



Promotion of cooperation through co-evolution of networks and strategy in a 2×2 game

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ABSTRACT

A 2×2 game model implemented by co-evolution of both networks and strategies is established. An existing link between two agents is killed through network adaptation, which then establishes a new link to replace it. Strategy is defined as an offer of “cooperation” (C) or “defection” (D) by an agent. Both networks and strategies are synchronously renovated in each simulation time step. After killing the link with the most disadvantageous neighbor, we consider network adaptations that involve rewiring to (1) a randomly selected agent, (2) a proportionally selected agent (through a roulette selection process based on the degrees of respective agents), (3) an agent randomly selected among a set of neighbors of the neighbors, excluding the most disadvantageous neighbor. Several numerical experiments considering various 2×2 game classes, including Prisoner’s Dilemma (PD), Chicken, Leader, and Hero, reveal that the proposed co-evolution mechanism can solve dilemmas in the PD game class. The result of solving a dilemma is the development of mutual-cooperation reciprocity (R reciprocity), which arises through the emergence of several cooperative hub agents, which have many links in a heterogeneous and assortative social network. However, the co-evolution mechanism seems counterproductive in the case of the Leader and Hero game classes, where alternating reciprocity (ST reciprocity) is more demanding. It is also suggested that the assortative and cluster coefficients of a network affect the emergence of cooperation for R reciprocity.

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1. Introduction

The emergence of cooperation in overcoming a dilemma has been explained by several theories. With regard to evolutionary game theory, Nowak [1] identified five mechanisms that make cooperation (C) evolve instead of defection (D)—kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. Network reciprocity relies on two effects; the first is limiting the number of game opponents (depressing anonymity), which leads to a rise in mutual cooperation, and the second is a local adaptation mechanism, in which an agent copies a strategy from a neighbor linked by a network. These explain how C-agents survive in a network game of Prisoner’s Dilemma (PD), even though it requires agents to use only the simplest strategy—either C or D.

In the last few years, many studies have dealt with network reciprocity. Studies on spatial PD played on small-world (SW) topologies (e.g., Refs. [2–5]), and on scale-free (SF) networks (e.g., Refs. [6–10]) have drawn wide interest, because these networks are more complex and seem more realistic than the lattice, random graph, and complete graph networks. In general, in a heterogeneous network, network reciprocity enhances cooperation, except for cases considering game participation cost, as Masuda [11] highlighted. In particular, an SF network works as a strong amplifier of altruism. For

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Table 1Payoff matrix for a 2×2 game.

		Opponent	
		Cooperation	Defect
Ego	Cooperation (C)	<i>R</i>	<i>S</i>
	Defect (D)	<i>T</i>	<i>P</i>

dilemma games on a fixed network, Ohtsuki and colleagues [12–14] showed that network reciprocity is mostly determined by average degree in the case of a Donor–Recipient game (a special type of PD), irrespective of the type of network assumed.

These previous studies are based on a framework where agents are initially allocated in a fixed network. Zimmermann et al. [15] demonstrated a variant of a co-evolution system in a networking game. Their model can consider simultaneous evolution of networks and strategies. The co-evolution model might enable more robust cooperation than fixed network games, and shed some light on what network type is adaptively appropriate for cooperation to emerge. In fact, by applying this model to several PDs, they observed a stable cooperation phase when a cooperative hub agent (which they called C-Leader) emerged, resulting in C-chains. Another significant study, by Pacheco et al. [16,17] deals with simultaneous network and strategy adaptations. They are analytically formulated for situations where the network-updating time scale is much smaller than that for strategy updating; this can also be evaluated by replicator dynamics using a 2×2 game matrix that is revised from the original.

