Cooperation in *n* – player Prisoner’s Dilemma threshold game

Boza, G. [1,4], Könnyű, B. [1] and Számadó, Sz. [2,3]

1-Department of Plant Taxonomy and Ecology, Eötvös Loránd University

2-HAS Research Group of Ecology and Theoretical Biology, Eötvös Loránd University

3-Collegium Budapest, Institute for Advanced Study

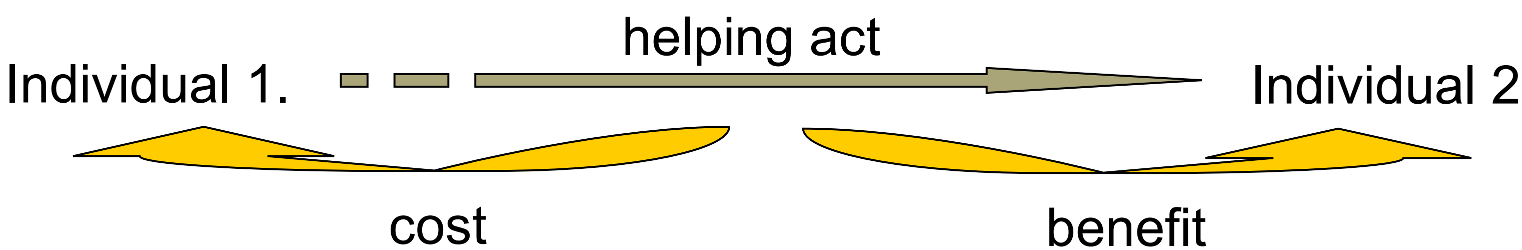
Budapest, Hungary

4-IIASA, International Institute for Applied Systems Analysis

Laxenburg, Austria

Studying cooperation

<u>Reciprocal altruism</u>



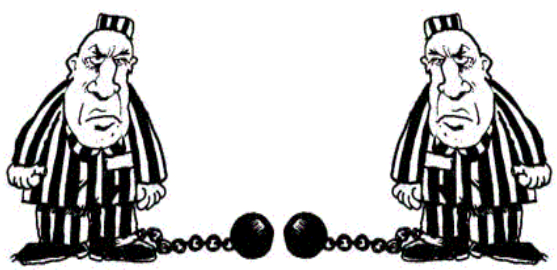
if cost $>$ benefit $\longrightarrow$ X

cost $<$ benefit $\longrightarrow$

Game theory: a tool for studying cooperation

<u>The Prisoner's Dilemma Game</u>

- conflict of interest: the dilemma
- two players



		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3 Reward	0 Sucker
	Defect	5 Cheat	1 Punishment

