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Evolutionary stability in the n-person iterated prisoner's dilemma ☆

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Abstract

The iterated prisoner's dilemma game has been used extensively in the study of the evolution of cooperative behaviours in social and biological systems. The concept of evolutionary stability provides a useful tool to analyse strategies for playing the game. Most results on evolutionary stability, however, are based on the 2-person iterated prisoner's dilemma game. This paper extends the results in the 2-person game and shows that no finite mixture of pure strategies in the n-person iterated prisoner's dilemma game can be evolutionarily stable, where $n>2$. The paper also shows that evolutionary stability can be achieved if mistakes are allowed in the n-person game.

Keywords: Iterated prisoner's dilemma; Evolutionarily stable strategies; Evolutionary analysis

1. Introduction

The 2-person prisoner's dilemma game (2PD) is a 2×2 non-zerosum non-cooperative game, where 'non-zerosum' indicates that the benefits obtained by a player are not necessarily the same as the penalties received by another player and 'non-cooperative' indicates that no preplay communication is permitted between the players (Rapoport, 1966; Colman, 1982). The 2PD was first explicitly formulated and given the name 'Prisoner's Dilemma' by A.W. Tucker (see also Colman, 1982). If the 2PD is played repeatedly for many rounds between two players, it is known as the 2-person iterated (or repeated) prisoner's dilemma game (2IPD). The 2IPD has been used widely in studying the evolution of cooperation in biological and social systems since the 1980s (Axelrod and Hamilton, 1981; Axelrod 1984; Axelrod and Dion, 1988). Two of the most interesting issues in the evolution of cooperation are how cooperation can evolve from a group of selfish individuals and whether there exist stable cooperative strategies which can persist in a system.

Evolutionary stability has been proposed by Maynard Smith (1974, 1982) to analyse stable strategies. An evolutionarily stable strategy is one

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which cannot be invaded by other strategies or combinations of strategies (Maynard Smith, 1974; 1982). It has been shown that no finite mixture of pure strategies can be evolutionarily stable in the 2IPD (Boyd and Lorberbaum, 1987; Farrell and Ware, 1989) where a pure strategy is a complete plan of action, specifying in advance what moves (choices) a particular player will make in all possible situations that might arise during the course of the game (Colman, 1982, p. 7). This negative result was interpreted by Farrell and Ware (1989) as a suggestion that evolutionary stability might be too demanding a criterion. Nevertheless, evolutionary stability provides us with a useful theoretical tool in analysing different strategies.

In addition to the theoretical work in analysing evolutionarily stable strategies, a lot of computational studies have also been carried out (Axelrod, 1987; Chess, 1988; Lindgren, 1991; Fogel, 1991, 1993; Darwen and Yao, 1995, 1994). These studies not only investigated the conditions that promote the evolution of cooperation but also demonstrated the computational approach to evolve and design novel strategies for playing games.

The 2-person prisoner's dilemma game can be illustrated by the payoff matrix in Fig. 1. For the single round game, each player is always better off to defect, but they will be worse off by mutual defection than by mutual cooperation. If the game is repeated for many rounds, i.e., in a 2IPD, there is more room for cooperation to evolve.

While the 2IPD has been studied extensively for more than three decades, there are many real world problems, especially many social and economic ones, which cannot be modelled by the

		player B	
		D	C
player A	C	S	R
	D	P	T

Fig. 1. The payoff matrix of the 2-person prisoner's dilemma game. S, P, R, T are payoff values player A gets when playing against player B, where two conditions must be satisfied: (1) $T > R > P > S$; and (2) $R > (S + T)/2$. The payoff matrix is symmetric for each player.

2IPD. Hardin (1968) described some examples of such problems. More examples can be found in Colman's book (1982, pp. 156– 159). The n-person iterated prisoner's dilemma (NIPD) is a more realistic and general game which can model those problems. In comparing the NIPD with the 2IPD, Davis et al. (1976, p. 520) commented that 'The N-person case (NPD) has greater generality and applicability to real-life situations. In addition to the problems of energy conservation, ecology, and overpopulation, many other real-life problems can be represented by the NPD paradigm.'