In the co-evolution model for strategies and networks, network adaptation consists of two procedures—severing an existing link and building a new one to replace it. Zimmermann et al. [15] assume that the disconnection of a D–D link occurs with a certain probability, and connection of a new link to a randomly selected agent takes place. They reported that the emergence of cooperation by C-leader hub agents produces a heterogeneous network similar to SF with a power-law degree distribution. By building upon the study by Zimmermann et al. [15], Tanimoto [18] studied other procedures for severing and connecting processes. He insisted that the disconnection of both D–D and D–C links has a wider game area where dilemma can be converted into mutual cooperation. He also found that the preference connection (where higher-degree agents attain even higher degrees, as in the B–A algorithm; Albert & Barabasi [19]) makes emergent networks more heterogeneous. He indicated that the co-evolution model works well for producing *R* reciprocity (valid for PD, Chicken, and Stag Hunt (SH), where mutual cooperation is relevant), but not for *ST* reciprocity (valid for Leader and Hero, where alternating reciprocity (where a focal agent offers C and opponent offers D, and they change roles in the next turn) is relevant [20]). Another study on the co-evolution model by Santos et al. [21] also deals with several dilemma game classes including PD, Chicken, and SH. They comprehensively investigated the influences of time scales and selection pressures of both strategy and network adaptation processes. Feng et al. [22] observed a robust cooperation in their co-evolution model, where the connection is defined to link with a randomly selected neighbor of the disconnected neighbor. This mechanism most likely entails a higher cluster coefficient than preference connections similar to either random connection or B–A, although they did not mention this. However, it seems to be more appropriate that “a new link with a randomly selected neighbor of the neighbors, other than the disconnected neighbor” is created, rather than “a new link with a randomly selected neighbor of the disconnected neighbor”, from the human, real-world viewpoint. In addition, Rong et al. [23] imply that Newman’s assortative coefficient [24] is influential in making cooperation emergent in spatial PDs (note that they only evaluated fixed heterogeneous network games). They discovered that the cooperative phase is more attainable in a heterogeneous network having a negative assortative coefficient than in a network with a positive one. Newman also reported [24] that human social networks, built through intentional human interactions (e.g., sharing relevant information, highly sophisticated relationships in terms of intelligence), have positive assortative coefficients, while natural network systems (including unintentional human social networks) feature negative assortative mixing. Then, the point initially presented by Rong and his colleagues of how assortative mixing affects on cooperation in dilemma games seems very interesting, which can lead us to solve a substantial open question of how and why human cooperation differs from other animal species. To approach the question, the scope should be expanded considering several points. The most important one is to consider dynamics of network, since various networks observed in human and animal societies evolve to adapt themselves to respective environments. To the end, we think that it makes sense to shed some light on the co-evolution model for both strategy and networks covering various network adaptation procedures and dilemma game classes.

In short, these previous studies do not provide a holistic picture of the co-evolution model in terms of (1) which connecting procedure is more efficient, (2) whether *ST* reciprocity can be encouraged by the co-evolution model, (3) what feature is observed in emergent networks when cooperation is obtained, etc. Explaining these is the aim of this paper.

2. Model

Let us assume a 2×2 game, and let the number of agents in the society be N . Table 1 indicates respective payoffs determined by strategies of both focal and opponent agents (either C or D).

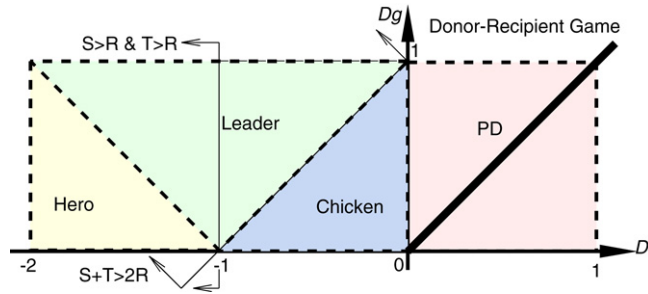


Fig. 1. 2×2 game area used in the experiment. R reciprocity is valid for PD and Chicken, while ST reciprocity is valid for Leader and Hero (in particular, Leader and Hero games for $S > R$ and $T > R$) because $S + T > 2R$. Thick black line indicates Donor–Recipient game, a special class of PD, being satisfied with $D_r = D_g$.

2.1. Game description

We define a 2×2 game space as follows:

$$\begin{aligned}
 P &= -0.5, & (1-1) \\
 R &= 0.5, & (1-2) \\
 S &= P - D_r, & (1-3) \\
 T &= R + D_g & (1-4).
 \end{aligned} \tag{1}$$

We vary the game structure within the limits $-2 \leq D_r \leq 1$ and $0 \leq D_g \leq 1$ in Eq. (1); this includes games such as PD, Chicken, Leader, and Hero (Fig. 1). ST reciprocity is particularly beneficial in the area corresponding to Leader and Hero for $S > R$ and $T > R$. D_g and D_r are game structural parameters that quantify a 2×2 game class [25].

2.2. Agents

To maximize payoff, an agent evolves both his/her strategy and network, and these two adaptation processes operate synchronously. Each agent plays 2×2 games with all agents connected by his/her links. The total payoff is evaluated by summing all games played by a certain agent at a certain time step. Therefore, the more links an agent has, the higher payoff he/she can possibly earn. The average degree of an agent (i.e., average number of links) is denoted by $\langle k \rangle$.

2.3. Strategy adaptation

Each agent deterministically copies the strategy (either C or D) from the neighbor (agents connected by his/her links) who obtained the highest payoff in the previous time step. This is called *imitation dynamics*.

