What diffraction limit?

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Several approaches are capable of beating the classical 'diffraction limit'. In the optical domain, not only are superlenses a promising choice: concepts such as super-oscillations could provide feasible alternatives.

Research on artificial photonic materials engineered on the subwavelength scale was stimulated a few years ago by the intriguing opportunity to develop media that refract light in the opposite direction to normal media, and that therefore were branded 'negative-index materials'. The term 'metamaterial' quickly came into widespread use to describe this general class of artificial media. Today its meaning encompasses not only negative-index materials[1,2] but also manmade media with all sorts of unusual functionalities that can be achieved by artificial structuring smaller than the length scale of the external stimulus (such as the optical wavelength). This includes metamaterials with exceptionally high[3] or zero[4] refractive indices and high-permeability materials, or optical-frequency 'superconductors' that repel the magnetic field of optical waves in the same way as conventional superconductors repel static magnetic fields[5]. Metamaterials have proved to show fairy-tale properties such as 'invisibility' (as introduced by H. G. Wells in *The Invisible Man*)[6,7], the ability to 'hide' objects in the miraculous manner of Harry Potter's cloak[8,9], and puzzling transmission properties rivalling that of Lewis Carroll's looking-glass[10]. They can act as science-fiction-like electromagnetic force shields, as recently proposed by Graeme Milton, and in flying 'magic carpets' with the use of quantum levitation[11]. However, it was the incredible promise of a Veselago–Pendry optical negative-refraction superlens[12], capable of resolving features beyond the wavelength limit, and possibly even revealing the structure of individual molecules, that was the impetus to mobilize the best research laboratories in the world to work at the interface between 'nano' and 'meta' photonics.

MODERN MICROSCOPES PUSH THE BOUNDARIES

The wide interest in super-resolution was not in itself a surprise, because the increasing finesse of observational instruments has for many centuries been one of the main engines of science and technology. The invention of the compound optical microscope by Hans and Zacharias Janssen in 1590 and its improvement by Robert Hooke, Anthony van Leeuwenhoek and Ernst Karl Abbe revolutionized all aspects of science, especially biology, when it became possible to see bacteria and blood cells, for instance. Since then the Nobel Prize has been awarded several times for the development of new imaging techniques: in 1903 to Richard Zsigmondy for the slit optical ultramicroscope and the study of colloids, in 1953 to Frits Zernike for the phase-contrast biological microscope; and in 1986 to Ernst Ruska for the electron microscope and to Gerd Binnig and Heinrich Rohrer for the scanning tunnelling microscope.

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Recent advances in high-resolution X-ray microscopy with sub-15-nm spatial resolution were underpinned by the development of a sophisticated zone plate for X-ray applications[13]. In the domain of optical instruments, a key step was the invention in 1984 of the scanning near-field optical microscope (SNOM) by Dieter Pohl, Aaron Lewis and co-workers, allowing subwavelength near-field optical resolution for the first time. Here, an image of a structure is created by scanning a physical probe with a subwavelength aperture in close proximity to the illuminated specimen. In fact, the stethoscope, a near-field acoustic imager, has been an indispensable tool of medical practitioners since its invention in 1816 by René Laennec. It also seems that high-resolution near-field scanning-aperture optical imaging is a much earlier invention by E. H. Synge, who described a method "which makes the attainment of a resolution of 0.01μ and even beyond" possible. He published the idea in 1928 (ref. 14), being encouraged by Albert Einstein. Suggested implementation involved "a miniature aperture, whose diameter is approximately 10^{-6} cm, … that is illuminated intensely from below, and is placed immediately beneath" the imaged sample so the transmitted light is than detected by a photoelectric cell and registered by a telephotographic apparatus, synchronized with the motion of the aperture. Recently, the wide application of lasers has also led to the development of several high-resolution nonlinear optical techniques such as two-photon luminescence microscopy and stimulated emission depletion microscopes. Unfortunately these optical microscopy techniques work only with narrow classes of specimens.

Indeed, it has not been possible so far to look inside a living cell, small biological objects or other specimens non-destructively with subwavelength resolution by using low-intensity light and without dependence on specific molecular absorption resonances. Electron microscopy techniques can provide such resolution, but living cells cannot survive the required vacuum, exposure to intense electron beams or the often necessary sample metallization. High-resolution tunnelling and optical scanning microscopes are not capable of seeing internal sections of a living object — or indeed any object — without destroying it: their operation depends on the presence of probes a few nanometres from the feature being imaged. Optical subwavelength resolution is not possible for objects more than about a fraction of a wavelength away from the probe of a scanning instrument. This is why so much interest is being shown in the development of new concepts of optical super-resolution in which it will be possible to image an object located at a somewhat remote distance from the imaging instrument, perhaps even a few tens of micrometres away.

