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Axial superposition of Gaussian spherical beams

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Abstract

A novel model of beam formed by coherent superposition of two correlated Gaussian spherical beams with different centers is presented. This kind of beam has a transverse spreading smaller than a fundamental Gaussian beam. The beam quality factor is smaller than one. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Laser beam propagation; Laser resonators

It was derived by Durnin [1] that the Bessel beam is an exact solution of the scalar-wave equation. Bessel beams are not subject to transverse spreading when it propagates through free-space. In this sense, it has been called 'diffraction-free beams'. The ideal Bessel beam has finite energy density, but infinite energy. Therefore, the ideal Bessel beam cannot be realized in practice. Nevertheless, the quasi-Bessel beams with finite energy and finite beam size have been obtained through various methods [2–7].

On the other hand, it has been shown that the Cosine beam is another kind of 'non-diffraction beam' in rectangular coordinates [8]. Similar to Bessel beams, Cosine beams both of one- and two-dimension, are the exact solutions of the scalar wave equation [9]. Due to the same reason, the ideal Cosine beam cannot be realized. However, the quasi-Cosine beam is demonstrated experimentally [8] and analyzed theoretically [10].

Because of the existence of side lobe of the Bessel and Cosine beams, the beam quality factors of both Bessel and Cosine beams are greater than one. In this paper, we are going to introduce a new model of beam, the Cosine–Gaussian beam, which has a smaller transverse spreading than the Gaussian beam. This is, to our knowledge, the first type of beam with beam quality factor smaller than one in function and mode.

Let us start with the time-independent Helmholtz wave equation in free-space:

$$(\nabla^2 + k^2)E = 0, \tag{1}$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength. Under the condition of far field, i.e., when the distance between the observation point and the source is much larger than the wavelength and the source dimension, the following solution can be derived from Eq. (1) [11]:

$$E(x,\ y,\ z) = E_0 \frac{z}{r^2} \exp(ikr) \exp\left(-\frac{1}{\alpha^2} \frac{x^2 + y^2}{r^2} \right), \tag{2}$$

where α is the divergence angle of the Gaussian spherical beam, $r = \sqrt{x^2 + y^2 + z^2}$. Eq. (2) represents the Gaussian spherical wave with the center at $r = 0$. Eq. (2) is valid even for the large divergence angle α. For an unconfined case, the maximum value of α is 65.5° [11].

If apertures to make the divergence angle a small quantity confine the Gaussian spherical beam, Eq. (2) can be simplified according to paraxial approximation, expressed as:

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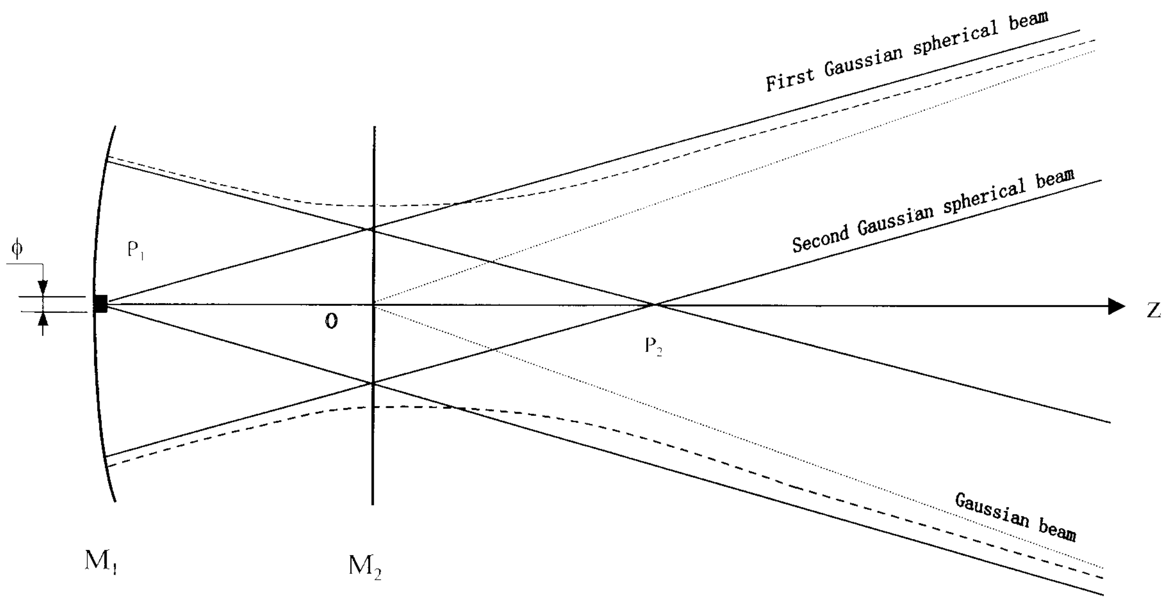