Colman (1982, p. 142) and Glance and Huberman (1993, 1994) have also indicated that the NIPD is 'qualitatively different' from the 2IPD and that '... certain strategies that work well for individuals in the Prisoner's Dilemma fail in large groups.'

The n-person prisoner's dilemma game can be defined by the following three properties (Colman, 1982, p. 159): (1) each player faces two choices between cooperation (C) and defection (D); (2) the D option is dominant for each player, i.e., each player is better off choosing D than C no matter how many of the other players choose C; (3) the dominant D strategies intersect in a deficient equilibrium. In particular, the outcome if all players choose their non-dominant C strategies is preferable from every player's point of view to the one in which everyone chooses D, but no one is motivated to deviate unilaterally from D. Similar definition can also be found in Schelling's book (1978, p. 218). The payoff matrix of the n-person game is illustrated by Fig. 2.

This paper is organised as follows. Section 2 discusses evolutionarily stable strategies in the NIPD when individuals do not make any mistakes. An individual who intends to cooperate actually cooperates. An individual who intends to defect actually defects. This situation can be viewed as a noise-free version of the NIPD. We will show in this section that every finite history of interactions among n players occurs with positive probability in any evolutionarily stable mixture of pure strategies in the infinite NIPD where the probability of further interaction is sufficiently high. As a result, no finite mixture of pure strategies in the infinite NIPD can be evolutionarily stable.

Section 3 discusses a more realistic version of the NIPD in which mistakes are allowed. The mistakes are modelled by some probability that an individual who intends to cooperate defects and some probability that an individual who intends to defect cooperates. It is shown that evolutionary stability can be achieved in this noisy version of the NIPD. A simple strategy ALLD is given as an example of evolutionarily stable strategies.

Finally, Section 4 concludes with some discussions.

2. Evolutionary stability when individuals do not make mistakes

Evolutionary stability is an important concept in analysing strategies for playing the NIPD game. We are interested in knowing whether there exists a finite mixture of pure strategies which are evolutionarily stable, that is, no other strategies can invade such a mixture. Evolutionary stability was originally introduced by Maynard Smith (1974; 1982) for 2-person games. This section first introduces the concept of evolutionary stability in the n-person game and then presents our main result.

2.1. Evolutionary stability in the NIPD

The NIPD considered in this paper has an infinite population of players. There are a finite number of distinct strategies in the population. Strategy S_i occurs in the population with frequency f_i. A game is played by n players randomly selected from the population. The payoff from one round of play is determined by Fig. 2. The game continues to play the second round with probability w, the third round with w^2, etc. In general, the game plays for t ($t \geq 1$) rounds with probability w^{t-1}. The only information available to each player at round t is his/her complete history of previous $t-1$ interactions with other $n-1$ players in the game.

Number of cooperations among the remaining n-1 players

		0	1	2		$n-1$
player A	C	C_0	C_1	C_2	$\cdots$	C_{n-1}
	D	D_0	D_1	D_2	$\cdots$	D_{n-1}

Fig. 2. The payoff matrix of the n-person prisoner's dilemma game, where the following conditions must be satisfied: (1) $D_i > C_i$ for $0 \leq i \leq n-1$; (2) $D_{i+1} > D_i$ and $C_{i+1} > C_i$ for $0 \leq i < n-1$; (3) $C_i > (D_i + C_{i-1})/2$ for $0 < i \leq n-1$. The payoff matrix is symmetric for each player.

The essence of Maynard Smith's definition of an evolutionarily stable strategy (ESS) in the 2-person game is the uninvadability of an ESS by other strategies (Maynard Smith, 1982). Let $E(S_i | S_j)$ be the expected payoff of strategy S_i playing against strategy S_j in the 2-person game. Then the first condition for S_i to be an ESS is that for all S_j,

$$E(S_i | S_i) \geq E(S_j | S_i) \tag{1}$$

That is, S_i is the best reply to itself.