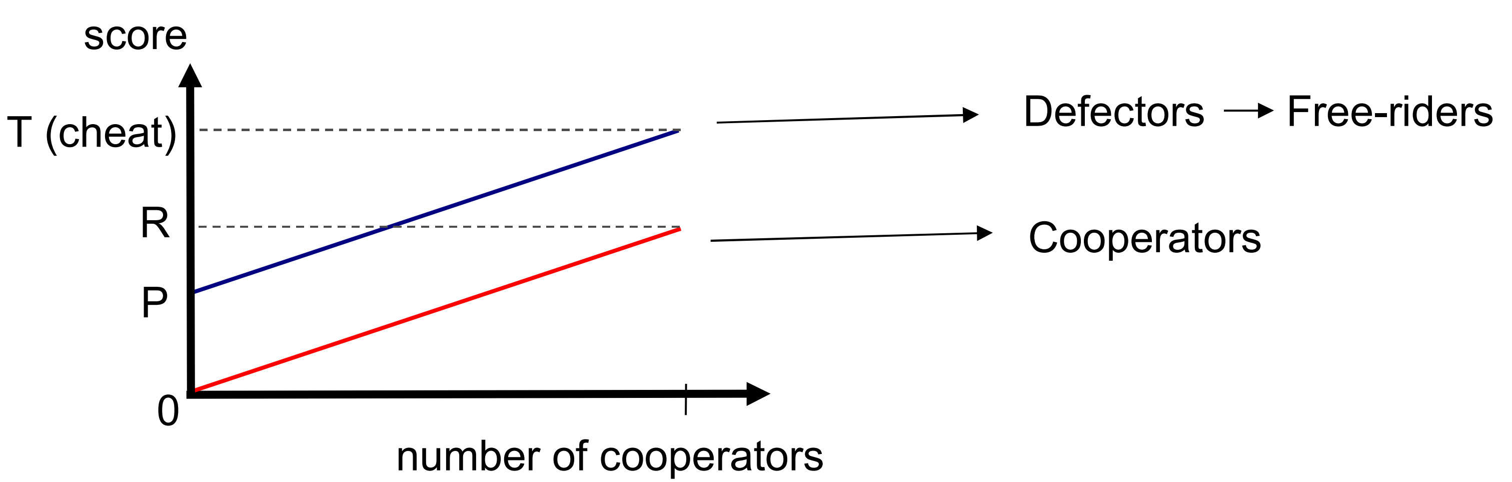
n-players game: Public Goods Game

conflict of interest

n-players (n > 2)

public good: *non-excludable, non-rival resource*

benefit function (fitness function)