Fig. 1. Schematic of a half-confocal optical resonator with $\lambda/2$ phase plate on the spherical mirror.

$$E(x,\ y,\ z) = E_0\frac{\exp(ikz)}{z}\exp\left(i\frac{k(x^2+y^2)}{2z}\right)\exp\left(-\frac{1}{\alpha^2}\,\frac{x^2+y^2}{z^2}\right). \tag{3}$$

The center of the Gaussian spherical beam given by Eq. (3) is located at $z = 0$. In the following, we will show how to get the confined Gaussian spherical beam in practice. Let us consider a special half-confocal laser resonator whose structure is shown in Fig. 1. This resonator consists of one spherical mirror (M_1) with a radius of curvature R and one plane mirror (M_2). The separation between the two mirrors is $z_0 = R/2$. On the central part of the spherical mirror, there is small phase-step area with a thickness of $\lambda/2$ and a transverse size of $\phi = 0.1$ mm. By using this setup, a CO_2 laser with very good beam quality was achieved [12].

It is easy to check that the light coming from P_1 whose axial coordinate is $-(z_0-\lambda/2)$ reflected by M_2, and then by M_1 will come back to P_1 after two round trips. In other words, the spherical wave with its center located at P_1 is self-reproducing. The Gaussian spherical beam centered at P_1 can be expressed as:

$$E_1(x,\ y,\ z) = E_0\frac{\exp[ik(z+z_0-\lambda/2)]}{z+z_0}\exp\left[i\frac{k(x^2+y^2)}{2(z+z_0)}\right]\exp\left[-\frac{1}{\alpha^2}\,\frac{x^2+y^2}{(z+z_0)^2}\right]. \tag{4}$$

The Gaussian spherical beam centered at P_2 whose axial coordinate is $z_0-\lambda/2$ can be expressed as:

$$E_2(x,\ y,\ z) = E_0\frac{\exp[ik(z-z_0+\lambda/2)]}{z-z_0}\exp\left[i\frac{k(x^2+y^2)}{2(z-z_0)}\right]\exp\left[-\frac{1}{\alpha^2}\,\frac{x^2+y^2}{(z-z_0)^2}\right]. \tag{5}$$

Actually, P_2 is the image of P_1 by mirror M_2. Both $E_1(x,\ y,\ z)$ and $E_2(x,\ y,\ z)$ exist with the resonator. Taking into account the phase relation of them, we can get the total output beam to be:

$$E(x,\ y,\ z) = \tfrac{1}{2}\{E_1(x,\ y,\ z) + E_2(x,\ y,\ z)\exp[ik(2z_0-\lambda/2)]\}. \tag{6}$$

The intensity at distance $z = mz_0$ will be:

$$I(\rho,\ m) = \frac{I_0}{4}\left\{\frac{1}{(m+1)^2}\exp\left(-\frac{2\rho^2}{(m+1)^2\alpha^2 z_0^2}\right) + \frac{1}{(m-1)^2}\exp\left(-\frac{2\rho^2}{(m-1)^2\alpha^2 z_0^2}\right) + \frac{2}{|m^2-1|}\cos\left(\frac{k\rho^2}{(m^2-1)z_0}\right)\exp\left(-\frac{2\rho^2(m^2+1)}{(m^2-1)^2\alpha^2 z_0^2}\right)\right\}, \tag{7}$$

where $I_0 = (E_0^2/z_0^2)$, $\rho = \sqrt{x^2+y^2}$ stands for the transverse coordinate. It should be noted that the points of $m = \pm 1$ are singular, where Eq. (7) is not applicable.

