , $h(x, y)$ — Degradation Function

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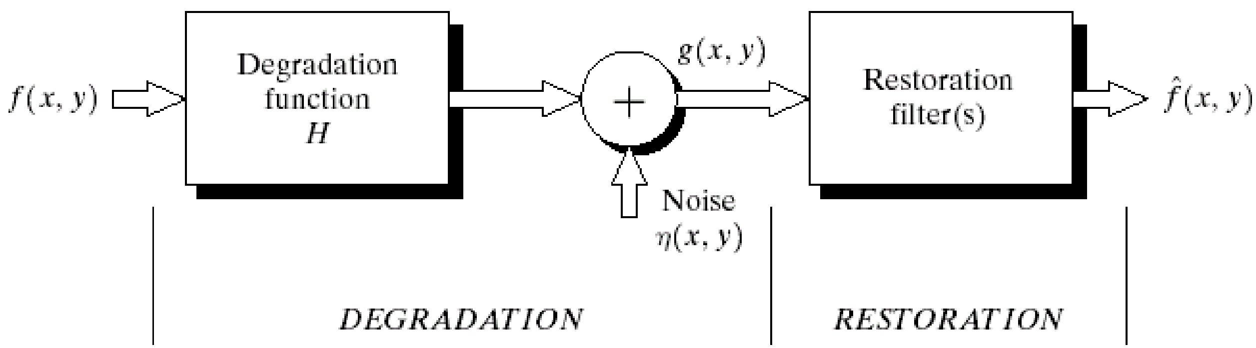

FIGURE 5.1 A model of the image degradation/ restoration process.

According to Fourier Transform and the Convolution Theory, we have:

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

5.2 Noise Models

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注意学习一种建模处理方法

如前面所述, 图像的污染源分为两部分, 一部分是噪声（加性）, 一部分是退化（乘性）。先考虑加性噪声的情形. 处理噪声, 首先要做的是建立噪声模型。

重要假定: 除周期噪声之外，这里假设噪声是与图像和坐标无关的随机变量。对于随机变量, 概率密度函数是描述其行为的主要工具。

5.2 Noise Models

wanted image

Outline

Gained image

Problem to solve here: we have

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y) = f(x, y) + \eta(x, y)$$

Our purpose is to recover $f(x, y)$ from the noise image $g(x, y)$, which is almost the same as to remove noise $\eta(x, y)$ from $g(x,y)$ if we don't consider the impact of $h(x, y)$.

- **To remove noise efficiently, it is better to know the noise model first;**
- **To build a model for an unknown noise image, it is better to know all the existing and widely used noise models.**

注：所谓建模这里指的就是建立噪声的分布或者概率密度函数

5.2 Noise Models

一些和随机变量有关的参量和函数:

知道了分布函数或概率密度函数就相当于知道了噪声的属性

Distributed function: $F(z) = P\{ x \le z \}$

Probability Density Function: $p(z) = dF(z)/dz$

期望值相当于平均值、重心:

$$E[x] = \overline{x} = m = \sum_{i=1}^{N} x_i P(x_i)$$

Standard deviation: standard deviation squared is called the *variance*, which shows the deviation from the expected (mean) value of random variable

$$\boxed{\sigma^2 = E[(x-m)^2] = \sum_{i=1}^{N} (x_i - m)^2 P(x_i)}$$

5.2 Noise Models

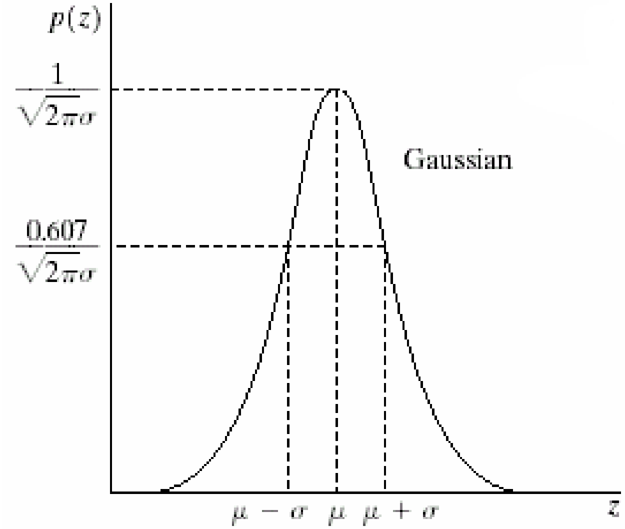
几种重要噪声的概率密度函数 **PDF (Probability Density Function)**

常用的工程方法：给出几种常用和易于处理的噪声模型, 以便于在实际问题中进行对比参考。(注意噪声看成是随机变量)

- **Gaussian Noise:**

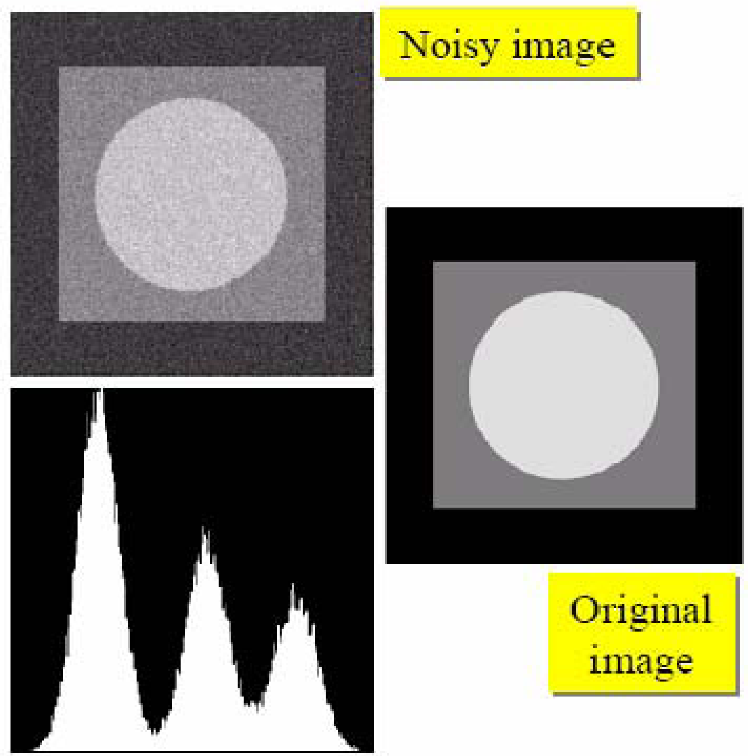
The probability density function (normalized histogram) $p(z)$ is given by

$$\boxed{p(z)=\frac{1}{\sqrt{2\pi\sigma}}e^{-(z-\mu)^2/2\sigma^2}}$$



z: gray level
μ: mean value
σ: standard deviation

其中标准差的平方σ^2称为z的方差. 当z服从这一分布时, 其值有70%落在$[\mu-\sigma, \mu+\sigma]$的范围内, 而95%的值落在$[\mu-2\sigma, \mu+2\sigma]$的范围内.



Histogram and p(z) of noise – triple Gaussian noises

The original image is designed to measure noise. The noise probability density p(z) can be measured by

Noise = noisy image – original image

Then the noise model parameters μ and σ^2 can be evaluated from the histogram.

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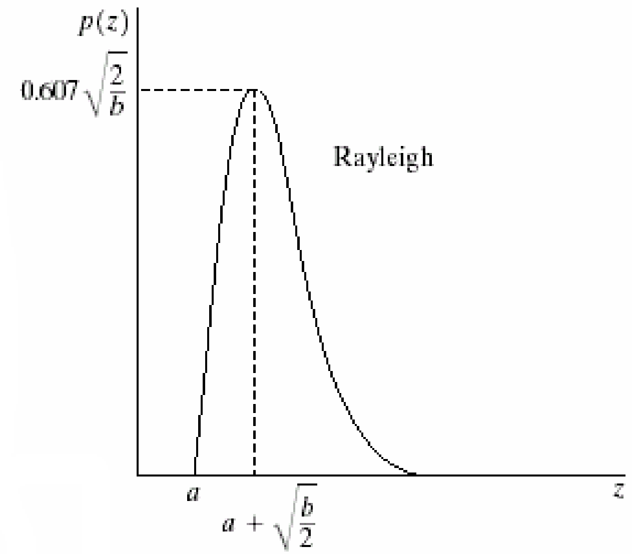
Research

5.2 Noise Models

- **Rayleigh noise**

Its density function is:

$$p(z)=\begin{cases}\dfrac{2}{b}(z-a)e^{-(z-a)^2/2b} & z\ge a\\ 0 & z<a\end{cases}$$



The mean and variance of this density are given by

$$\mu = a+\sqrt{\pi b/4}$$

$$\sigma^2=\frac{b(4-\pi)}{4}$$

注意此密度函数距原点的位移和图形的形状向右偏移的事实, 瑞利密度函数可以较好的近似那些形状偏移的直方图.

5.2 Noise Models

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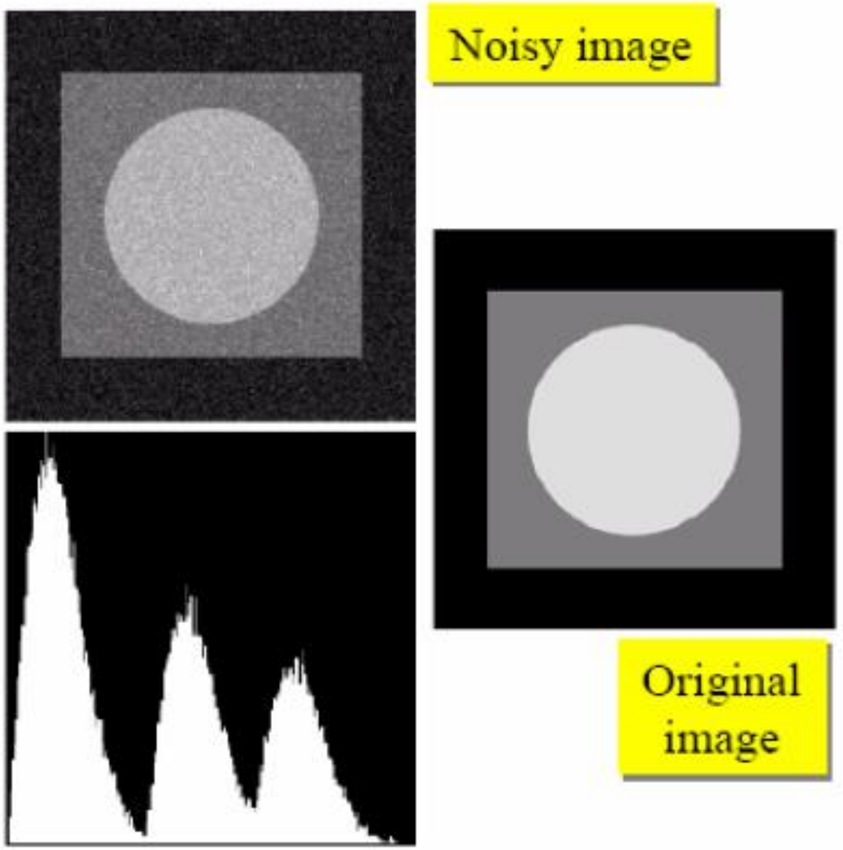


The histogram shows there are three peak of Rayleigh type noise, hence the noise model can assumed as

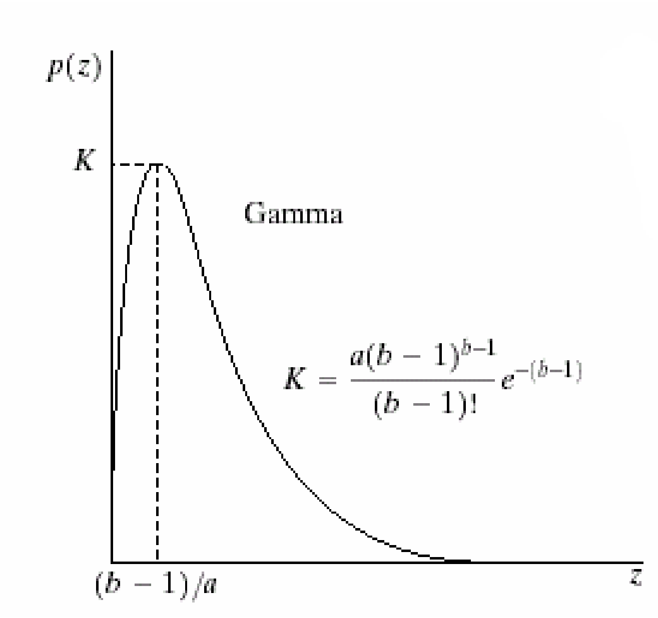
$$p(z)=\sum_{k=1}^{3}\frac{2}{b_k}(z-a_k)e^{-(z-a_k)^2/b_k}$$

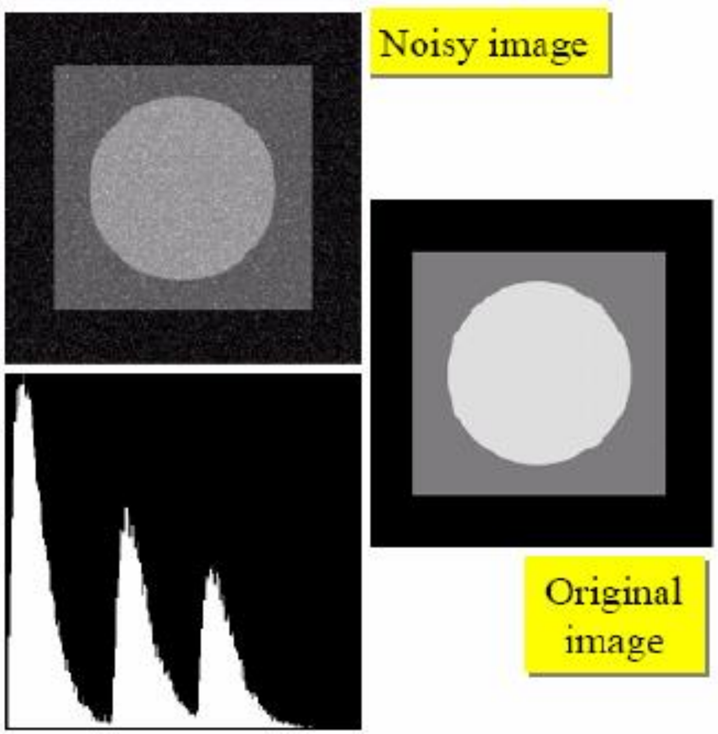
5.2 Noise Models

- **Erlang/Gamma noise**

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\mu = \frac{b}{a} \qquad \sigma^2 = \frac{b}{a^2}$$



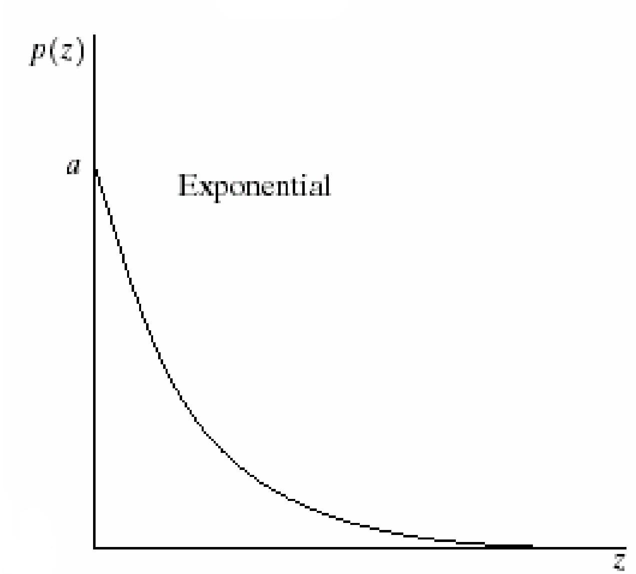


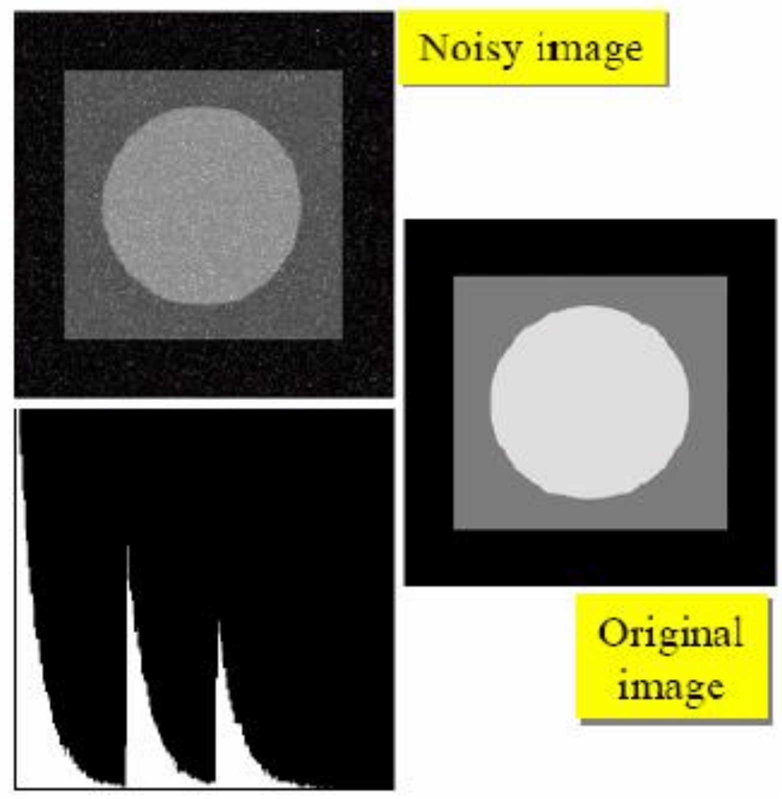
5.2 Noise Models

- **Exponential Noise**

$$p(z) = \begin{cases} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\mu = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$





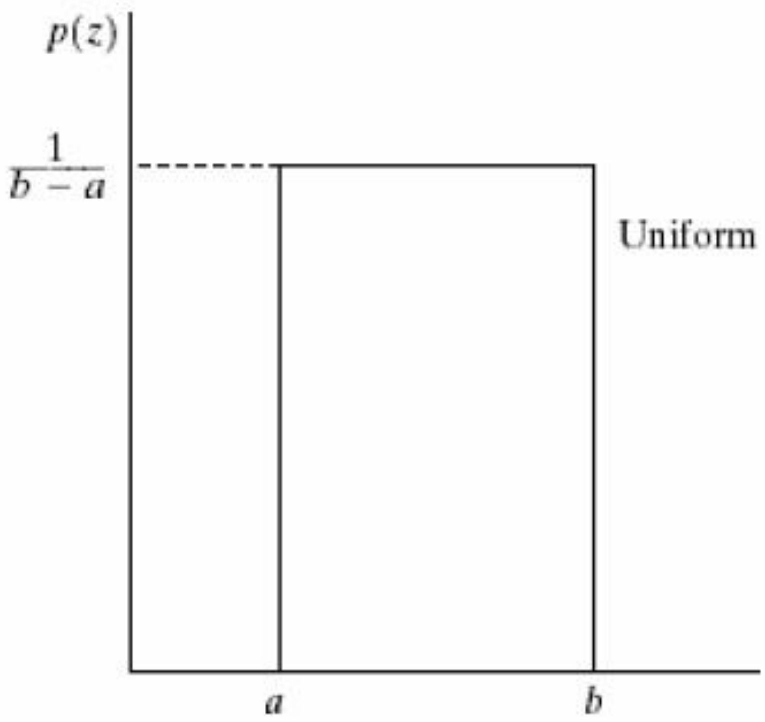
5.2 Noise Models

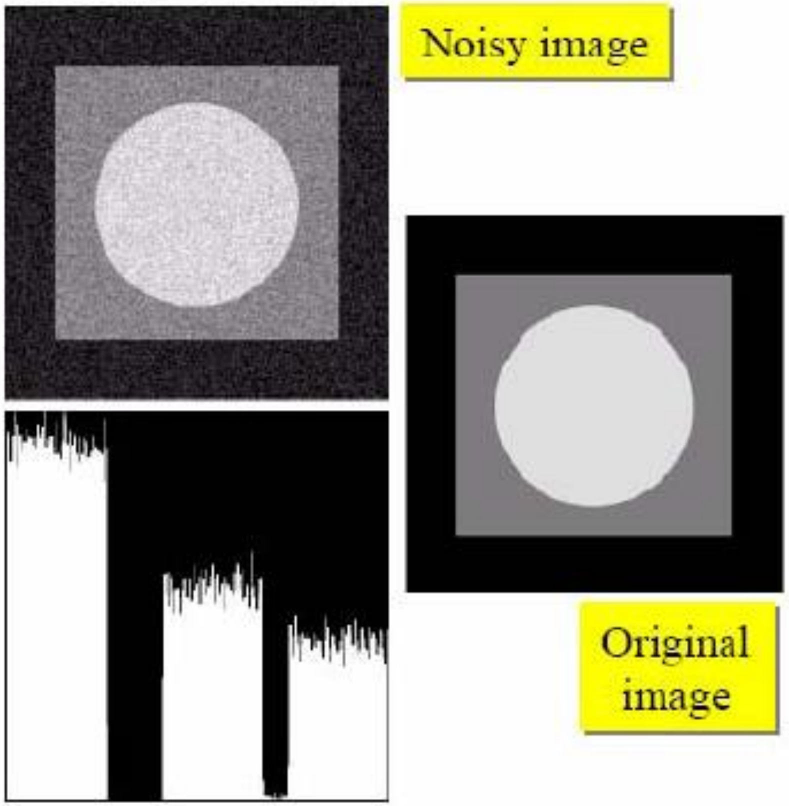
- Uniform noise

$$p(z)=\begin{cases}\dfrac{1}{b-a} & \text{for } a\le z\le b\\ 0 & \text{otherwise}\end{cases}$$

$$\mu=\frac{a+b}{2}$$

$$\sigma^2=\frac{(b-a)^2}{12}$$



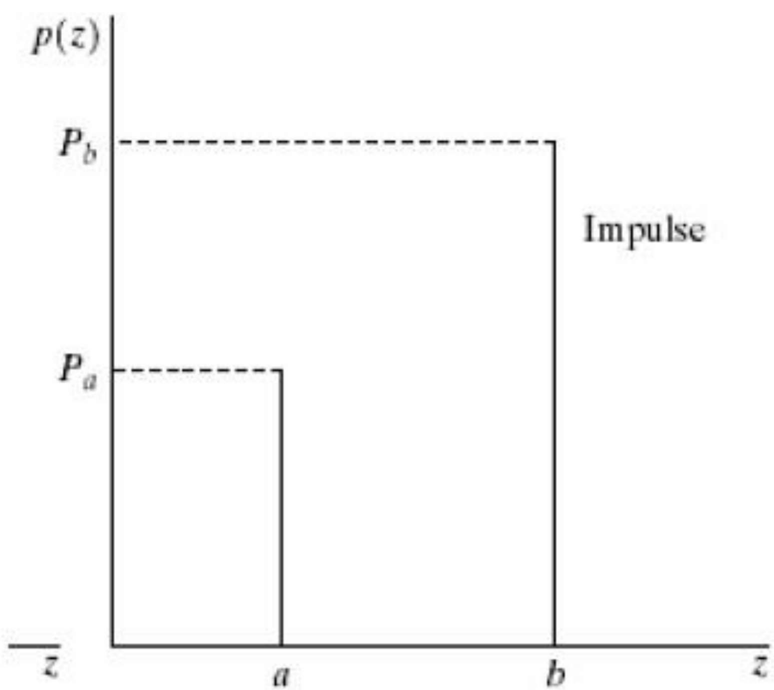


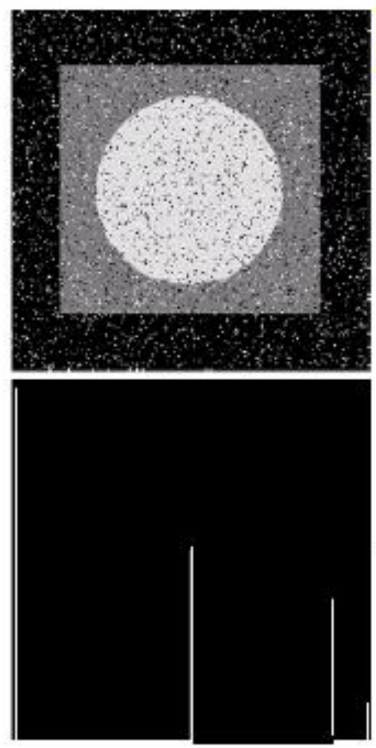
5.2 Noise Models

- Impulse (salt-and-pepper) noise

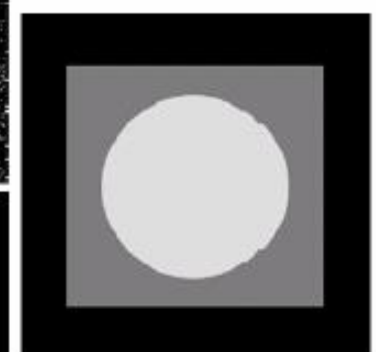
$$\boxed{p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}}$$

bipolar noise has two impulse peak





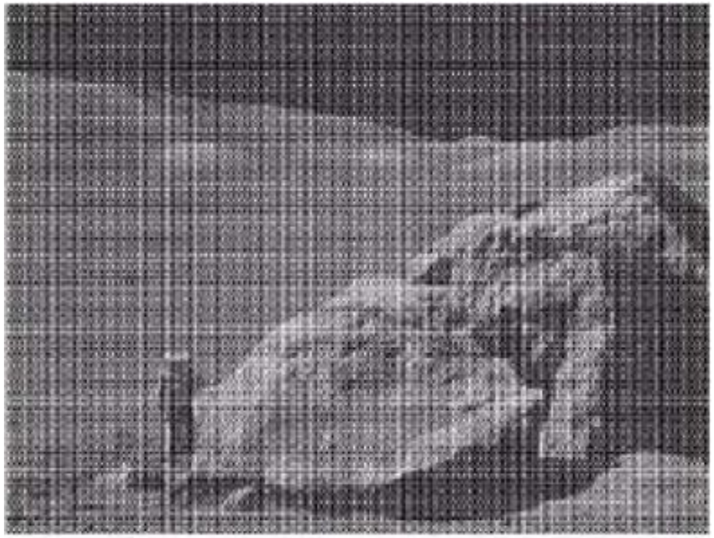
Noisy image



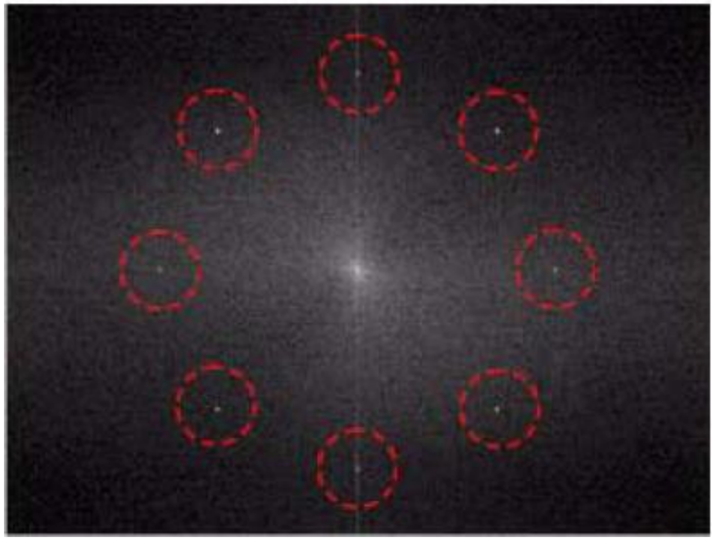
Original image

5.2 Noise Models

- **Periodic noise**



The image shows spatially periodic noises, corresponds to sine waves of various frequency



The DFT of the original image shows four conjugate pair of peaks indicating the frequencies of the periodic noise in the original image.