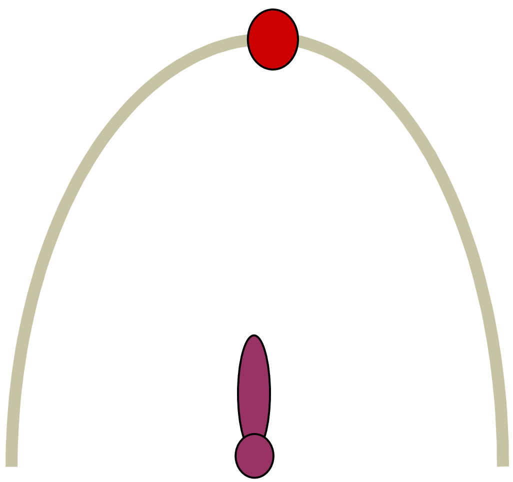


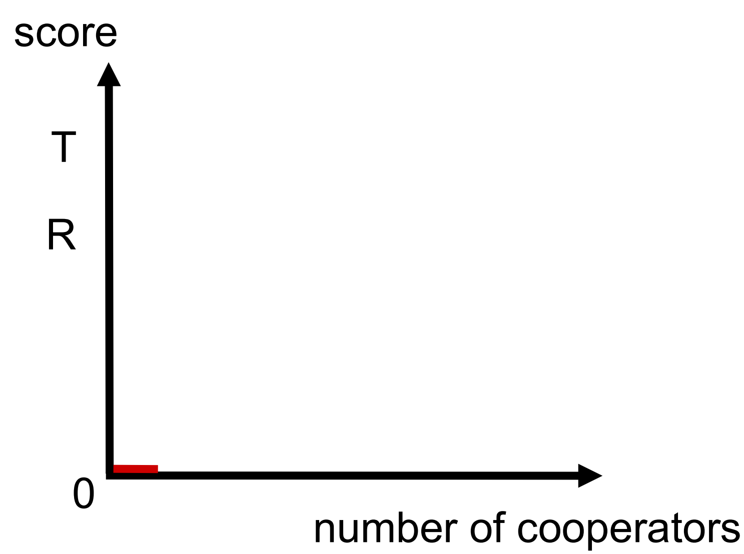
n-player games in natural systems

…on land…

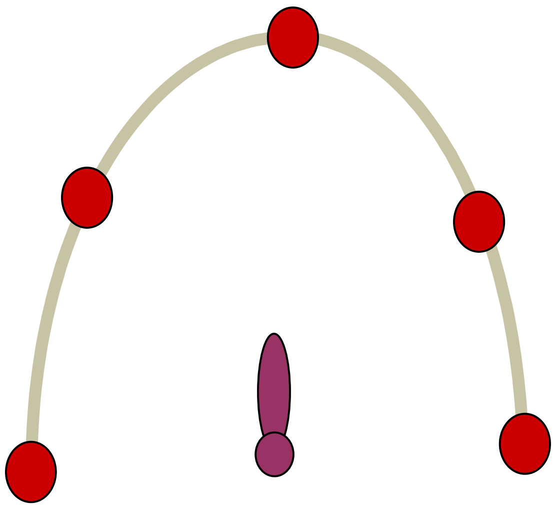
…and in water

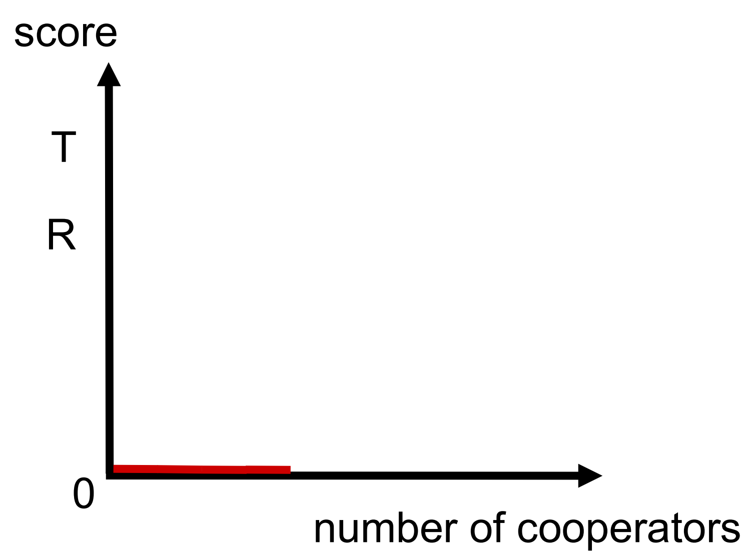
n-player Threshold Game



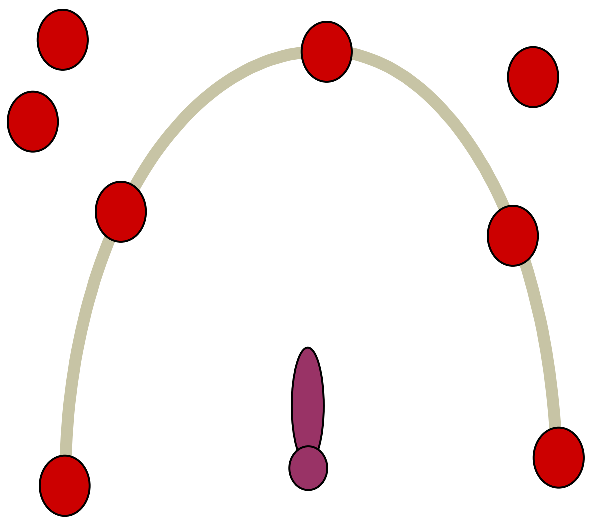


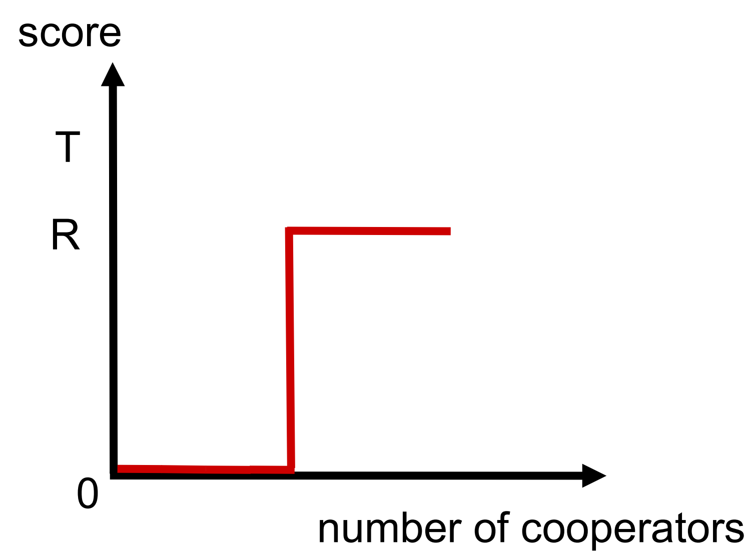
n-player Threshold Game



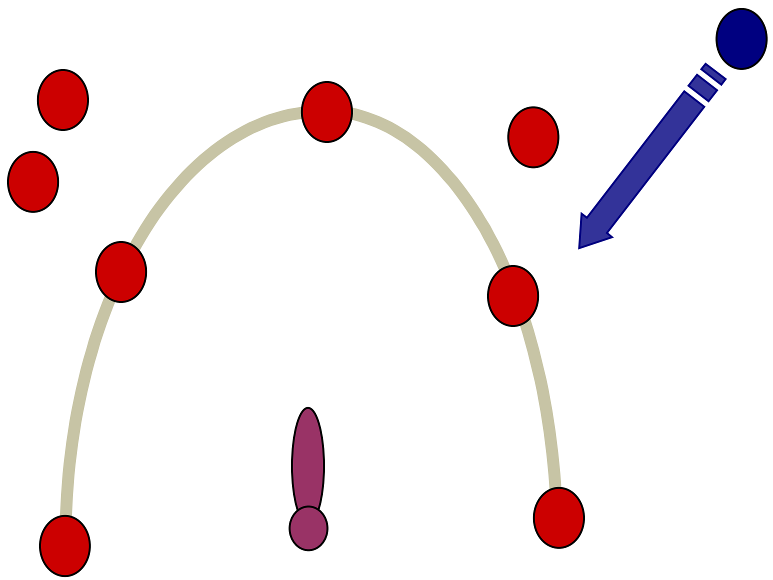


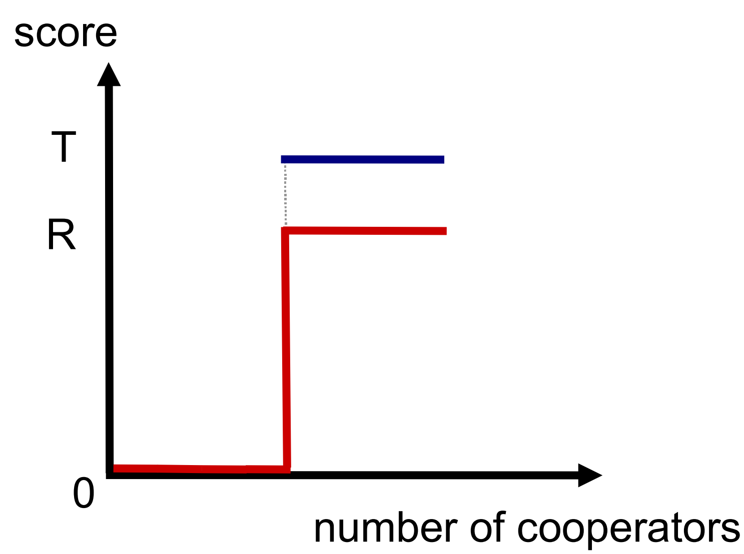
n-player Threshold Game





n-player Threshold Game





n-player Prisoner's Dilemma Threshold Game

