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Invisibility cloak without singularity

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An elliptical invisible cloak is proposed using a coordinate transformation in the elliptical-cylindrical coordinate system, which crushes the cloaked object to a line segment instead of a point. The elliptical cloak is reduced to a nearly circular cloak if the elliptical focus becomes very small. The advantage of the proposed invisibility cloak is that none of the parameters is singular and the changing range of all parameters is relatively small. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3026532]

Great attention has been paid to the electromagnetic (EM) cloaks due to the exciting property of invisibility.[1–15] The coordinate transformation method to control the EM fields was reported in Ref. 1. The cloaking principle was experimentally demonstrated using reduced constitutive parameters in the microwave regime.[2] Many further theoretical and numerical studies have been devoted in the past two years.[3–11] A big problem in the full-parameter cloak is that some parameter components exist singularity on the inner boundary of the cloak, which makes the full cloak difficult to achieve even by using metamaterials.[2] Recently, the elliptical cloaks have been designed and investigated in different coordinate systems.[12–14] However, the cloak parameters are fully anisotropic and some components still have singular points on the inner boundary. In principle, the above circular and elliptical cloaks try to crush the cloaked objects to a point, which results in the singular parameters.

In view of the difficulty to realize the full-parameter cloak and the imperfection of reduced-parameter cloak, a recently published theory has suggested a carpet cloak, which can hide any objects under a metamaterial carpet.[15] Different from the completely invisible cloak, the carpet cloak crushes the hidden object to a conducting sheet. The great advantage of the carpet cloak is that it does not require singular values for the material parameters. However, the carpet cloak can only hide objects placed under a conducting plane and cannot hide objects in free space.

In this work, we propose an elliptical invisible cloak using the coordinate transformation in the classical elliptical-cylindrical coordinate system. Instead of shrinking the cloaked object to a point, the proposed cloak crushes the object to a line segment, which avoids any singularities in the constitutive parameters. Closed-form formulations are derived for both permittivity and permeability tensors. If the focus of the ellipse is very small, the elliptical cloak approaches a circular cloak. The advantage of the proposed invisibility cloak is that none of the parameters is singular and the changing range of all parameters is relatively small.

We consider the elliptical-cylindrical cloak in two dimensions. Due to the elliptical shape of the cloak, we construct the coordinate transformation in the classical elliptically cylindrical coordinate system (ξ,η,z), whose relationship to the Cartesian coordinates (x,y,z) is written as

$$x = p\cosh\xi\cos\eta,\quad y = p\sinh\xi\sin\eta,\quad z = z, \tag{1}$$

in which $2p$ is the focus of the ellipse, as shown in Fig. 1. In the elliptical-cylindrical coordinate system, if we assume p to be constant, then isolines for ξ can be a series of elliptical-cylindrical shells with the same focus value. Then a spatial transformation from the elliptical region $\xi\in[0,\xi_2]$ to the annular region $\xi'\in[\xi_1,\xi_2]$ can be represented mathematically as

$$\xi' = \frac{\xi_2-\xi_1}{\xi_2}\xi+\xi_1,\quad \eta'=\eta,\quad z'=z, \tag{2}$$

where ξ_1 and ξ_2 are coordinate parameters of the inner and outer boundaries of the elliptical cloak. Let a_1 and a_2 denote the lengths of major axes for inner and outer shells of the cloak. The nonlinear relationship between coordinate parameters and the lengths of major axis can be expressed as $\xi_i = \ln(a_i/p+\sqrt{(a_i/p)^2-1})$, $i=1,2$. We remark that the inner and outer ellipses are of the same focus value $2p$. Hence the inner boundary will be crushed to the line segment $2p$ using the coordinate transformation.