5.2 Noise Models

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为什么周期噪声的富里叶变换会出现共轭对称亮点? 先看一个纯正弦函数的傅里叶变换: 设r_0是整数，$f(x) = A\sin(2\pi r_0 x/M)$。

根据欧拉公式

$$f(x) = A\sin\frac{2\pi r_0 x}{M} = \frac{A}{2j}\left(e^{j2\pi r_0 x/M} - e^{-j2\pi r_0 x/M}\right)$$

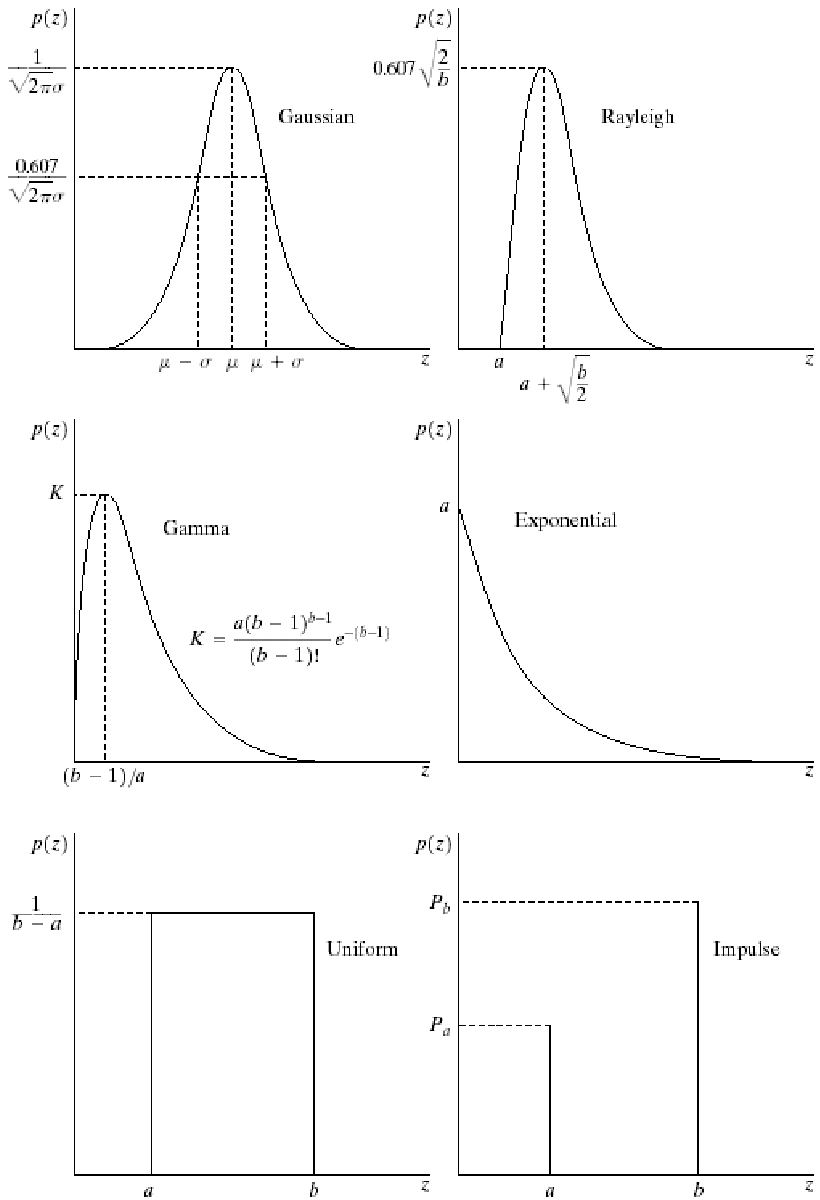
其傅里叶变换为（注意基底的正交性）:

$$F(u) = \frac{A}{2jM}\sum_{x=0}^{M-1}(e^{j2\pi r_0 x/M} - e^{-j2\pi r_0 x/M})e^{-j2\pi ux/M}$$

$$= \frac{A}{2jM}\sum_{x=0}^{M-1}(e^{\frac{j2\pi(r_0-u)x}{M}} - e^{\frac{-j2\pi(r_0+u)x}{M}})$$

易见上式只有当$u = r_0$和$u = M - r_0$时不为零（作业：证明）。$u = r_0$和$u = M - r_0$恰好是关于中点对称的两个点. 因此一维纯正弦函数的富里叶变换就是两个关于中心对称的两个脉冲（能否观察到与振幅强度有关）。

5.2 Noise Models



a b
c d
e f

FIGURE 5.2 Some important probability density functions.

5.2 Noise Models

Applications

Gaussian Noise—电子电路噪声、低照明度或高温引起的传感噪声

Rayleigh Noise—可用于刻划远程图象的噪声现象

Exponential/Gamma Noise—应用于激光成像中的噪声

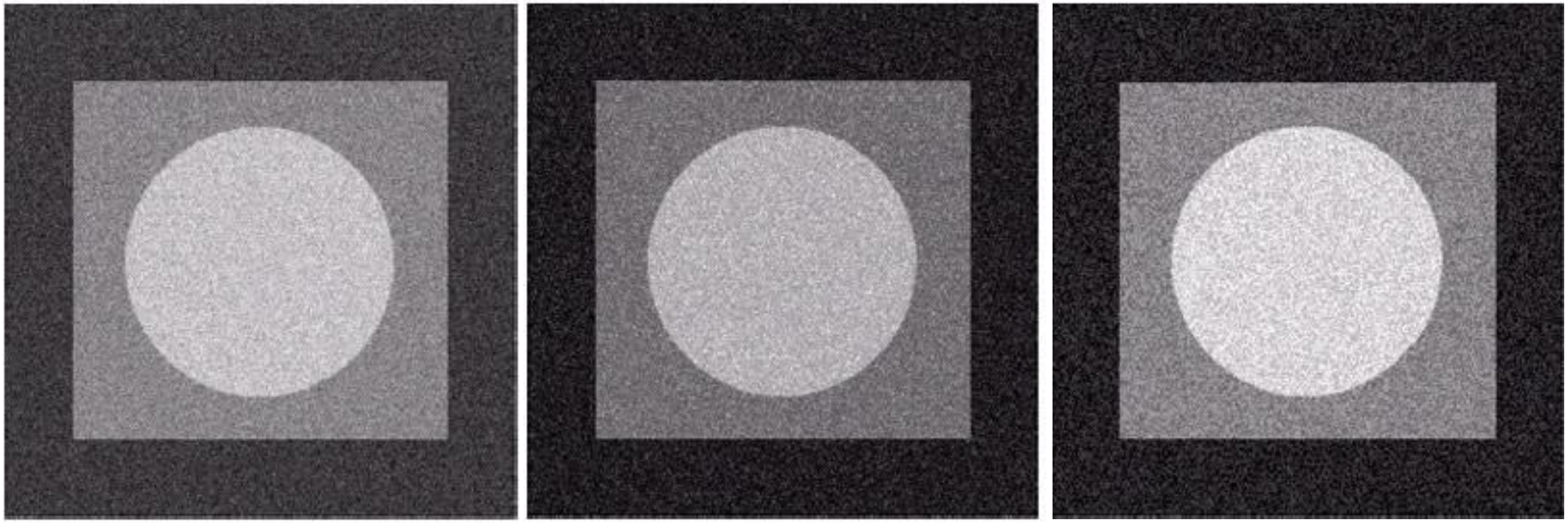
Impulse Noise—成像中短暂停顿(如错误的切换开关等)引起的噪声

Periodic Noise —唯一一种依赖于空间位置的噪声，常出现于电力或机电的干扰(图像扫描仪老化也会出现这种噪声)。处理这种噪声的主要工具是频谱分析, 也就是说在频率域处理此类噪声更为方便.

5.2 Noise Models

- **Estimation of noise parameters**

What are the right noise models for following noised images?



5.2 Noise Models

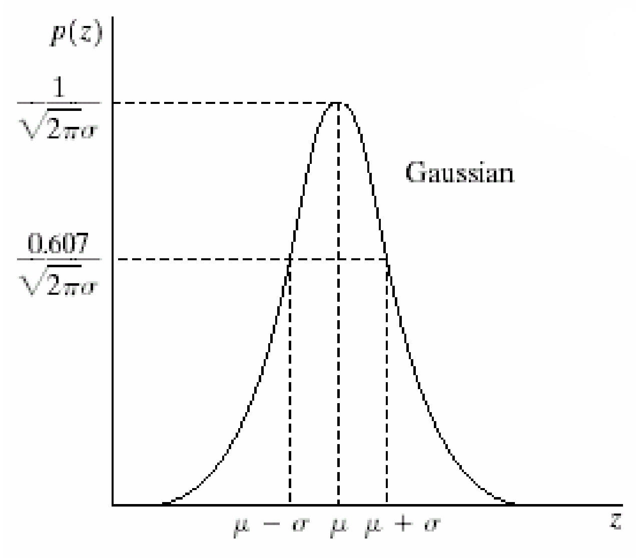
- **Estimation of noise parameters**

如何尽可能多的知道噪声模型的信息?

对周期噪声的估计, 主要是通过对图像频谱的分析. 周期噪声趋向于产生频率尖峰, 这些尖峰甚至由视觉分析也可以观察到.

对一般的随机噪声, 主要是以估计其**PDF**为主.两个要素：模型和参数（均值和方差）

$$\boxed{p(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(z-\mu)^2/2\sigma^2}}$$

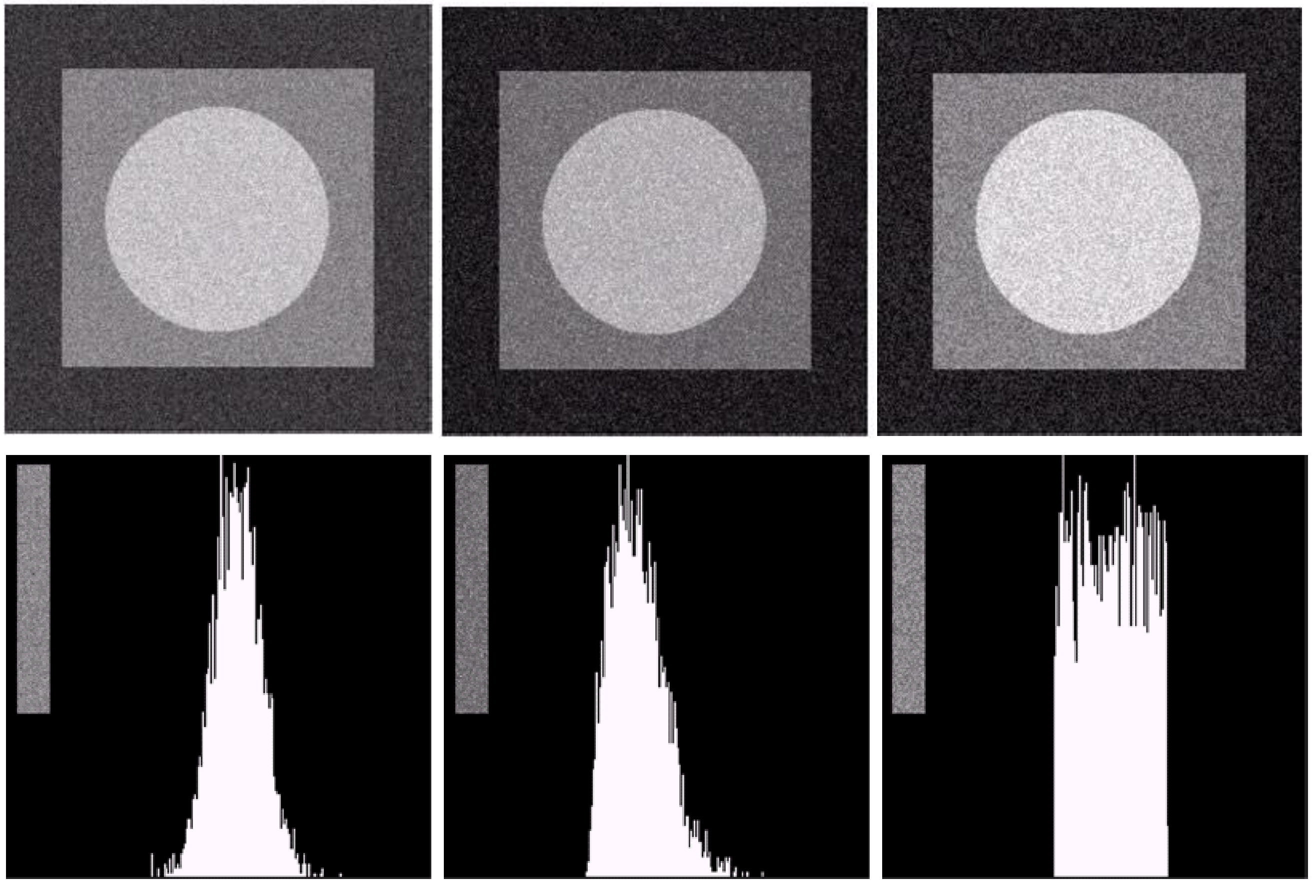


5.2 Noise Models

- 传感器产生的噪声, 其PDF参数可以从说明书上查到. 无法查到的成像系统, 下列两种方式可以作参考;
  - 若成像系统可用, 那么估计系统噪声特性最简单的方式就是在一组“平坦”(均匀光照, 单色背景)环境中成像, 截取一组以噪声为主的图像. 例如, 在光学传感器情况下, 就是简单的对一个固体的、光照均匀的灰度板成像. 成像后就是一个典型的系统噪声的指示器;
  - 如果无法利用成像系统得到噪声的参数, 而仅有传感器产生的图像可以利用时, 常常可以从图像中截取一小部分具有恒定灰度的区域来估计噪声的PDF. 首先计算这一小部分图像的直方图, 然后利用已知的噪声模型, 例如前面提到过的高斯、瑞利、脉冲等, 和计算出来的直方图作对比, 寻找最接近的形状(如图5.6所示). 再根据需要估计相应的参数, 如均值、方差等.

5.2 Noise Models

从图像的一小块“平坦”区域中提取噪声信息(值得借鉴的建模方式)



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

5.2 Noise Models

- Given the probability density function measured from the histogram of noise using the test pattern shown above, first the most-likely noise model is chosen (be it Gaussian, Rayleigh, Gamma or exponential) before the noise parameters are estimated.
- Then, we can use the image strip, denoted by S, to calculate the *mean and variance* of the gray level.

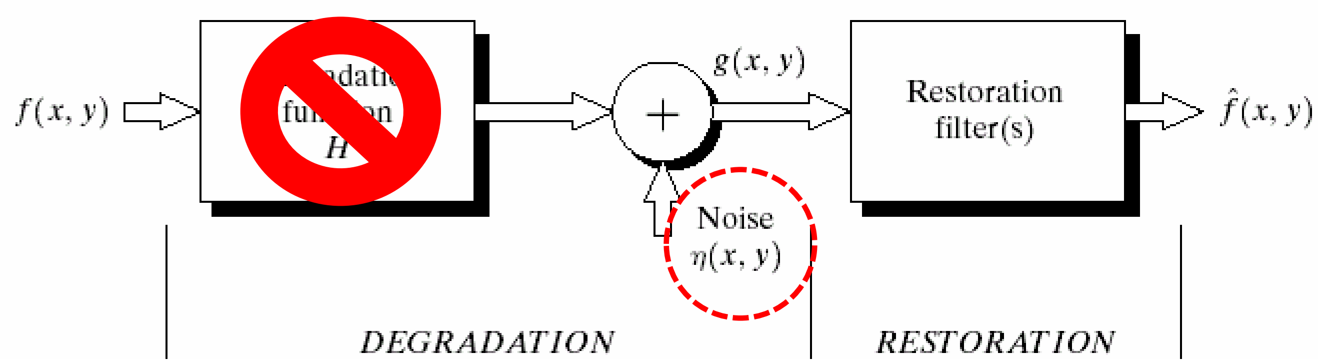
$$\boxed{\mu = \sum_{z_i \in S} z_i p(z_i)} \qquad \boxed{\sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 p(z_i)}$$

Here z_i *'s* are the gray-level values of pixels in S, and $p(z_i)$ are the corresponding normalized histogram values.

5.2 Noise Models

从直方图的形状找出最接近的匹配, 如高斯等. 那么其后计算出来的均值和方差便可以完全将这个高斯分布确定下来. 前边提到的其他类型, 也可以利用均值和方差解出*a* 和 *b*来, 从而将PDF确定下来. 脉冲（椒盐）噪声要用不同的方法来处理, 但一般脉冲噪声是唯一视觉可见的噪声, 比较容易区分.

5.3 Restoration in the Presence of Noise only-Spatial Filtering



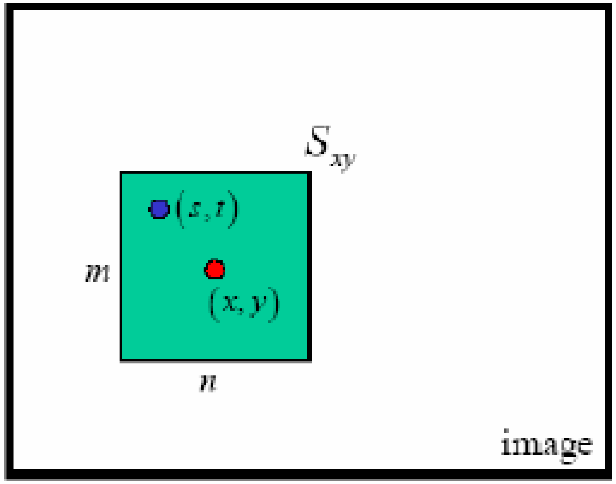
This section discusses various noise filters for image restoration when the degradation function H(u, v)=1 and the noises are position independent. That is, noise is the only source for degradation of the image.

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

Mean Filters

A mean filter uses a convolution mask of the size $m \times n$ to result in one pixel in the output image.



- ✓ **Arithmetic Mean Filter**

$$\boxed{\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)}$$

- ✓ **Geometric Mean Filter**

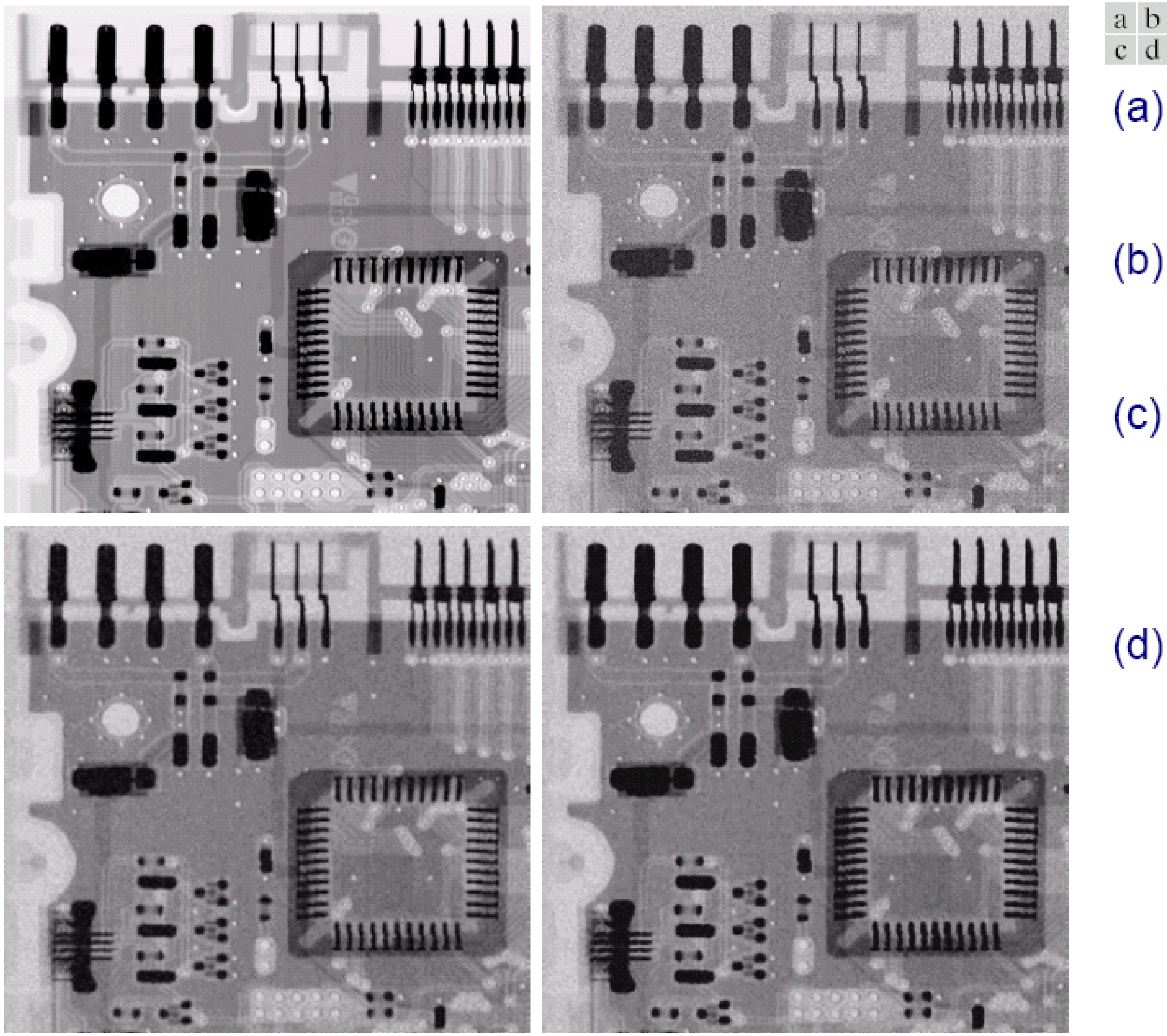
$$\boxed{\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}}$$

- In the above, (x, y) is the location of the output pixel and (s, t) is the location of a pixel in the mask.

◆ Arithmetic mean filter find the arithmetic average, while the other find the geometric average. Averaging causes blurring (low-pass filtering).