n-players from the population of *N* randomly chosen: Well – mixed population

group size (*n*) 3

threshold value (TV) 2

cost of cooperation (*c*)

benefit of cooperation (*b*) 1

	partners		
focal	**CC**	**CD**	**DD**
C	*b-c*	*b-c*	*-c*
D	*b*	*0*	*0*

willingness to cooperate (*x*), evolving trait

$x = 1 \longrightarrow$ *always cooperates*

$x = 0 \longrightarrow$ *always defects*

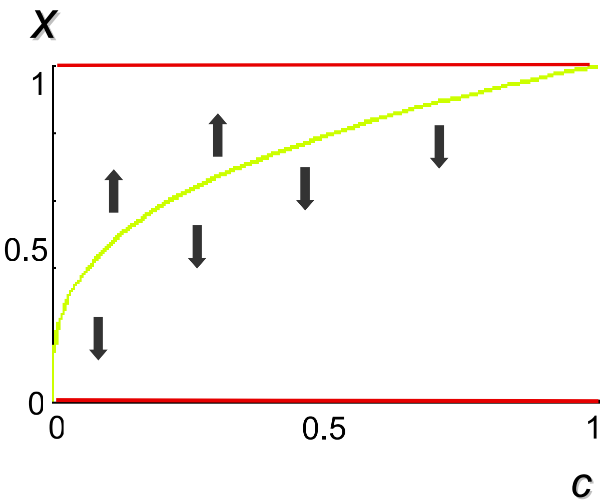
Bach, L. A., Helvik, T., Christiansen, F. B., (2006). The evolution of n-player cooperation – threshold games and ESS bifurcations, Journal of Theoretical Biology, 238: 426-434.