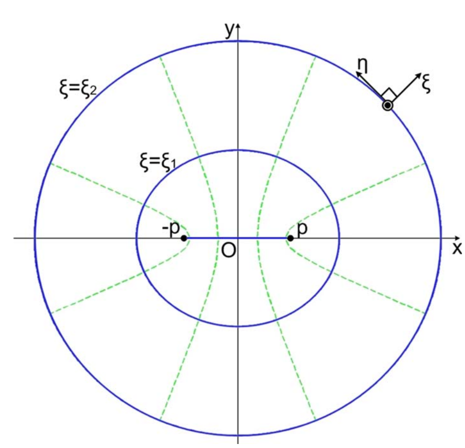

FIG. 1. (Color online) An elliptical cloak in the elliptical coordinate system. The blue (solid) and green (dashed) lines represent constant ξ and η contours, respectively. The cloaked object is crushed to a line segment $2p$.

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Similar to the procedure stated in Ref. 1, one can deduce the parameter tensors of the elliptical cloak. The relative permittivity and permeability tensors are expressed as

$$\varepsilon'_{\xi'} = \mu'_{\xi'} = \frac{\xi_2 - \xi_1}{\xi_2}, \tag{3}$$

$$\varepsilon'_{\eta'} = \mu'_{\eta'} = \frac{\xi_2}{\xi_2 - \xi_1}, \tag{4}$$

$$\varepsilon'_{z'} = \mu'_{z'} = \frac{\xi_2}{\xi_2 - \xi_1}\frac{\cosh^2 \beta - \cos^2 \eta'}{\cosh^2 \xi' - \cos^2 \eta'}, \tag{5}$$

in which $\beta = \xi_2(\xi' - \xi_1)/(\xi_2 - \xi_1)$, $\xi_1 \le \xi' \le \xi_2$, and $0 \le \eta' \le 2\pi$.

Equations (3)–(5) provide full design parameters for the elliptical cloak in the classical elliptically cylindrical coordinates. Clearly, the cloak is composed of inhomogeneous and uniaxially anisotropic metamaterials. For circularly cylindrical cloaks with full parameters, singular material parameters are distributed on the inner boundary,[4–11] which are difficult to realize in the actual applications.[2] For elliptical cloaks that shrink the cloaked objects to a point, singular values still exist on the inner boundary of the cloaks.[12–14] The material parameters for the proposed elliptical-cylindrical cloak that crushes the cloaked object to the line segment $2p$, however, have no singularity. This makes it possible to realize the full-parameter cloak using metamaterials.

In order to validate the elliptical cloak with the designed parameters, we make full-wave simulations based on the method of finite elements. Either TE-polarized or TM-polarized time-harmonic incident plane waves can be used. In the case of TE polarization, only μ_ξ, μ_η, and ε_z components of the material parameters are required for the simulations; in the case of TM polarization, however, we only need ε_ξ, ε_η, and μ_z components. In this work, we only consider the TM case for space reason. The working frequency is chosen as 9 GHz.

The example we considered is a perfectly electrical conducting (PEC) cylinder covered by the elliptical cloak, in which the lengths of major axes for inner and outer ellipses are 0.025 and 0.05 m, respectively, and half of the focus length is $p = 0.015$ m. We show that the magnetic fields are smoothly excluded from the interior region with the minimal scattering in any direction when the incident direction of plane waves is parallel to the long axis of the ellipse. When the incident waves are perpendicular to the long axis, however, a big scattering outside the cloaking region is observed. The physical reason for the incident angle dependence is that the cloaked object is crushed to the line segment $2p$, instead of a point. In the first case, the incident direction is parallel to the line segment, which produces the minimum scattering; in the second case, the incident direction is vertical to the line segment, which produces the maximum scattering.

When the focus length $2p$ of the elliptical cloak approaches zero, however, the line segment $2p$ would shrink to a point. As a result, the elliptical cloak would approach a circular cloak and the cloaking effect would become much better. Nevertheless, p cannot be zero due to the elliptical coordinate transformation (1). Hence we select p as a small value.