Although this condition was used solely to define collective stability by Axelrod (1984), it is actually an equilibrium condition. It states that in a population in which S_i is the only strategy used by all players nobody can gain by switching to another strategy. However, the equality in (1) implies that a player could switch to another strategy without loss of any payoff although not gaining anything. That is, the population could be invaded by other strategies. In order to prevent invasion by other strategies, a stricter condition is necessary to ensure evolutionary stability, i.e., for all strategy S_j ($S_j \neq S_i$), either

$$E(S_i | S_i) > E(S_j | S_i) \tag{2}$$

or

$$E(S_i | S_j) \geq E(S_j | S_i)$$

and if

$$E(S_i | S_i) = E(S_j | S_i)$$

then

$$E(S_i | S_j) > E(S_j | S_j) \tag{3}$$

Inequality 2 indicates that S_i is the only best reply to itself, while Inequality 3 implies that S_i is a weak best reply to itself.

Evolutionary stability in the n-person game is more complex than that in the 2-person case. We provide a general definition here based on the idea of uninvadability. Although there might be other similar definitions of the ESS in n-person games, our definition reflects the essence of the ESS and is sufficient to show our main results. Let $E(S_i | S_{j_1}, S_{j_2}, \ldots, S_{j_{n-1}})$ be the expected payoff of strategy S_i playing with $S_{j_1}, S_{j_2}, \ldots, S_{j_{n-1}}$ and f_k be the frequency of S_k in a population P. We can define

$$V(S_i|P) = \sum_{j_1, j_2, \ldots, j_{n-1}} f_{j_1}, f_{j_2} \cdots f_{j_{n-1}} E(S_i|S_{j_1}, S_{j_2}, \ldots, S_{j_{n-1}})$$

where the summation goes over all the different combinations of $n-1$ strategies. Then we define S_i as an ESS if the following inequality can be satisfied: i.e., for all S_j ($S_j \neq S_i$) in P,

$$V(S_i|P) > V(S_j|P) \tag{4}$$

It is clear from our definition that an ESS cannot be invaded by any other strategy or combination of strategies because of the strict inequality in (4).

2.2. Main result

In this section, we will use a game tree to show our main result that no finite mixture of pure strategies is evolutionary stable. A game tree in the NIPD is a 2^n-ary tree where each branch represents one possible combination of choices of the n players. Each node represents one possible game situation. The path from the root to a node represents the history of a particular sequence of interactions among n players. The level of a node represents the number of game steps (interactions) played so far. Using a game tree, it is not too difficult to prove the following result.

Theorem 2.1

In the infinite NIPD where the probability of further interaction is sufficiently high, every finite history of interactions among the n players occurs with positive probability in any evolutionarily stable mixture of pure strategies.

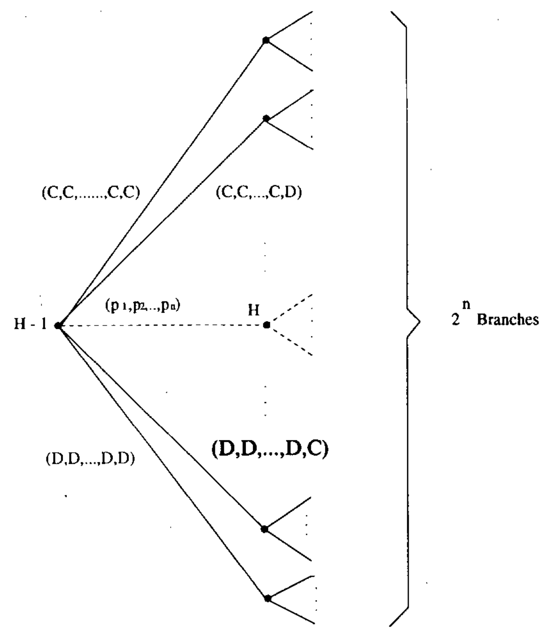

Fig. 3. A branch of the game tree for the NIPD.

Proof. Suppose there is at least one finite history that occurs with zero probability in an evolutionarily stable mixture of pure strategies in population P. We can construct a combination of n strategies, an invading strategy I and a sustained set of mutation strategies M of size $n-1$, such that I can invade population P when one-way mutation sustains M at a low frequency in the population.

By assumption, there is at least one finite history that occurs with zero probability in the NIPD, i.e., at least one sequence of interactions among n players which cannot be generated. Let H be a shortest such history. In the game tree, H can be regarded as a first unreached node. Fig. 3 illustrates the situation, where dashed lines represent unreached branches. $H-1$ can be reached through a sequence of interactions among a set of n strategies (used by n players). Denote one such set as $S_1, S_2, \ldots, S_n$.