2.4. Network adaptation

At the beginning of the simulation episode, agents are connected by a random network (based on the Erdős–Rényi graph (e.g., Ref. [26])) having $\langle k \rangle$ links. A double-connected link is prohibited.

We assumed that each link is defined by a directed graph. For example, in the case of a link connecting *agent A* to *agent B*, *agent A* can operate the network adaptation procedure but *agent B* can never manipulate this particular link. This assumption might be unrealistic for real social networks. However, in the absence of this assumption, there would be a large number of agents having no neighbors in a co-evolution model, which seems even more unrealistic. The assumption might be justified to some extent when we consider that an individual might have a relationship that he can control and also some that he cannot manipulate in a real human society.

Network adaptation consists of two particular procedures—severing a link with one neighbor and connecting a new link to an unknown agent.

We now define probability p_k for an agent spontaneously severing a certain link controlled by him/her, if it is the link with the minimum payoff, and its payoff is less than $(P + R + S + T)/4$. Following the disconnection, the agent creates a new link with a certain agent; however, the new link is never the same as any existing link. We presume three connecting methods.

Random: Each agent who has severed a link creates a new link with an agent selected randomly from the population.

Preference attachment: Each agent who has severed a link creates a new link with an agent proportionally selected through a roulette selection process, based on the average degrees of respective agents. On the grounds of similarity to the Barabasi–Albert algorithm, this method may encourage a power-law degree distribution, such as an SF network.