5.3 Restoration in the Presence of Noise Only

例5.2: 几何均值滤波器和算术均值滤波器的比较



(a) Original X-ray image

(b) Gaussian noise

(c) Result of arithmetic mean filter - blurring

(d) Result of geometric mean filter - blurring

5.3 Restoration in the Presence of Noise Only

- Harmonic mean filter 调和均值滤波——倒数均值的倒数

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

It works well for Gaussian noise and salt noise (white dots on the image), but fails with pepper noise (black dots)

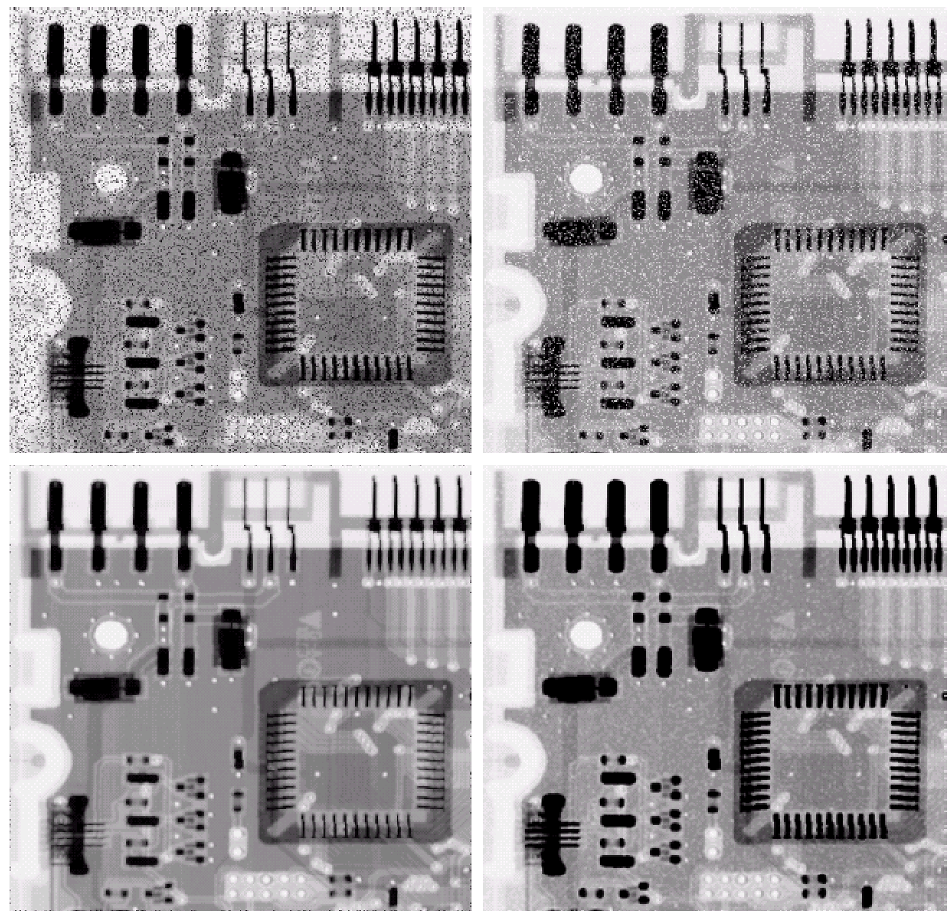
- Contra-harmonic mean filter (反调和均值滤波)

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- $Q > 0$: for pepper noise
- $Q < 0$: for salt noise
- $Q = 0$: arithmetic mean filter
- $Q = -1$: harmonic mean filter

5.3 Restoration in the Presence of Noise Only

总的来说，算术均值滤波器和几何均值滤波器更适合处理高斯或者均匀等随机噪声．谐波均值滤波器更适合处理脉冲噪声，但它的缺点是必须知道噪声是暗噪声(胡椒)还是亮噪声(盐)，以便于选择合适的Q符号．若符号选错，就会出现灾难的结果.



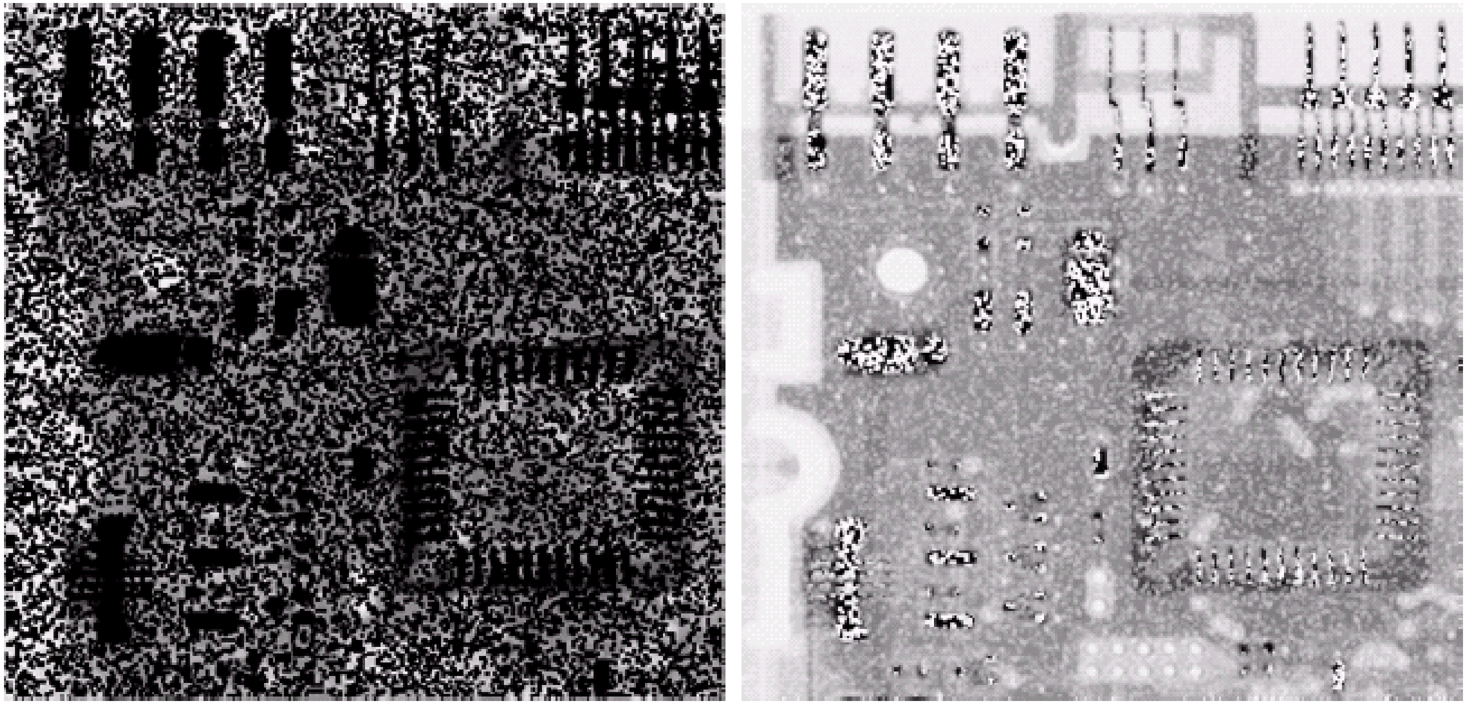
a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Correct parameters

5.3 Restoration in the Presence of Noise Only

Wrong parameters



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

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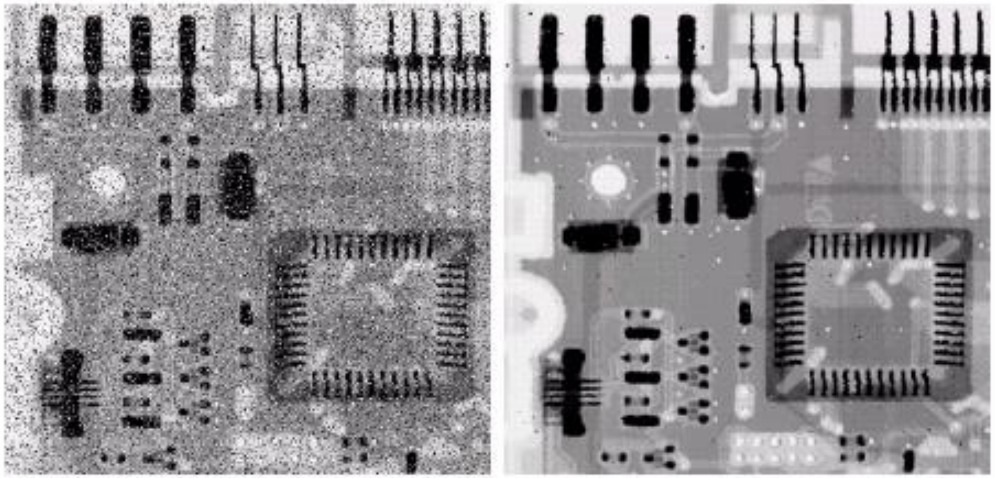
- **Order-statistic filters**

◆Median filter: is a typical order-statistic filter that determines the output pixel according to the order of pixel brightness in a mask.

Median filter is good for both bipolar and unipolar impulse noise

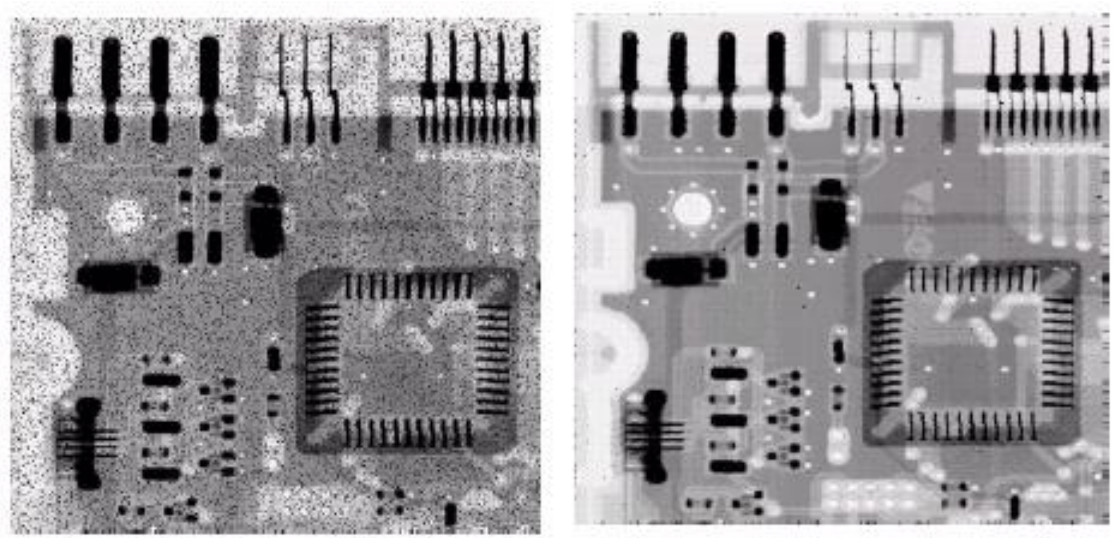
$$\boxed{\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}}$$



5.3 Restoration in the Presence of Noise Only

- Max filters : To filter out pepper noise (black dots)

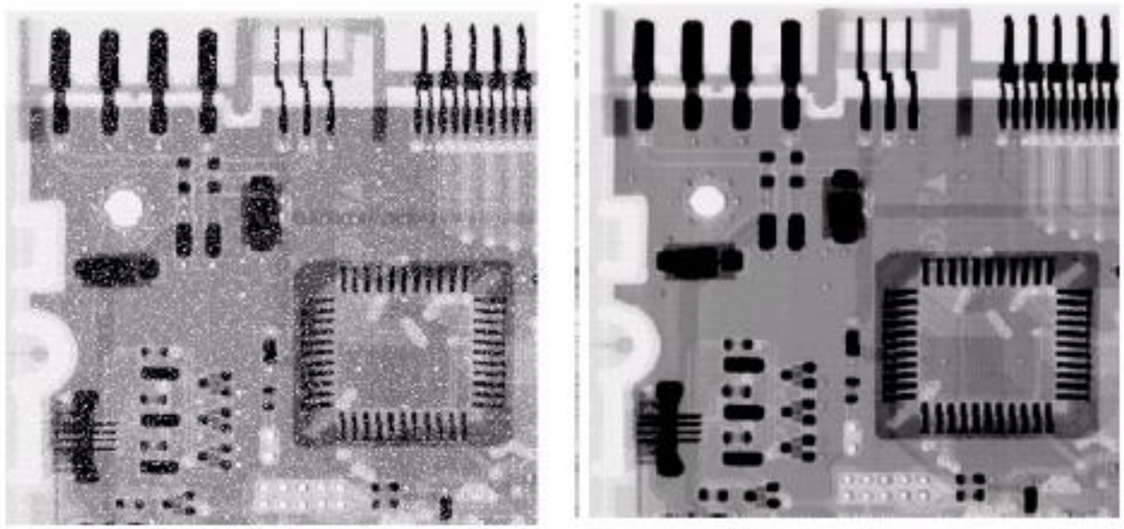
$$\boxed{\hat{f}(x,y)=\max_{(s,t)\in S_{xy}}\{g(s,t)\}}$$



5.3 Restoration in the Presence of Noise Only

- Min filters : To filter out salt noise (white dots)

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$



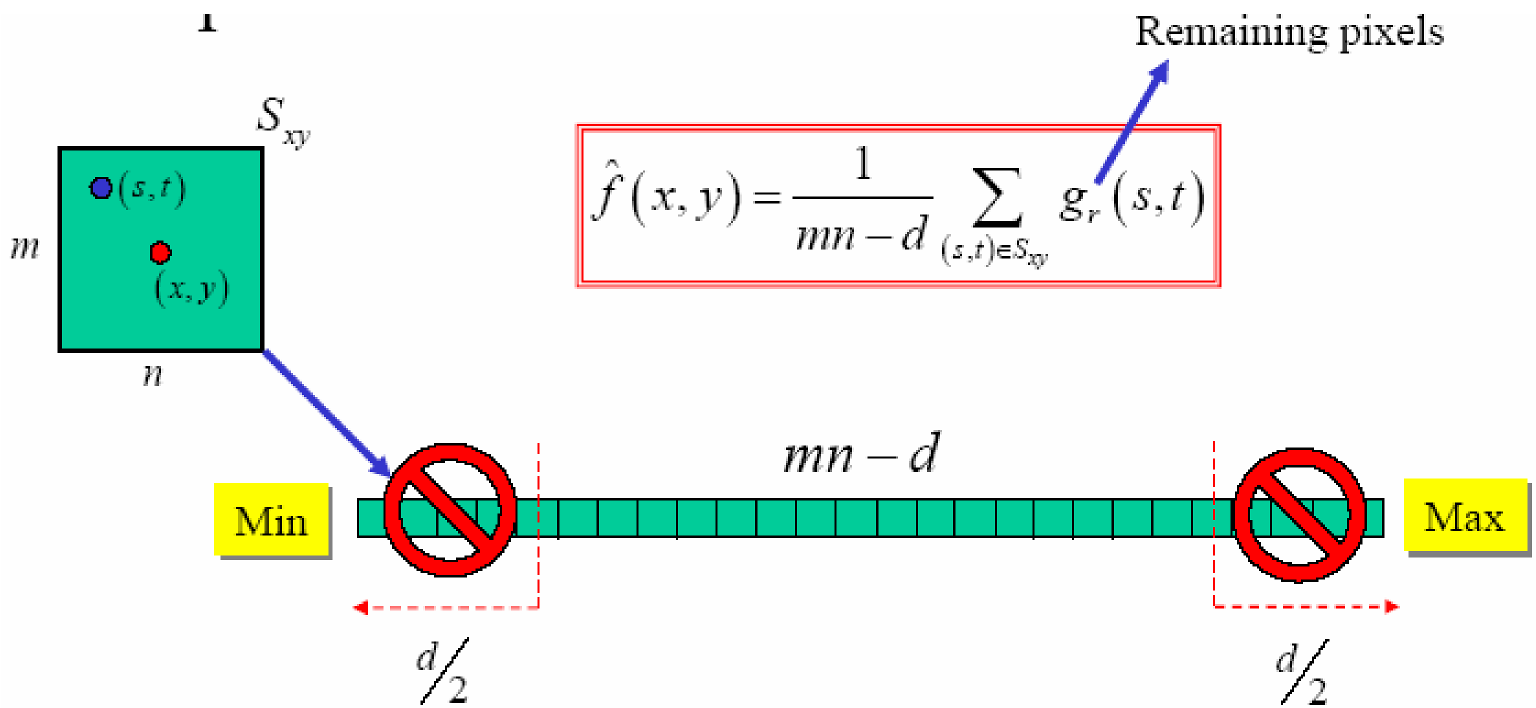
- Midpoint filters : Use the sum of Max and Min filter to remove pepper and salt noises, Gaussian noise and uniform noises.

$$\hat{f}(x,y) = \frac{1}{2}\left[\min_{(s,t)\in S_{xy}} \{g(s,t)\} + \max_{(s,t)\in S_{xy}} \{g(s,t)\}\right]$$

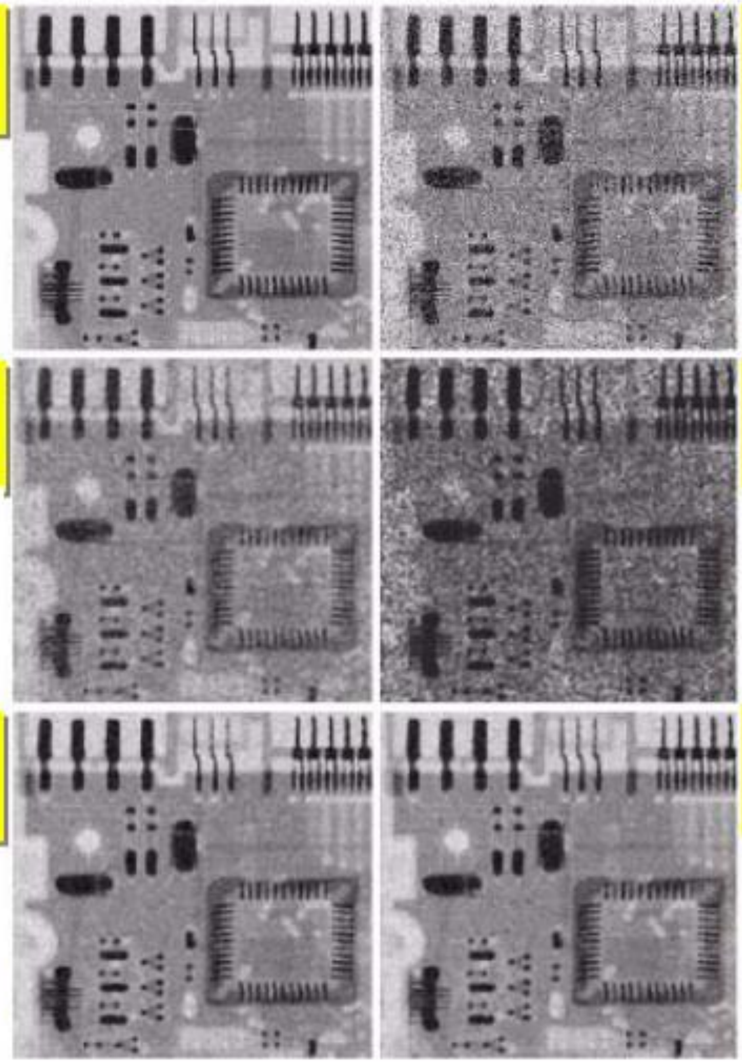
5.3 Restoration in the Presence of Noise Only

◆ Alpha-trimmed mean (ATM) filter ：for Gaussian + salt-and-pepper noise

This filter first remove the d/2 lowest and the d/2 highest grey level in the mask and leave mn-d pixels denoted by $g_r(s, t)$, them average these pixels to give the resultant output.



5.3 Restoration in the Presence of Noise Only



Averaging causes blurring

ATM filter = Arithmetic mean filter for d = 0

ATM filter = median filter for d = (mn-1)/2

- Adaptive filters

Let g(x, y) be the noisy image, σ_η^2 describes the global zero-mean noise of the image, m_L and σ_L^2 describes the local noise in the mask, then the output grey level at (x, y) is given by

Variance of the zero-mean noise

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Local variance

Local mean

The global variance σ_η^2 is usually unknown, but it is important to maintain $\sigma_\eta^2 < \sigma_L^2$ by some pretests.

An adaptive filter can remove Gaussian noise as effective as arithmetic and geometric mean filter, but with a minor effect of blurring (low-pass filtering).

- **Adaptive Filters**

优点：adaptive、local noise filtering；缺点：computing complexity increase

Only consider the case of additive noises. Suppose (x, y) is current pixel to deal with, and S_{xy} is its local region.

(*a*) $g(x, y)$—input (带噪声图像在(x, y)上的值)

(*b*) σ_η^2—噪声的方差

(*c*) m_L—S_{xy}内像素点的局部均值

(*d*) σ_L^2—S_{xy}内像素点的局部方差

除了σ_η^2之外, 其余的计算都是可行的。

5.3 Restoration in the Presence of Noise Only

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我们期望自适应滤波器有以下预期性能：

- 若$\sigma_\eta = 0$（没有噪声），滤波器返回$g(x, y)$的值;
- 若相对于σ_η^2, 局部方差σ_L^2较高, 则滤波器应该返回一个$g(x, y)$的近似值。原因是高局部方差通常和边缘有关, 这些边缘应该保留;
- 若两个方差相等, 表明局部区域的图像和整体图像有相同特性, 这时局部噪声可以用求平均来降低.希望滤波器返回区域上S_{xy}像素的算术平均值.

5.3 Restoration in the Presence of Noise Only

满足上述条件的自适应滤波器可以定义为:

$$\hat{f}(x,y)=g(x,y)-\frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y)-m_L\right]$$

该式的一个假设是$\sigma_\eta^2 \le \sigma_L^2$, 由于$S_{xy}$是$g(x, y)$的一个子集, 这个假设是合理的. 然而在实际应用中对σ_η^2知之甚少（通常是估计值或者尝试值）, 所以, 有时也会出现$\sigma_\eta^2 > \sigma_L^2$的情况. 因而在程序中可以设置一个条件判断, 当$\sigma_\eta^2 > \sigma_L^2$出现时将它们的比率值设置为1. 这时滤波器是非线性的.

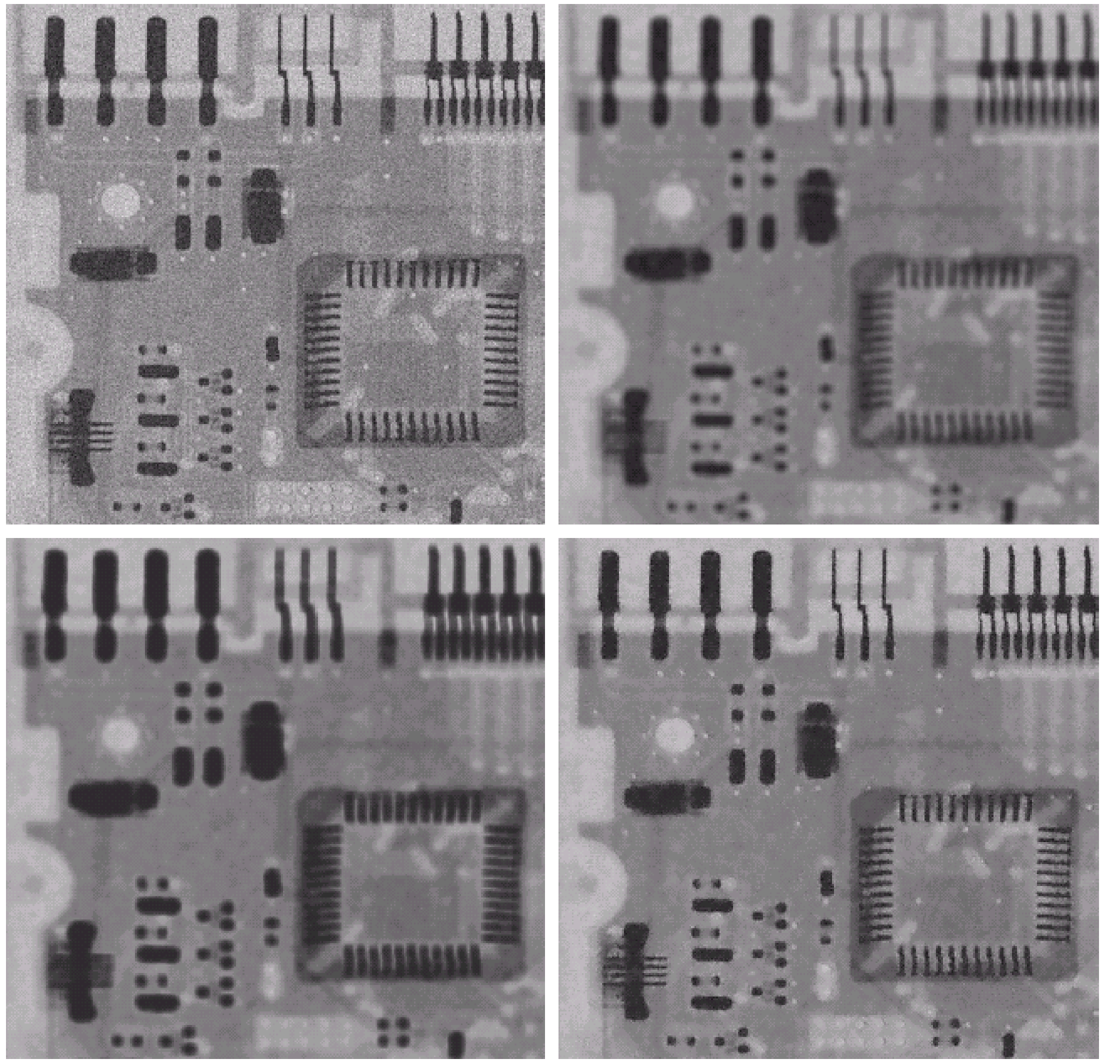
另一种选择是允许负值, 最后作图像标定, 但这样会损失图像的动态范围.

5.3 Restoration in the Presence of Noise Only

高斯噪声污染

a	b
c	d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.