Results with well-mixed population structure

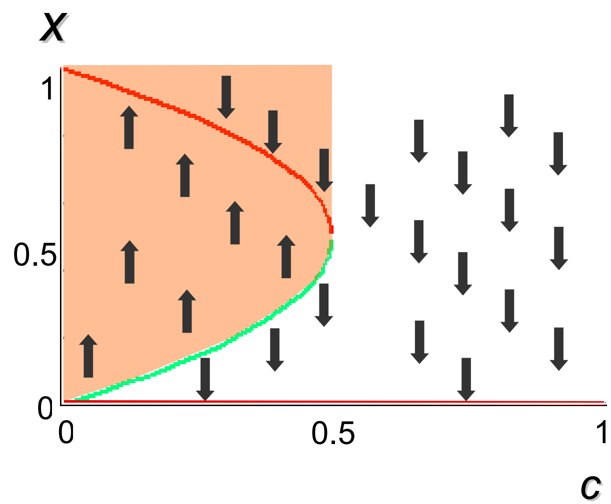
<u>Changing parameter:</u>

c – cost of cooperation

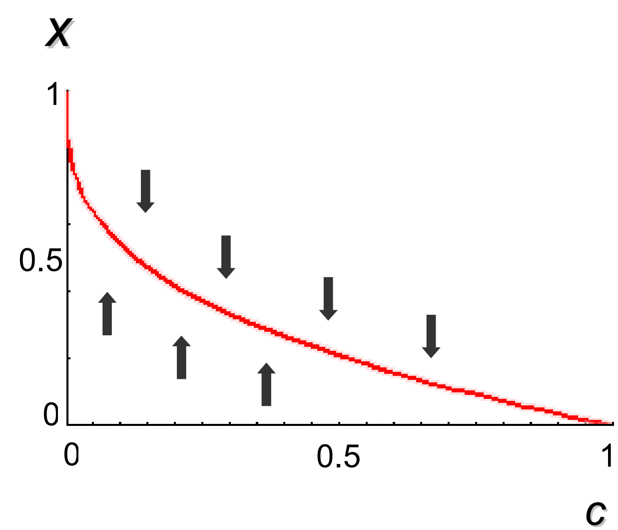
TV – threshold value



***TV* = 3 / 3**



***TV* = 2 / 3**



***TV* = 1 / 3**

Game with spatial population structure

cellular automaton

<u>von Neumann – neighborhood:</u>

focal individual +

4 closest individual on the grid

<u>Moore – neighborhood:</u>

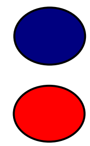
focal individual +

8 closest individual on the grid

asynchronous update

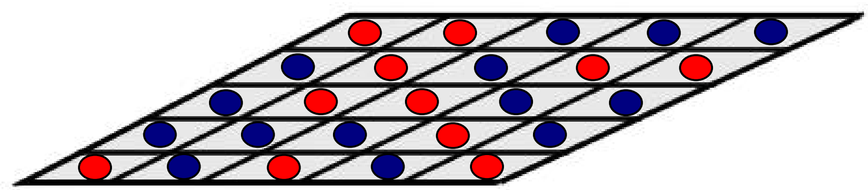
proportional update rule

$$p_i = \frac{d_i}{\sum_{j=1}^{n} d_j}$$



= defectors

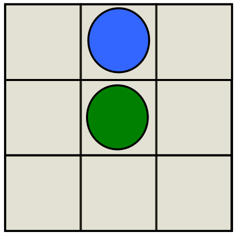
= cooperators



TWO GROUP FORMING SCENARIOS

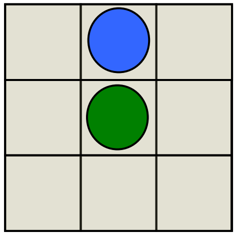
Results with spatial population structure

<u>Scenario 1: fixed group composition</u> (1 area – 1 group)



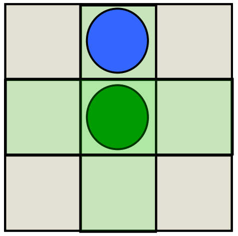
Results with spatial population structure

Scenario 1: fixed group composition (1 area – 1 group)



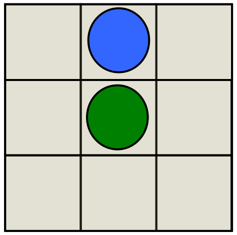
Scenario 2: group composition corresponding to focal individual

(focal individual – 1 focal's group + 8 neighbors' group)



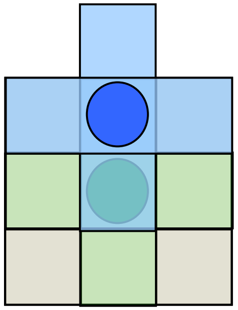
Results with spatial population structure