Claim. We can always pick $S_1, S_2, \ldots, S_n$ and H

such that H could be reached if we changed at least one move chosen by $S_2, \ldots, S_n$ at $H-1$, i.e., from D to C or from C to D.

The above claim can be shown as follows. Suppose the branch $(p_1, p_2, \ldots, pn)$ from $H-1$ leads to H. If there is a branch $(p_1, q_2, \ldots, q_n)$ which is followed by a set of n strategies, i.e., the branch leads to a reachable node in the game tree, where there is at least one of q_i, $2 \le i \le n$ which is different from p_i, $2 \le i \le n$ (the rest can be the same), then we can pick the strategy making the move p_1 as S_1 and the rest of strategies which make move $q_2, \ldots, q_n$ as $S_2, \ldots, S_n$.

If no such branch $(p_1, q_2, \ldots, q_n)$ exists, then it implies that none of the branches of the form $(p_1, *, \ldots, *)$ leads to a reachable node, where * means the move can be either D or C. Let p'_1 be the opposite move to p_1, i.e., $p'_1 = \mathrm{C}$ if $p_1 = \mathrm{D}$ or $p'_1 = \mathrm{D}$ if $p_1 = \mathrm{C}$. Then there must exist at least one branch of the form $(p'_1, *, \ldots, *)$ which leads to a reachable node from $H-1$ in the game tree, or else the game terminates after a finite number of steps. Let such a branch be $(p'_1, p'_2, \ldots, p'n)$. We can pick the node arrived at by branch $(p_1, p'_2, \ldots, p'_n)$ as H, the strategy making the move p'_2 as S_1 and the rest of strategies making move $p'_1, p'_3, \ldots, p'_n$ as $S_2, \ldots, S_n$. (In fact, we can pick any of the $p'_2, \ldots, p'_n$ as S_1.) Hence the claim is true.

Now we can construct a set M of $n-1$ mutation strategies $M_2, \ldots, M_n$ whose sustained presence makes it possible to design a successful invading strategy I. Let $M_2, \ldots, M_n$ imitate $S_2, \ldots, S_n$ respectively at all nodes except at $H-1$ and histories that begin with $H-1$. At $H-1$, $M_2, \ldots, M_n$ play differently from $S_2, \ldots, S_n$ in such a way that H will be reached if they play with S_1. This is guaranteed to be feasible because of the above claim. At H, $M_2, \ldots, M_n$ cooperate unconditionally. After H, $M_2, \ldots, M_n$ play according to the following description.

Let us construct the invading strategy I at the same time. Let I imitate S_1 at all nodes except for H and histories that begin with H. Since H is never reached in the given population P, H is never reached in the augmented population $P^* = P + \{I\}$. Because I always imitates S_1 when interacting with (evolutionarily stable) P, we have

$$V(I|P^*) = V(I|P) = V(S_1|P) = V(S_1|P^*) \tag{5}$$

I can invade P in the presence of $M_2, \ldots, M_n$ if and only if the expected payoff of I is greater than the expected payoff of S_1, that is,

$$\begin{aligned}
&\left(\prod_{k=2}^{n} f_{M_k}\right) E(I|M_2, \ldots, M_n) \\
&+ \sum_{j_1, j_2, \ldots, j_{n-1}} \left(\left(\prod_{k=1}^{n-1} f_{j_k}\right) E(I|X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}})\right) \\
&> \left(\prod_{k=2}^{n} f_{M_k}\right) E(S_1|M_2, \ldots, M_n) \\
&+ \sum_{j_1, j_2, \ldots, j_{n-1}} \left(\left(\prod_{k=1}^{n-1} f_{j_k}\right) E(S_1|X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}})\right)
\end{aligned} \tag{6}$$

where f values are frequencies of the strategies in the whole population consisting of P^* and M_j values, and each X_{j_k} is an M_j, an S_j or I. Inequality (6) can be satisfied by the following continued construction of I and $M_2, \ldots, M_n$.

At H, let I do the opposite of what S_1 would do. Thereafter I plays defection all the time unconditionally (i.e., ALLD). After H, let $M_2, \ldots, M_n$ play ALLD if they play with a strategy acting like S_1 at H, and play ALLC (cooperation all the time) otherwise. Now we've finished all our construction.

