Available online at www.sciencedirect.com
SCIENCE DIRECT
Social Networks 26 (2004) 309–321
SOCIAL NETWORKS
www.elsevier.com/locate/socnet

Defectors' niches: prisoner's dilemma game on disordered networks

Masaki Tomochi*

Faculty of Policy Studies, Chuo University, 742-1 Hachioji, Tokyo 192-0393, Japan

Abstract

Effects of disordered networks on evolution of cooperation are studied based on the prisoner's dilemma game on a random regular relational graph. As a parameter q ($0 \le q \le 1$) that controls a degree of randomness of the network varies from zero to unity, the initial two-dimensional square lattice network becomes more relationally randomized while regularity of the network is kept. It is shown that random connections that make possible cooperator jump into defectors' clusters ironically trigger the formation of defectors' niches, in which defectors impose upon cooperators and do not have incentive to change their strategy.

JEL classification: C62; C72; D74

Keywords: Prisoner's dilemma; Spatial game; Random network; Small world

1. Introduction

There exists conflict of interest almost everywhere. For example, Luce and Raiffa (1957, p. 1) says that "[i]n all man's written record there has been a preoccupation with conflict of interest; possibly the topics of God, love, and inner struggle have received comparable attention." Game theory, which separately originated from Borel in 1921 and von Neumann in 1928 and formalized by von Neumann and Morgenstern in 1944, has been regarded as one of the best mathematical theories to analyze situations of conflict of interest (see Luce and

* Tel.: +81 426 74 4171; fax: +81 426 74 4118.
E-mail address: mtomochi@fps.chuo-u.ac.jp.

0378-8733/$ – see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.socnet.2004.08.003

Raiffa, 1957, pp. 2 and 3). Especially, in the game theory, the prisoner's dilemma (PD) game has received much attention due to its wide range of applicability. The PD game, which was found in the experiment conducted by Flood and Dresher in 1950 and formalized by Tucker in 1950, is often utilized to describe a situation where there exist free riders who exploit common properties (see Poundstone, 1992, p. 21; Hardin, 1968). However, at the same time, in the real world, there also exist those who act altruistically even in a situation that can be described by the PD game.

In order to explain coexistence of egoistic and altruistic players in the real world, for example, spatial prisoner's dilemma (SPD) game, in which players who are placed on a network repeatedly play PD games with their neighbors who are located closed to each of them, has been proposed (Axelrod, 1984; Pollock, 1989; Nowak and May, 1992, 1993; Huberman and Glance, 1993; Nowak et al., 1994a, 1994b, 1996; Oliphant, 1994; Herz, 1994; May et al., 1995; Mukherji and Slagle, 1996; Watts, 1999, pp. 199–222; Tomochi and Kono, 2002; Tomochi, 2003, pp. 27–44; Masuda and Aihara, 2003). The SPD game has been especially utilized since human beings as well as many other creatures may be regarded as those who live in either physically or abstractly limited territory in which they interact with their neighbors most likely. This setting is represented by introducing network structure into game theory.

Axelrod (1984) is thought to be the first social scientist who pointed out the importance of the SPD game. Since Axelrod (1984) first suggested the ideas of PD games on a lattice network, SPD games have received much attention. In SPD games, it is possible to observe coexistence of cooperation and defection mainly depending on the elements of the payoff matrix of the PD, and it has been shown that clusters of cooperators play an important role (Axelrod, 1984; Pollock, 1989; Nowak and May, 1992, 1993; Huberman and Glance, 1993; Nowak et al., 1994a, 1994b, 1996; Oliphant, 1994; Herz, 1994; May et al., 1995; Mukherji and Slagle, 1996; Watts, 1999, pp. 199–222; Tomochi and Kono, 2002; Tomochi, 2003, pp. 27–44; Masuda and Aihara, 2003).

There are a number of studies on the effects of several types of orderly neighborhood structures such as the von Neumann and Moore neighborhood (Axelrod, 1984; Pollock, 1989; Nowak and May, 1992, 1993; Huberman and Glance, 1993; Nowak et al., 1994a, 1994b, 1996; Oliphant, 1994; May et al., 1995; Mukherji and Slagle, 1996; Tomochi and Kono, 2002). Neighborhood structure on a hexagonal lattice network has been also examined (Herz, 1994). These studies suggest that there is not much significant difference in the effects between these orderly neighborhood structures.