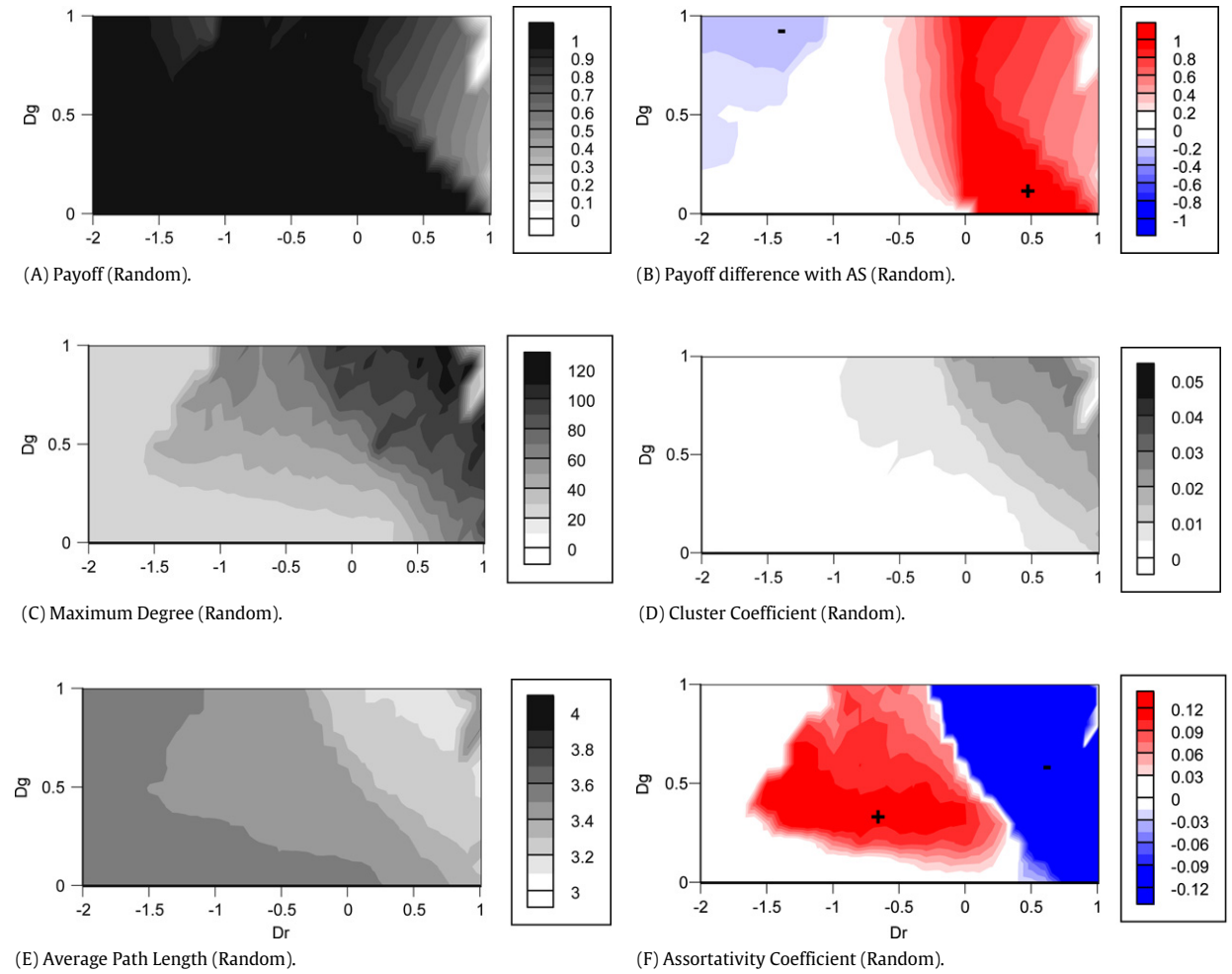


Fig. 2. Results of numerical experiments obtained within limits $-2 \leq D_r \leq 1$ and $0 \leq D_g \leq 1$ in the case of *random* connection; $N = 3000$, $\langle k \rangle = 12$, and $p_k = 0.05$. (A) Average payoff normalized by $(R-P)$, (B) payoff difference using analytic solution with replicator dynamics, (C) maximum degree among agents, (D) averaged cluster coefficient, (E) averaged path length, and (F) assortative coefficient defined by Newman [24].

Neighbor's neighbor: Each agent who has severed a link creates a new link with an agent selected randomly from the neighbors of his neighbors, excluding the disconnected agent. This method may encourage a high cluster coefficient.

3. Numerical procedure

The assumed experimental parameters are $N = 3000$, $\langle k \rangle = 12$, and $p_k = 0.05$. The initial distribution of C , imposed at the beginning state of every simulation episode, is assumed to be 0.5. The results discussed below were confirmed to be robust for these parameters. The contours are drawn by ensemble averages of ten equilibrium trials (quasi-steady state of the dynamics) for respective game structures (we confirmed that a ten-ensemble average is acceptable to observe the general tendency discussed in the following text).

One simulation episode runs until the time-variations of social-averaged cooperation fraction and payoff can be regarded sufficiently small after 6000 time steps, which is equivalent to an asymptotic equilibrium.

4. Results

4.1. Variables in the equilibrium

Fig. 2 shows the results for random connection, including data for (A) average payoff normalized by $(R - P)$, (B) payoff difference with analytic solution, (C) maximum degree among agents, (D) averaged cluster coefficient, (E) averaged path length, and (F) assortative coefficient defined by Newman [24]. Each payoff indicates payoff per single game. An analytical

