Comment on "Hawking Radiation from Ultrashort Laser Pulse Filaments"

The authors of [1] claimed to have observed the analog of Hawking radiation created by light pulses in silica glass. They detected radiation in a frequency range in which what they called "phase horizons" existed. The experiment is very interesting and a refined setup might indeed provide the first observation of spontaneous Hawking emission. However, we feel that the cause of the observed radiation is not understood, and it is thus not justified to call this a detection of Hawking radiation.

Hawking radiation is thermal with the temperature being determined by the geometry. It is created in a time-independent system and caused by the (quasi)exponential tearing apart $k_{\rm out} \lll k_{\rm in}$ of the quantum fluctuations near the black-hole horizon. (For white holes it would be compression.) Since the part of the (torn apart) wave packet beyond the horizon can have negative energy, the other part may acquire positive energy and constitutes the Hawking radiation. Out of these five conditions, only the last one (i.e., the negative energy beyond the phase horizon which is related to the Landau criterion, see below) applies to the experiment [1].

Even in the frame of the pulse, the system is not time independent on the time scale $\tau = \mathcal{O}(10\ \text{ps})$ set by the Hawking temperature, see below. The pulse itself lasts at most a few τ and has much faster space-time–dependent substrucutre (as the phase velocity of the pulse differs from the pulse speed). In the pulse frame, comoving photons satisfying Eq. (1) in [1] approach zero frequency because $\omega_{\rm frame}^{\rm pulse} \approx \omega_{\rm frame}^{\rm lab} - \boldsymbol{v}_{\rm pulse} \cdot \boldsymbol{k} \to 0$, making the creation of particles by that time dependence easy from an energetic point of view. (Momentum conservation is another matter.) This is closely related to the Landau criterion for particle creation.

Second, there is no exponential tearing (nor compaction) by the "horizon." Crucially, the group velocity of the photons under consideration is always smaller than the pulse speed [1] and it just passes rapidly over them (i.e., there is no "group velocity horizon"). Consider the dispersion relation $\omega^2 = c^2k^2 + m^2c^4/\hbar^2$ of a massive particle, for example. As in [1], the phase velocity $\omega/k > c$ exceeds the group velocity $d\omega/dk < c$ and a perturbation δc moving with $v > c$ can have phase horizons but no group horizons. In a suitable Lorentz frame, this corresponds to an instantaneous, time-dependent perturbation with no horizon.

Third, the condition $\omega_{\rm frame}^{\rm pulse} \to 0$ applies only to waves (in the frequency range of interest) which are almost colinear with the pulse. Thus, Hawking radiation would occur only in this direction. However, the authors of [1] observed photons perpendicular to this direction. The unpolarized nature of the observed radiation seems to rule out scattering of such comoving radiation as a source.

Fourth, the interpretation of this emission as Hawking radiation yields far too low Hawking temperatures. Even if the spectrum was deformed by dispersion and no longer Planckian, the following estimates for the energy and number of emitted photons would still yield the correct orders of magnitude. In the pulse frame, the Hawking temperature would be given by $T_{\rm Hawking} = \hbar|\partial c/\partial x|/(2\pi k_B)$ which yields $\hbar|\partial \delta n/\partial x|/(2\pi k_B n^2)$, where $n = 1/c$ is the refractive index and δn its variation. Since the Kerr nonlinearity δn is quite small $\delta n \approx 10^{-3}$, this temperature could only create the observed photons at around 800 nm if one assumed that δn changes on a sub-nm length scale, which is unrealistic. The transformation to the lab frame increases the frequency (though not the number) of the photons emitted in forward direction, but this Doppler shift does not apply to any photons emitted to the side.

As Hawking radiation is thermal, the number of particles goes as $\sigma A T_{\rm Hawking}^3 (\Delta\vartheta)^2$. Here $A \sim L^2$ is the horizon area and $\Delta\vartheta$ the solid angle into which the radiation is created. Both are very small: from the phase horizon condition, one has $\Delta\vartheta = \mathcal{O}(10^{-2}\ \text{rad})$ and the core size L of the pulse is a few μm. With $T_{\rm Hawking} \propto \delta n \approx 10^{-3}$, one obtains estimates for the number of created particles [2] which are several orders of magnitude too small to explain the data [1].

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[1] F. Belgiorno *et al.*, Phys. Rev. Lett. **105**, 203901 (2010).
[2] An even simpler perturbation theory estimate for the maximum number of photons emitted per unit time scales with $(\delta n)^2$, showing serious difficulties with the interpretation as a stationary quantum process.