Although a lot of studies on SPD games with orderly neighborhood networks have been conducted, not much work has been implemented on random networks (1996; Watts, 1999, pp. 199–222; Tomochi, 2003, pp. 27–44; Masuda and Aihara, 2003). After all, it is thought to be natural to think that networks of people often contain a certain degree of randomness, and simplification with regard to utilizing lattice network is to be relaxed.

Watts (1999, pp. 199–222) introduced a series of gradually and randomly rewired one-dimensional lattice network (a.k.a. ring graph) into a SPD game in chapter 8 of his book: "Small World: The Dynamics of Networks between Order and Randomness." Through the randomization process, Watts and Strogatz (1998) and Watts (1999) have found that the initial lattice graph becomes small-world graph that is characterized by the following two properties when the network contains only a few random connections. First, the clustering coefficient of a small-world graph, which measures the degree of local clusterness, is as high as that of the initial lattice graph, that is, the density of the edges is locally high. Second, the characteristic path length of a small-world graph, which is a global measurement of the average distance (shortest step) between any two vertices in the graph, is as short as that of a random graph. The coexistence of these two properties in a graph delivers a small-world graph. According to Watts and Strogatz (1998) and Watts (1999), many of networks existing in the real world possess the properties of small-world graphs. One of the main contributions Watts (1999, pp. 199–222) has made toward SPD games is to provide a new parameter, which is the degree of randomness of the network connection, with the framework of SPD games. Watts (1999, pp. 199–222) showed that cooperative strategies such as generalized Tit-for-Tat and ALL C can dominate the network as long as the degree of randomness is kept low when update dynamics such as “Copycat” and “Win-Stay-Lose-Shift” are used. He also showed that the time the system needs in order to converge to its stable or quasi-stable state becomes shorter as the degree of randomness of the network increases and pointed out that there may exist functional relationship between the fraction of cooperation and the clustering coefficient as well as the speed of convergence and the characteristic path length. He also suggested that variance of the degrees of the vertices (neighbors) in his model, which increases as the initial lattice network is randomized, might cause the decrease of cooperation in the SPD game.

Tomochi (2003, pp. 27–44) used a two-dimensional square lattice network as an initial network and randomly rewired its edges with keeping the degree of all the vertices constant. The graph used can be called as a random regular relational graph (triple R graph), on which the structure of each player’s neighbors with whom s/he plays PD games is disordered. The number of neighbors for each player, however, is the same fixed number for all players, that is, the network constructed by the players contains regularity. The regularity of the graph was used because it helped to amplify the effects of random connections, not of irregularity. The probability of choosing neighbors is uniformly distributed, that is, the graph used is a relationally randomized graph. Utilizing the method of relational randomization enable one to see effects of small-world graphs on the evolution of cooperation based on SPD games. The numerical results Tomochi (2003, pp. 27–44) has obtained show that the fraction of cooperation decreases as the network is more randomized and that the fraction can be expressed as a linear function of the clustering coefficient when the payoffs of the PD game are set as being advantageous for clustered cooperators in the SPD-game setting.

Masuda and Aihara (2003) applied the same type of random-rewiring process as Tomochi (2003, pp. 27–44) toward not only two but also one-dimensional lattice network and also observed decrease of the fraction of cooperation as the network is more randomized when the payoffs of the PD game are set as being profitable for cooperators forming clusters. They also observed that the necessary period for the system to reach its stable or quasi-stable state becomes shorter as the characteristic path length becomes shorter. Masuda and Aihara (2003) concluded that rapid convergence of slightly suboptimal states in which many cooperators survive can be realized on small-world networks.

In this paper, effects of disordered networks on evolution of cooperation are studied based on the PD game on a triple R graph. The main goal of the study is to explain the mechanism behind the observed result that the fraction of cooperation decreases as the network gets randomized and, more precisely, that the fraction can be expressed as a linear function of the clustering coefficient. It will be revealed that random connections that make possible cooperation jump into defectors' clusters ironically cause the formation of defectors' niches around the cooperators. This lead to decrease of cooperators at the system's quasi equilibrium.

In the next section, the model based on a SPD game is presented. In Section 3, the algorithm of generating a triple R graph is explained. Results of the simulations and the investigations on the results are given in Section 4. Discussions are provided in the last section.