According to our construction, both S_1 and I reach H when playing with $M_2, \ldots, M_n$. Because I imitates S_1 at all nodes except H and histories that begin with H, it performs equally well as S_1 up to H (not including H) when playing with $M_2, \ldots, M_n$. At H, I may do worse than S_1 if I cooperates while S_1 defects. That is, the worst possible loss for I at H is

$$w^{H-1}\,(D_{n-1} - C_{n-1}) \tag{7}$$

However, I does strictly better than S_1 after H because $M_2,\ldots,M_n$ play ALLC with I while play ALLD with S_1, i.e.,

$$E(I|M_2, \ldots, M_n) - E(S_1|M_2, \ldots, M_n) \geq \frac{w^H}{1-w}\,(D_{n-1} - D_0) > 0 \tag{8}$$

and because I plays ALLD and no strategy can do better than it, i.e.,

$$E(I|X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) - E(S_1 \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) \geq 0 \tag{9}$$

where X_{j_k} is an M_j, an S_j or I.

Hence, Inequality (6) can be satisfied, i.e., I can invade P if

$$\frac{w^H}{1-w}\,(D_{n-1} - D_0) - w^{H-1}(D_{n-1} - C_{n-1}) > 0$$

i.e.,

$$w > \frac{D_{n-1} - C_{n-1}}{2D_{n-1} - D_0 - C_{n-1}} \tag{10}$$

A better bound can be obtained by more detailed computation of Inequality (9).[2]

It has been noted that, while the set of nodes in the game tree is countably infinite, it is nevertheless mathematically possible to put non-zero probability on each node (Farrell and Ware, 1989). The following corollary gives the result we are most interested in.

Corollary 2.2

No finite mixture of pure strategies in the infinite NIPD can be evolutionarily stable if the probability of further interaction w is sufficiently high.

Proof. Because any finite mixture of pure strategies can only follow a finite number of paths with a positive probability, there will be infinitely many unreachable nodes in the game tree. According to Theorem 2.1, such strategies cannot be evolutionarily stable.

Although the theoretical result given by Theorem 2.1 appears to be negative, it should be applied with care to real world situations. We assume that individuals never make mistakes. An individual who intends to cooperate cooperates. An individual who intends to defect defects. This is certainly not a very realistic assumption in the real world. Human beings make mistakes, and so do other animals. In reality, an individual who intends to cooperate may defect instead and vice versa. We will show in the following section that mistakes allow evolutionary stability.

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[2] Let

$$E(I \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) = E(I \mid Y_{j_1}, Y_{j_2}, \ldots, Y_{j_{n-1}}) + E(I \mid Z_{j_1}, Z_{j_2}, \ldots, Z_{j_{n-1}}) \tag{11}$$

and

$$E(S_1 \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) = E(S_1 \mid Y_{j_1}, Y_{j_2}, \ldots, Y_{j_{n-1}})\; E(S_1 \mid Z_{j_1}, Z_{j_2}, \ldots, Z_{j_{n-1}}) \tag{12}$$

where Y_{j_k}, $1 \leq k \leq n-1$, is either an M_j or an S_j and Z_{j_k}, $1 \leq k \leq n-1$, is either an I or S_j. There is at least one and at most $n-2$ M_j values in Y_{j_k}, $1 \leq k \leq n-1$. Hence we have after H

$$E(I|Y_{j_1}, Y_{j_2}, \ldots, Y_{j_{n-1}}) - E(S_1 \mid Y_{j_1}, Y_{j_2}, \ldots, Y_{j_{n-1}}) \geq \frac{w^H}{1-w}\,(D_{y+1} - D_y) > 0 \tag{13}$$

and

$$E(I \mid Z_{j_1}, Z_{j_2}, \ldots, Z_{j_{n-1}}) - E(S_1 \mid Z_{j_1}, Z_{j_2}, \ldots, Z_{jn-1}) \geq 0 \tag{14}$$

where $y (0 \leq y < n-1)$ is the number of cooperators from P appeared in Y_{j_k}, $1 \leq k \leq n-1$. Inequality (13) is true because there is at least one M_j in Y_{j_k}, $1 \leq k \leq n-1$, and they will cooperate with I but defect against S_1. Hence, Inequality (6) can be satisfied, i.e., I can invade P if