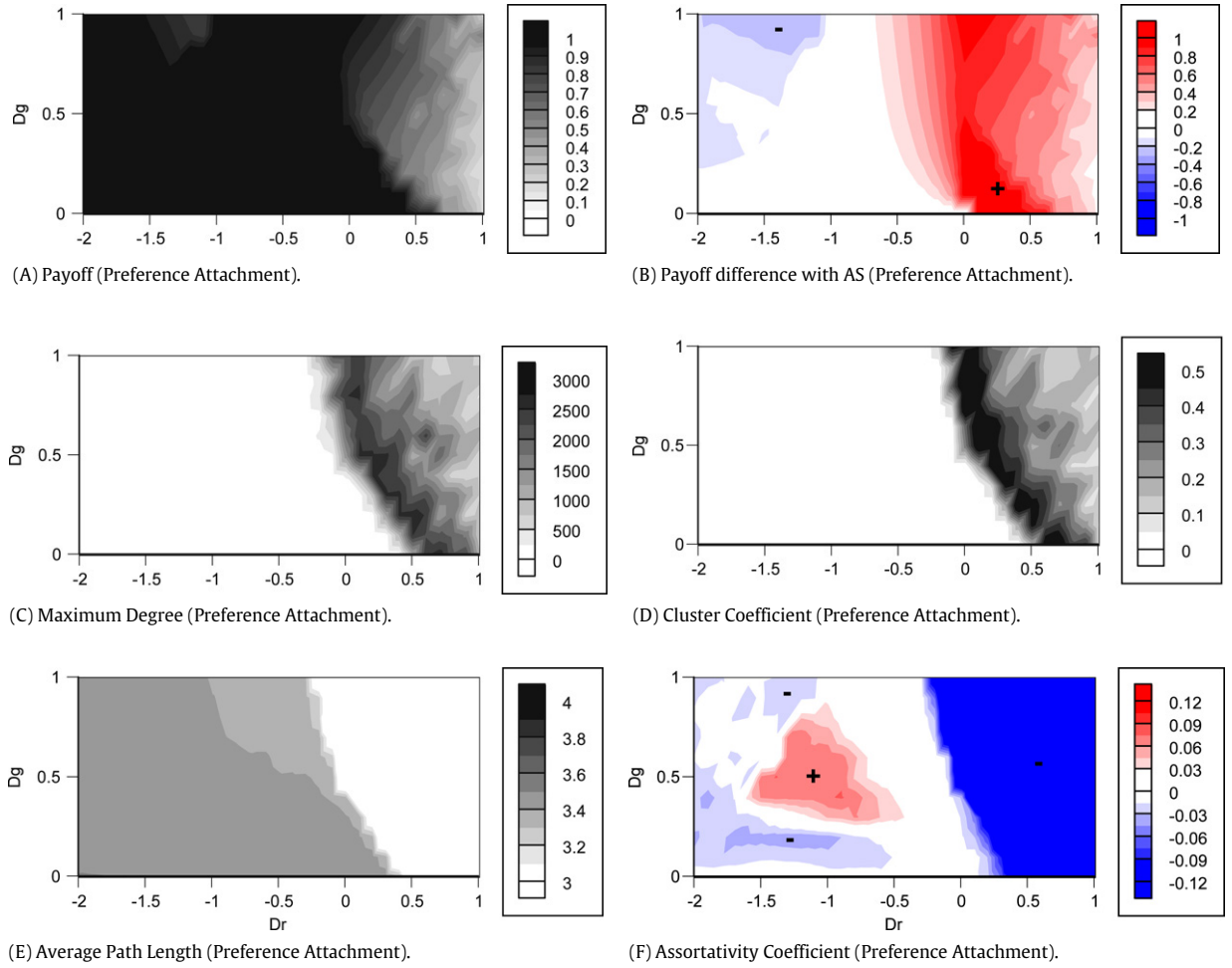


Fig. 3. Results of numerical experiments obtained within limits $-2 \leq D_r \leq 1$ and $0 \leq D_g \leq 1$ in the case of *preference attachment* connection. $N = 3000$, $\langle k \rangle = 12$, and $p_k = 0.05$.

solution (AS) is obtained from the replicator dynamics for a 2×2 game; this means that the control case is obtained without any supporting cooperation mechanisms (such as game iteration, network, memory, and punishment). Based on the AS (e.g., Ref. [25]), every 2×2 game with an infinite population and no supporting cooperation mechanisms can be classified as C-dominant (a trivial game that contains no dilemma), D-dominant (a PD game having both D_g and D_r positive), polymorphic (Chicken-type dilemma game including Leader and Hero only having positive D_g), or bi-stable (an SH-type dilemma game having only D_r).

Figs. 3 and 4 are counterparts of Fig. 2 for preference attachment and neighbor's neighbor connection.

As mentioned above, we assume a directed graph, where only one of the two vertices (agents) on a certain link can sever the connection. This assumption means that there are very few (less than five among 3000) agents who have no links, and therefore, cannot play a game. In absence of this assumption, there can be a large number of solitary agents and few active agents with a large number of links to other active agents, thus creating intensive game plays among a few agents. If an agent can arbitrarily sever any link connected to the agent because the agent is being exploited by his/her opponents, the agent must keep on disconnecting links, which inevitably causes the agent to become solitary, and therefore, preventing the agent from playing the game. In particular, in a stronger dilemma game, this type of two-class situation arises, wherein there are many solitary agents (obtaining no payoff) and few active agents (obtaining large payoffs) at the same time, which seems unrealistic. Therefore, our assumption of each link being a directed graph seems appropriate.

4.2. Degree distribution

Fig. 5 shows the degree distribution of one of ten trials for the neighbor's neighbor connection at $D_r = 0.7$, $D_g = 1.0$ (see Fig. 4(B)). In this particular example, the equilibrium allows co-existing C- and D-agents; however, the society as a whole seems cooperative (cooperation fraction $P_C = 0.675$). D-agents have relatively fewer links while C-agents maintain a relatively higher degree. The higher-degree range of C-agent distribution shows a power-law (SF) tendency.