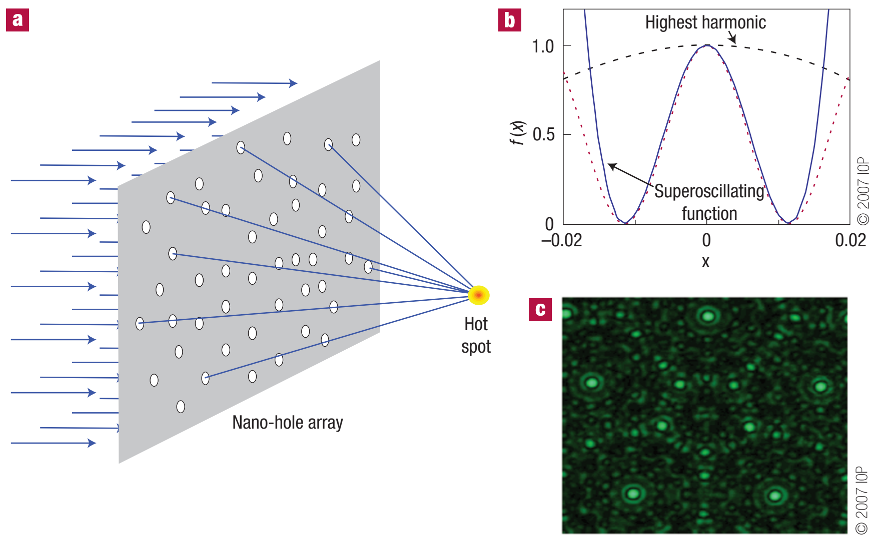

Figure 1 Focusing light with a nanohole array. **a**, An array of nano-holes in a screen as a generator of a superoscillating field. It can create a subwavelength hot-spot when illuminated by a plane monochromatic wave. **b**, A function (equation (1)) superoscillating at $x = 0$. **c**, An example of a 'photonic carpet' generated by quasi-periodic array of holes when subwavelength superoscillating hot spots were observed. Parts **b** and **c** reprinted with permission from ref. 26.

THE SUPERLENS

The remarkable superlens proposed by John Pendry and Victor Veselago is exactly this kind of device allowing super-resolution imaging of a distance object into a far-field image. The superlens is based on the recovery of the quickly fading evanescent fields close to the object by amplifying them in a slab of a negative-index material. Such evanescent, non-propagating fields are commonly believed to be the necessary components for forming subwavelength field concentrations and to achieve subwavelength resolution. Indeed, it is accepted by the photonics community that the resolving power of optical instruments imaging objects located in the far-field, where evanescent waves have faded, cannot be far from that given by the well-known Abbe–Rayleigh rule, according to which the smallest distance between two points that can be distinguished with a lens is about the wavelength of light.

A review by Xiang Zhang and Zhaowei Liu in this issue[15] gives a representative and captivating account of the current status of optical superlens research, from the negative-index superlens and beyond. The bulk negative-index material required for the superlens should simultaneously exhibit negative permeability μ and a negative electric permittivity ε. Apparently, achieving super-resolution when the object and its image are placed in the subwavelength vicinity of the 'lens' is a much simpler task than imaging a remote object. In this case only one material property (ε or μ) of the 'lens' needs to be negative, an eased constraint that led John Pendry to brand this 'lens' a "poor-man's superlens". It was the silver nanolayer-film "poor-man's superlens" that was used in the independent demonstrations of subwavelength resolution in the optical part of the spectrum by the groups of Richard Blaikie[16] and Xiang Zhang[17]. However, this remarkable achievement had limited practical importance: the object and the image would have to be restrictively close, within nanometres, to the metal film. This is why much effort is being concentrated on the development of proper negative-index materials — ones that show negative values of both ε and μ. Despite recent numerous successful demonstrations of such double-negative optical materials, there is still substantial scepticism that such materials can, in the near future, be developed for use in the manufacturing of practical superlenses. The resonance nature of the negative index that is coupled to the problem of losses inheritably limiting the optical bandwidth and transmission of the superlens is the main fundamental obstacle. For this reason, many researchers are now seeking alternatives to the negative-index superlens. For instance, the group of Xiang Zhang came up with an ingenious idea of decoupling evanescent waves on the image side of the "poor-man's superlens" with a grating to achieve a near-field to far-field imaging device[16]. Another detour from the use of bulk negative-index materials is to use anisotropic materials with hyperbolic dispersion: when evanescent waves enter such anisotropic media, their wavevectors are gradually compressed until they become propagating waves that could project a magnified image into the far-field[16]. Although the 'NIM detour' superlens has demonstrated the unique ability of overcoming the 'diffraction limit' the main limitation of all such designs is that the object still has to be in the near-field of the superlens.

LESSONS FROM MICROWAVES

However, there is a solution that can provide subwavelength concentrations of light beyond the near-field. As noted by G. Toraldo di Francia as early as 1952, "fortunately it appears that microwave researchers were not very much concerned, or perhaps even acquainted, with the old well-established theorems of wave optics [on the Abbe/Rayleigh resolution criterion] … As a result, an entirely new theory has been set up, which contains many revolutionary implications"[18]. For several decades the microwave community contemplated the idea of constructing antennas that beat the diffraction limit for directivity. In fact as early in 1922, Oseen, with reference to Einstein's radiation 'needle stick', proved that an arbitrarily large fraction of the emitted energy can be sent into an arbitrarily small solid angle[19].