2. The model

The size of the population is given by N, and each player is labeled by an integer i where it holds $1 \leq i \leq N$. Each player is supposed to play one-shot PD games with her/his neighbors at each unit time, and therefore, the strategy of a player i at a discrete time t denoted by $\sigma_i(t)$ is either cooperate or defect:

$$\sigma_i(t) = \begin{cases} +1 & \text{if } i \text{ cooperates} \\ -1 & \text{if } i \text{ defects} \end{cases} \tag{1}$$

for all i. The payoff function for i in a game with a player j if denoted as $f_i(\sigma_i(t), \sigma_j(t))$, that is, $f_i(+1, +1) = x$, $f_i(+1, -1) = 0$, $f_i(-1, +1) = 1$, and $f_i(-1, -1) = y$ by utilizing the payoff matrix of the PD game in Table 1. Due to the definition of the PD game, the following inequalities should hold among the four elements in the payoff matrix:

$$0 < y < x < 1 \tag{2}$$

Note that as long as the inequalities (Eq. (2)) hold, Table 1 provides a general form of a payoff matrix of a PD game because only relative values between the four elements in the payoff matrix matter. Now the parameter space for the payoff matrix elements, $\{x, y\}$, is shown in Fig. 1 as the shaded area excluding the boundary. Any point inside the shaded triangle in Fig. 1 lets Table 1 be a payoff matrix of a PD game.

The score of a player i at time t, $u_i(t)$, is defined as the sum of the outcomes of the games with her/his all neighbors denoted by $\tilde{n}(i)$:

$$u_i(t) = \sum_{j \in \tilde{n}(i)} f_i(\sigma_i(t), \sigma_j(t)) \tag{3}$$

Note that the number of the neighbors, denoted as $|\tilde{n}|$, is invariable for all i.

Table 1
Payoff matrix of the PD game with inequalities (2)

	Cooperate	Defect
Cooperate	x, x	0, 1
Defect	1, 0	y, y

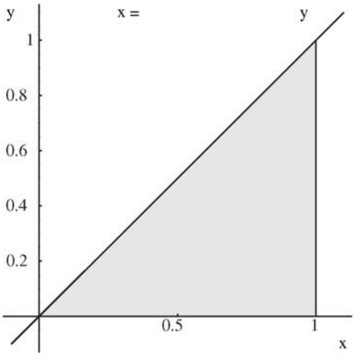

Fig. 1. The parameter space for the payoff matrix elements $\{x, y\}$ (the shaded area excluding the boundary).

The updating rule utilized in this paper is called copycat where each player imitates a strategy of their most successful neighbors' in terms of their scores in Eq. (3). The copycat is adopted as the updating mechanism in this paper because it is commonly observed that people try to imitate a strategy of their most successful neighbors (Axelrod, 1984). Due to the copycat, i's strategy at time $t + 1$ is given as follows:

$$\sigma_i(t+1) = \{\sigma_j(t) | u_j(t) = \max_{j \in n(i)} u_j(t)\} \tag{4}$$

$$= \mathrm{sign}[\max_{j \in c(i)} \{u_j(t)\} - \max_{j \in d(i)} \{u_j(t)\}] \tag{5}$$

where sign[0] = $\sigma_i(t)$ is assumed. The symbol $n(i)$ in Eq. (4) represents i's neighborhood that includes $\tilde{n}(i)$ and i, and $c(i)$ and $d(i)$ in Eq. (5) stand for cooperators and defectors in $n(i)$, respectively.

In the next section, the algorithm, according to which a triple R graph is generated, is presented.

3. The algorithm

The triple R graph used in this paper is constructed by the following algorithm. On the triple R graph, the structure of each player's neighbors with whom s/he plays PD games is disordered depending on a parameter q defended later in this section. The number of neighbors for each player, however, is the same fixed number for all players, that is, the network constructed by these players contains regularity. The randomizing procedure begins with a pre-rewired graph, which is a two-dimensional $nv \times nv$ square lattice graph, and then the edges are randomly rewired as follows:

1. Pick two vertices v_a and v_b randomly. These two vertices are to be picked independently because the graph considered here is a relational graph. The two vertices, v_a and v_b, however, should not be each other's neighbors or share common neighbors.

2. Pick v_a's one of neighbors, v_a', and v_b's one of neighbors, v_b', randomly.
3. Cut the existing edges between v_a and v_a' and v_b and v_b', and newly connect v_a and v_b and v_a' and v_b'. (Note that regularity is preserved at the end of this step.)
4. Repeat the above steps from 1 to 3, Q times, without allowing double rewiring.

In the algorithm introduced above, two edges are always simultaneously rewired after each process. Four distinct vertices are always to be picked to make rewiring two edges possible. The reason why v_a and v_b should not be each other's neighbor or share common neighbors in the step 1 is to make sure that four distinct vertices are always picked during the above random-rewiring procedure.