When p is chosen as a small value, the elliptical cloak is nearly a circular cloak with outer radius R_2 and inner radius R_1. In such a case, ε_ξ becomes ε_r and ε_η becomes ε_ϕ, indicating the radian and angular components of the permittivity. In order to compare with the ordinary circular cloak,[4–11] in the following simulations, we choose the shape of cloak as an exact circle, while the material parameters are given by

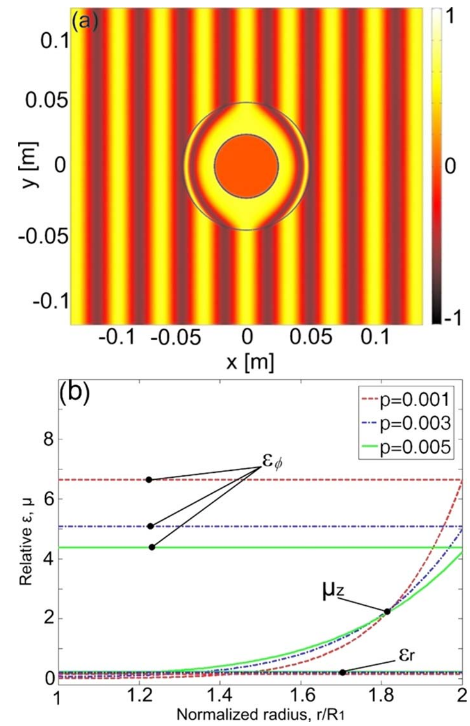

FIG. 2. (Color online) (a) The distributions of magnetic fields inside circular cloaks with $R_1 = 0.025$ m, $R_2 = 0.05$ m, and $p = 0.001$ m. (b) The distributions of ε_ξ, ε_η, and μ_z components in the cloak region.

$$\varepsilon'_r = \mu'_r = k, \tag{6}$$

$$\varepsilon'_\phi = \mu'_\phi = \frac{1}{k}, \tag{7}$$

$$\varepsilon'_z = \mu'_z = \frac{\cosh^2 \beta}{k \cosh^2 \xi}, \tag{8}$$

in which $k = (\xi_2 - \xi_1)/\xi_2$, $\beta = (\xi - \xi_1)/k$, $\xi = \ln(r/p + \sqrt{(r/p)^2 - 1})$, and $\xi_i = \ln(R_i/p + \sqrt{(R_i/p)^2 - 1})$, $i = 1, 2$.

The full-wave simulation result for a circular TM cloak with $p = 0.001$ m is illustrated in Fig. 2(a). We observe that the phase fronts are bent smoothly around the PEC object inside the cloak, and the fields are smoothly excluded from the interior region with the tiny scattering. Similar phenomena are observed for other small values of p, for example, $p = 0.003$ and 0.005 m (not shown).

All material parameters for the above circular cloaks have relatively small ranges. Figure 2(b) illustrates the parameter distributions inside the TM cloaks when $p = 0.001$, 0.003, and 0.005 m. Obviously, for each p, ε_r and ε_ϕ are constants, and μ_z ranges from 0 to a small constant, all of which can be realized using metamaterials. From Fig. 2(a), we clearly observe that the proposed cloak with smaller p can achieve better cloaking performance because the cloaked object is crushed nearly to a point, which results in tiny scattered fields.

In the earlier example, the material parameters are gradiently distributed in the circular region, which are difficult