In 1943, S. A. Shelkunoff published an analysis of the radiation pattern of a linear array of dipoles and proved that, by properly adjusting the individual radiating elements, it is possible to achieve a much narrower radiation pattern than that of a conventionally uniform array[20]. Soon after that Bouwkamp and Bruijn[21], and then Woodward and Lawson[22], were able to prove that there were no theoretical limits to directivity whatsoever.

However, in spite of several cunning suggestions for super-gain antenna designs, the initial enthusiasm for the development of extremely narrow-beam antennas was soon replaced by sober scepticism. The sceptics argued that the extreme sensitivity of the super-directive illumination function to changes in the array design and feed characteristics would make the necessary manufacturing tolerance difficult to achieve and would drastically limit the antenna's bandwidth. Moreover, the sharp increase in the proportion of reactive to radiated power that would be required to achieve super-directivity could mean that the gain improvement might well be offset by the need to provide an even higher increase in power to the antenna to maintain the signal level, thus rendering the concept impracticable.

SUPEROSCILLATIONS

However, the idea of achieving super-resolution without evanescent fields had an independent revival recently in the domain of optics: Berry and Popescu, starting from earlier studies on quantum mechanics, predicted that diffraction on a grating structure could create subwavelength localizations of light that propagate farther into the far field than more familiar evanescent waves[23] (Fig. 1). They related this effect to the fact that band-limited functions are able to oscillate arbitrarily faster than the highest Fourier components that they contain, a phenomenon now known as superoscillation[24]. The superoscillation idea challenges the well-established belief that a function whose Fourier spectrum is bounded can vary no faster than its highest-frequency component.

This astonishing claim is clearly counterintuitive to many and goes against all common experience. However, many examples of simple superoscillating functions have been identified. For instance, a limited series with five components only

$$f(x) = \sum a_n \cos(2\pi \times nx) \qquad (1)$$

can generate superoscillating functions relevant to optical scattering and microwave emission. Thus, $f(x)$ with $a_0 = 1$, $a_1 = 13295000$, $a_2 = -30802818$, $a_3 = 26581909$, $a_4 = -10836909$, $a_5 = 1762818$ and $a_n = 0$ is a superoscillating function. It is plotted in Fig. 1b (solid curve) alongside the highest-frequency component (dashed curve). At $x = 0$ the function has a feature that oscillates nearly nine times faster than its highest-frequency component.

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Aside from the fact that the super-gain antenna aims at creating a narrow beam of electromagnetic radiation while the superoscillation generator seeks to achieve subwavelength localization of light at a distance from the grating, both ideas have the same underlying physics: the tailored interference of several coherent sources (see Fig. 1a,c). However, the task of designing superoscillation in optics could be a much easier problem than designing a super-gain microwave antenna. An array of nano-holes may be used in such a way that superoscillation is achieved a few tens of micrometres away from it by the tailored interference of light penetrating through the holes.

Indeed, microscopy can tolerate much higher losses than communications applications can. If the photon throughput inefficiency of the system is the price to pay for improved resolution, one can reasonably work with only a few detected photons per second, giving about 19 orders of magnitude to play with (a 1-watt laser generates about 10^{19} photons per second). Such a power reserve will in fact be needed because the cost of a decrease in the hot-spot size is a polynomial increase in the power going into the sidebands. The relative intensity and phase stability of emitters coherently excited by one light source are easy to maintain in the optical system. The only serious barrier to the development of optical superoscillation generators is manufacturing accuracy. Today, a hole in a thin metal film on a silica or silicon substrate can be fabricated with a positional accuracy of a few nanometres, which should be sufficient to create optical superoscillation generators that beat the diffraction limit several times over. An improvement by a factor of only a fewfold would be a revolutionary step for observational instrumentation.

An optical generator of superoscillating fields has recently been demonstrated with the use of a Penrose type quasi-crystal array of nano-holes in a thin metal film[25]. When illuminated with a coherent light source it creates a complex diffraction pattern on the other side of array — a few tens of micrometres away. At certain distances these patterns show well-defined, sparsely distributed subwavelength light localizations. Moreover, because such subwavelength localizations are formed by propagating far-fields, they can be projected to the far-field by a conventional lens[26] or used as a subwavelength source in a scanning imaging device for imaging an object located far beyond the near-field area. The question now is whether such a pattern, or for that matter any superoscillating grating-type field generator, could be used as a proper far-field to far-field super-resolution lens and whether it can achieve a subwavelength resolution.

In conclusion, as is often the case, science develops in circles: what the microwave community has understood about loopholes in wave theory but failed to apply in practical super-directive antenna designs, could well be a sensible proposition for optical microscopy: one day, thanks to nanotechnology, a schoolboy will be able to screw a nano-array lens to his science class microscope and see a DNA molecule.

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Acknowledgements

I thank Nader Engheta, Fumin Huang, Mark Dennis, Kishan Dholakia and Javier Garcia de Abajo for discussions.