In this paper, nv is usually set as 101, therefore N is given as 101^2. The periodic boundary condition is used so that we can neglect edge effects. According to one of the most important theorems in the field of random graph theory by Erdös and Rényi (1959), almost any random graph with the condition expressed by the following inequality (6) is guaranteed to have no separated components:

$$k \gtrsim \ln N \tag{6}$$

where k $(= |\tilde{n}|)$ and N denote the number of edges for each vertex (the number of neighbors) and the number of all vertices (the number of the whole population), respectively. In the following, $|\tilde{n}|$ is set as 12 so that the inequality (6) can hold when N is set as 101^2.

A parameter q $(0 \leq q \leq 1)$, which denotes a ratio of rewired edges to all the edges, controls a degree of randomness of the network, that is, as q goes from zero to unity, the initial two-dimensional square lattice network becomes more randomized. Since two edges are always simultaneously rewired, q is given by:

$$q = \frac{2Q}{Nk/2} = \frac{4Q}{Nk} \tag{7}$$

The maximum number of Q, $Q_{\max}$, which is given when $q = 1$ $(=q_{\max})$, is obtained as half the number of all edges, $(Nk/2)/2 = 30{,}603$. However, due to the restriction in the step 1, the actual value of $Q_{\max}$, Q'_{max}, in a simulation is approximately 18,500 that sets a possible value of $q_{\max}$, q'_{max}, as approximately 0.6.

One of the most important indexes of a random graph is a clustering coefficient of a vertex v, γ_v, that is defined as follows:

$$\gamma_v = \frac{|E(\Gamma_v)|}{\binom{k_v}{2}} \tag{8}$$

where $|E(\Gamma_v)|$ denotes the number of total edges in a subgraph Gamma_$\{v\}$ that consists of vertices adjacent to v excluding v itself. The k_v stands for the number of edges of the vertex v, therefore:

$$\binom{K_v}{2}$$

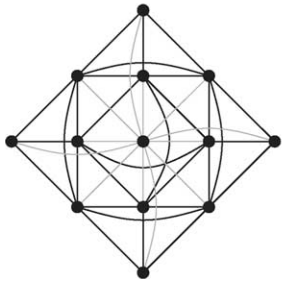

Fig. 2. There are 30 edges in the cluster of 12 nearest neighbors. The clustering coefficient of this case is equal to $30/66 = 0.\dot{4}\dot{5}$.

gives the total number of possible edges in Γ_v. The overall average value of the clustering coefficient, γ, of the pre-rewired two-dimensional square lattice graph with $k_v = k = 12$ and $q = 0$ is therefore given by:

$$\gamma(q = 0) = \frac{30}{66} = 0.\dot{4}\dot{5} \tag{9}$$

(see Fig. 2). The triangular dots in Fig. 3 show typical example of the semilog-plotted average value of the scaled clustering coefficient, $\gamma(q)/\gamma(0)$, of the triple R graph as a function of q. The average value of the clustering coefficient remains only a little less than its maximum value, which is given when q is set as zero, for q up to about 0.01 and starts remarkably dropping down after $q \simeq 0.01$.

Another important index of a random graph is a characteristic path length, L, that is an average distance between any two vertices in a graph. The square dots in Fig. 3 show typical example of the scaled L as a function of q, $L(q)/L(0)$, which is obtained by utilizing random sampling techniques.

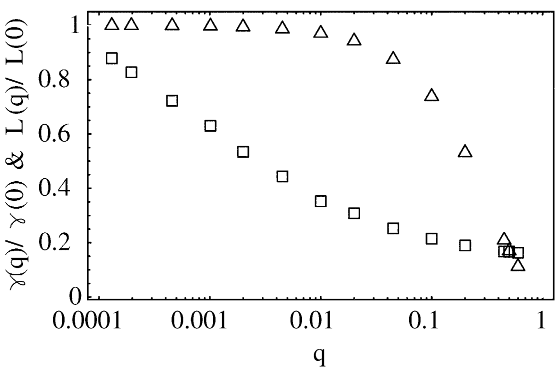

Fig. 3. The average value of scaled clustering coefficient $\gamma(q)/\gamma(0)$ (triangular dots) and of scaled characteristic path length $L(q)/L(0)$ (square dots) is shown as a function of q. The small-world phenomena can be observed in the graph used in this study when q is around 0.01.

According to Watts and Strogatz (1998) and Watts (1999), small-world phenomena are observed when a large value of γ and a small value of L coexist. From Fig. 3, one can say that small-world phenomena can be observed in the triple R graph used in this study when q is around 0.01.