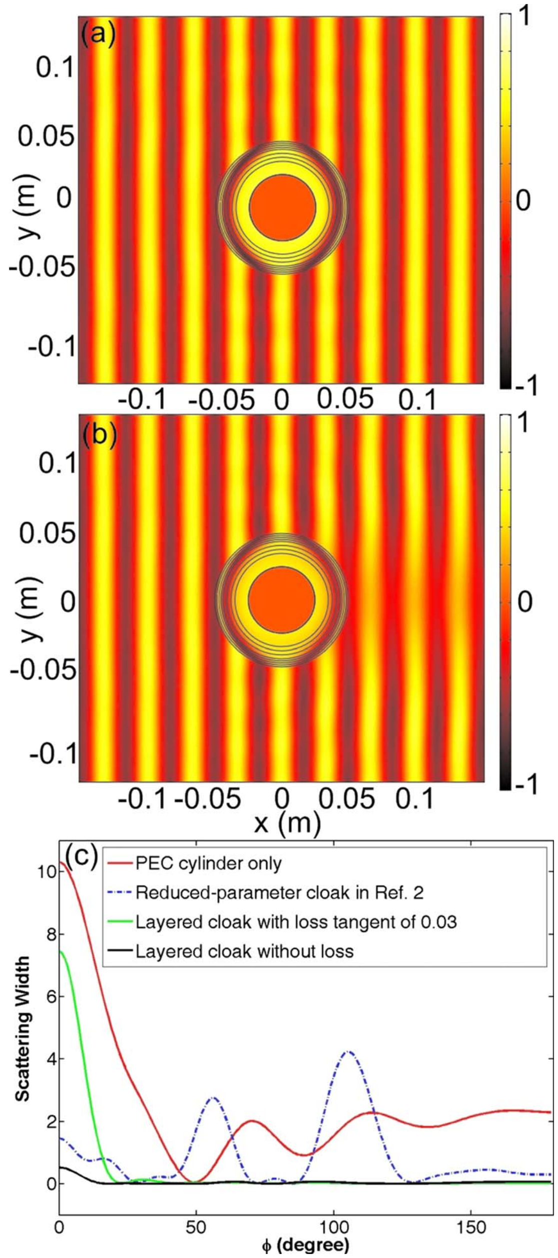

FIG. 3. (Color online) (a) The full-wave simulation results for a layered lossless homogeneous cloak, in which $R_1=0.025$ m and $R_2=0.05$ m. (b) The full-wave simulation results for the layered homogeneous cloak with electric and magnetic-loss tangents of 0.03. (c) Scattering width for the PEC cylinder only, the PEC cylinder with the reduced-parameter cloak (Ref. 2), the layered homogeneous cloak, and the lossy layered cloak.

to implement in real experiments. In order to design a practical cloak, we consider a layered homogeneous circular cloak that is divided into eight layers. When we choose $p=0.001$ m, the permittivity components in all layers are constants, $\varepsilon_r=0.151$ and $\varepsilon_\phi=6.641$, while the permeability component μ_z is given by 0.025, 0.193, 0.466, 1.063, 1.90, 2.756, 3.95, and 5.60 in all layers. We remark that we choose different thicknesses for the eight layers due to the nonlinear distribution of the permeability.

Figure 3(a) illustrates the simulation results of the layered cloak. We clearly observe a good performance of the invisible cloaking although we applied the designed parameters for a nearly circular cloak to a layered circular cloak. From Fig. 3(a), the cloak forces the incoming plane waves to propagate around the inner cloaked region, and such waves return to their original propagation directions without distorting the waves outside the cloak.

In real applications, artificial metamaterials are always lossy. Hence it is important to study the lossy effect of cloak on the invisible property. Figure 3(b) shows the magnetic-field distributions for the cloak with electric- and magnetic-loss tangents of 0.03. From this figure, although the inhomogeneous cloak has been partitioned into eight homogeneous layers and both the permittivity and permeability have a relatively large loss, we still observe good overall invisibility property except in the forward-scattering direction.

The present cloak can be realized experimentally by designing proper metamaterial structures. To achieve the constant ε_r, we can use the electric resonant structures that align in the r-z plane. To realize the varying μ_z, we can use the split-ring resonators that align in the r-ϕ plane. The constant $\varepsilon_\phi=6.641$ can be obtained using nonresonant structures such as wires or I shapes. It will be a hard work to optimize the overall design to make a compact layout of cylindrical cloak.

Compared with the full-parameter cloak,[1] the advantage of the proposed invisibility cloak is that none of the parameters is singular and the changing range of all parameters is relatively small. To compare with the reduced-parameter cloak,[2] we compute the scattering width for the PEC target only and the PEC target covered by the reduced cloak,[2] the proposed layered cloak without loss, and the layered cloak with electric- and magnetic-loss tangents of 0.03, as shown in Fig. 3(c). Clearly, the proposed cloak has a much better overall performance of invisibility.

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