Scenario 1: fixed group composition (1 area – 1 group)



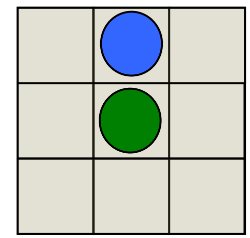
Scenario 2: group composition corresponding to focal individual

(focal individual – 1 focal's group + 8 neighbors' group)



Results with spatial population structure

<u>Scenario 1: fixed group composition</u> (1 area – 1 group)



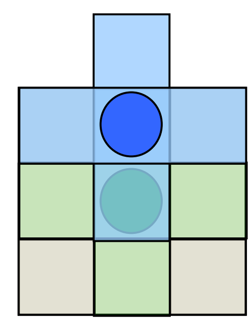
Defectors overtake

<u>Scenario 2: group composition corresponding to focal individual</u>

(focal individual – 1 focal's group + 8 neighbors' group)

Cooperators prevail more

than in well-mixed population

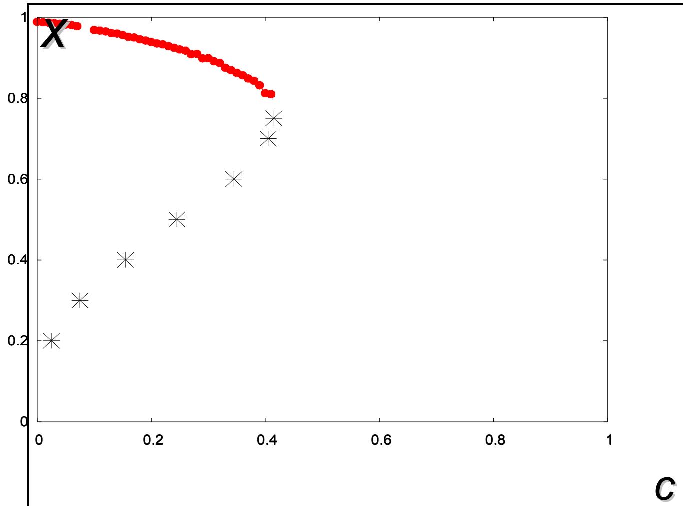


Results with spatial population structure II.