4. Numerical results

For a moment, coupling topology for the players is fixed as the pre-rewired two-dimensional square lattice graph ($q = 0$) with $N = 21^2$ and $k = 12$ in order to microscopically see how defection spreads among cooperators on the two-dimensional square lattice network. In Fig. 4, at $t = 1$, a single defector, who is colored dark gray, is placed at a certain vertex of the pre-rewired lattice graph that is filled with cooperators, who are colored light gray. Defection can keep spreading when $\{x, y\}$ is in the case of Fig. 4a $\{0.6, 0.2\}$, while it cannot when $\{x, y\}$ is in the case of Fig. 4b $\{0.8, 0.2\}$ and Fig. 4c $\{0.8, 0.1\}$. Moreover, when $\{x, y\}$ is set as in Fig. 4c, the number of defectors even decreases sometimes. The parameter space of $\{x, y\}$ for the case of Fig. 4c can be obtain with the aid of Fig. 5a where the scores of the players at $t = 2$ are depicted. The parameter space to be realized is determined by the condition that the scores of the defectors are less than those of cooperators, therefore the defectors are converted to cooperators at the next generation. Hence, from Eq. (5), a boundary dividing the parameter space into that for the case of Fig. 4c and for the

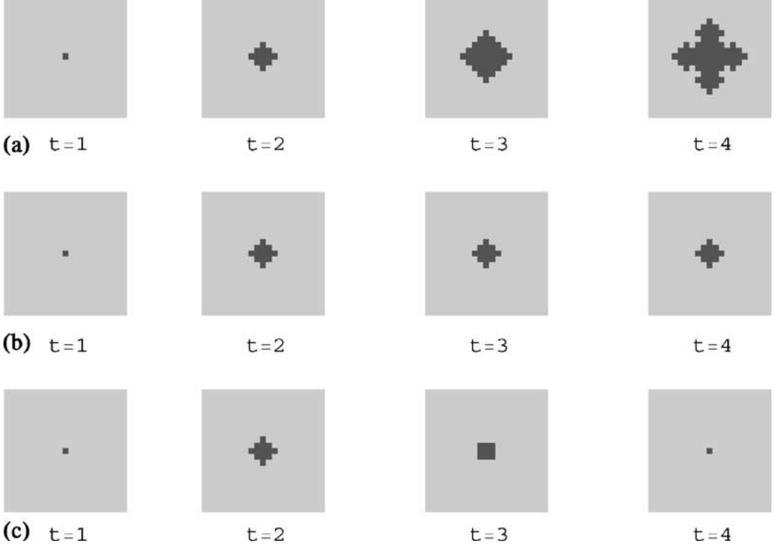

Fig. 4. A single defector is placed at the center of the pre-rewired two-dimensional square lattice graph, which is filled by cooperators. Light gray and dark gray represent cooperators and defectors, respectively. The payoff matrix elements $\{x, y\}$ are given as (a) $\{0.6, 0.2\}$, (c) $\{0.8, 0.2\}$, and (d) $\{0.8, 0.1\}$.

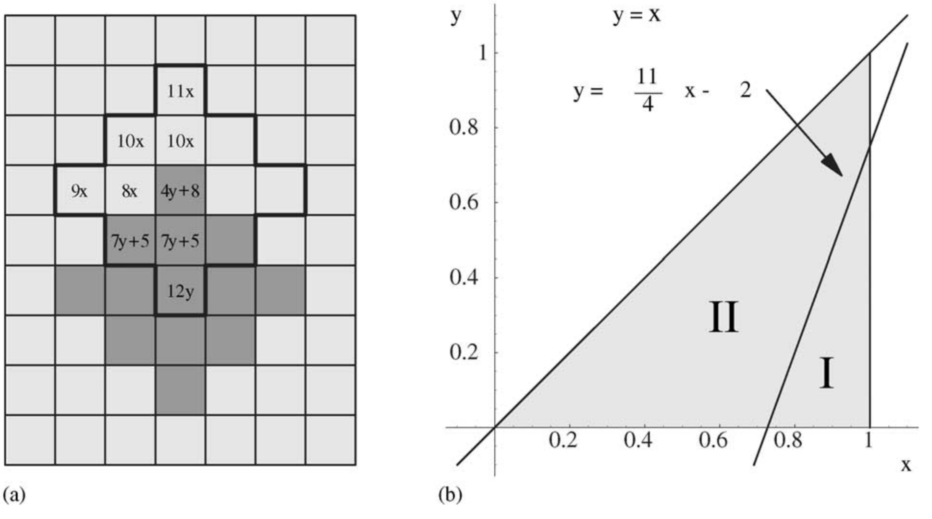

Fig. 5. (a) Players' scores at the second generation in the case of Fig. 4. (b) The parameter space for $\{x, y\}$ is divided into two regions, I and II, by the line $4y + 8 = 11x$.