算术平均

几何平均

自适应算法

- **自适应中值滤波器**

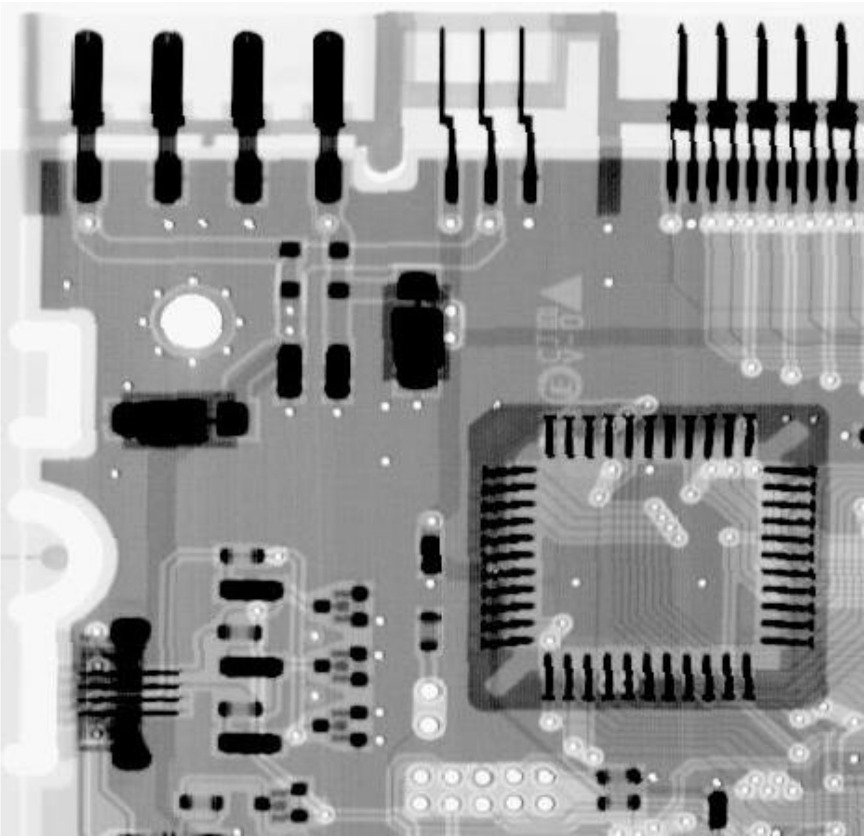
Median filter——

优点：除去椒盐噪声、平滑其他非冲击噪声；

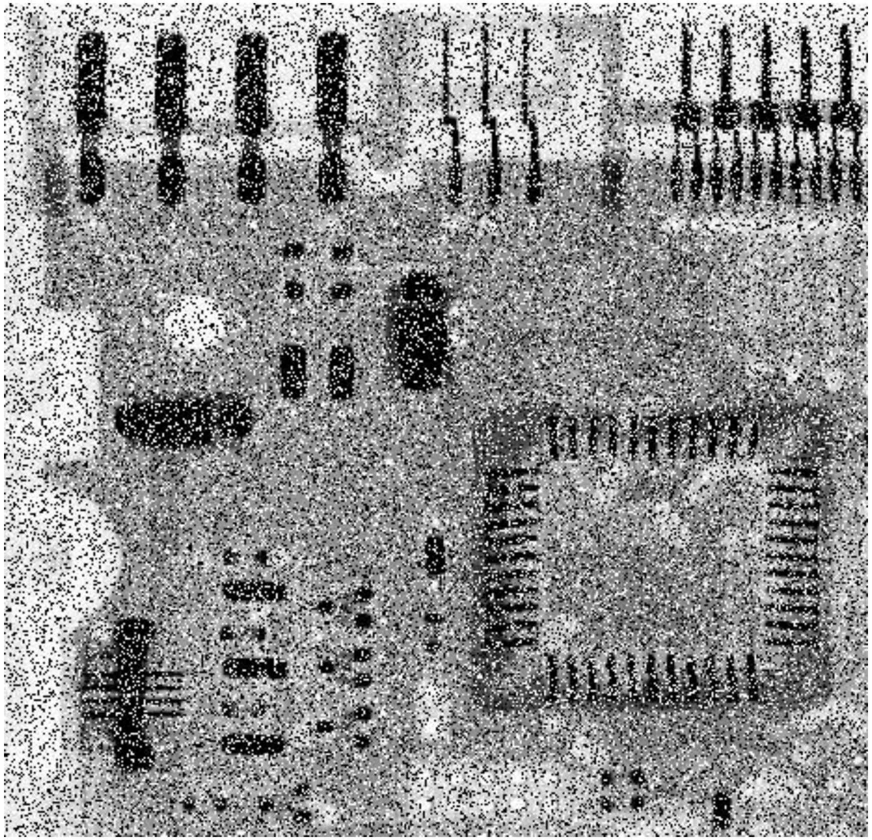
缺点：物体边缘有细化或粗化等失真，去噪效果还依赖于窗口的大小。

See an example of different effects for different fixed window sizes.

5.3 Restoration in the Presence of Noise Only



Original



Salt-and-pepper noise

(P_a=P_b=0.16)

PSNR=9.64

5.3 Restoration in the Presence of Noise Only

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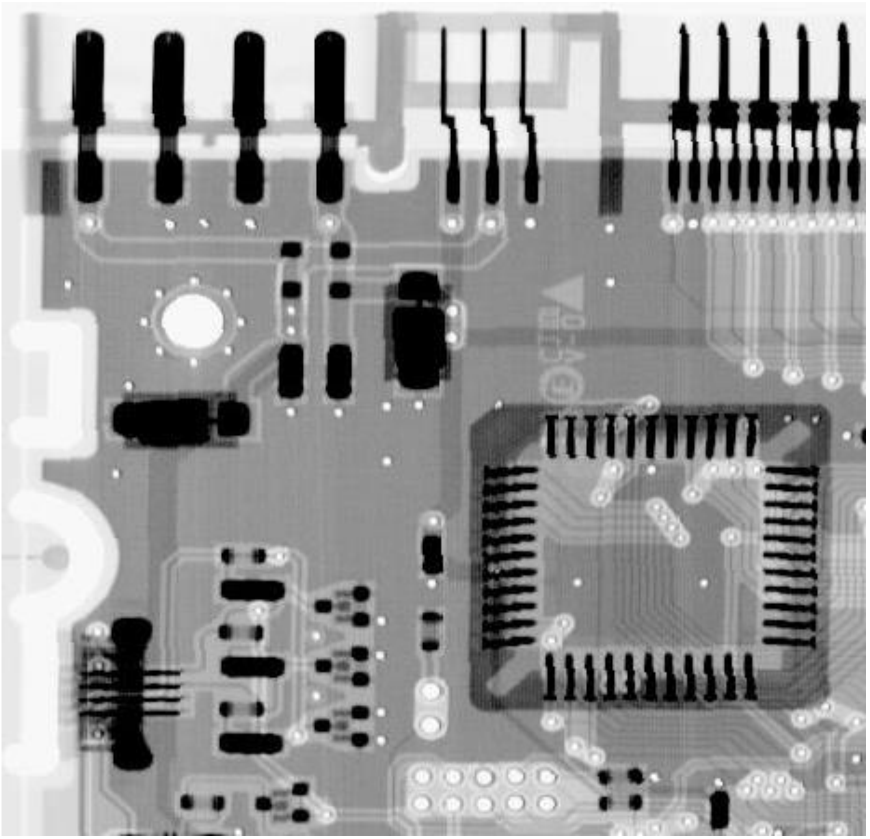
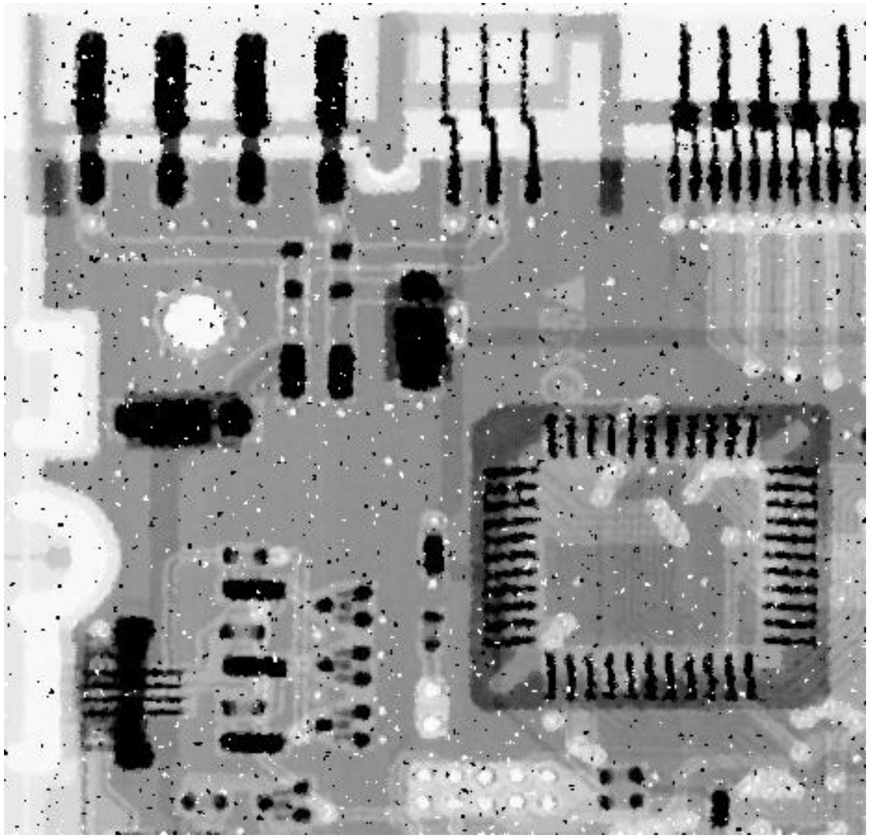


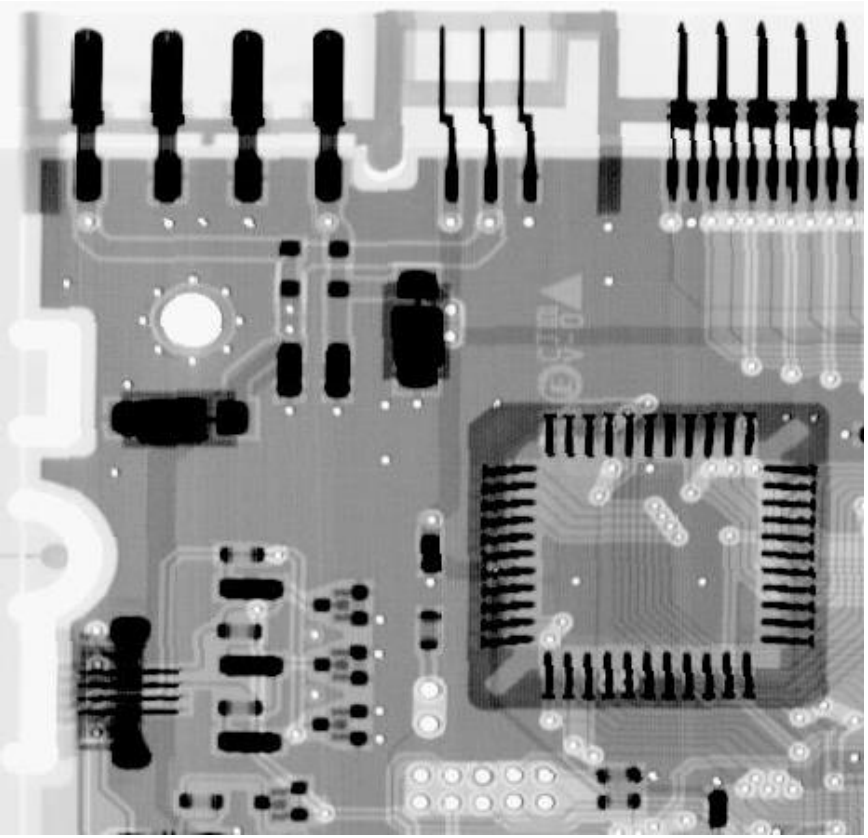
Original



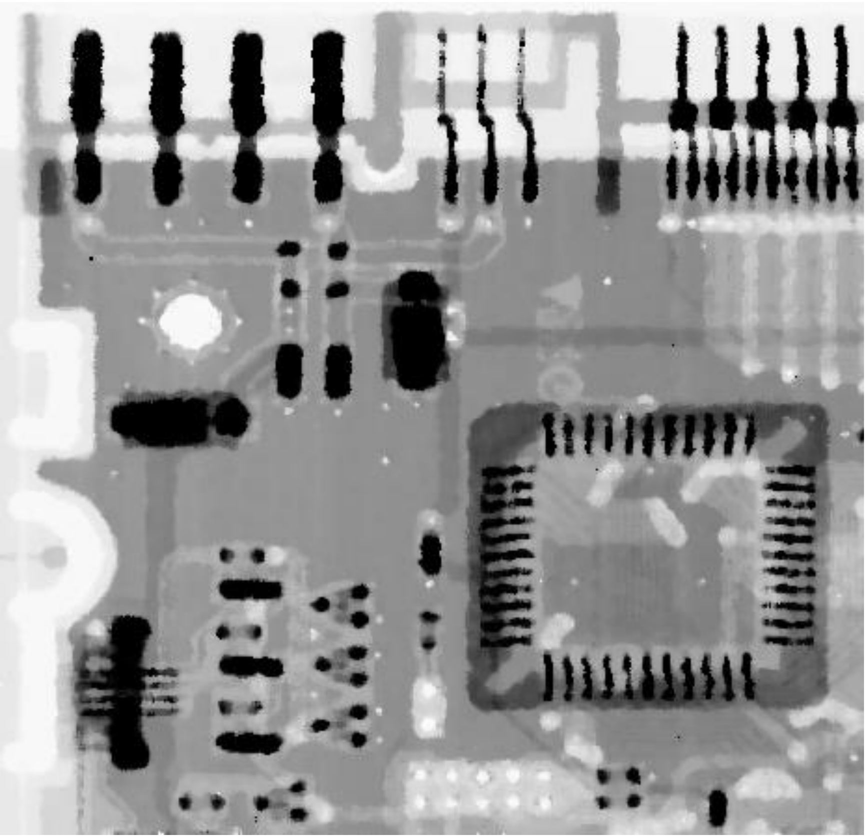
3 x 3

PSNR=20.17

5.3 Restoration in the Presence of Noise Only



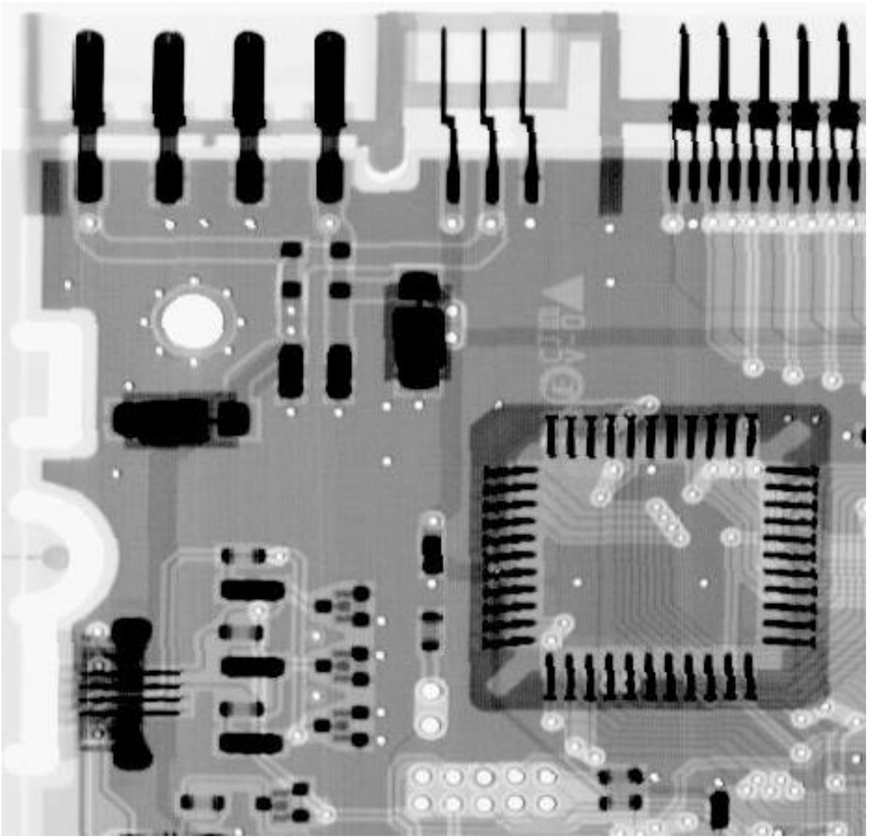
Original



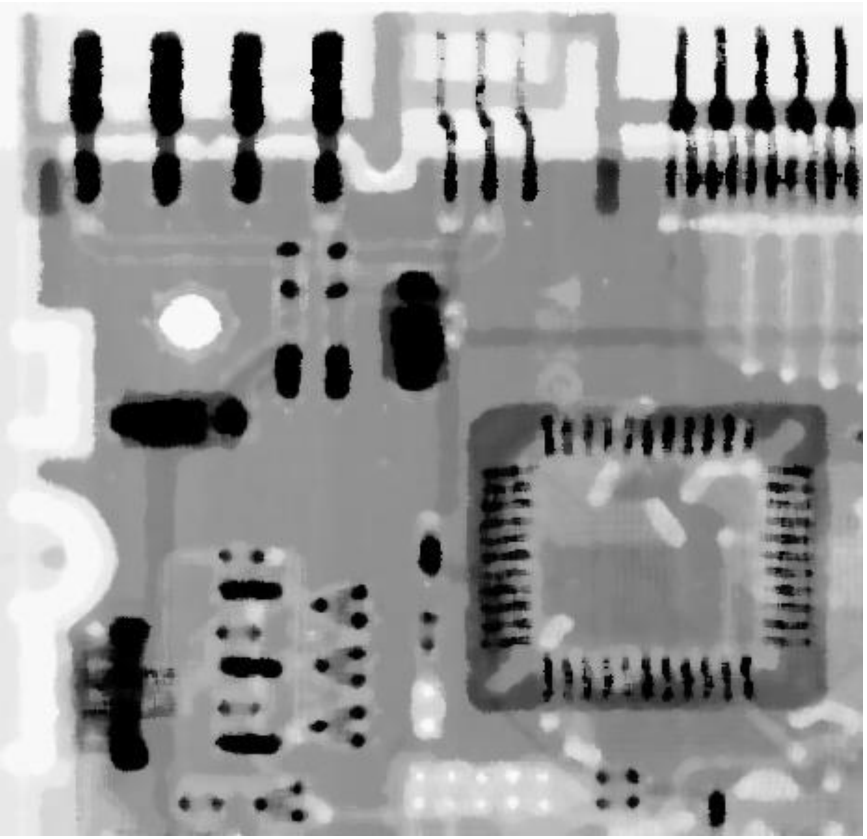
5 x 5

PSNR=23.02

5.3 Restoration in the Presence of Noise Only



Original



7 x 7

PSNR=21.25

5.3 Restoration in the Presence of Noise Only

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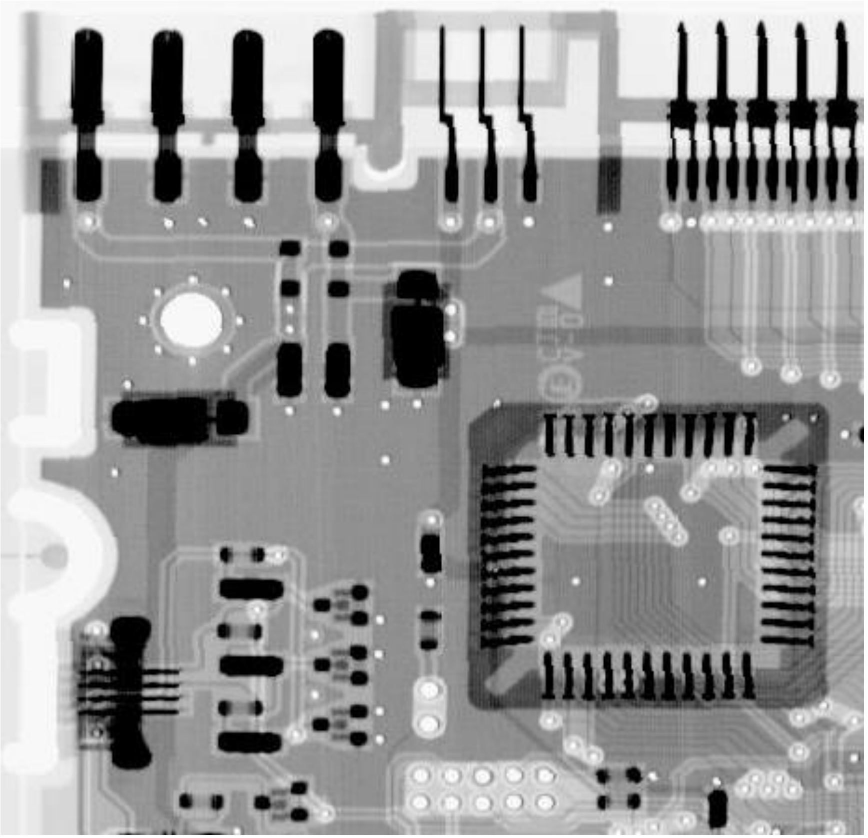
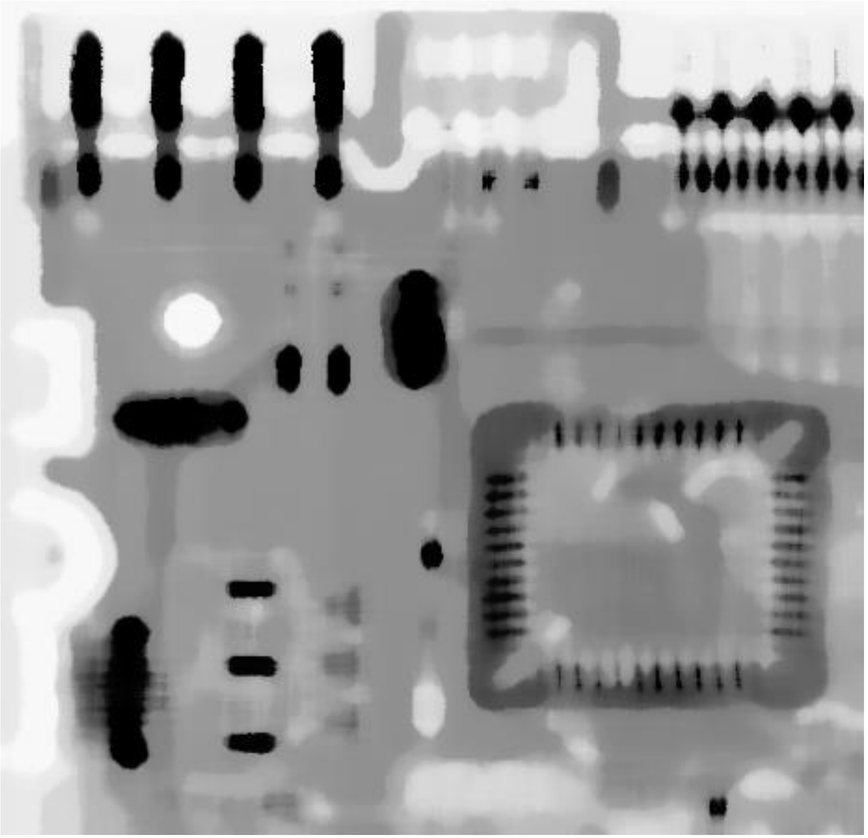


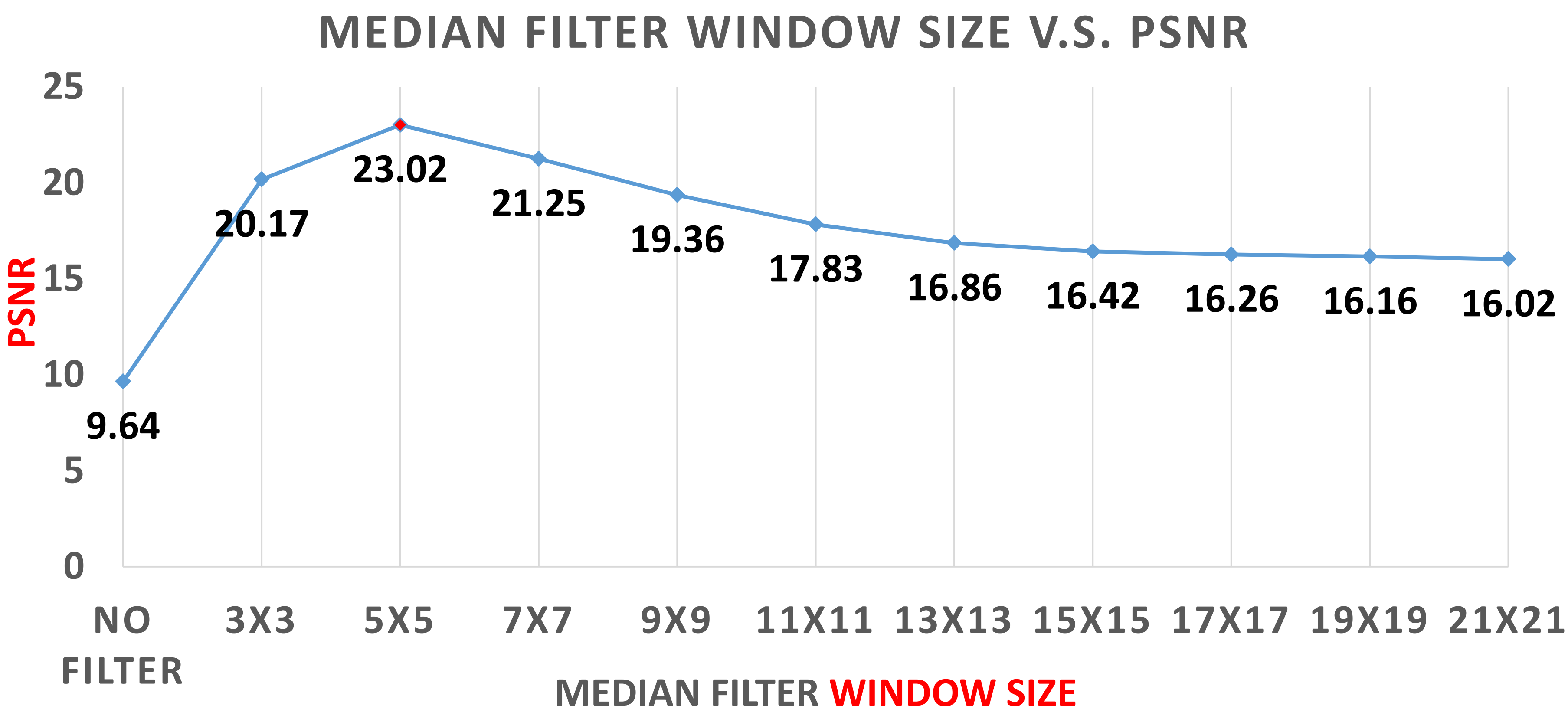
Original



15 x 15

PSNR=16.42

5.3 Restoration in the Presence of Noise Only



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Question: could we adaptively choose the window size accordingly to meet the different requirement in different local area?