Population structure

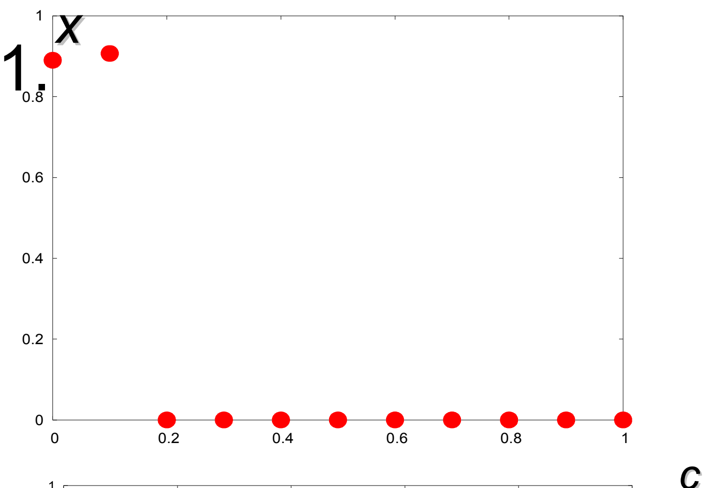
TV=4

Well –mixed

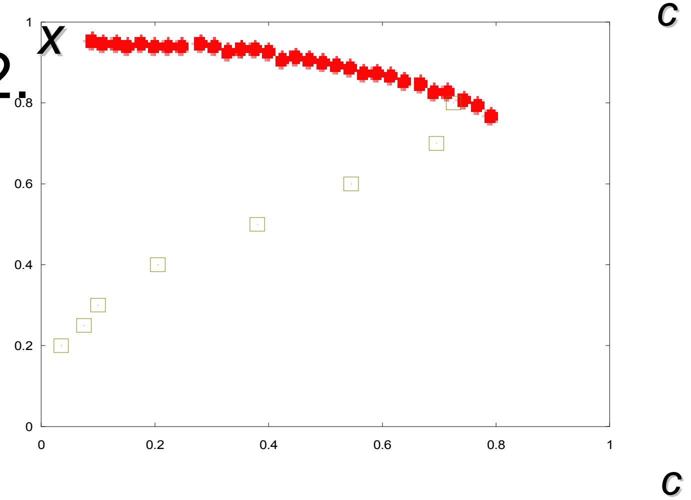


Spatial

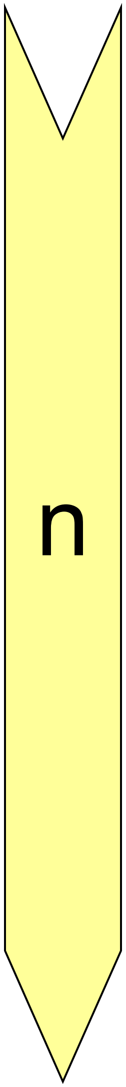
1.

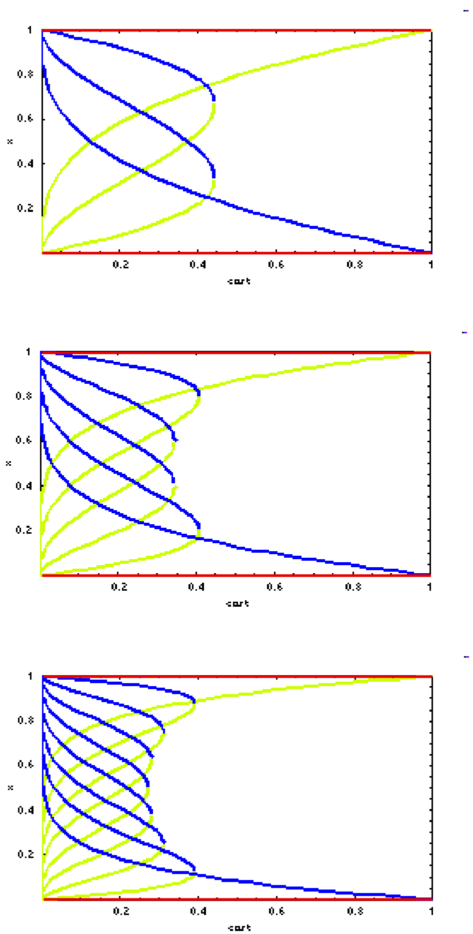


2.



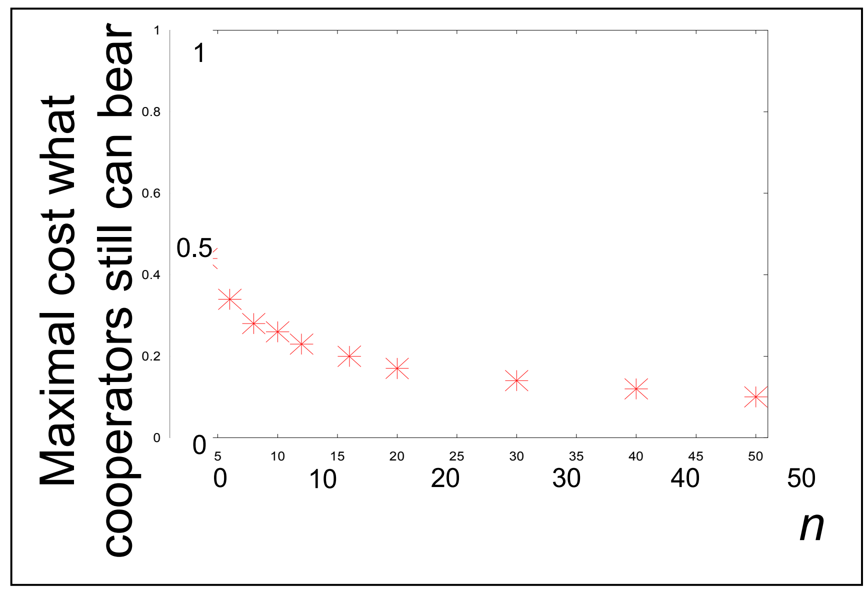
Results with well-mixed population structure II.





<u>Changing parameter:</u>

n – group size



Conclusions

High levels of cooperation can evolve in *n*-player Prisoner's Dilemma Threshold Game

What matters: …..

- the size of the group
- the threshold value
- interpretation of (localized) interaction group
- Update rule (competition rule)

Thank you for your attention !