other cases is given as:

$$\max_{j \in d(i)} \{u_j(t)\} = \max_{j \in c(i)} \{u_j(t)\} \Rightarrow 4y + 8 = 11x \tag{10}$$

In Fig. 5b, when $\{x, y\}$ is chosen from the region named I (and the region II for the remaining parameter space), cooperators cannot be overwhelmed by defectors since the scores of cooperators who are interacting with sufficiently many cooperators to be able to survive are always larger for the payoff, and defectors who are dealing with sufficiently many defectors to suffer a loss and to be forced to change their mind are eventually converted into cooperators.

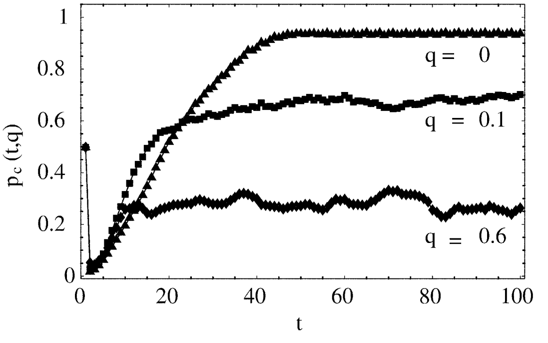

Fig. 6. The evolution of the ratio of cooperators as a function of time t, $p_C(t, q)$, is shown when q is set as various values. The $p_C(1, q)$ and $\{x, y\}$ are given as 0.5 and $\{0.8, 0.1\}$, respectively.

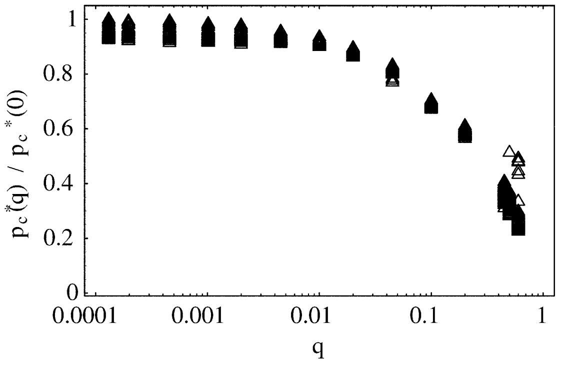

Fig. 7. The scaled value of $p^*_C(q)$, which is obtained by dividing $p^*_C(q)$ by $p^*_C(0)$. One can see that $p^*_C(q)/p^*_C(q)$ and $\gamma(q)/\gamma(0)$ are almost identical.

Since now, N is set as 101^2 again. Fig. 6 shows a typical example of the evolution of the fraction of cooperators as a function of time t, $p_C(t, q)$, when q is set as various values such as 0, 0.1, and 0.6. Initial configuration between cooperators and defectors on the network is randomly determined, and its initial ration between them is given as 1:1, namely, $p_C(1, q) = p_D(1, q) = 0.5$. The payoff elements are set as $\{0.8, 0.1\}$ that is chosen to be in the region I in Fig. 5b, otherwise evolution of cooperation cannot been observed since if $\{x, y\}$ is selected from the region II, then defectors take over the whole network anyhow. In Fig. 6, one can see that $p_C(t, q)$ sooner or later reaches its quasi-stable point and fluctuates after a transition period. Note that the behavior of $p_C(t, q)$ with various values of q has been checked up to $t = 20{,}000$, and it has been numerically confirmed that the fluctuation keeps staying around its quasi-stable point. As Watts (1999, pp. 199–222) and Masuda and Aihara (2003) have observed, it can be seen that the time needed for the system to converge or asymptotically converge is proportion to the characteristic path length that is a decreasing function of q.

Fig. 7 shows the scaled values of the quasi-stable point of the fraction of cooperators, $p^*_C(q)/p^*_C(0)$, with various values of q. Fifty different initial configurations has been attempted in order to obtain Fig. 7. The value $p^*_C(q)$ is obtained as:

$$p^*_C(q) = \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t=t_2} p_C(t, q) \tag{11}$$

where t_1 and t_2 are given as 51 and 100, respectively, namely this gives an average value of $p_C(t, q)$ over time after its transition period given as t_1. Interestingly enough, it is observed that $p^*_C(q)/p^*_C(0)$ linearly decreases as q increases, and in the following, the reason why it does is explained.