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- 自适应中值滤波器（自动选取窗口大小）

三个主要目的: 除去椒盐噪声、平滑其他非冲击噪声、减少物体边缘细化或粗化等失真.

前面给出的中值滤波器在脉冲噪声空间密度不大的情况下(P_a和P_b的值小于0.2)有较好的结果。考虑适应噪声密度范围更广的滤波器，并仍然考虑像素点(x, y)和其邻域S_{xy}（可变大小）

定义下述符号:

Z_{xy} = 当前要处理的像素(x, y)的灰度值

Z_{min} = S_{xy}中灰度的最小值

Z_{max} = S_{xy}中灰度的最大值

Z_{med} = S_{xy}中灰度的中值

S_{max} = S_{xy}允许的最大尺寸

5.3 Restoration in the Presence of Noise Only

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自适应中值滤波器分为两个层次:

A层:（确定Z_{med}是否为脉冲）

$$A_1 = Z_{med} - Z_{min},\ A_2 = Z_{med} - Z_{max}$$

1). 如果$A_1 > 0$且$A_2 < 0$, 转到B层; 否则增大窗口尺寸. (检验是否满足$Z_{min} < Z_{med} < Z_{max}$, 若满足，则$Z_{med}$不可能是脉冲点；若不满足（有可能是），扩大搜索范围再作比较和判别)

2). 如果窗口尺寸 $\le S_{max}$, 重复A层(包括重新计算Z_{min}、Z_{med}、Z_{max}); 否则输出Z_{xy}。

从A层出来后, 总有Z_{xy}输出.

B层:（检测要处理的象素点（x, y）是否为脉冲）

$$B_1 = Z_{xy} - Z_{min},\quad B_2 = Z_{xy} - Z_{max}$$

如果$B_1 > 0$且$B_2 < 0$, 输出Z_{xy}, 否则输出Z_{med}(检验是否满足$Z_{min} < Z_{xy} < Z_{max}$, 即不是脉冲点时保留原值以减少失真. 若不满足表示是一个局部极值, 输出中值是一个不错的选择)

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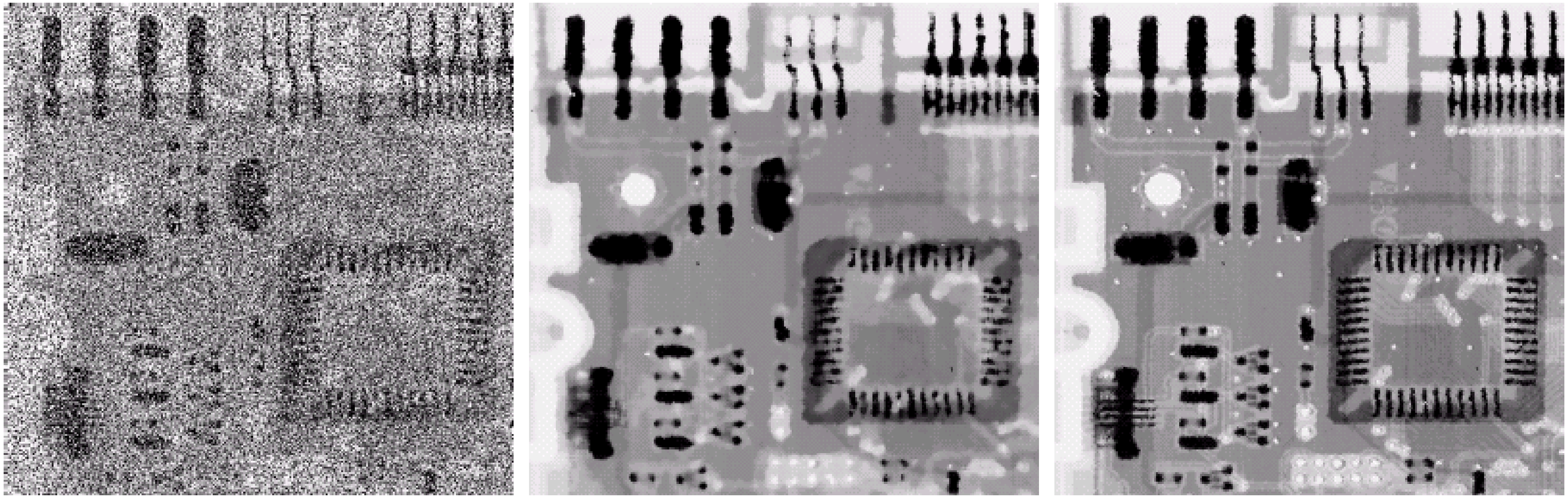


a b c

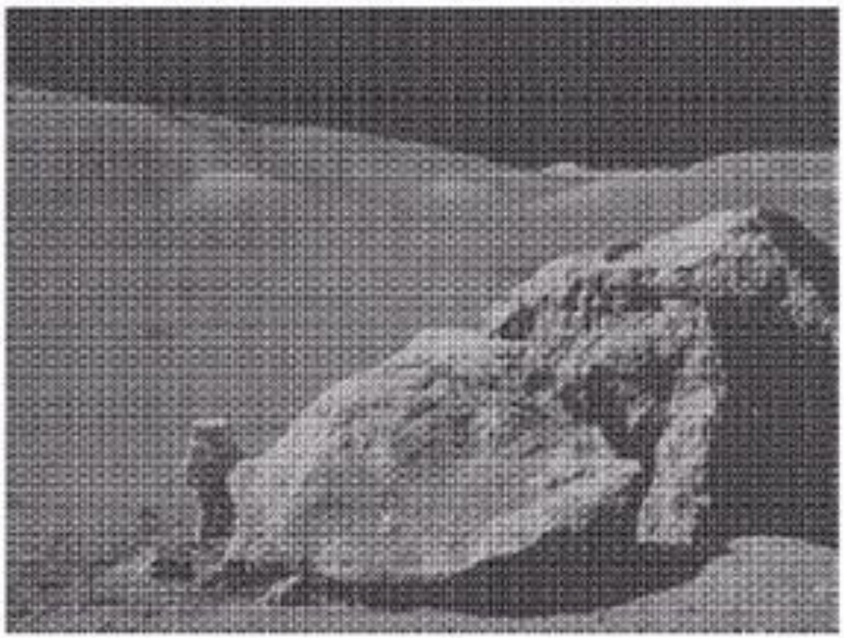
FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

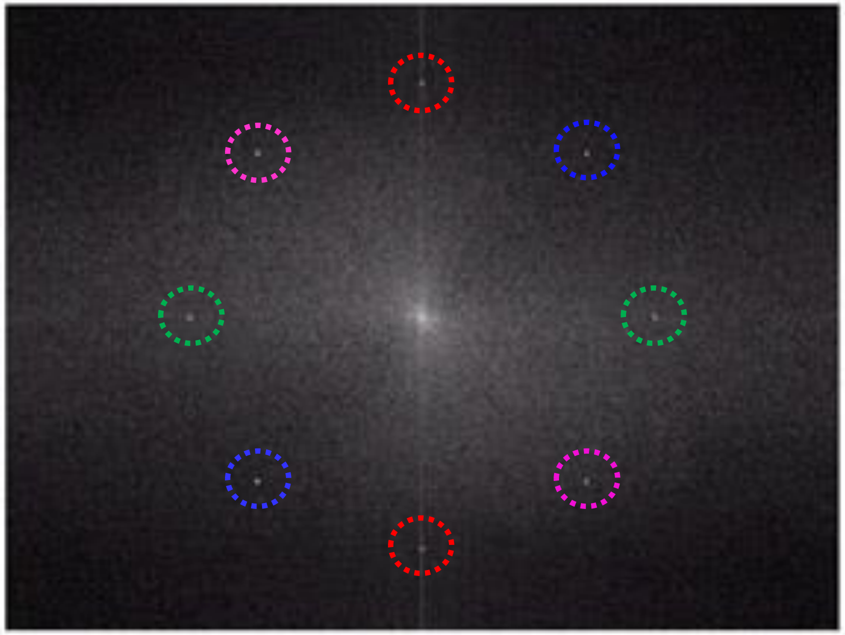
(*a*) 是被“椒盐”噪声干扰过的电路图, 噪声概率为0.25(很高). 模糊了大部分图像;

(*b*) 7×7的中值滤波器, 去除了大部分噪声, 但细节有较大的损失;

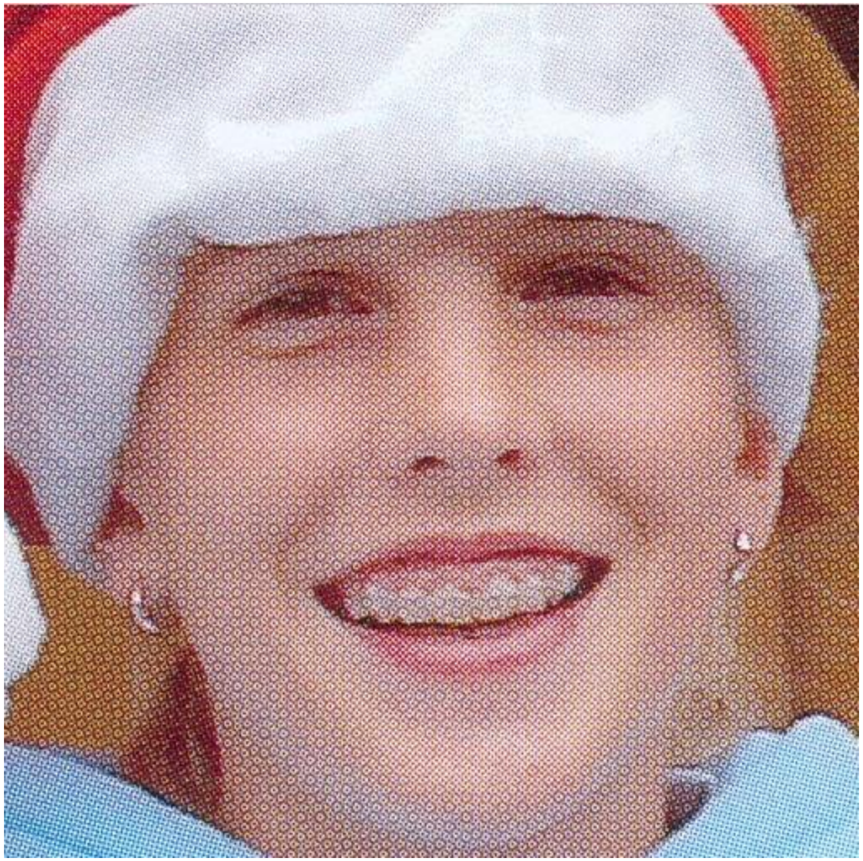
(*c*) 自适应算法的结果.允许最大窗口的选择和应用有关, 初始窗口的选择可从标准中值滤波器的实验结果中得到（自己动手重复这一结果）。

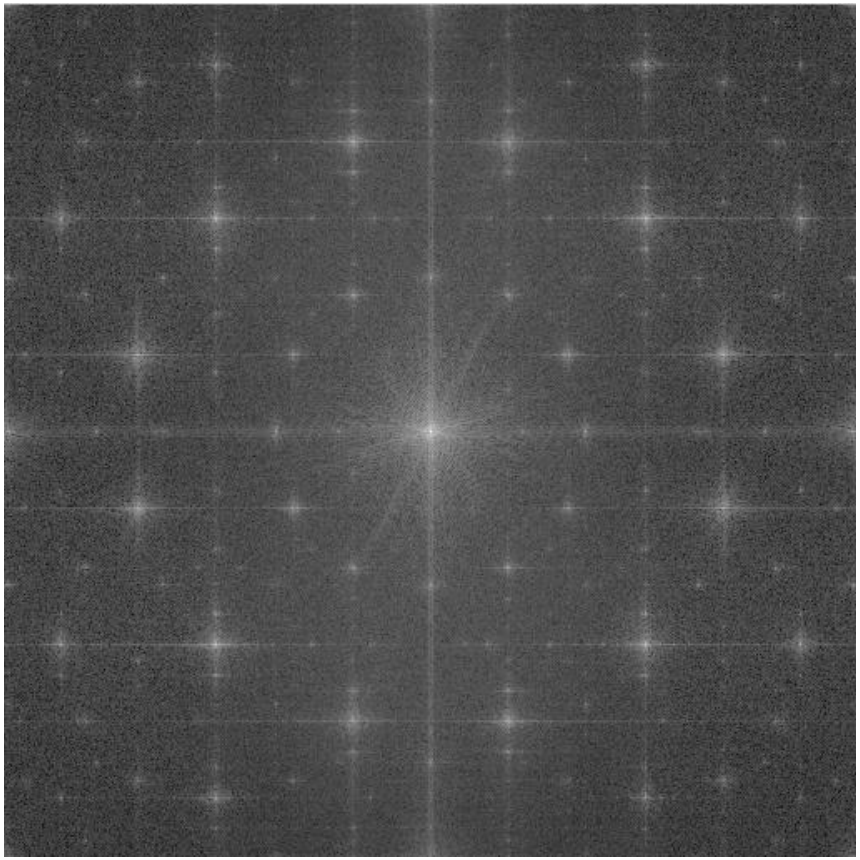
5.4 Periodic Noise Reduction by Frequency Domain Filtering



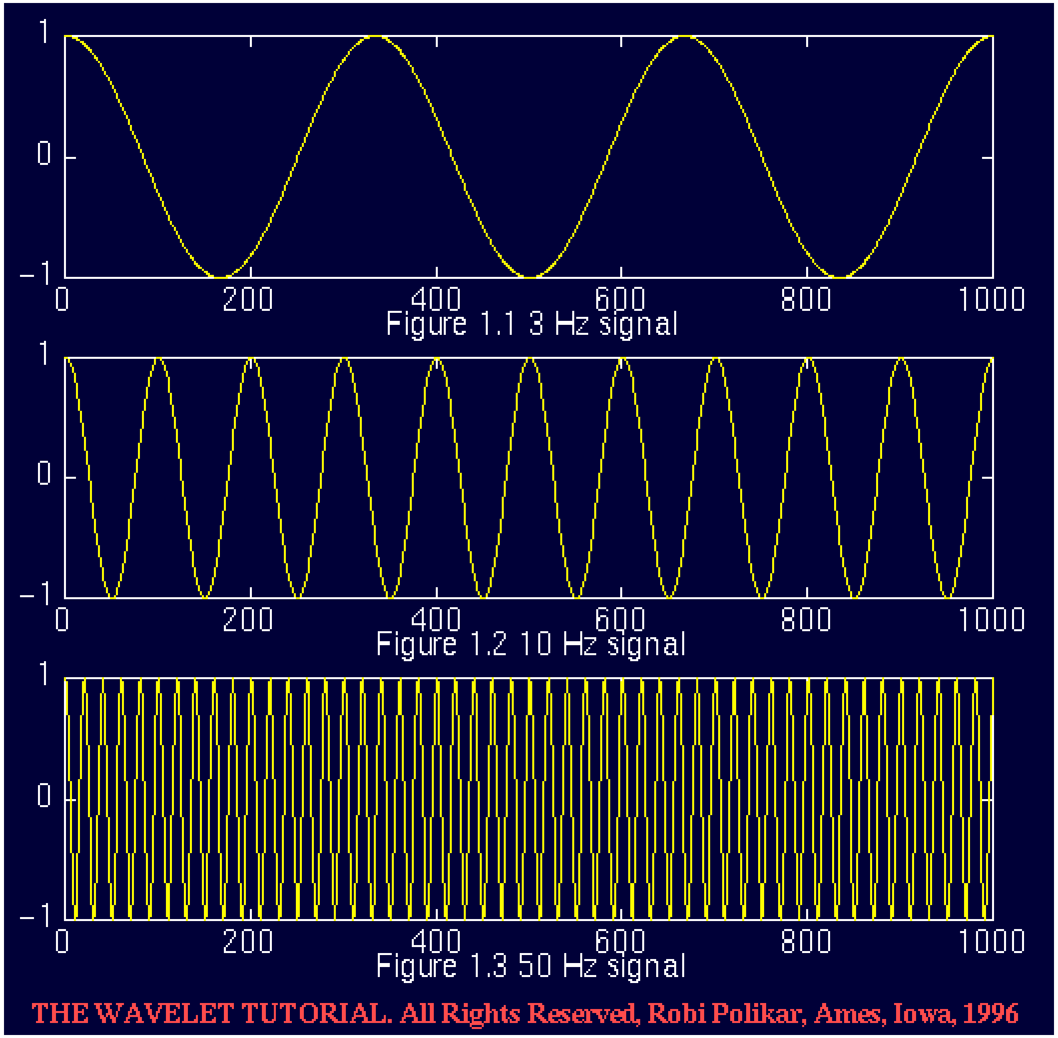


5.4 Periodic Noise Reduction by Frequency Domain Filtering





5.4 Periodic Noise Reduction by Frequency Domain Filtering



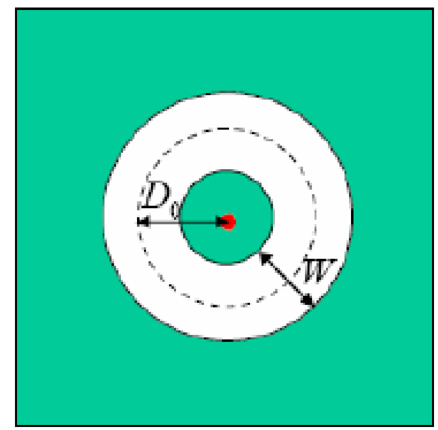
5.4 Periodic Noise Reduction by Frequency Domain Filtering

由于周期噪声在频率域有较为特征化的表示, 所以通常都在频率域处理这类问题.

- Band-reject filters

  ideal band-reject filter

$$H(u,v)=\begin{cases}1, & D(u,v)<D_0-\dfrac{W}{2}\\ 0, & D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\ 1, & D(u,v)>D_0+\dfrac{W}{2}\end{cases}$$

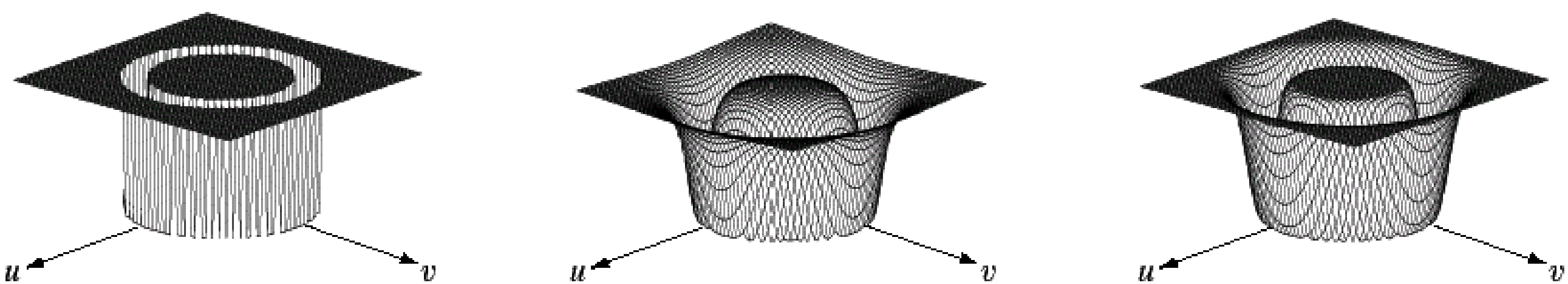


Where : $D(u, v) = [(u - u_0)^2 + (v - v_0)^2]^{1/2}$ is the distance from the origin of the centered frequency rectangle. W is the band width and D_0 is its radial center.

Butterworth band-reject filter and Gaussian band-reject filter

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

5.4 Periodic Noise Reduction

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Example 5.6: **rejected band of Butterworth filter**

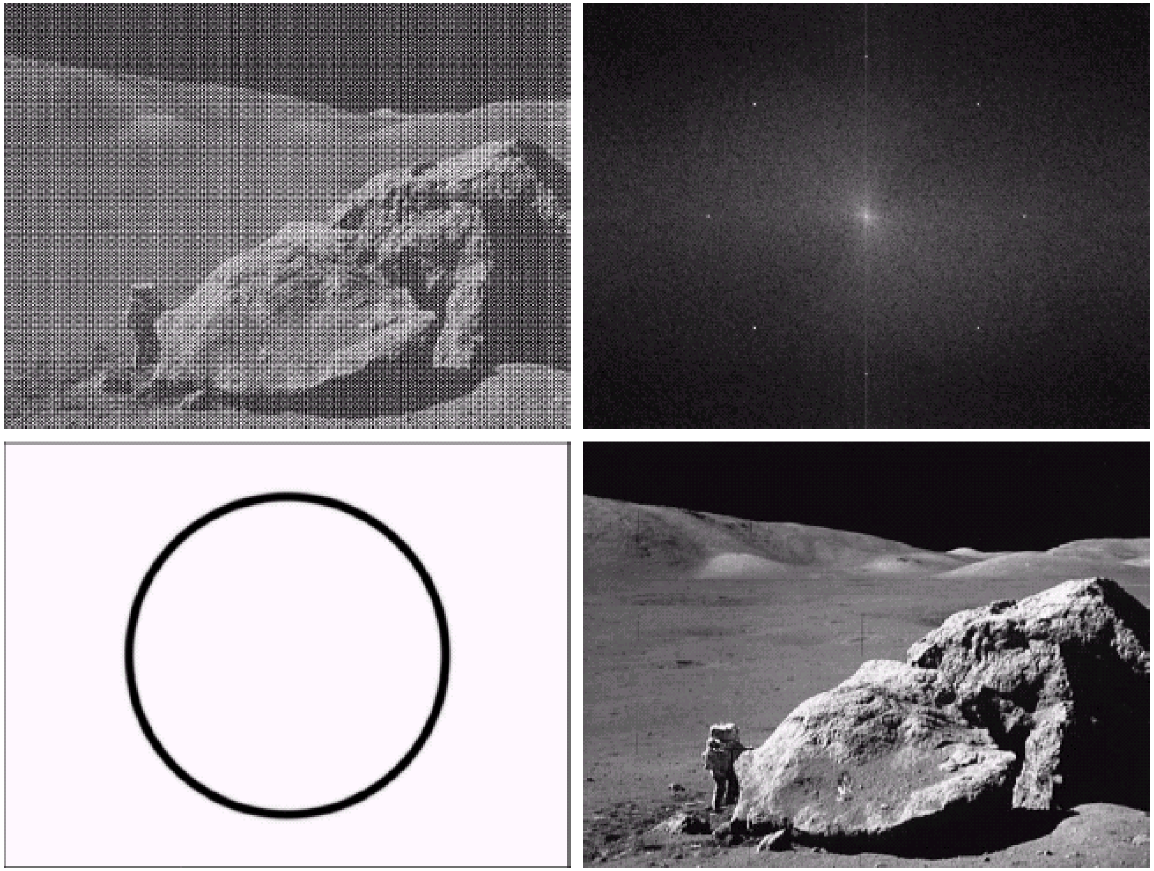


Image with periodic noise and its Fourier transform

Butterworth band- reject filter and its result

5.4 Periodic Noise Reduction

- Band-pass filters

A band-pass filter HBP(u, v) can be obtained from a band-reject filter HBR(u, v):

$$H_{bp}(u, v) = 1 - H_{br}(u, v).$$

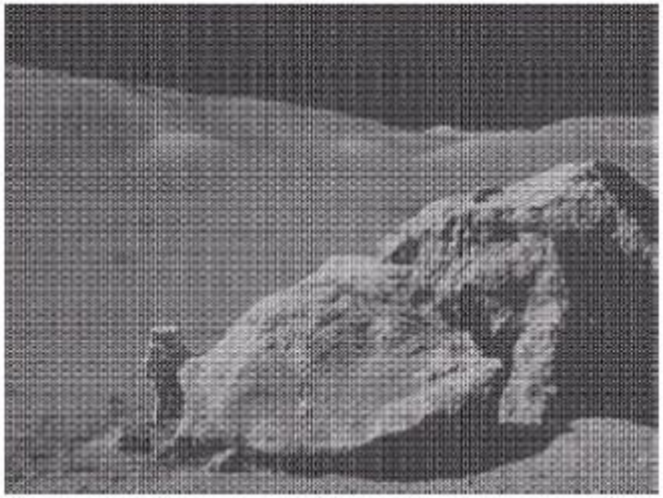
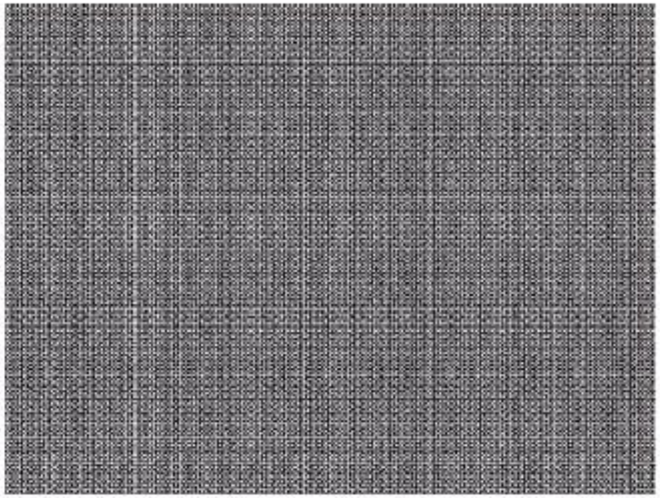