$$w > \frac{D_{n-1} - C_{n-1}}{2D_{n-1} - D_0 - C_{H-1} + D_{y+1} - D_y}$$

3. Evolutionary stability when individuals make mistakes

It has been shown by Boyd (1989) that mistakes allow evolutionary stability when certain conditions are met in the 2IPD game. We show in this section that Boyd's results can be extended to the NIPD game despite the qualitative difference between the 2IPD and the NIPD. We also give a concrete example of ESS in the NIPD, i.e., ALLD.

The major reason why there exists an ESS in the NIPD if mistakes are allowed is the existence of a strategy which is the strictly best reply (i.e., the uniquely best reply) to $n-1$ copies of itself. This strategy can be shown to be evolutionarily stable if certain conditions about the probability of making mistakes are met, i.e., there is a positive probability of both types of mistakes (C for D or D for C) on every turn regardless of the node in the game tree and the probability is independent of players. Such assumptions about the probability, which are not unrealistic, imply that every node in the game tree has a positive probability of being reached. Furthermore, any group of n strategies must have a positive probability of reaching every node in the game tree because there is a positive probability of both types of errors.

Let us introduce the concept of 'strong perfect equilibrium' in the NIPD, which extends Boyd's (1989) definition in the 2IPD. In the NIPD, suppose player A knows that the other $n-1$ players have all committed to using strategy S_e. What will be the optimal strategy for player A in order to obtain the maximum expected payoff? Strategy S_e is said to be a *strong perfect equilibrium* against itself if player A's optimal strategy is the same as S_e.

Lemma 3.1

If a strategy S_e is a strong perfect equilibrium against itself and if there is a positive probability of both types of mistakes on every turn regardless of the node in the game tree and the probability is independent of players, then for any other strategy S_d ($S_d \neq S_e$),

$$E(S_e|\underbrace{S_eS_e, \ldots, S_e}_{n-1}) > E(S_d|\underbrace{S_eS_e, \ldots, S_e}_{n-1}) \quad (15)$$

Proof. Because any group of n strategies have a positive probability of reaching every node in the game tree and S_d is distinct from S_e, S_d must behave differently from S_e at some node and thereafter (i.e., all histories from that node). Since S_e is a strong perfect equilibrium against itself, S_d must get lower expected payoff than that S_e gets at the node and thereafter. Thus Inequality (15) is true.

Theorem 3.2

A strategy S_e satisfying Lemma 3.1 cannot be invaded by any finite mixture of other distinct strategies if the invading strategies are sufficiently rare. In other words, S_e is evolutionarily stable if the invading strategies are sufficiently rare.

Proof. Let a population have strategy S_e and m other invading strategies $I_1, I_2, \ldots, I_m$, and the frequency of each strategy in the population be f_{S_e} and $f_{I_1}, f_{I_2}, \ldots, f_{I_m}$. Then S_e can resist invasion by rare strategies $I_1, I_2, \ldots, I_m$ if it has higher expected payoff than any of them, i.e., for each I_i:

$$f_{S_e}^{n-1} E(S_e \mid \underbrace{S_e, S_e, \ldots, S_e}_{n-1}) + \sum_{j_1, j_2, \ldots, j_{n-1}} \left(\left(\prod_{k=1}^{n-1} f_{j_k} \right) E(S_e \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) \right)$$

$$> f_{S_e}^{n-1} E(I_i \mid \underbrace{S_e, S_e, \ldots, S_e}_{n-1}) + \sum_{j_1, j_2, \ldots, j_{n-1}} \left(\left(\prod_{k=1}^{n-1} f_{j_k} \right) E(I_i \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) \right)$$

where there are at most n-2 S_e values in X_{j_k}, $1 \leq k \leq n-1$ and each X_{j_k} is either S_e or an I_j. Let the number of S_e values in X_{j_k}, $1 \leq k \leq n-1$ be x. The above inequality can be rewritten as

$$f_{S_e}^{x}\Bigg(f_{S_e}^{n-x-1}\Bigg(E(S_e \mid \underbrace{S_e, S_e, \ldots, S_e}_{n-1}) - E(I_i \mid \underbrace{S_e, S_e, \ldots, S_e}_{n-1})\Bigg) + \sum_{j_1, j_2, \ldots, j_{n-1}} \Bigg(\Bigg(\prod_{k=1,\, j_k \neq S_e}^{n-1} f_{j_k}\Bigg) (E(S_e \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) - E(I_i \mid X_{j1}, X_{j2}, \ldots, X_{jn-1}))\Bigg)\Bigg) > 0 \qquad (16)$$