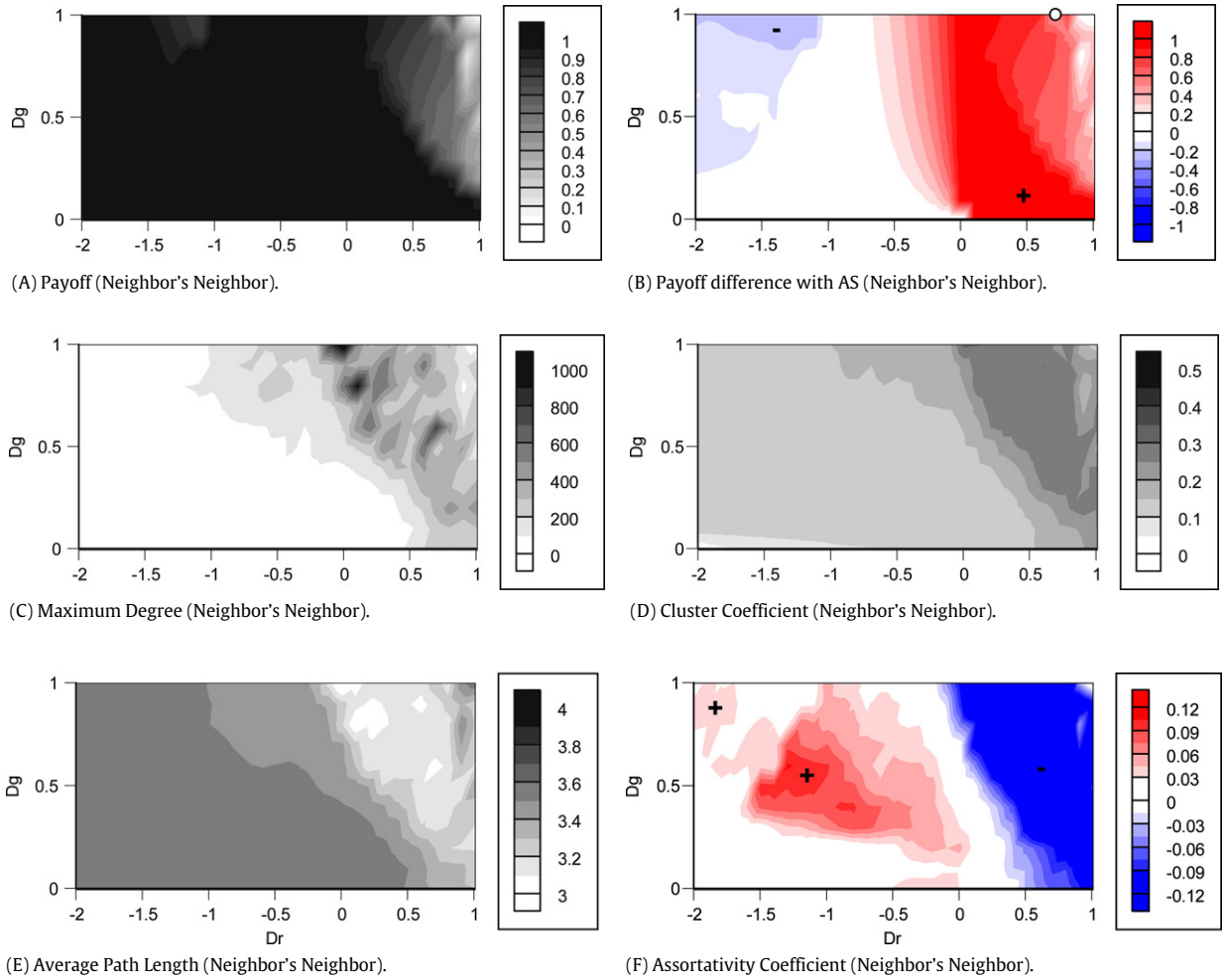


Fig. 4. Results of numerical experiments obtained within limits $-2 \leq D_r \leq 1$ and $0 \leq D_g \leq 1$ in the case of *neighbor's neighbor* connection. $N = 3000$, $(k) = 12$, and $p_k = 0.05$. Open circle in (B) indicates conditions of $D_r = 0.7$, $D_g = 1.0$, for which degree distributions are shown in Fig. 5.

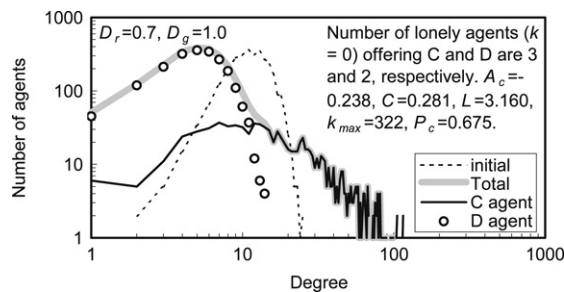


Fig. 5. Degree distribution in the case of *neighbor's neighbor* connection for $D_r = 0.7$, $D_g = 1.0$ (see Fig. 4(B)). Dotted line indicates initial distribution of all agents. Thick gray line, thin black line, and open-circle plot indicate distributions of all agents, C-agents, and D-agents in equilibrium, respectively. Distribution is for only one of the ten trials. Three C-agents and two D-agents are lonely ($k = 0$). Values of assortativity coefficient (A_c), cluster coefficient (C), average path length (L), maximum degree (k_{max}), and cooperation fraction (P_c) at equilibrium are also listed.