Image with periodic noise

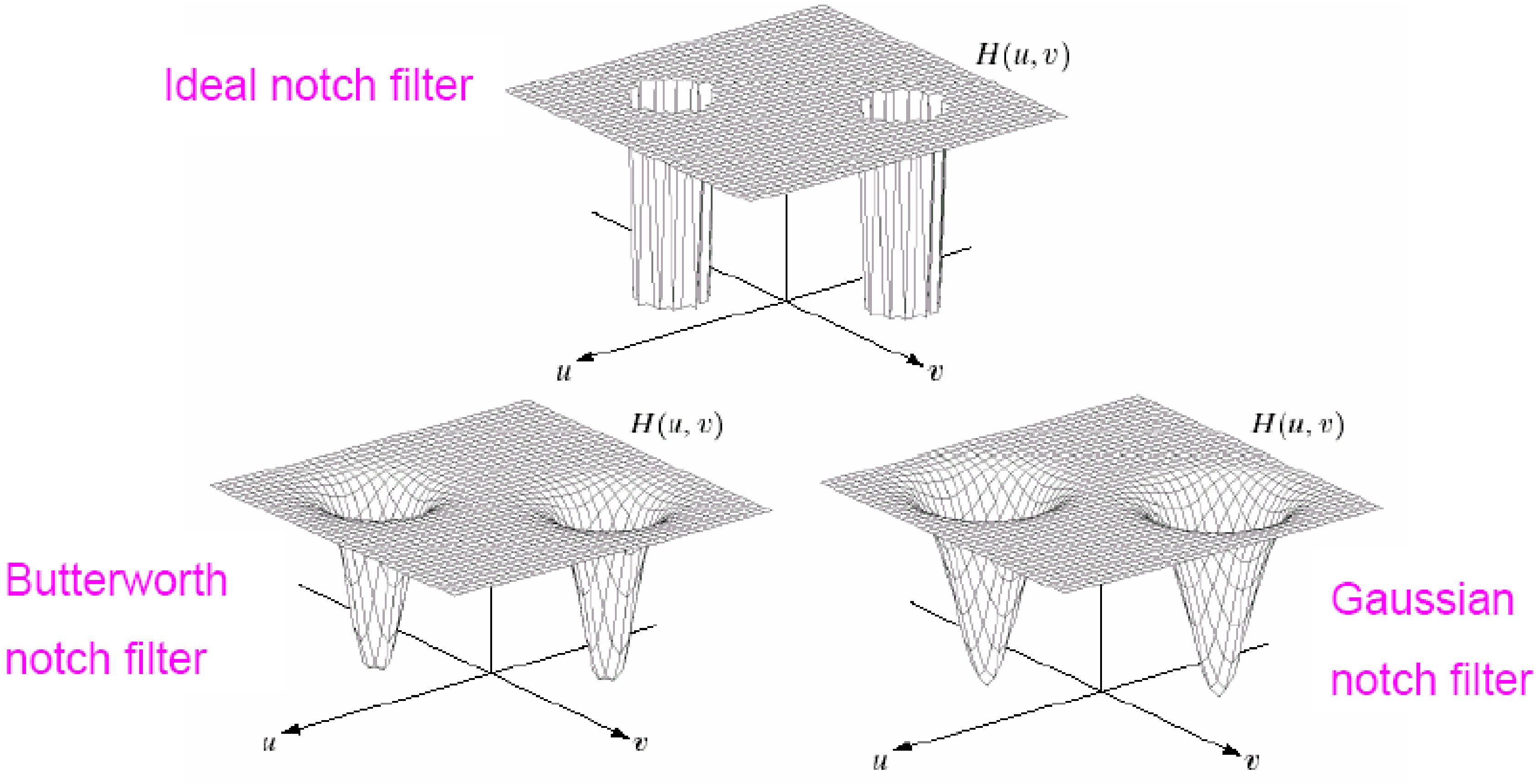


periodic noise obtained by band-pass filter

5.4 Periodic Noise Reduction

- **Notch filter**

  A notch filter rejects frequencies near prescribed frequency points.



5.4 Periodic Noise Reduction

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Ideal notch filter

$$H(u,v)=\begin{cases}0 & \text{if } D_1(u,v)\le D_0 \text{ or } D_2(u,v)\le D_0\\ 1 & \text{otherwise}\end{cases}$$

where

$$D_1(u,v)=\sqrt{\left(u-u_0-\frac{M}{2}\right)^2+\left(v-v_0-\frac{N}{2}\right)^2} \text{ and } D_2(u,v)=\sqrt{\left(u+u_0-\frac{M}{2}\right)^2+\left(v+v_0-\frac{N}{2}\right)^2}$$

Butterworth notch filter

$$H(u,v)=\frac{1}{1+\left[\dfrac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

Gaussian notch filter

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]}$$

5.4 Periodic Noise Reduction

中山大學

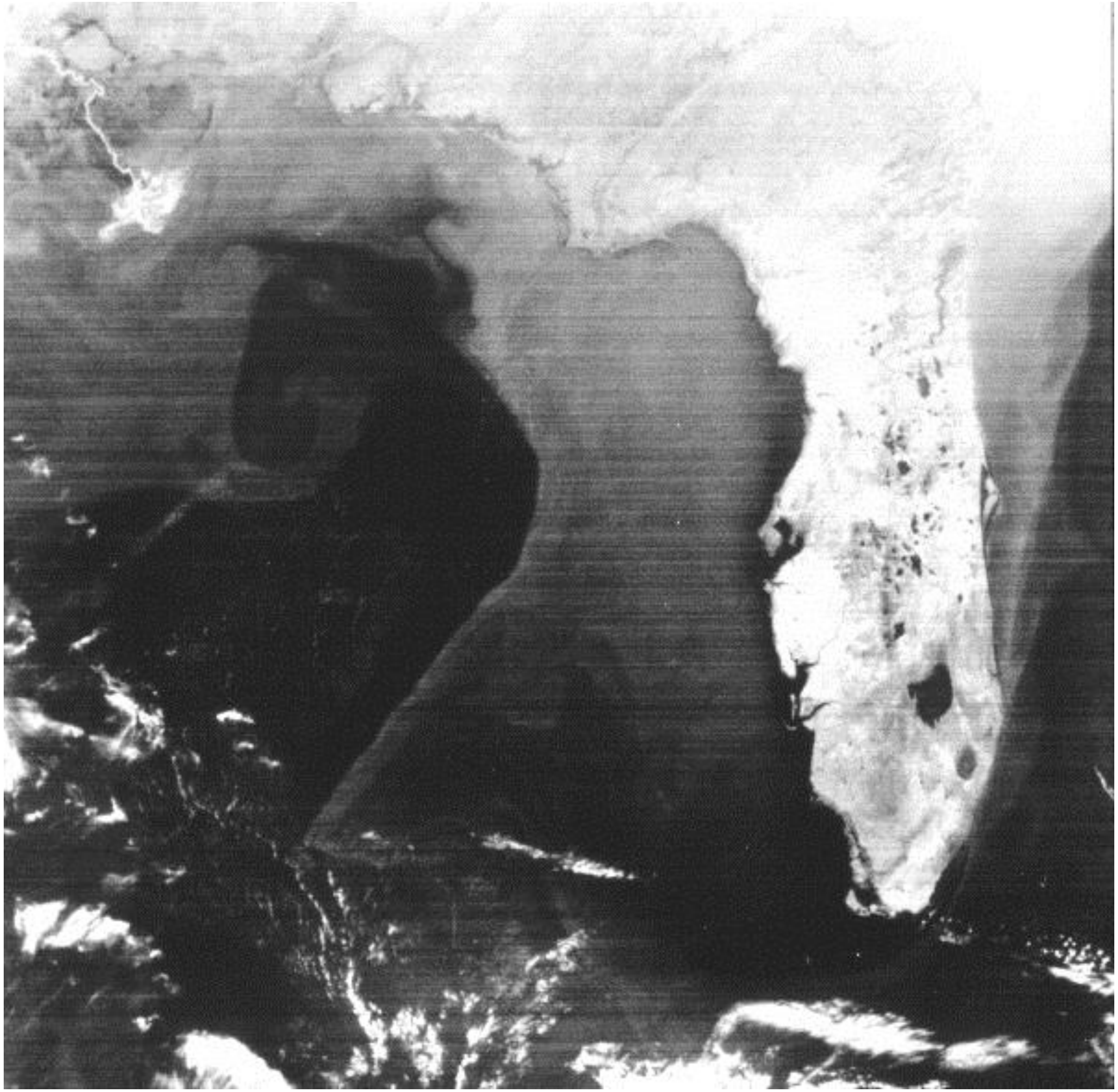
例5.8： 使用陷波滤波器消除周期性噪声

卫星拍摄的佛罗里达和墨西哥海湾的图片, 其上有水平方向的干扰, 期望在频率域的噪声谱的分布集中在垂直轴附近。但由于噪声没有起主导作用, 所以干扰模式不是很清晰. 然而, 如果我们沿傅里叶变换的垂直轴建立一个简单的陷波带通滤波器, 就可以开展近似的噪声分析. 再利用傅里叶反变换可以得到噪声的空间表示. 反复调整带通陷波滤波器, 可以得到较接近的噪声模型.

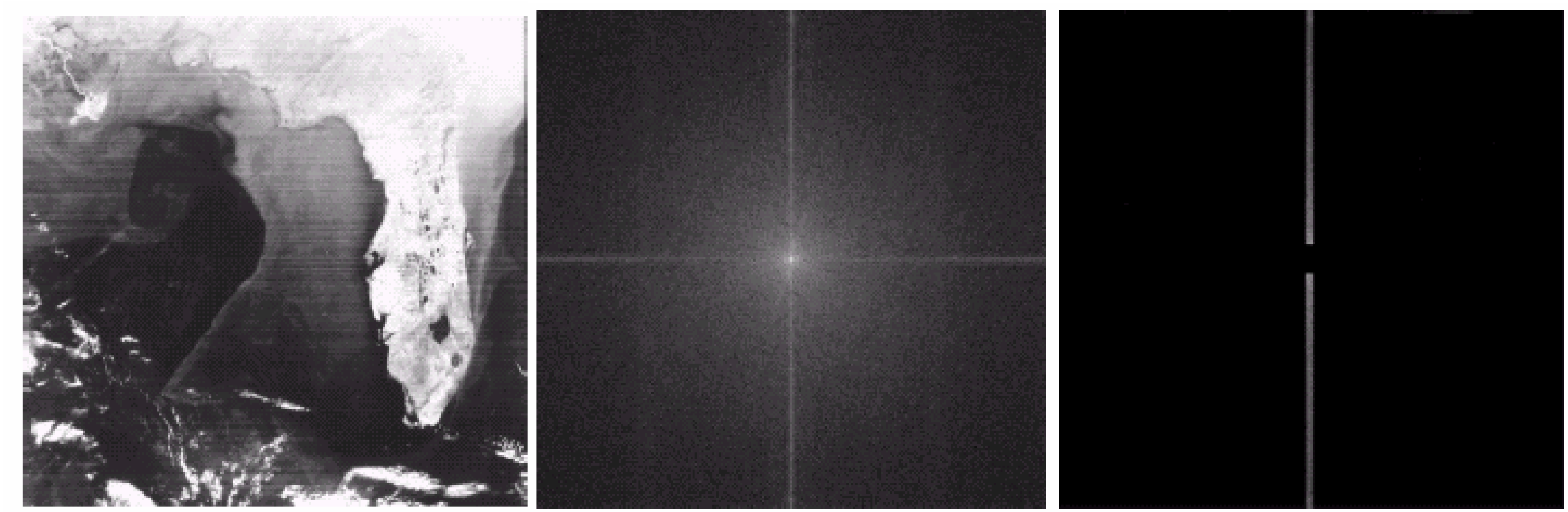
有了上面的结果, 再构造相应的陷波带阻滤波器, 就能去除图像上的类似的周期噪声.

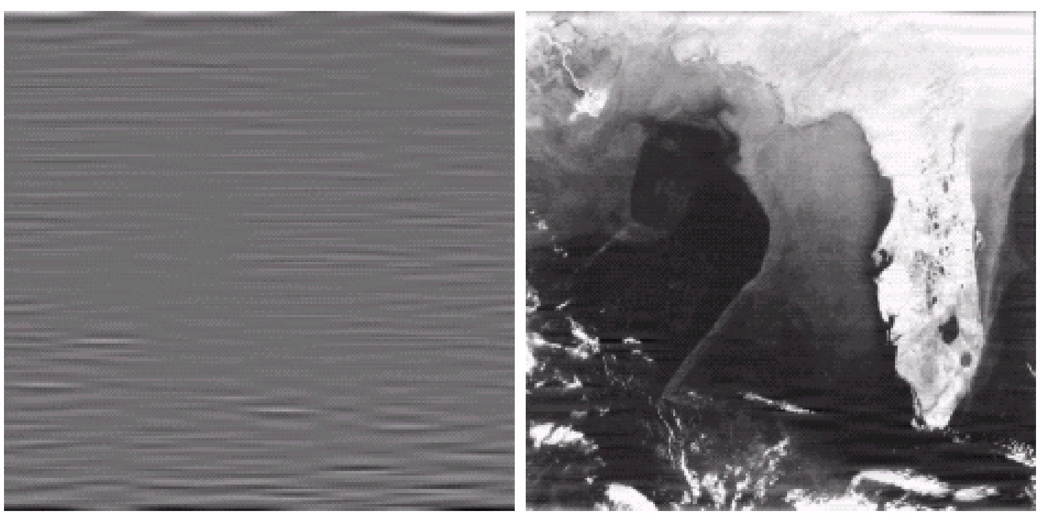
图像中的噪声模式并不明显, 反应在频谱域垂直方向也是如此. 所以先了解噪声的模式是重要的一步.

5.4 Periodic Noise Reduction



5.4 Periodic Noise Reduction





The original image (a) has horizontal scan lines (d), which can be removed by the band-like notch filter (c) to result in (e)

5.4 Periodic Noise Reduction

步骤:

- 构造一个沿垂直方向的理想陷波(带通)滤波器, 对图像的频谱图滤波;
- 将上面的结果作傅立叶反变换得到近似的噪声模型;
- 调整陷波(带通)滤波器使得噪声模型更接近原始噪声;
- 根据前面陷波带通滤波器的结果, 建立相应得陷波带阻滤波器, 屏蔽噪声, 得到滤波后原图像的傅立叶变换, 然后再做反变换就得到了滤波后的图像.

问题:

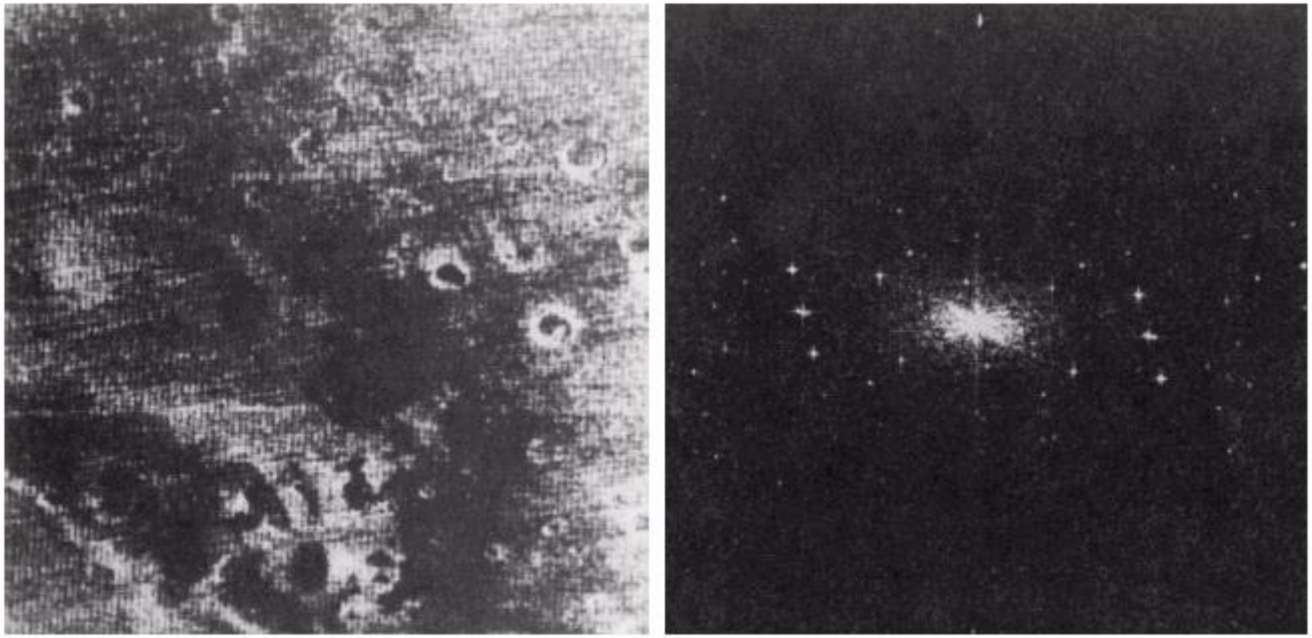
- 能否在空间域构造类似的滤波器?

5.4 Periodic Noise Reduction

- Optimum Notch Filter

The need of an optimum notch filter arises from the fact that clear noise pattern in the Fourier transformed plane are not common.

Consider the image shown below from a spacecraft, the start like bright spots in the Fourier transformed plane (on the right) are not all due to only one type of noises. Instead, it is the combination of several types of noises. In such a case, previous approaches fail.



a b

FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

水手6号拍摄的火星地形图片, 有不止一个正弦分量

5.4 Periodic Noise Reduction

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Let $\eta(x, y)$ be the noise pattern, N(u, v) be its Fourier transform, G(u, v) be the Fourier transform of the noise corrupted image, and a filter H(u, v) is designed to allow only the noise pattern to pass, that is

$$N(u,v) = H(u,v)G(u,v) \qquad (5.4.11)$$

Accordingly, the noise pattern $\eta(x, y)$ can be reconstructed from

$$\eta(x, y) = \mathscr{F}^{-1}\{H(u,v)G(u,v)\} \qquad (5.4.12)$$

However, in many cases, $\eta(x, y)$ can not be reconstructed exactly. In such a case, the image $\hat{f}(x, y)$ is to be reconstructed from the weighted noise

$$\hat{f}(x, y) = g(x, y) - w(x, y)\eta(x, y) \qquad (5.4.13)$$

where $w(x, y)$ is a position dependent weighting function to minimize the local

5.4 Periodic Noise Reduction

中山大學

variance of $\hat{f}(x,y)$, denoted as $\sigma^2(x,y)$, in an neighbor around (x, y) of the size $(2a+1)$x$(2b+1)$

$$\sigma^2(x,y) \triangleq \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{s=-b}^{b} \left[\hat{f}(x+s,y+t) - \overline{\hat{f}}(x,y) \right] \qquad (5.4.14)$$

$$\overline{\hat{f}}(x,y) \triangleq \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{s=-b}^{b} \hat{f}(x+s,y+t)$$

$$= \text{the average value of } \hat{f}(s,t) \text{ in the region} \qquad (5.4.15)$$

$$s \in [-a+x,\ a+x] \text{ and } t \in [-b+y,\ b+y]$$

Following the derivations given in page 251 of the textbook, it can be shown

$$\boxed{w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)}}$$

學

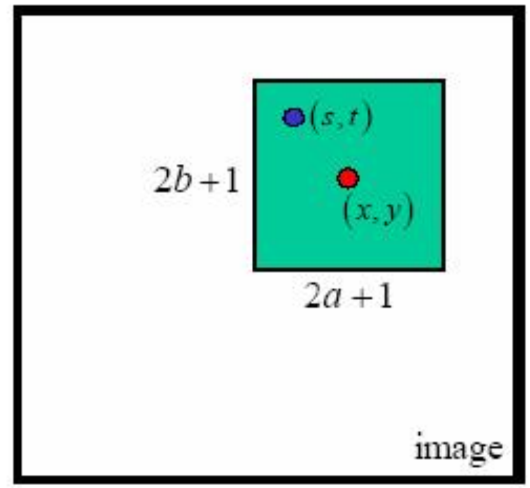
where $\overline{g(x,y)\eta(x,y)}$, $\overline{g}(x,y)$, $\overline{\eta}(x,y)$, $\overline{\eta^2}(x,y)$

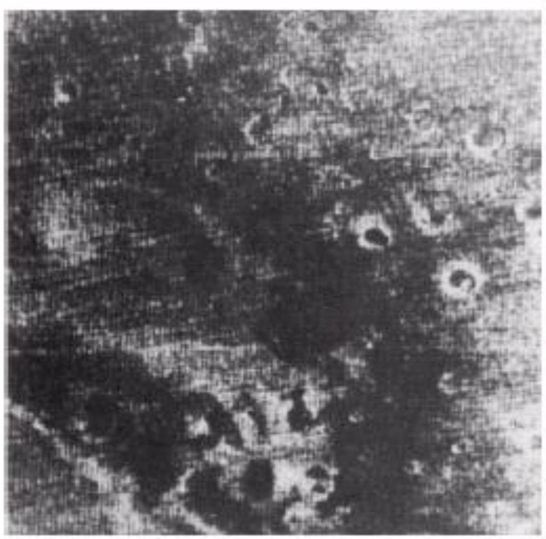
and $\overline{\eta}^2(x,y)$ are, respectively, the average

values of $g(x,y)\eta(x,y)$, $g(x,y)$, $\eta(x,y)$, $\eta^2(x,y)$

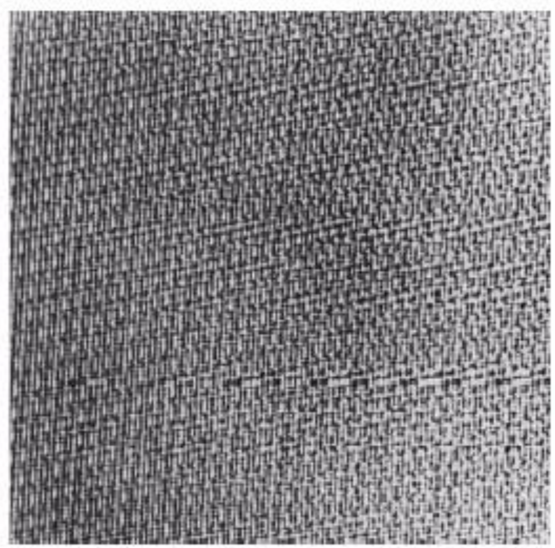
and $\overline{\eta}(x,y)\bullet\overline{\eta}(x,y)$ in the region

$s\in[-a+x,\ a+x]$ and $t\in[-b+y,\ b+y]$.

