Saving Private Jermann and Quadrini [2012]

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This note is based on Jonas' previous notes. The problems are in recursive form to save space.

1 Firm's Problem

$$V(s;k,b) = \max_{d,n,k',b'} \{d + \mathbb{E}m'V(s';k',b')\}$$

subject to

$$\begin{aligned}
(1-\delta)k + F(z,k,n) + \frac{b'}{R} &= wn + \varphi(d) + k' + b \\
\xi\left(k' - \frac{b'(1-\tau)}{R-\tau}\right) &\geq F(z,k,n)
\end{aligned}$$

Let λ and μ be Lagrange Multipliers associated with firm's budget constraint and financial constraint, respectively. The FOCs, besides complementary slackness condition, are given by:

$$\begin{aligned}
\lambda &= \frac{1}{\varphi_d(d)} \\
w &= \left(1 - \frac{\mu}{\lambda}\right) F_n(z,k,n) \\
\lambda - \mu\xi &= \mathbb{E}m'\{\lambda'(1-\delta) + (\lambda' - \mu')F_k(z',k',n')\} \\
\frac{\lambda}{R} - \mu\xi\frac{1-\tau}{R-\tau} &= \mathbb{E}m'\lambda'
\end{aligned}$$

2 Household's Problem

$$\max_{\{n_t,c_t,b_{t+1},s_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t,n_t)$$

subject to

$$w_t n_t + s_t(d_t + p_t) + b_t = \frac{b_{t+1}(1-\tau)}{R_t - \tau} + s_{t+1}p_t + c_t + T_t.$$

The FOCs are given by:

$$\begin{aligned}
w_t &= \frac{-U_n(c_t,n_t)}{U_c(c_t,n_t)} \\
U_c(c_t,n_t) &= \beta\frac{R_t-\tau}{1-\tau}\mathbb{E}_t U_c(c_{t+1},n_{t+1}) \\
p_t U_c(c_t,n_t) &= \beta\mathbb{E}_t(d_{t+1}+p_{t+1})U_c(c_{t+1},n_{t+1})
\end{aligned}$$

3 Government's Problem

... is to balance the budget period-by-period:

$$T_t = \frac{b_{t+1}}{R_t} - \frac{b_{t+1}}{1+r_t}$$

where

$$R_t = 1 + r_t(1-\tau)$$

is the effective return in the perspective of the firm. The government will collect tax today, store them somehow and deliver them as subsidy to the firm tomorrow.

4 Equilibrium Conditions

Assuming the following parametric forms:

$$U(c,n) = \log(c) + \alpha \log(1-n)$$
$$\varphi(d) = d + \kappa(d-\bar{d})^2$$
$$F(z,k,n) = zk^\theta n^{1-\theta},$$

we can write down the equilirium conditon explicitly:

Definition of λ:

$$\lambda = \frac{1}{1+2\kappa(d-\bar{d})}$$

Capital Demand:

$$\frac{1}{c}(\lambda - \mu\xi) = \mathbb{E}\beta\frac{1}{c'}\left[\lambda'(1-\delta) + (\lambda' - \mu')\theta z'(k')^{\theta-1}(n')^{1-\theta}\right]$$

Bond Supply:

$$\frac{1}{c}\left(\frac{\lambda}{R} - \mu\xi\frac{1-\tau}{R-\tau}\right) = \mathbb{E}\beta\frac{1}{c'}\lambda'$$

Bond Demand:

$$\frac{1}{c}\frac{1-\tau}{R-\tau} = \mathbb{E}\beta\frac{1}{c'}$$

Labor Market Clearing:

$$\frac{\alpha c}{1-n} = (1-\frac{\mu}{\lambda})(1-\theta)zk^\theta n^{-\theta}$$

Resource Constraint:

$$(1-\delta)k + zk^\theta n^{1-\theta} = c + k' + \kappa(d-\bar{d})^2$$

Household's Budget Constraint:

$$\frac{ac}{1-n}n + b + d = c + \frac{b'}{R}$$

Complementary Slackness:

$$\mu \geq 0$$
$$\mu\left[\xi k' - \xi\frac{b'(1-\tau)}{R-\tau} - zk^\theta n^{1-\theta}\right] = 0$$