5. Discussion

From Figs. 2–4, we can confirm that our co-evolution model produces *R* reciprocity mainly in the PD area (obtaining more payoff than the AS) but can be counterproductive for *ST* reciprocity in Leader and Hero areas (obtaining less payoff than the AS). This is consistent with Tanimoto [18], although he did not consider the neighbor's neighbor connection, and his severing method was defined differently from ours. We confirmed that even the co-evolution model using the neighbor's neighbor connection, and producing a higher cluster coefficient, cannot help *ST* reciprocity in Leader and Hero areas for

$S > R$ and $T > R$. As mentioned above, ST reciprocity requires two agents alternating between C and D to share S and T alternately; this seems more complex than R reciprocity, which demands only always-offering C. Tanimoto [27] reported that ST reciprocity requires a more sophisticated intelligent system than R reciprocity for obtaining time-variable actions by alternately offering C and D. He tried two sets of a 4-bit finite state machine (FSM) to support reciprocity, which can be a model for primitive communication. It is worth considering that ST reciprocity would be possible if co-evolution allowing a time-variable network is implemented. However, in the present study, it is observed that network reciprocity cannot produce ST reciprocity even though we assume a network adaptation system.

Comparing Figs. 2(B), 3(B), and 4(B), the co-evolution mechanism, based on the procedure of connecting with a neighbor's neighbor, shows the best performance in solving the PD dilemma. It is notable that preference attachment shows inferior performance to the neighbor's neighbor connection for stronger dilemmas (close to $D_r = 1$). In this particular area, the neighbor's neighbor connection produces a high cluster coefficient, whereas the preference attachment produces a smaller one. This leads us to suppose that there is a relationship between a high network cluster coefficient and the emergence of R reciprocity in the PD area.

The "solved" PD dilemma area entails a relatively larger cluster coefficient, larger maximum degree, and relatively smaller average path length compared to other game areas, not only for the neighbor's neighbor connection, but also the other two cases. The most remarkable point in our results is that this particular area in PD is consistent with the area having a negative assortativity coefficient (Figs. 2(F), 3(F), and 4(F)). In short, the co-evolution model, whether based on random connection, preference attachment, or neighbor's neighbor connection, can produce R reciprocity to oppress a PD-type dilemma. Such an evolved network features (1) a negative assortativity coefficient, (2) a larger cluster coefficient, (3) a smaller average path length, and (4) a larger maximum degree. This implies that the evolved network with R reciprocity in PD is very heterogeneous. In fact, Fig. 5 implies that the society supports cooperation by causing hub-C-agents to emerge, which is consistent with Zimmermann et al. [15] and Tanimoto [18]. Santos et al. [21] reported that a robust cooperation can be preserved even in the stronger PD area when assuming an appropriately frequent network adaptation instead of strategy adaptation. This dilemma-solved area is consistent with the area having heavily heterogeneous degree distribution, which perhaps implies that the emerged hub-C-agents support cooperation.

6. Conclusions

We established a co-evolution model for strategy and network adaptation. In the network adaptation process, we considered three methods of establishing a connection after disconnecting the most disadvantageous link for the agent—random connection, preference attachment, and neighbor's neighbor connection.

1. The co-evolution model works well for PD with emerging R reciprocity, and the neighbor's neighbor connection shows the best performance.
2. The co-evolution model never works for dilemma games requiring ST reciprocity.
3. Attaining R reciprocity through the co-evolution model features (i) a negative assortativity coefficient, (ii) a larger cluster coefficient, (iii) a smaller average path length, and (iv) a larger maximum degree; this implies that the network evolved by the model is very heterogeneous.

Santos et al. [21] also suggested that high performance of R reciprocity emerges with a larger maximum degree (i.e., in a heterogeneous network). They also viewed various game classes including dilemma games requiring ST reciprocity. However, they used the cooperation fraction for evaluating their co-evolution model. Since the cooperation fraction cannot grasp its effectiveness appropriately, measuring how much payoff the co-evolution system can obtain compared with the analytic solution should be used instead of the cooperation fraction, as used in the present study.

Concerning the relationship between R reciprocity and assortative mixing, Pusch et al. [28] have recently reported that assortative degree–degree correlations can substantially enhance cooperation, which is contradictory to our result even after considering that their model assumes a game on a fixed SF network. In this sense, our result presented here seems to be consistent with that of Rong et al. [23].

Considering the above points, our future study will focus on the time evolution characteristic of an emerging network.