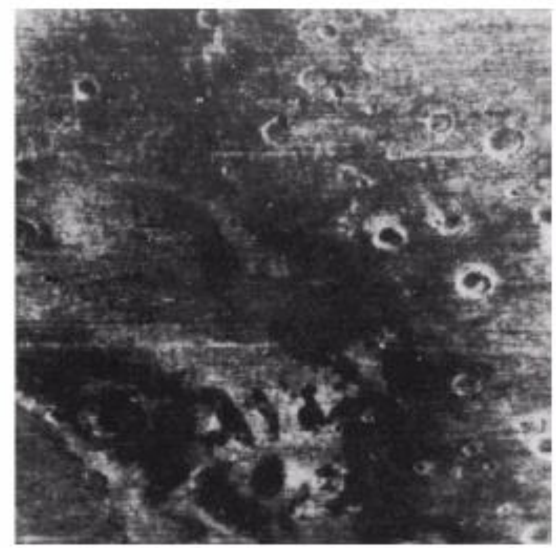




original image



noise pattern



optimum notch filter

5.4 Periodic Noise Reduction

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$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a}\sum_{t=-b}^{b} \{[g(x+s,y+t)$$

Derivation→

$$-\omega(x,y)\eta(x+s,y+t)]$$

$$-[\bar{g}(x,y)-\omega(x,y)\bar{\eta}(x,y)]\}^2$$

To minimize $\sigma^2(x, y)$, we solve

$$\frac{\partial\sigma^2(x,y)}{\partial\omega} = 0$$

→

$$\omega(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

5.4 Periodic Noise Reduction

到此为止，利用最优陷波方法，我们就可以如下获得复原图像的估计值 ：

- 利用陷波方法得到参考噪声的模式
- 对每一(x, y)，利用（5.4.21）计算加权因子的值；
- 利用（5.4.13）得到在坐标 (x, y)处的复原图像值

$$\hat{f}(x,y) = g(x,y) - \omega(x,y)\eta(x,y)$$

$$\omega(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

如果假设在某一邻域内 $\omega(x,y)$ 为常量，则这个邻域内所有点的复原图像值可同时处理。

5.4 Periodic Noise Reduction

图5.20(a)的傅里叶谱(没有做中心化处理)

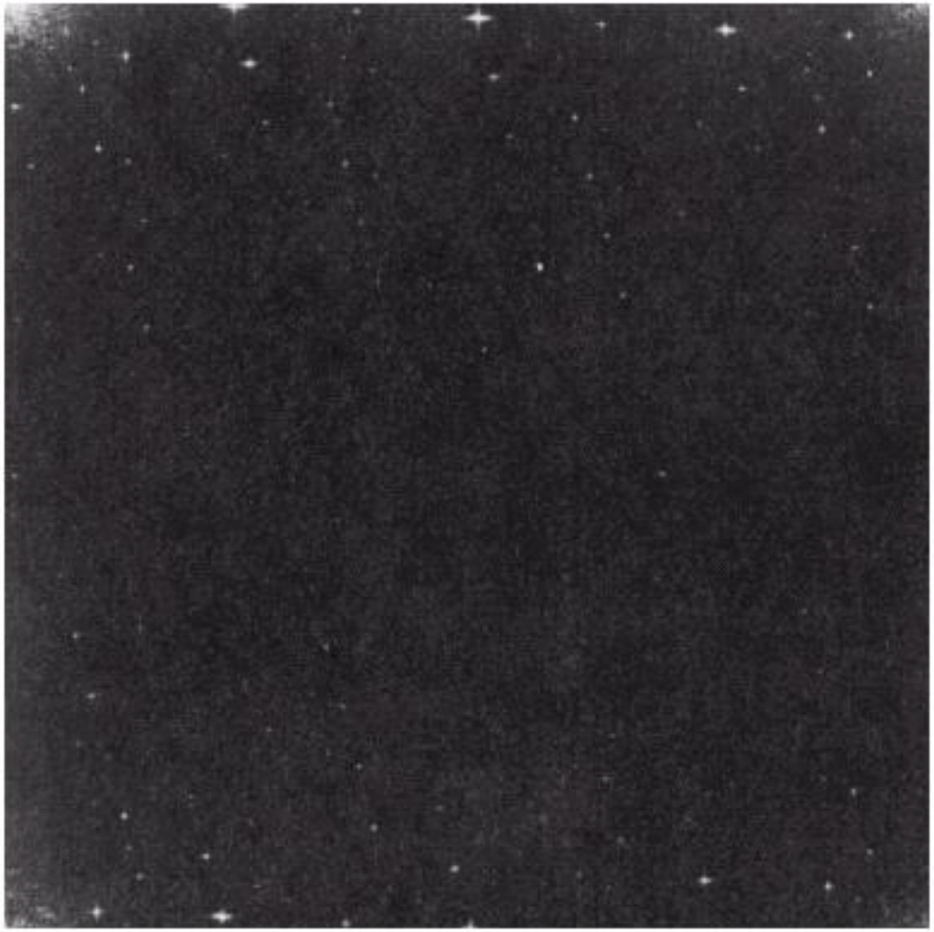
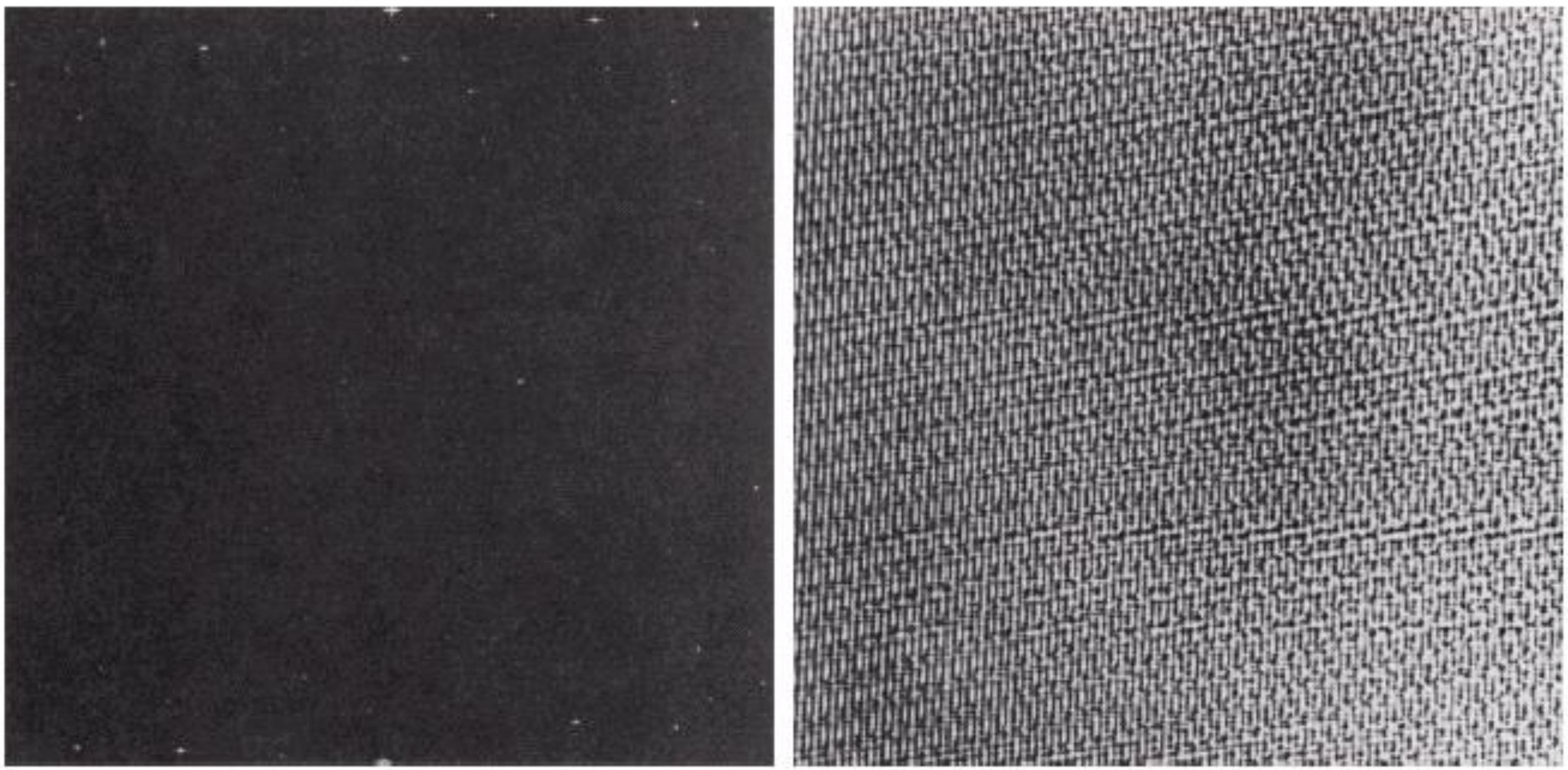


FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

5.4 Periodic Noise Reduction

在每个尖峰处设一个陷波带通滤波器, 这是滤波后的频谱以及空间噪声干扰模式.



a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

5.4 Periodic Noise Reduction

处理后的结果

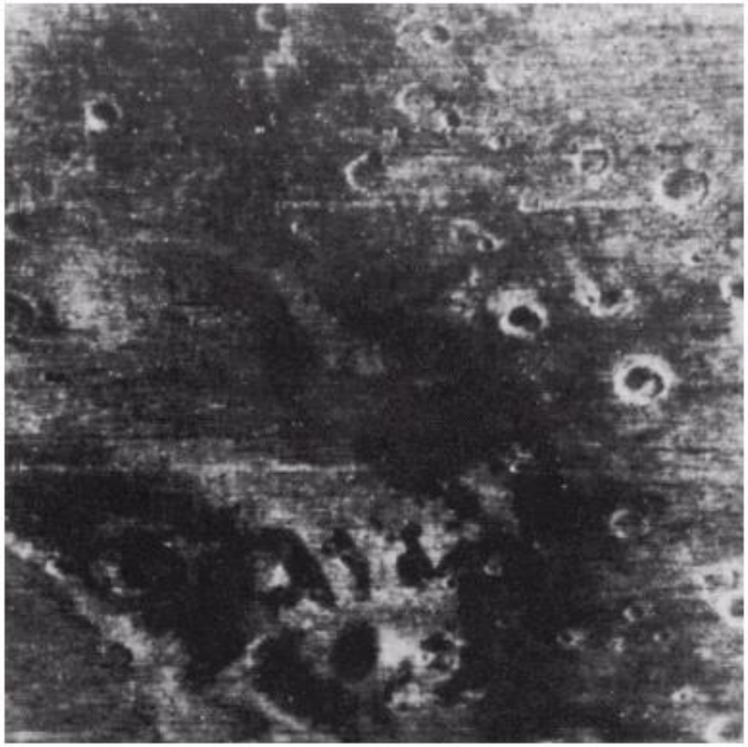


FIGURE 5.23 Processed image. (Courtesy of NASA.)

结论: 这种方法分为两部分: 第一部分是将图像的干扰的主要部分孤立分析, 提取噪声的模式; 第二部分是将噪声模式乘上一个加权因子从干扰图像中减去, 加权因子的选取要使得去噪后的图像在某种意义(例如方差最小)上是最佳的.

5.5 Linear, Position-Invariant Degradation

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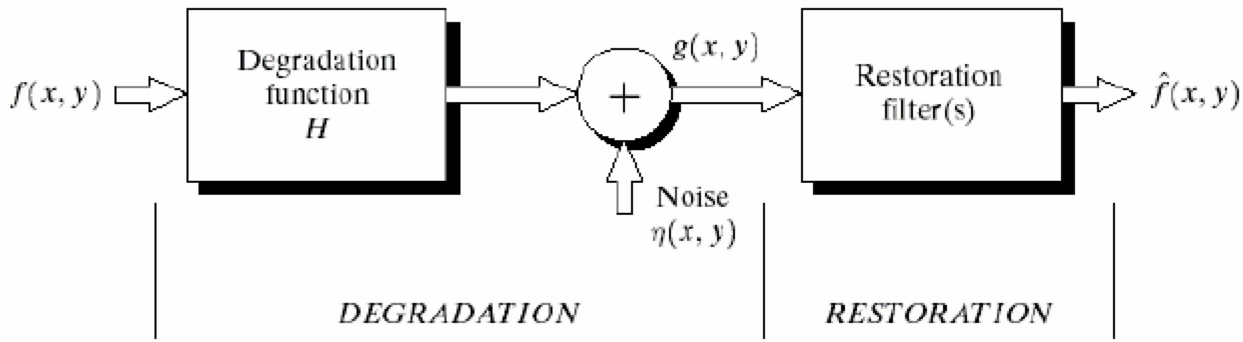

At beginning of this Chapter, we assume that the image degradation model we deal with here only is expressed as:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

or

$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v)$$

We will explain why this assumption(skip). (下面将从成像系统出发，说明上述图像退化模型假设的根据)。

5.5 Linear, Position-Invariant Degradation

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- **Definition of linear, position-invariant (LSI) degradation**

Consider the degradation operator H in the presence of noise , we have

$$g(x,y) = H\left[f(x,y)\right] + \eta(x,y) \qquad (5.5.1)$$

The degradation operator H is linear IFF it satisfies super-position principle

$$H\left[\alpha f_1(x,y) + \beta f_2(x,y)\right] = \alpha H\left[f_1(x,y)\right] + \beta H\left[f_2(x,y)\right] \qquad (5.5.4)$$

Furthermore, let $g(x, y)=H[f(x, y)]$, the linear degradation operator H is position-invariant IFF it satisfies: for any $f(x,y)$, α and β

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta) \qquad (5.5.5)$$

A linear, position invariant system is also called a linear, space invariant (LSI) system.

5.5 Linear, Position-Invariant Degradation

讨论线性、位置不变系统的数学表达

Review the definition of an impulse function δ(*x*, *y*), we have

$$\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} f(\alpha,\beta)A\delta(x-\alpha,y-\beta)=Af(x,y)$$

Similarly, a continuous function also can be represented by an impulse function:

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)\mathrm{d}\alpha\mathrm{d}\beta$$

令A=1，代入(5.5.1)（无加性噪声）利用系统*H*的线性特性, 得到

$$g(x,y)=\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} f(\alpha,\beta)H[\delta(x-\alpha,y-\beta)]$$

or

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)H\left[\delta(x-\alpha,y-\beta)\right]\mathrm{d}\alpha\mathrm{d}\beta \qquad (5.5.10)$$

其中式*h*(*x*, α, *y*, β) = *H*[*δ*(*x*-α, *y*-β)]=*h*(*x*-α, *y*-β) (LSI)称为系统*H*的脉冲响应.也就是说, (5.5.10)中的两个式子都可以写成（5.5.1）的形式(“图像去卷积”就是线性图像复原的技术术语)：

$$g(x,y)=h(x,y)*f(x,y)$$

5.5 Linear, Position-Invariant Degradation

- **Point spread function (PSF) of a camera** （点扩散函数）

A typical example of LSI degradation is the blurring effect due to the finite radius of the camera diaphragm: the smaller the radius the sharper the step edge of an image. To capture the blurring/distortion of a camera, assuming that there is a impulse light source at a position (α, β) and the corresponding intensity at a position (x, y) due to this light source be $h(x, y, \alpha, \beta)$, then the distortion (degradation) operator H can be expressed as

$$h(x, y, \alpha, \beta) \triangleq H\left[\delta(x-\alpha,\ y-\beta)\right] \quad (5.5.10)$$

where $h(x, y, \alpha, \beta)$ acts very similar to the 'impulse response' in linear time-invariant system, and is called 'point spread function' (PSF) of a camera.

点扩散函数的模糊程度是由光学成像系统的质量所确定的，它也反映了设备的质量。

5.5 Linear, Position-Invariant Degradation

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It is a common practice to assume that the PSF of a camera be position –invariant, in such a case, Eq. (5.5.10) reduces to

$$h(x,y,\alpha,\beta) \triangleq H\left[\delta\left(x-\alpha,\ y-\beta\right)\right] = h(x-\alpha,\ y-\beta) \qquad (5.5.12)$$

Let f(x, y) be a 'perfect' image free of distortion and noise, the actual image g(x, y) subject to camera distortion H and noise $\eta(x,y)$ will be given by

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,\ y-\beta)d\alpha d\beta + \eta(x,y) \qquad (5.5.15)$$

convolution integral

$$g(x,y) = \sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} f(\alpha,\beta)h(x-\alpha,y-\beta)+\eta(x,y)$$

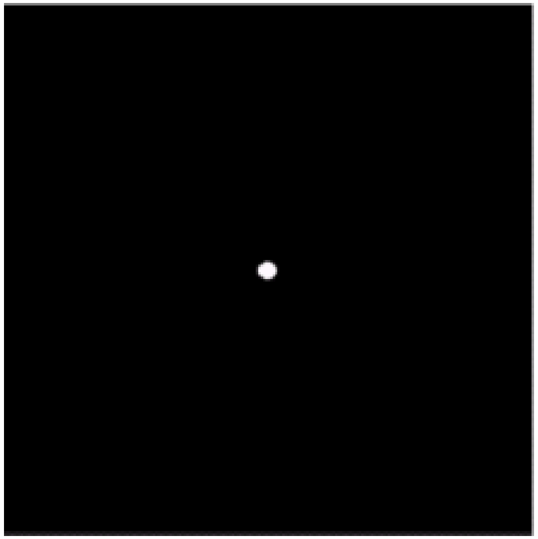
and, accordingly, the Fourier transform of Eq. (5.5.15) yields

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \qquad (5.5.17)$$

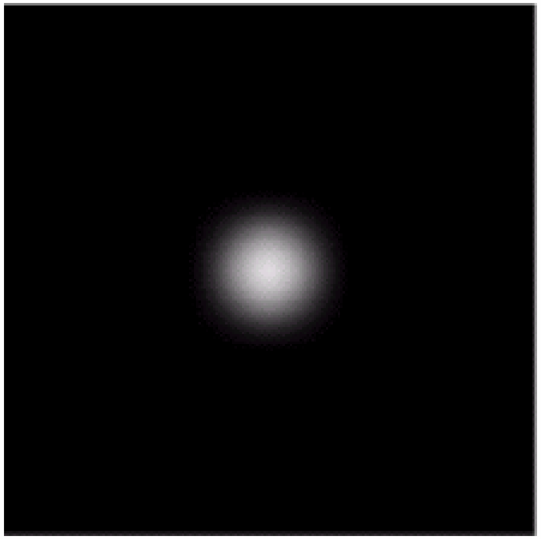
5.5 Linear, Position-Invariant Degradation

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v)$$



point light source



blurred image showing PSF

5.6 Estimating the Degradation Function

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v)$$

求解上述问题有两个要点：估计H(u,v)和逆滤波

先看成像系统问题：H(u, v)通常也是成像系统的点扩散函数(PSF)或脉冲响应函数，找到了PSF就确定了图像退化模型

There are three approaches to estimate (model) the PSF of a camera or, in a more general sense, the degradation function:

- Observation（观察法）, informal and not to be discussed
- Direct experimentation for $H(u, v)$（实验法）
- Mathematical modeling（模型估计法）

5.6 Estimating the Degradation Function

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- **Estimation by experimentation for H(*u*, *v*)** （实验法）

A typical approach is to design a point light source with the intensity A and capture its image g(*x*, *y*), which is the PSF. Assuming that noise is negligible, the degradation operator *H* can be obtained from the Fourier transform of the PSF g(*x*, *y*)

$$H(u,v) = \frac{G(u,v)}{A} \qquad (5.6.2)$$

Then H(*u*, *v*) can be recorded using an approximate mathematical form or look-up table.

模型估计法

- Estimation by physical-law-based mathematical model（利用已知物理模型估计成像系统函数）

In this approach, a physical law is established, and the experiments is used to determine only the model parameters in the physical model.

For instance, Hufnagel and Stanley (1964) has established a degradation model due to atmospheric turbulence

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}} \tag{5.6.3}$$

where k is a parameter to be determined by experiments because it changes with the nature of the turbulence.

As the other example, Gaussian function is usually adopted to model mild, uniform blurring.

5.6 Estimating the Degradation Function

a	b
c	d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

大气湍流模型:

a. 可忽略的湍流;

b. 剧烈湍流, k=0.0025;

c. 中等湍流, k = 0.001;

d. 轻微湍流, k = 0.00025

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- **Modeling blurring due to linear motion**
  从基本原理开始推导

Assume that the exposure time of an image be T, during that period the input image f(x, y) undergoes a planar motion $x_0(t)$ and $y_0(t)$, then the resultant image g(x, y) will be blurred as follows

$$g(x,y)=\int_0^T f\left[x-x_0(t),\ y-y_0(t)\right]dt \tag{5.6.4}$$

In the special case where the motion is carried out at a constant speed so that $x_0(t)=at/T$ and $y_0(t)=bt/T$, the Fourier transform of g(x, y) is given by

$$G(u,v)\triangleq\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{\int_0^T f\left[x-x_0(t),\ y-y_0(t)\right]dt\right\}e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_0^T\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left[x-x_0(t),\ y-y_0(t)\right]e^{-j2\pi(ux+vy)}dxdy\right\}dt$$

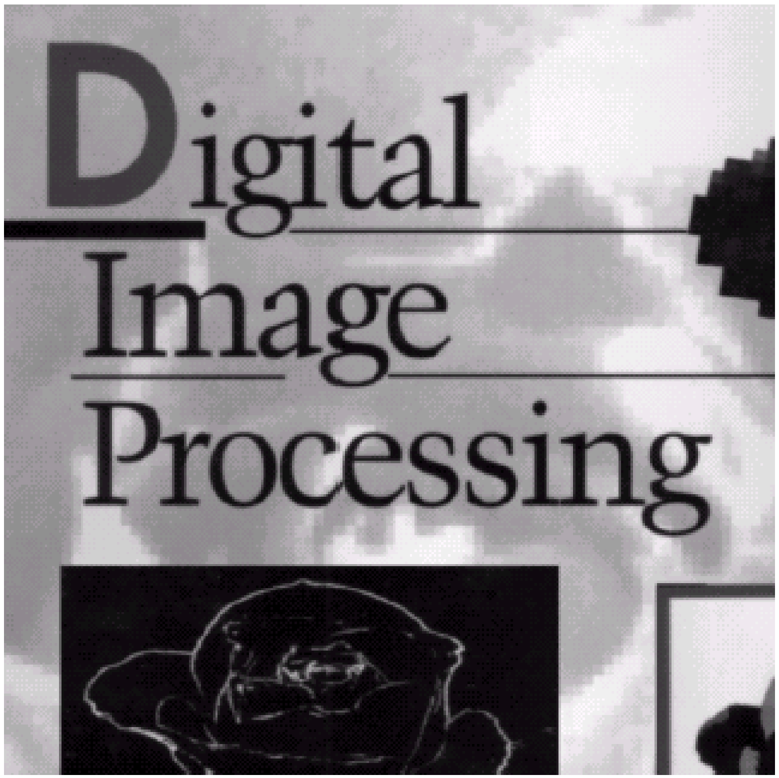
Recalling the position shift theorem (resembles time shift theorem) of Fourier transform, we have

$$
\begin{aligned}
G(u,v) &= \int_0^T \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left[x - x_0(t),\ y - y_0(t)\right] e^{-j2\pi(ux+vy)} dxdy \right\} dt \\
&= \int_0^T \left\{ F(u,v) e^{-j2\pi\left[ux_0(t)+vy_0(t)\right]} \right\} dt \\
&= F(u,v) \int_0^T \left\{ e^{-j2\pi\left[uat/T+vbt/T\right]} \right\} dt \\
&= F(u,v) \frac{T}{\pi(au+bv)} \sin\left[\pi(au+bv)\right] e^{-j\pi\left[ua+vb\right]}
\end{aligned}
\qquad (5.6.11)
$$

Since $\sin(n\pi)=0$ for all n, G(u, v) vanishes at all frequencies (u, v) where $au+bv=n$ for all integers n = 0, 1, 2, … etc.

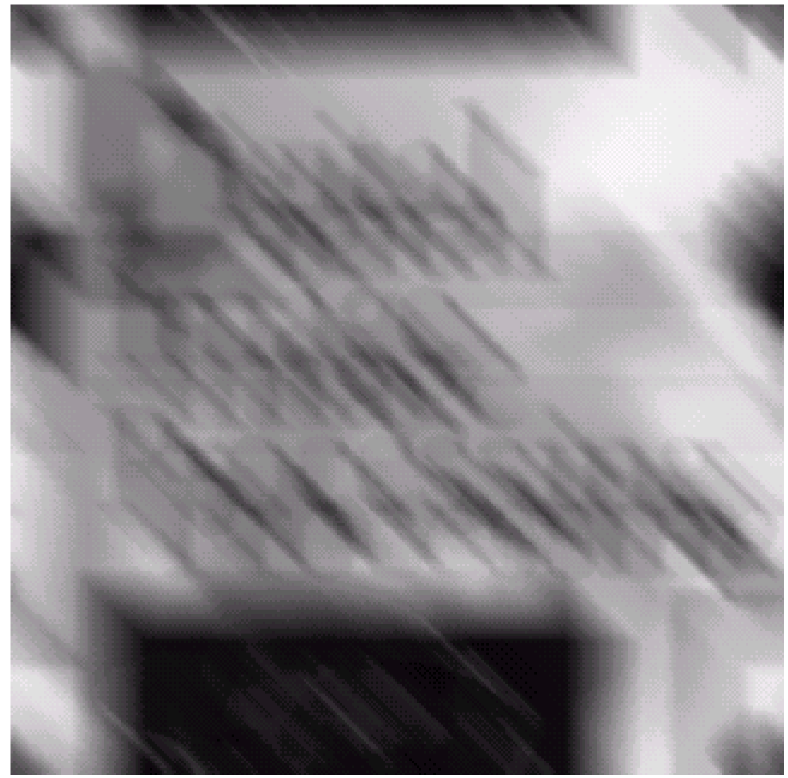
5.6 Estimating the Degradation Function

例5.10: T = 1, $a = b = 0.1$时的运动模糊结果



Original image

f(x, y)



blurring due to linear motion

$$g(x,y) = \Im^{-1}\{F(u,v)H(u,v)\}$$

5.7 Inverse Filtering

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逆滤波的基本公式(由假设模型推导出)

$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v) \qquad (5.5\text{-}17)$$

令

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

代入(**5.5-17**)则有

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

问题: 首先, $N(u, v)$一般是未知的. 其次, 当$H(u, v)$的任何元素为零或者值很小时, 上式在计算机中运算就有可能出问题. 这个问题是不适定的. 这种方式由于问题的不适定性, 在实际中并不可行.

如何解决这类不适定的问题?

一种方式是限制滤波频率使其难以接近原点值, 从而减少遇到零值的可能性.