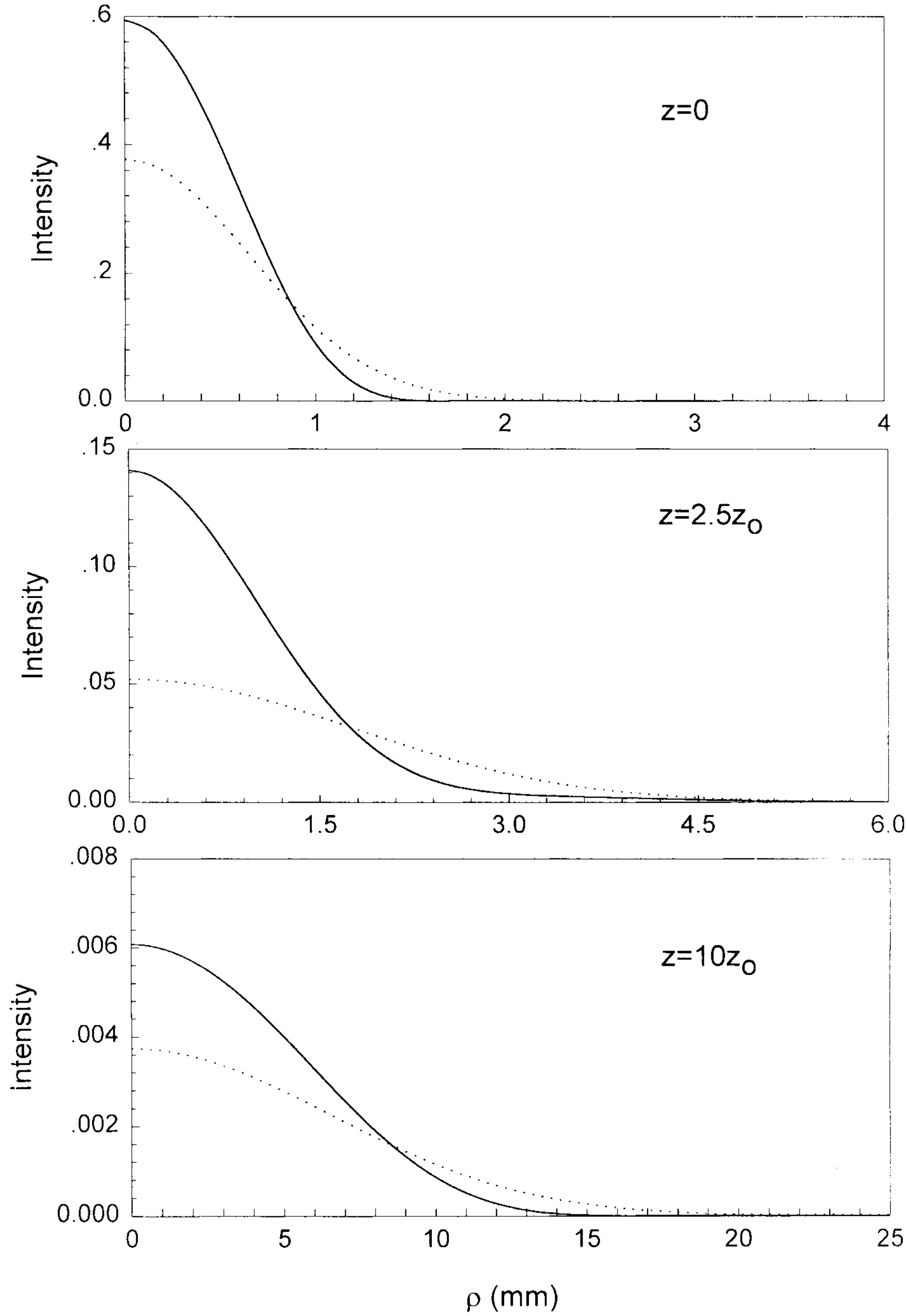

Fig. 2. Transverse intensity distribution of Cosine–Gaussian beam (solid line) and the theoretical Gaussian beam (dotted line) at several propagation distances.