References

- [1] M.A. Nowak, Five rules for the evolution of cooperation, *Science* 314 (2006) 1560–1563.
- [2] N. Masuda, K. Aihara, Spatial prisoner's dilemma optimally played in small-world network, *Physics Letters A* 313 (2003) 55–61.
- [3] C. Hauert, G. Szabo, Game theory and physics, *American Journal of Physics* 73 (2005) 405–414.
- [4] M. Tomochi, Defectors' niches: Prisoner's dilemma game on disordered network, *Social Network* 26 (2004) 309–321.
- [5] J. Ren, W.X. Wang, F. Qi, Randomness enhances cooperation: A resonance-type phenomenon in evolutionary games, *Physical Review E* 75 (2007) #045101.
- [6] J. Gomez-Gardenes, M. Campillo, L.M. Floria, T. Moreno, Dynamical organization of cooperation in complex topologies, *Physical Review Letters* 98 (2007) #108103.
- [7] F. Santos, J. Pacheco, Scale free networks provide a unifying framework for the emergence of cooperation, *Physical Review Letters* 95 (2005) #098104.
- [8] F. Santos, J. Rodrigues, J. Pacheco, Graph topology plays a determinant role in the evolution of cooperation, *Proceedings of the Royal Society B* 273 (2006) 51–55.
- [9] C.L. Tang, W.X. Wang, X. We, B.H. Wang, Effects of average degree on cooperation in networked evolutionary game, *European Physical Journal B* 53 (2006) 411–415.

- [10] J. Poncela, J. Gomez-Gardenes, L.M. Floria, Y. Moreno, Robustness of cooperation in the evolutionary prisoner's dilemma on complex networks, *New Journal of Physics* 9 (2007) #184.
- [11] N. Masuda, Participation costs dismiss the advantage of heterogeneous networks in evolution of cooperation, *Proceedings of the Royal Society B* 274 (2007) 1815–1821.
- [12] H. Ohtsuki, M.A. Nowak, Evolutionary games on cycles, *Proceedings of the Royal Society B* 273 (2006) 2249–2256.
- [13] H. Ohtsuki, M.A. Nowak, The replicator equation on graphs, *Journal of Theoretical Biology* 243 (2006) 86–97.
- [14] H. Ohtsuki, C. Hauert, E. Lieberman, M.A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks, *Nature* 441 (2006) 502–505.
- [15] M.G. Zimmermann, V.M. Eguiluz, Cooperation, social networks, and the emergence of leadership in a prisoner's dilemma with adaptive local interactions, *Physical Review E* 72 (2005) #056118.
- [16] J.M. Pacheco, A. Traulsen, M.A. Nowak, Active linking in evolutionary games, *Journal of Theoretical Biology* 243 (2006) 437–443.
- [17] J.M. Pacheco, A. Traulsen, M.A. Nowak, Coevolution of strategy and structure in complex networks with dynamical linking, *Physical Review Letters* 97 (2006) #258103.
- [18] J. Tanimoto, Dilemma-solving effects by the coevolution of both networks and strategy in a 2×2 game, *Physical Review E* 76 (2007) #021126.
- [19] R. Albert, A.-L. Barabasi, Statistical mechanics of complex networks, *Review of Modern Physics* 74 (2002) 47–97.
- [20] J. Tanimoto, H. Sagara, A study on emergence of Coordinated Alternating Reciprocity in a 2×2 game with 2-memory length strategy, *BioSystems* 90 (3) (2007) 728–737.
- [21] F.C. Santos, J.M. Pacheco, T. Lenaerts, Cooperation prevails when individuals adjust their social ties, *PLoS Computational Biology* 2 (10) (2006) 1284–1291.
- [22] F. Feng, X. Chen, L. Liu, L. Wang, Promotion of cooperation induced by the interplay between structure and game dynamics, *Physica A* 383 (2007) 651–659.
- [23] Z. Rong, X. Li, Z. Wang, Roles of mixing patterns in cooperation on a scale-free networked game, *Physical Review E* 76 (2007) #027101.
- [24] M.E.J. Newman, Assortative mixing in networks, *Physical Review Letters* 89 (20) (2002) #208701.
- [25] J. Tanimoto, H. Sagara, Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game, *BioSystems* 90 (1) (2007) 105–114.
- [26] B. Ballobas, *Random Graphs*, Academic Press, London, 1985.
- [27] J. Tanimoto, Emergence of cooperation supported by communication in a one-shot 2×2 game, in: 2007 IEEE Congress on Evolutionary Computation, 2007, pp. 1374–1381.
- [28] A. Pusch, S. Weber, M. Porto, Impact of topology on the dynamical organization of cooperation in the prisoner's dilemma game, *PRE* 77, #036120, 2008.