When a positive value of q is introduced into a simulation, cooperation can jump into clusters of defectors from clusters of cooperators, that is, some cooperators emerge in some clusters of defectors. This is because the defectors, who belong to the clusters of defectors, can observe and copy the cooperators, who belong to the clusters of cooperators, through the random connections between these two types of players. Note that the observed cooperators in the cooperators' cluster make more score than the observed defectors in the defectors'

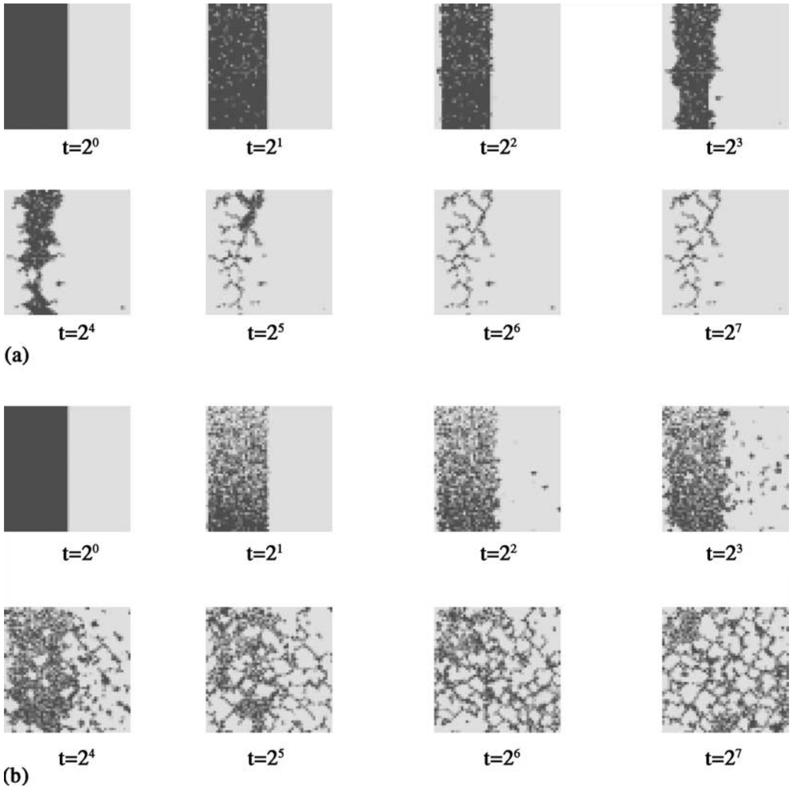

Fig. 8. Snapshots of the game fields with (a) $q = 0.01$ and (b) $q = 0.1$. As the degree of random connection increases, more defectors' niches are formed.

cluster because of Eq. (3) that defines *i*'s score with the given payoff set in the region I. As a result, due to the emerged cooperators, ironically, defectors' niches, in which defectors can exploit cooperators and do not have incentive to change their strategy since their scores are close to the highest available score ($=12T$), are formed in between cooperators' clusters (see Fig. 8a, $q = 0.01$). Note that since the payoff set is in the region I, the number of the event that defectors' niches are formed is proportion to the number of cooperators jumping into the defectors' clusters that is proportion to q. In Fig. 8a, initially cooperators and defectors are placed at the right and left hand side, respectively, since it is easy to see that the formation of the defectors' niches are triggered by the cooperators jumped into the defectors' clusters.

On the other hand, defection can also jump into clusters of cooperators, but in this case, from defectors' niches, that is, some defectors can emerge in some cooperators' clusters, because the cooperators in the cooperators' clusters can observe and copy the defectors in the defectors' niches through random connections between those two types of players (see Fig. 8b, $q = 0.1$). The number of the emergence of defectors is proportion to q. Here, note that defection that jumps into the cooperators' clusters can only stay there and form blinkers such as the one shown in Fig. 4c, but cannot expand due to Eqs. (3) and (5) with the given payoff set in the region I. When q is relatively set as high value, the number of blinkers increase, and this can explain the relatively large fluctuation observed, for example, in the case of $q = 0.6$ in Fig. 6.

Now we know $p^*_C(q) \propto -q$ and $\gamma(q) \propto -q$ that give the linear-functional relationship between $p^*_C(q)$ and $\gamma(q)$. In fact, the correlation coefficient between the scaled values of $p^*_C(q)$ and those of $\gamma(q)$ is nearly equal to unity.