Because the first term on the left hand side of the above inequality is greater than 0 according to Lemma 3.1, the inequality can be satisfied if f_{I_i} values are sufficiently small, i.e., the second term in the inequality is sufficiently small. Note that the second term in Inequality (16) could be negative since we do not assume that

$$E(S_e \mid X_{j_1}, X_{j_2}, \ldots, X_{j_{n-1}}) > E(S_d \mid X_{j1}, X_{j2}, \ldots, X_{jn-1})$$

where S_d is a distinct strategy from S_e and there is at least one X_{j_k} that is distinct from S_e. In a sense, a strong perfect equilibrium in the NIPD is not as strong as it is in the 2IPD .

Lemma 3.1 and Theorem 3.2 have shown a theoretical result about evolutionary stability in the NIPD when mistakes are allowed. We now give a concrete example of ESS, i.e., ALLD. Consider the optimal strategy player A should use when he/she faces $n-1$ ALLD players. Because an ALLD player never intends to cooperate regardless of what others are doing, player A is always better off to defect (even if other players mistakenly cooperate). Thus the best strategy for player A to play with $n-1$ ALLD players is ALLD. Any other distinct strategy from ALLD will have lower expected payoff than ALLD has. This claim can be shown as follows. Let X be a distinct strategy from ALLD. Then there must be some sequence of interactions (i.e., some node in the game tree) that would induce it to cooperate. Because defection is better off than cooperation, ALLD must do better than X at that node and do no worse than X thereafter. Since every sequence of interactions occurs with a positive probability as a result of a positive error probability at each node, the expected payoff of ALLD must be higher than X. Hence ALLD is a strong perfect equilibrium and is the strictly best strategy against $n-1$ other ALLD players. According to Theorem 3.2, ALLD in the NIPD is an ESS if those conditions mentioned in the theorem and Lemma 3.1. are satisfied.

The evolutionary stability of ALLD is rather interesting. It seems to indicate that players would eventually end up with defecting against each other in a group and stay that way. However, this is only true when invading strategies are sufficiently rare according to Inequality (16), a condition that may not be satisfied easily in the real world. Another interesting research issue is to investigate reciprocal strategies which are evolutionarily stable when mistakes are allowed. This is our next step.

4. Concluding remarks

This paper has shown two new theoretical results about ESS in the NIPD. Assuming that no individual makes mistakes in the NIPD, it is shown that no finite mixture of pure strategies can be evolutionarily stable if the probability of further interactions is sufficiently high. In fact, a more general result, i.e., Theorem 2.1 is proven.

The paper also investigates a more realistic model of the NIPD where individuals make mistakes. An individual who intends to cooperate may defect instead and vice versa. Assuming that there is a positive probability of both types of mistakes (D for C or C for D) on every turn regardless of the node in the game tree and the probability is independent of players, we have shown that there exist an ESS which can resist invasion of any finite mixture of other distinct strategies provided these invading strategies are sufficiently rare. ALLD was shown to be such an ESS. Although we believe that there exists an evolutionarily stable reciprocal strategy as well if mistakes are allowed, we are unable to prove it at this stage. This will be a topic for our future research.

The NIPD is a much richer game than the 2IPD. There are a lot of ways to study various issues in the NIPD. Glance and Huberman (1994) have used tools from statistical physics to analyse the NIPD and presented an excellent discussion about the sudden emergence of cooperation in a group where mistakes are allowed. They also studied the relationship between the group size and the evolution of cooperation. Albin (1992) has used cellular automata to model each player in the NIPD. Each player directly interacts only with his/her neighbours who are a subset of the n players. One of our future research topics is to investigate how such neighbourhood structures affect evolutionary stability.

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