4.1 Today Not Binding

If somehow we know today the financial constraint is not binding, we know immediately

$$\mu = 0$$

$$\xi\left(k' - \frac{b'(1-\tau)}{R-\tau}\right) > F(z,k,n)$$

Then we can rewrite the equilibrium conditons in the Adrian's representation (only true when today is not binding):

Definition of λ:

$$\lambda = \frac{1}{1+2\kappa(d-\bar{d})}$$

Capital Demand:

$$\frac{1}{c}\lambda = \mathbb{E}\beta\frac{1}{c'}m'_k$$

Definition of m_k:

$$m_k = \frac{1}{c}\lambda(1-\delta+\theta z k^{\theta-1}n^{1-\theta})$$

Bond Supply:

$$\frac{1}{c}\frac{\lambda}{R} = \mathbb{E}\beta m'_b$$

Definition of m_b:

$$m_b = \frac{1}{c}\lambda$$

Bond Demand:

$$\frac{1}{c}\frac{1-\tau}{R-\tau} = \mathbb{E}\beta m'_c$$

Definition of m_c:

$$m_c = \frac{1}{c}$$

Labor Market Clearing:

$$\frac{\alpha c}{1-n} = (1-\theta)zk^{\theta}n^{-\theta}$$

Resource Constraint:

$$(1-\delta)k + zk^{\theta}n^{1-\theta} = c + k' + \kappa(d-\bar{d})^2$$

Household's Budget Constraint:

$$\frac{ac}{1-n}n + b + d = c + \frac{b'}{R}$$

Suppose for the moment we know the augmented state $(k, b, z, \xi, m_k, m_b, m_c)$, and then we should be able to find the control variables. However there are two problems.

4.1.1 Problem 1: Ratio of m's

From the equilibrium conditons above, we know from labor market clearing condition and the definition of m_c we find the values of c, n. However $\frac{m_k}{m_b} = 1 - \delta + \theta z k^{\theta-1} n^{1-\theta}$ also pins down the value of n. In practice it's hard to guarantee the augmented state is consistent with this "double determination".

4.1.2 Problem 2: Indetermined b' and R

Suppose problem 1 is not a problem. Then from the augmented state we can directly find c, λ, d, n from the defintion of shadow variables. We can backout investment decision from the resource constraint

$$k' = (1-\delta)k + zk^\theta n^{1-\theta} - c - \kappa(d - \bar{d})^2.$$

We can only find $\frac{b'}{R}$ from the household's budget constraint, but there's no equation to pin down b' and R separately.

4.2 A Solution to Problems

A solution, suggested by Craig and Cosmin during my prospectus presentation is to add a tiny bit of adjustment cost to bond. I tried many things but only this works. We modify the household's budget constraint to be:

$$\frac{ac}{1-n} n + b + d = c + \frac{b'}{R} + \kappa_b \left(\frac{b}{R} - \frac{\bar{b}}{\bar{R}}\right)^2$$

This changes the resource constraint and defintion of m_c:

Resource Constraint:

$$(1-\delta)k + zk^\theta n^{1-\theta} = c + k' + \kappa(d - \bar{d})^2 + \kappa_b \left(\frac{b}{R} - \frac{\bar{b}}{\bar{R}}\right)^2$$

Definition of m_c:

$$m_c = \frac{1}{c}\left[1 - \frac{2\kappa_b}{R}\left(\frac{b}{R} - \frac{\bar{b}}{\bar{R}}\right)\right]$$

This way we can solve the two problems above. If we set κ_b sufficiently close to zero we should be very close to the Jerman Quadrini model. The disadvantage is that when the constraint is binding, finding the control variables given the augmented state involves nonlinear equation solving.

References

Urban Jermann and Vincenzo Quadrini. Macroeconomic effects of financial shocks. *The American Economic Review*, 102(1):238–271, 2012.