5. Discussion

In this paper effects of disordered networks on the evolution of cooperation have been studied based on the PD game on a triple R graph. As q increases, that is, as the network is getting more disordered, the values of the quasi-stable points of the fraction of cooperators are observed to linearly decrease. One might have to say that it is ironical for those who become cooperators in defectors' clusters due to the random connections since there are defectors who impose upon these cooperators and find their niches. The more the network gets randomized, the larger the defectors' niches grow.

For a future work, a study of the effects of spatial graph in which the probability of choosing a vertex v_b after a vertex v_a is chosen is a function of the metric between these two vertices is also thought to be interesting. The effects of spatial graph where interacting oscillators are embedded according to the Gaussian distribution function is observed by Niebur et al. (1991). According to them, more rapid and robust phase locking is possible on spatially randomized graph. Perhaps, a model where the effects of both relational and spatial graph coexist would give more intriguing results.

Acknowledgements

I am obliged to many people for helpful comments and discussion, but especially to Professor John Paul Boyd at the University of California, Irvine Professor Mitsuo Kono at the Chuo University.

References

Axelrod, R., 1984. The Evolution of Cooperation. Basic Books, New York.

Erdös, P., Rényi, A., 1959. On random graphs I. Publicationes Mathematicae (Debrecen) 6, 290–297.

Hardin, G.R., 1968. The Tragedy of the Commons. Science 162, 1243–1248.

Herz, A.V.M., 1994. Collective phenomena in spatially extended evolutionary games. Journal of Theoretical Biology 169, 65–87.

Huberman, B.A., Glance, N.S., 1993. Evolutionary games and computer simulations, Proceedings of the National Academy Sciences, USA, pp. 7716–7718.

Luce, R.D., Raiffa, H., 1957. Games and Decisions. Wiley, New York.

Masuda, N., Aihara, K., 2003. Spatial prisoner's dilemma optimally played in small-world networks. Physics Letters A 313, 55–61.

May, R.M., Bohoeffer, S., Nowak, M.A., 1995. Spatial games and evolution of cooperation. Advances in Artificial Life 929, 749–759.

Mukherji, A.R.V., Slagle, J.R., 1996. The use of artificially intelligent agents with bounded rationality in the study of economic markets. Artificial Intelligence and Eighth Innovative Applications of Artificial Intelligence Conference, AAAI 96, IAAI 96. AAAI Press/The MIT Press, 1, 102–107.

Niebur, E., Schuster, H.G., Karmann, D.M., Koch, C., 1991. Oscillator-phase coupling for different two-dimensional network connectivities. Physical Review A 44 (10), 6895–6904.

Nowak, M.A., Bohoeffer, S., May, R.M., 1994a. More spatial games. International Journal of Bifurcation and Chaos 4 (1), 33–56.

Nowak, M.A., Bohoeffer, S., May, R.M., 1994b. Spatial games and the maintenance of cooperation, Proceedings of the National Academy Sciences, USA, pp. 4877–4881.

Nowak, M.A., Bohoeffer, S., May, R.M., 1996. Robustness of cooperation. Nature 379, 125–126.

Nowak, M.A., May, R.M., 1992. Evolutionary games and spatial chaos. Nature 359, 826–829.

Nowak, M.A., May, R.M., 1993. The spatial dilemma of evolution. International Journal of Bifurcation and Chaos 3 (1), 35–78.

Oliphant, M., 1994. Evolving cooperation in the non-iterated prisoner's dilemma: the importance of spatial organization, Proceedings of the Fourth Artificial Life Workshop. MIT Press, Cambridge, Massachusetts, pp. 349–352.

Pollock, G.B., 1989. Evolutionary stability of reciprocity in a viscous lattice. Social Networks 11, 175–212.

Poundstone, W., 1992. The Prisoner's Dilemma. Doubleday, New Jersey.

Tomochi, M., 2003. Three Models of Spatial Games: Dynamic Payoffs and Disordered Networks. University of California, Irvine, California.

Tomochi, M., Kono, M., 2002. Spatial prisoner's dilemma games with dynamic payoff matrices. Physical Review E 65 (2), 026112-1-6.

Watts, D.J., 1999. Small World: The Dynamics of Networks between Order and Randomness. Princeton University Press, Princeton, New Jersey.

Watts, D.J., Strogatz, S.H., 1998. Collective dynamics of "small-world" networks. Nature 393, 440–442.