The properties of these two points will be discussed in another paper. In Eq. (7) we have taken into account the axial phase change across the focal point $z=z_0$. Let $m = 0$, we get the intensity distribution at output plane $z = 0$, the result is:

$$I(\rho,\ 0)=I_0\ \cos^2\left(\frac{k\rho^2}{2z_0}\right)\exp\left(-\frac{2\rho^2}{\alpha^2 z_0^2}\right). \tag{8}$$

It is a combination of the Cosine and Gaussian function. That is the reason we call it the Cosine–Gaussian beam.

The intensity distribution of the Cosine–Gaussian beam is shown in Fig. 2 for several propagation distances. In the calculation the parameters are chosen as: $\alpha=0.0018$, $z_0=500$ mm, $\lambda=0.0106$ mm. Where α is determined by the confining aperture in the resonator. The oscillation and competition of modes to compress

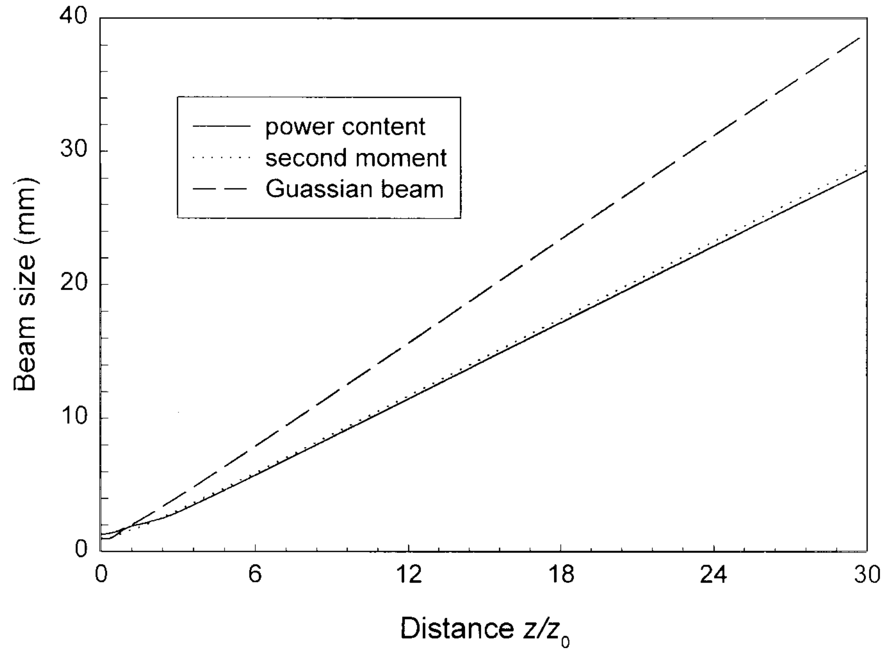

Fig. 3. Beam size of Cosine–Gaussian beam and Gaussian beam under definitions of power content and second moments vs different propagation distances.

the Gaussian mode form a pair of Gaussian spherical beams. For comparison, we draw the theoretical Gaussian beam with beam waist size $w_0 = 1.3$ mm in the same figure. The beam waist size w_0 is calculated by the configuration of the resonator but without a phase-step area.