5.7 逆滤波图像复原

例5.11: 对图5.25分别进行全滤波*(a)*、以及不同半径的限制滤波

(a) inverse filtering without low-pass filtering

(b) cutoff frequency =40

(c) cutoff frequency =70

(d) cutoff frequency =85

5.8 最小均方误差滤波(维纳滤波)

Minimum Mean Square Error (Wiener) Filtering

- 上面的讨论告诉我们, 即使知道了退化函数的表达式, 由于计算的误差和传递函数对噪声的高度敏感性(也就是问题的不适定性), 直接用逆滤波的方式通常都不能解决问题.
- 如果对图像本身的信息有更多的了解, 可以采取其他方式来解决这个问题. 比较常用的方式是把问题转化为求某一个物理量极小值的问题, 或者在某种约束下求极小的问题. 例如所谓的维纳(Wiener)滤波方法, 把原问题转化为: 求原图象 f 的估计值 $\hat{f}$, 使得它们之间的均方误差最小. 均方误差在这里定义为:

$$e^2 = E\{(f - \hat{f})^2\}. \qquad (5.8.1)$$

5.8 最小均方误差滤波(维纳滤波)

Minimum Mean Square Error (Wiener) Filtering

在一定的条件下, 问题(5.8.1)的解可以用下式表示（N. Wiener [1942]）

$$\hat{F}(u,v)=\left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2+S_\eta(u,v)}\right]G(u,v)$$

$$=\left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2+S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \qquad \textbf{(5.8.2)}$$

$$=\left[\frac{1}{H(u,v)}\cdot\frac{\left|H^*(u,v)\right|^2}{\left|H(u,v)\right|^2+S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

其中的各项如下所示

$H(u,v)$——退化函数

$H^*(u,v)$——$H(u,v)$的复共轭

$$|H(u,v)|^2 = H^*(u,v)H(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2$——噪声的功率谱（要求知道这个量是维纳滤波的限制）

$S_f(u,v) = |F(u,v)|^2$——未退化图像的功率谱（这个量一般也是未知的）

若式中的噪声功率谱和未退化图像的功率谱是未知的, (5.8.2)可以用下列公式近似替代:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H^*(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

这里K是一个特殊常数, 通常在复原图象时要进行调整. (注意在整个过程中的思维方式)

5.8 Wiener Filtering

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逆滤波和维纳滤波的比较

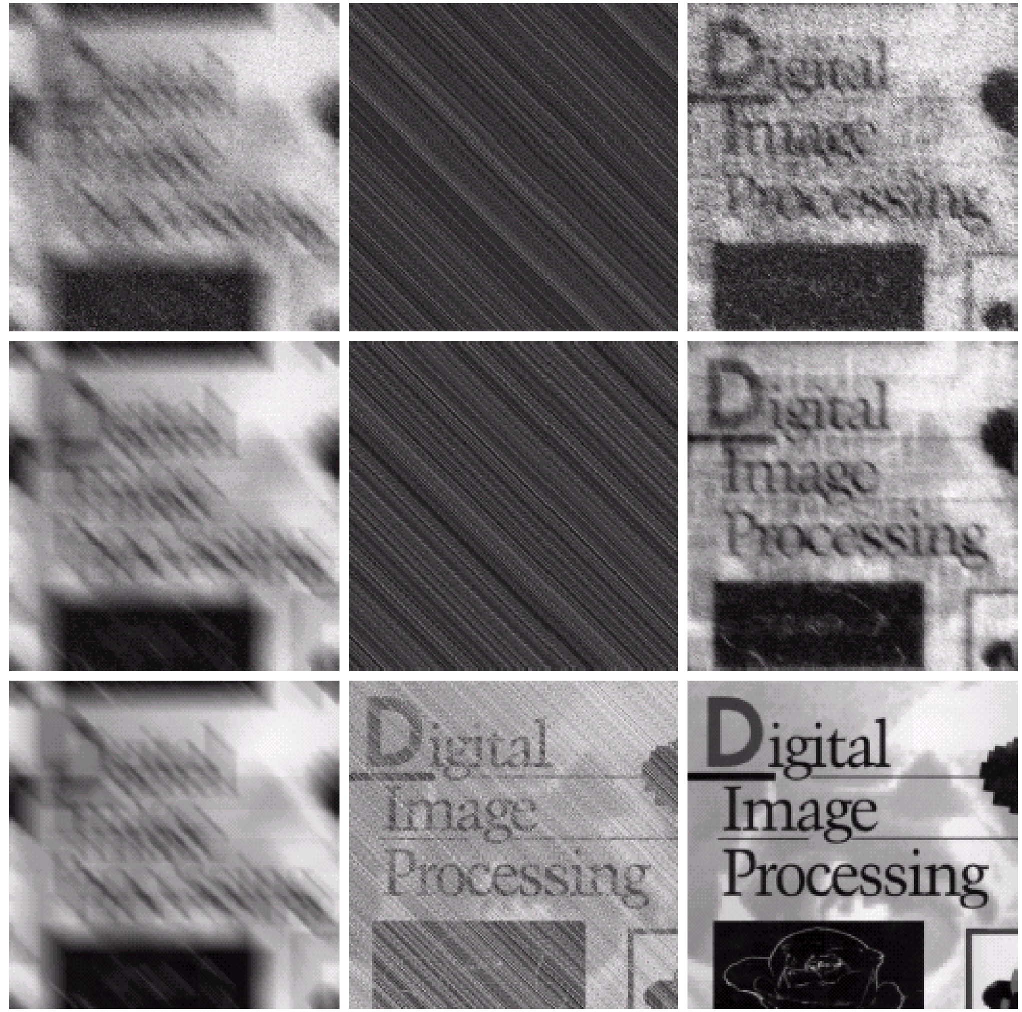
a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

5.8 Wiener Filtering

进一步的比较:

处理运动模糊及加性噪声的图像



a b c
d e f
g h i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.

5.9 Constrained Least Square Filtering

约束最小二乘滤波器

这种方法是将原来的问题转化为: 求未退化函数(原图象)f的估计值 $\hat{f}$, 在约束$\| g - H\hat{f} \|^2 = \|\eta\|^2$之下, 使得下列准则函数达到最小:

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\nabla^2 f(x,y)\right]^2$$

不加证明指出, 在频率域内这个最佳问题的解由下述表达式

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2}\right] G(u,v)$$

其中γ是一个需要调整选择的参数, 当$\gamma = 0$时, 上式退变成逆滤波. **$P(u, v)$**是函数**$p(x, y)$**的傅里叶变换, 式中的γ一般是在一个迭代过程中调整. 见课本.

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

5.9 Constrained Least Square Filtering

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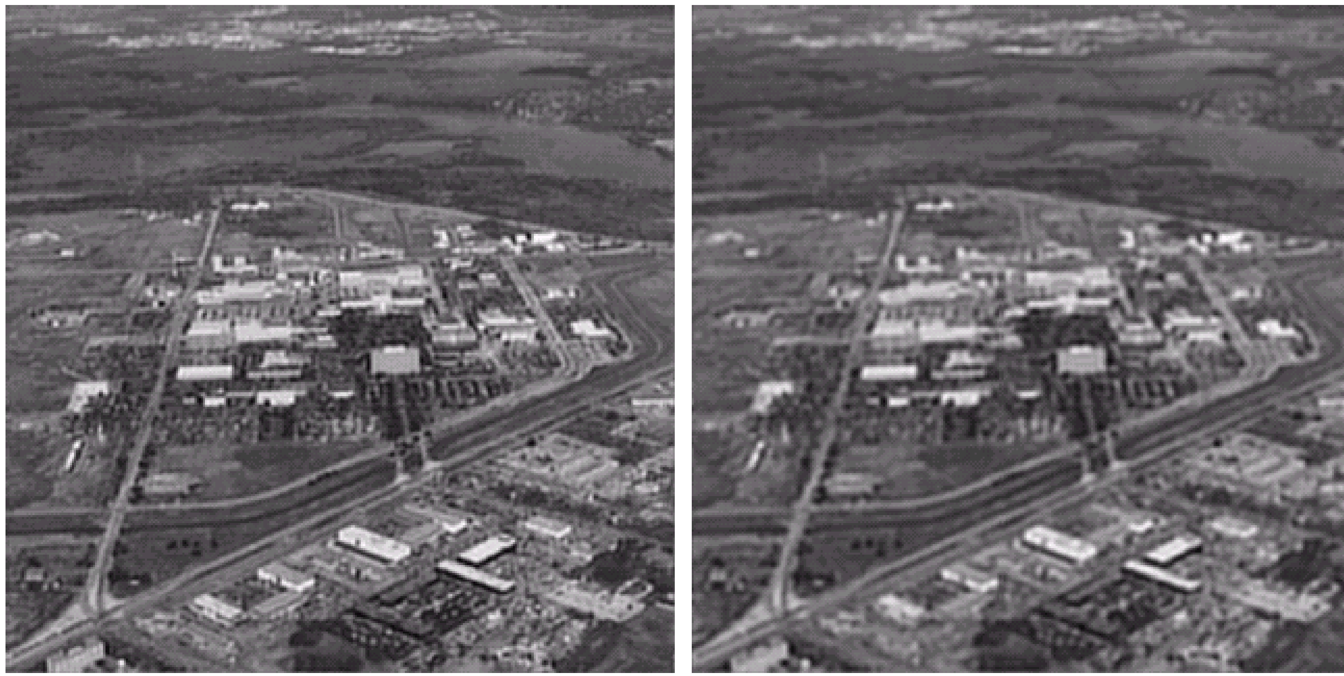
例5.14: 维纳和最小二乘方滤波的比较(用图5.29*c*, *f*, *i*中的结果和下图中的*a*, *b*, c作比较)

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

应该指出的是, 这类不适定问题的求解(包括图像复原), 至今仍然是一个开放的研究领域, 还有许多需要研究的问题.

5.9 Constrained Least Square Filtering

最优约束下最小二乘迭代估计



a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

5.11 几何变换—Geometric Transformations

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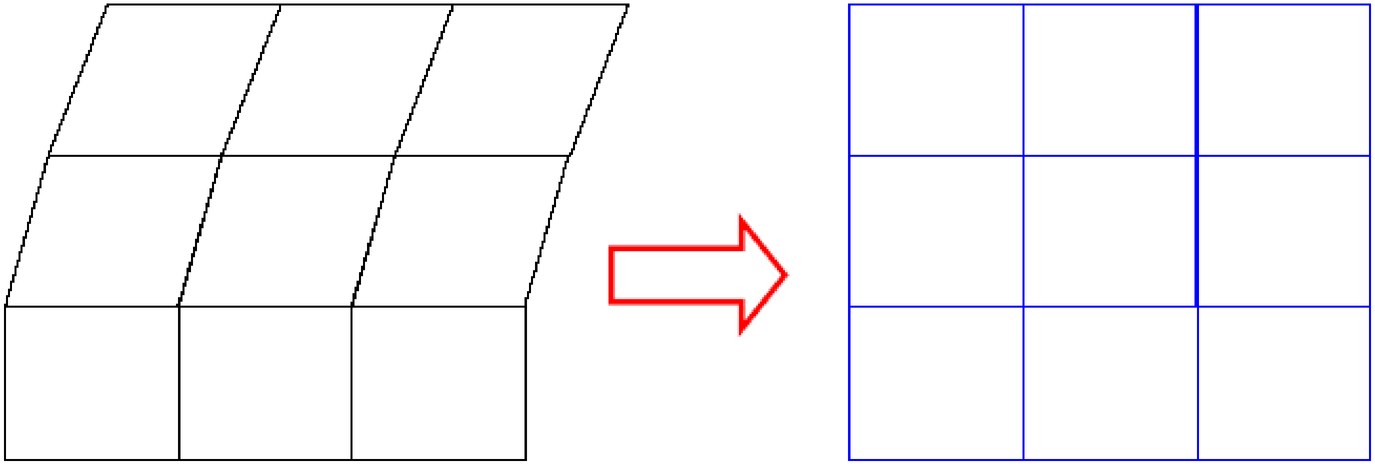
The last approach in image restoration is geometric transformation



Distorted image　　　　restored image

Such an restoration scheme usually involves case-dependent interpolation functions to map the distorted image into a restored image.

5.11 几何变换—Geometric Transformations

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For instance, assume that an image f(x, y) undergoes geometric distortion to result in an image g(x', y') where the coordinate has been distorted by

$$x' = r(x, y) = c_1 x + c_2 y + c_3 xy + c_4$$

and

$$y' = s(x, y) = c_5 x + c_6 y + c_7 xy + c_8$$

To restore the image, first we need to solve for the transformation (i.e., the eight coefficients c_i). To do so, find four pair of corresponding pixels in f(x, y) and g(x', y') to give

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & x_1 y_1 & 1 \\ x_2 & y_2 & x_2 y_2 & 1 \\ x_3 & y_3 & x_3 y_3 & 1 \\ x_4 & y_4 & x_4 y_4 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & x_1 y_1 & 1 \\ x_2 & y_2 & x_2 y_2 & 1 \\ x_3 & y_3 & x_3 y_3 & 1 \\ x_4 & y_4 & x_4 y_4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix}$$

and

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & x_1y_1 & 1 \\ x_2 & y_2 & x_2y_2 & 1 \\ x_3 & y_3 & x_3y_3 & 1 \\ x_4 & y_4 & x_4y_4 & 1 \end{bmatrix} \begin{bmatrix} c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & x_1y_1 & 1 \\ x_2 & y_2 & x_2y_2 & 1 \\ x_3 & y_3 & x_3y_3 & 1 \\ x_4 & y_4 & x_4y_4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix}$$

Once the coefficients are obtained, the restoration can be performed by using

$$x' = r(x, y) = c_1x + c_2y + c_3xy + c_4$$

and

$$y' = s(x, y) = c_5x + c_6y + c_7xy + c_8$$

for any (x, y).

5.11 几何变换—Geometric Transformations

◆处理成像中几何变形的问题，这种现象在相机成像中(例如现在的数码相机的广角成像、扫描仪扫描书本时)会经常出现.

几何变换由两个基本操作组成:

(1)空间变换—对图像平面的像素做了重新安排;

(2)灰度级插补—处理空间变换后图像中像素灰度级的赋值.

空间变换

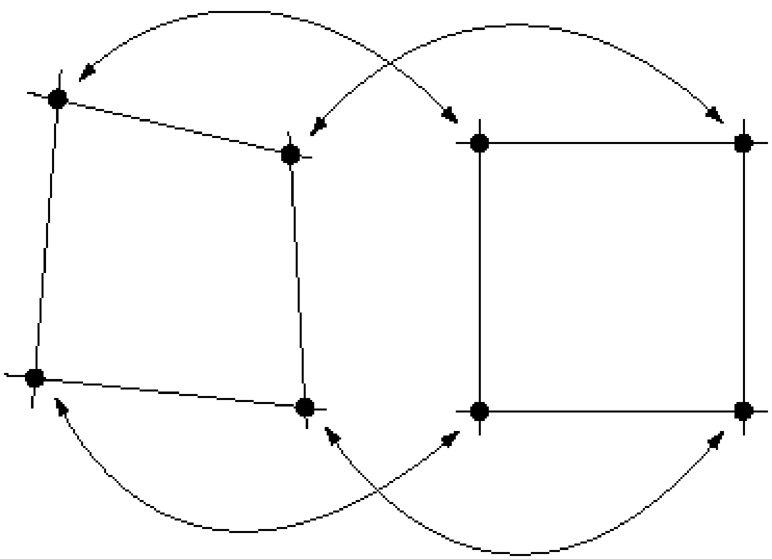

FIGURE 5.32 Corresponding tiepoints in two image segments.

5.11 Geometric Transformations

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设f是一幅数字图像, 象素点坐标为(x, y); 经过几何失真得到了另一幅图像g, 像素点的坐标为(x', y'). 变换公式可以形式地写为:

$$x' = r(x, y) \qquad y' = s(x, y)$$

如果上述公式已知, 空间变换的问题就解决了。但通常这是不可能的。解决的办法就是用一些“连接点”来重新定位。这些连接点在失真图和校正图中的位置是精确知道的。设r和s有下列表达式

$$x' = r(x, y) = c_1 x + c_2 y + c_3 xy + c_4$$
$$y' = s(x, y) = c_5 x + c_6 y + c_7 xy + c_8$$

若有四对八个“连接点”就可以求出上式的8个系数, 然后失真四边形中所有的点都可以对应算出相应校正后的坐标。

连接点可以通过很多方式得到, 比如图像中有明显标志的点集等等。

5.11 Geometric Transformations

5.11.2 灰度级插补

空间变换可能导致非整数的(x', y')值, 但由于失真图像g是数字图像, 它的像素值只定义在整数坐标.需要做灰度插补.

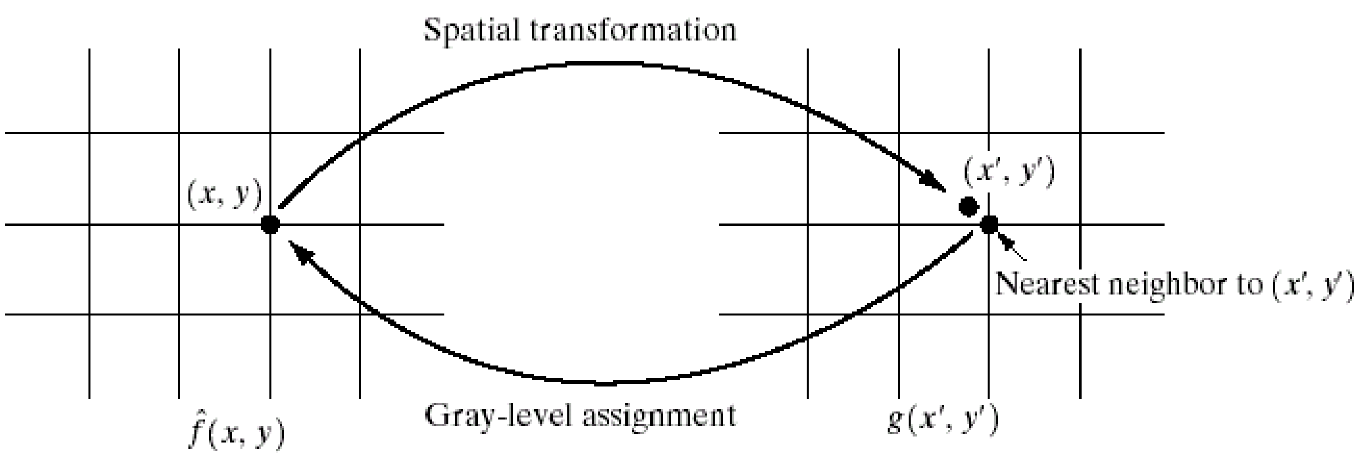


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

5.11 Geometric Transformations

几何变换的说明

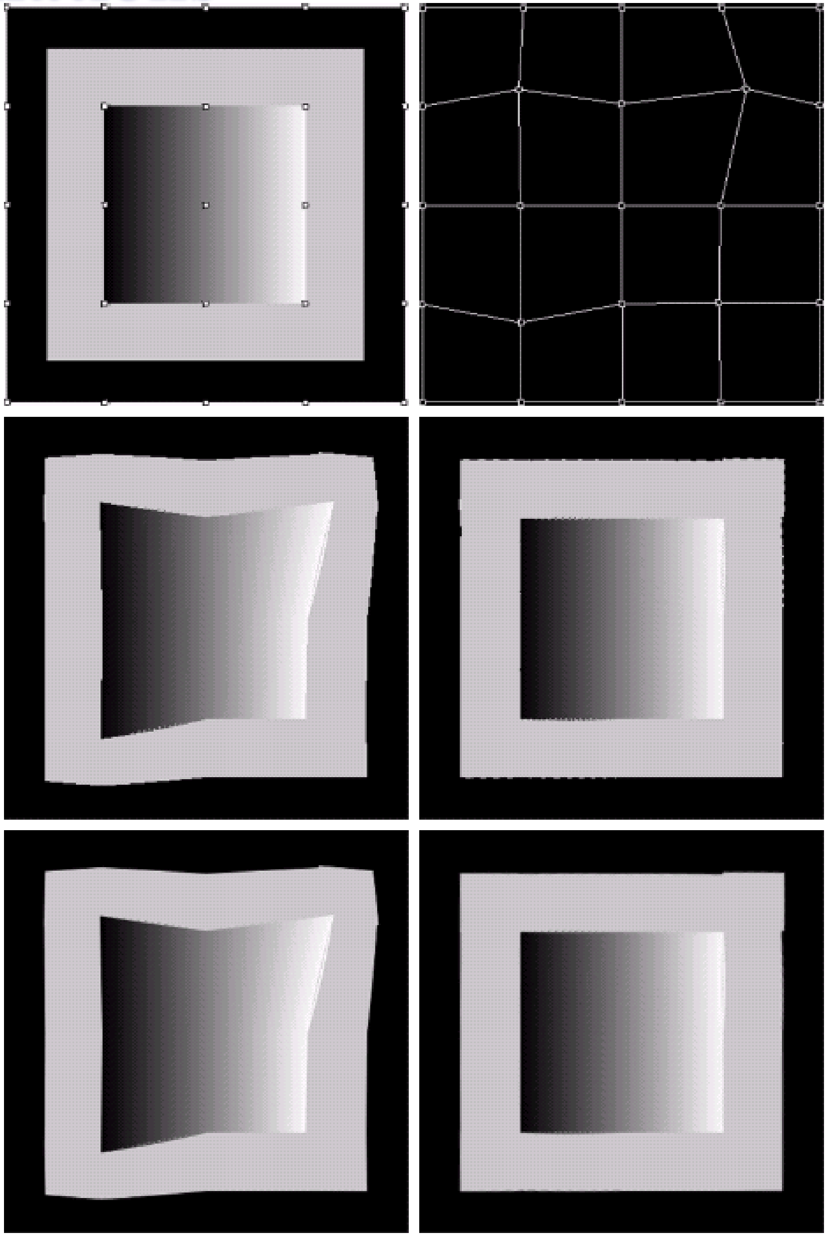
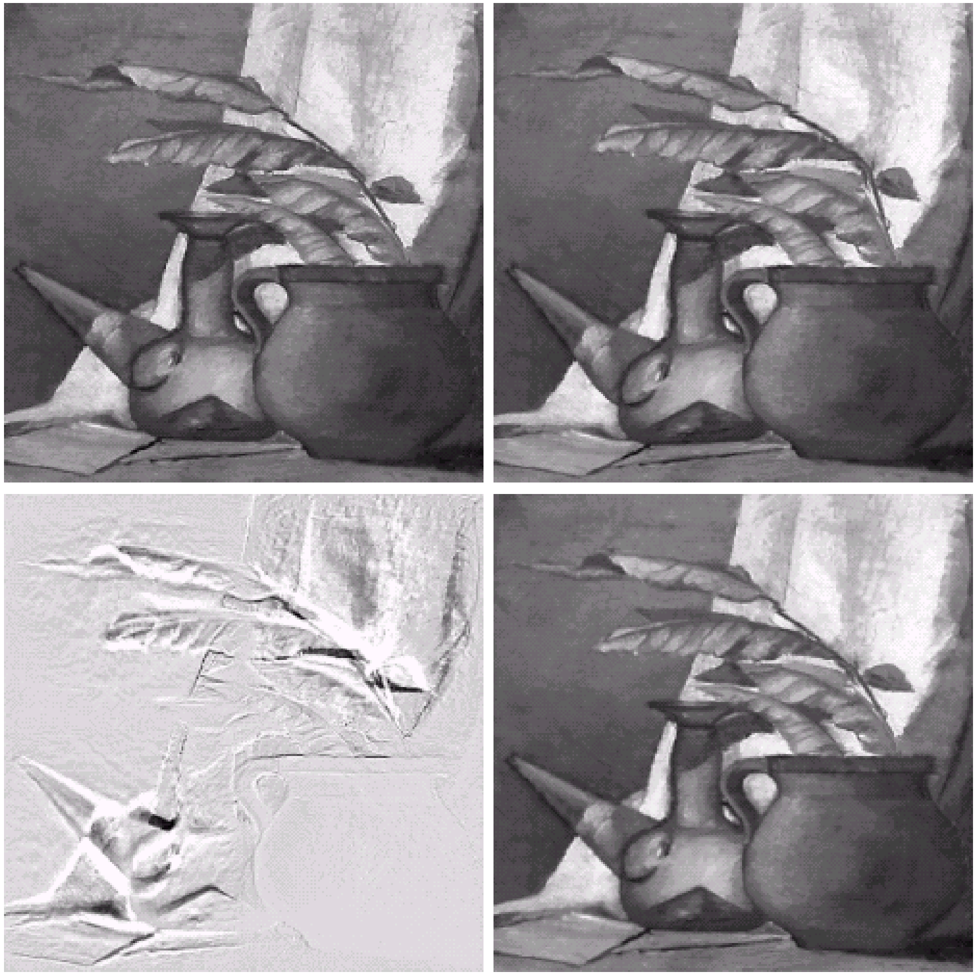


FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

5.11 Geometric Transformations

应用实例:



a b
c d

FIGURE 5.35 (a) An image before geometric distortion. (b) Image geometrically distorted using the same parameters as in Fig. 5.34(e). (c) Difference between (a) and (b). (d) Geometrically restored image.

End of Chapter 5

例1：设仅利用像素点（x，y）的4-近邻像素（不用点（x，y））组成一个低通滤波器。
（1）给出它在频域的等价滤波器H（u，v）；
（2）证明所得结果确实是一个低通滤波器。

例2：利用像素点（x，y）和其4-近邻像素组成一个高通滤波器：

$$g(x,y)=4f(x,y)-f(x-1,y)-f(x+1,y)-f(x,y-1)-f(x,y+1)$$

（1）给出它在频域的等价滤波器H（u，v）；
（2）证明所得结果确实是一个高通滤波器。

（1）滤波后的函数为（设各系数均为1）

$$g(x,\ y)=\frac{1}{4}\{f(x+1,y)+f(x,y+1)+f(x-1,y)+f(x,y-1)\}$$

对其进行傅里叶变换（借助平移性质）得

$$G(u,v)=\frac{1}{4}\{F(u,v)\exp(j2\pi u/N)+F(u,v)\exp(-j2\pi u/N)\}$$

$$+\frac{1}{4}\{F(u,v)\exp(j2\pi v/N)+F(u,v)\exp(j2\pi v/N)\}$$

$$=\frac{1}{2}F(u,v)[\cos(2\pi u/N)+\cos(2\pi v/N)]$$

所以频域的等价滤波器为

$$H(u,v)=\frac{1}{2}[\cos(2\pi u/N)+\cos(2\pi v/N)]$$

（2）上述滤波器 以N为周期，在u=0,v=0时取到最大值，在一个周期内随着频率值 的增加其幅值逐渐减小，这表明该滤波器的功能相当于一个低通滤波器。