$$\theta(x) := \sum_{n=-\infty}^{\infty} e^{-n^2\pi x} = \frac{1}{\sqrt{x}}\,\theta(\frac{1}{x}); \qquad \theta(1) = T_1 := \frac{1}{\sqrt{2\pi}}\,\frac{\Gamma(\frac{1}{4})}{\pi^{1/4}}$$

$$Y(x) := \frac{\theta(x)}{T_1} = \frac{1}{\sqrt{x}} Y(\frac{1}{x}) \qquad \Longrightarrow\; Y_1 := Y(1) = 1$$

$$Y'(x) = -\frac{1}{2}\frac{1}{x\sqrt{x}} Y(\frac{1}{x}) - \frac{1}{x^2\sqrt{x}}\,Y'(\frac{1}{x}) \qquad \Longrightarrow\; Y_1' = -\frac{1}{4}$$

$$Y''(x) = \frac{3}{4}\frac{1}{x^2\sqrt{x}} Y(\frac{1}{x}) + \left(\frac{1}{2}+\frac{5}{2}\right)\frac{1}{x^3\sqrt{x}} Y'(\frac{1}{x}) + \frac{1}{x^4\sqrt{x}}\,Y''(\frac{1}{x}) \qquad \Longrightarrow\; 0 = \frac{3}{4} + 3Y_1' = \frac{3}{4} - \frac{3}{4} = 0$$

$$Y'''(x) = -\frac{15}{8}\frac{1}{x^3\sqrt{x}} Y(\frac{1}{x}) - \frac{45}{4}\frac{1}{x^4\sqrt{x}} Y'(\frac{1}{x}) - \frac{15}{2}\frac{1}{x^5\sqrt{x}}\,Y''(\frac{1}{x}) - \frac{1}{x^6\sqrt{x}}\,Y'''(\frac{1}{x}) \qquad \Longrightarrow\; Y_1''' = \frac{15}{32} - \frac{15}{4}Y_1''$$

$$Y^{iv}(x) = \frac{105}{16}\frac{1}{x^4\sqrt{x}} Y(\frac{1}{x}) + \frac{105}{2}\frac{1}{x^5\sqrt{x}} Y'(\frac{1}{x}) + \frac{105}{2}\frac{1}{x^6\sqrt{x}}\,Y''(\frac{1}{x}) + \frac{14}{x^7\sqrt{x}}\,Y'''(\frac{1}{x}) + \frac{1}{x^8\sqrt{x}}\,Y^{iv}(\frac{1}{x})$$

$$\Longrightarrow\; 0 = -\frac{105}{16} + \frac{105}{2}Y_1'' + 14Y_1''' = -\frac{105}{16} + \frac{105}{2}Y_1'' + \left(\frac{105}{16} - \frac{105}{2}Y_1''\right) = 0$$

$$Y^{(n)}(x) = \sum_{k=0}^{n} a_{n,k}\frac{1}{x^{n+k}\sqrt{x}} Y^{(k)}(\frac{1}{x}); \qquad a_{n.k} = -a_{n-1,k-1} - \left(n+k-\frac{1}{2}\right)a_{n-1,k}; \quad a_{0,0} = 1$$

$$\left(Y^2Y''' - 15(Y')^3 - 15Y'R\right)^2 + 32R^3 = \pi^2\,T_1^8 Y^{10}R^2; \qquad R := YY'' - 3(Y')^2$$

$$R_1 = Y_1'' - \frac{3}{16}; \quad \left(\frac{15}{32} - \frac{15}{4}Y_1'' + \frac{15}{64} + \frac{15}{4}\left(Y_1'' - \frac{3}{16}\right)\right)^2 + 32R_1^3 = 32R_1^3 = \pi^2\,T_1^8 R_1^2$$

$$\Longrightarrow\; R_1 = \frac{\pi^2\,T_1^8}{32} = \frac{M}{16}; \quad Y_1'' = \frac{1}{16}(3+M); \quad Y_1''' = -\frac{15}{64}(1+M); \quad M := \frac{\pi^2\,T_1^8}{2}$$