Fig. 3 shows the beam size of the Cosine–Gaussian beam and the Gaussian beam at several propagation distance under definitions of power content and second moments, respectively. For the Gaussian beam, these two definitions lead to the same results. Using the concept of the beam quality factor M^2 defined by Siegman [13],

$$M^2 = w\theta\frac{\pi}{\lambda}, \tag{9}$$

where w and θ are the beam waist size and far field divergence angle under the definition of second

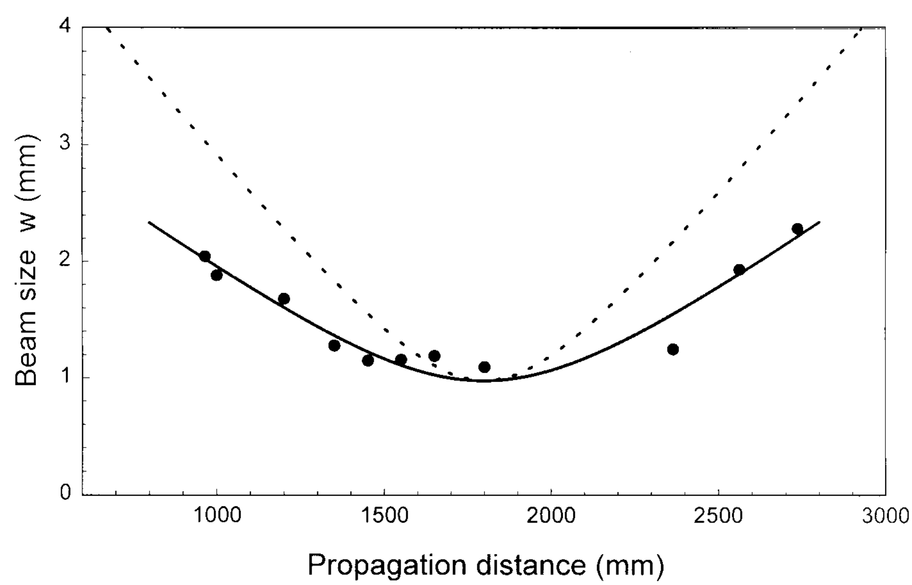

Fig. 4. Measured beam size by variable aperture method (dark circular points) and its hyperbolic regression (solid line) after transformation through a thin lens with focal length 1 m, dotted line is the hyperbolic of an ideal Gaussian beam with the same waist size.

moments, respectively. For the Cosine–Gaussian beam, the beam size $w = 0.97$ at $z = 0$, and $\theta = 0.0019$. Therefore, $M^2 = 0.55$ for the Cosine–Gaussian beam under consideration.

Because the output beam is not pure Gaussian, it is more practical to use the definition of 86.5% power content to measure the beam size. In order to compare the beam quality of the Cosine–Gaussian beam and the Gaussian beam, we use the equivalent beam quality factor M_e^2 defined as follows

$$M_e^2 = w_{86.5}\theta_{86.5}\frac{\pi}{\lambda}, \tag{10}$$

where $w_{86.5}$ and $\theta_{86.5}$ are the equivalent beam waist size and far field divergence angle under the definition of 86.5% power content, respectively. The equivalent beam size for the Cosine–Gaussian beam is $w_{86.5} = 0.96$ mm at $z = 0$, and the equivalent far-field divergence angle is $\theta_{86.5} = 0.0019$. So the equivalent beam quality factor of the Cosine–Gaussian beam is 0.54. These results coincide with the measurement of a CO_2 laser [12] based on the ISO standard [14].

According to previous knowledge, the beam quality factor M^2 must be greater than or equal to unity. It seems contradictory to the results of this paper. However, this is because the previous results are based on second intensity moments which do not take into account the coherent superposition of beams. In fact, the clipped Gaussian beams also have the properties of $M^2 < 1$ [15–17].

Fig. 4 is the measured results of the new beam CO_2 laser based on the ISO standard, which was done by the Chinese Academy of Metrology [18]. The measured beam quality factor is 0.62, which coincides with our simulation in this paper. For the sake of convenience, and comparison, the propagation of an ideal Gaussian beam with the same waist size is depicted in this figure.

In conclusion, we have given a novel model of a laser beam, the Cosine–Gaussian beam, which has a small transverse spreading than the ideal Gaussian beams. The beam quality factor of the Cosine–Gaussian beam can be smaller than one. This model of a beam can explain the existing experimental results satisfactorily. The Cosine–Gaussian beams have advantages in many applications such as cutting, drilling, alignment, etc., due to their small divergence angle.

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