Fully parallel, high-speed incoherent optical method for performing discrete Fourier transforms

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An incoherent optical data-processing method is described, which has the potential for performing discrete Fourier transforms of short length at rates far exceeding those afforded by both special-purpose digital hardware and representative coherent optical processors.

We report here on an incoherent optical method for performing discrete Fourier transforms (DFT's), which has the potential for an extremely high data-throughput rate. The DFT operation may be viewed as a process of multiplying an input vector $\mathbf{f}$ (consisting of N possibly complex-valued input samples) times an $N \times N$ matrix $\mathcal{H}$ [the n,mth element being $\exp(-j2\pi nm/N)$] to yield an output vector $\mathbf{g}$ (consisting of the N complex Fourier coefficients); thus we desire to perform

$$\mathbf{g} = \mathcal{H}\mathbf{f}. \tag{1}$$

Two separate issues must be addressed in describing the method of interest here: (1) How do we perform the matrix product in a highly parallel and fast way? (2) How do we perform complex arithmetic using incoherent light, for which only nonnegative and real quantities (intensities) can be manipulated?

To address the first issue, suppose that the elements of $\mathbf{f}$ and $\mathcal{H}$ are nonnegative and real. Then the system shown in Fig. 1 can be used to perform the matrix–vector product. The elements of $\mathbf{f}$ are entered in parallel by controlling the intensities of N light-emitting diodes (LED's). Lenses L_1 and L_2 image the LED array horizontally onto the matrix mask M while spreading the light from any single LED vertically to fill an entire column of the matrix mask. Lens L_3 is a field lens. The matrix mask M consists of $N \times N$ subcells, each containing a transparent area proportional to one of the matrix elements. Lens L_4 is a cylindrical lenslet array, which is not essential to the operation of the system but which can be used to improve light efficiency. Lens combination L_5 collects all light from a given row and brings it to focus on one element of a vertical array of N photodetectors. Each photodetector measures the value of one component of the output vector $\mathbf{g}$.

To permit the multiplication of a matrix $\mathcal{H}$ with complex elements times a vector $\mathbf{f}$ with complex elements, we decompose each of these quantities as follows[1,2]:

$$\mathbf{f} = \mathbf{f}^{(0)} + \mathbf{f}^{(1)} \exp(j2\pi/3) + \mathbf{f}^{(2)} \exp(j4\pi/3),$$
$$\mathcal{H} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} \exp(j2\pi/3) + \mathcal{H}^{(2)} \exp(j4\pi/3), \tag{2}$$

where $\mathbf{f}^{(0)}$, $\mathbf{f}^{(1)}$, and $\mathbf{f}^{(2)}$ each consist of N real and nonnegative elements, and $\mathcal{H}^{(0)}$, $\mathcal{H}^{(1)}$, and $\mathcal{H}^{(2)}$ consist of $N \times N$ real and nonnegative elements. If the output vector $\mathbf{g}$ is similarly decomposed, then we find that the overall matrix–vector product can be expressed as

$$\begin{bmatrix} \mathbf{g}^{(0)} \\ \mathbf{g}^{(1)} \\ \mathbf{g}^{(2)} \end{bmatrix} = \begin{bmatrix} \mathcal{H}^{(0)} & \mathcal{H}^{(2)} & \mathcal{H}^{(1)} \\ \mathcal{H}^{(1)} & \mathcal{H}^{(0)} & \mathcal{H}^{(2)} \\ \mathcal{H}^{(2)} & \mathcal{H}^{(1)} & \mathcal{H}^{(0)} \end{bmatrix} \begin{bmatrix} \mathbf{f}^{(0)} \\ \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \end{bmatrix}. \tag{3}$$

Thus, complex operations can be performed at a price of a factor of 3 in the length of the input and output vectors.

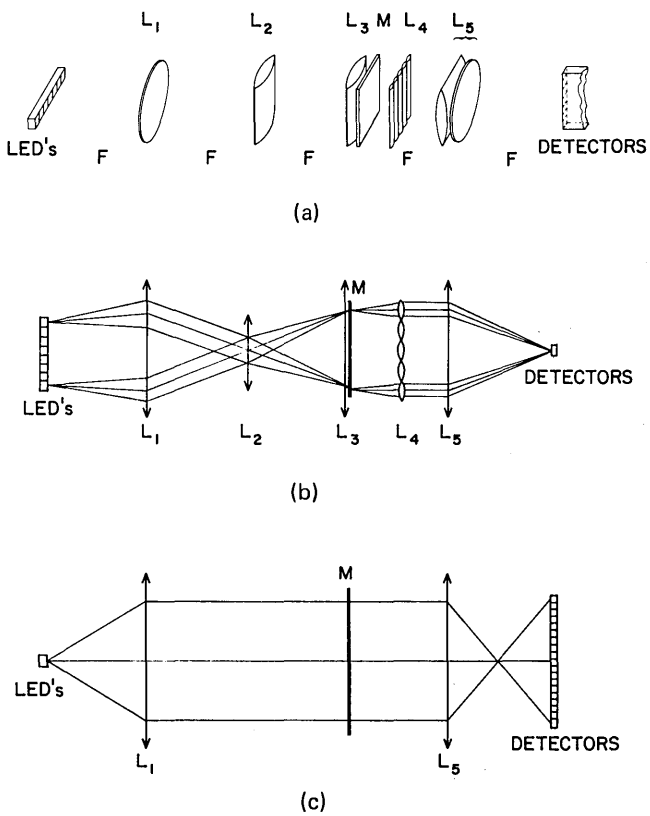

Fig. 1. Incoherent optical processor configuration. (a), pictorial view; (b), top view; (c), side view.

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Simple electronic circuits for producing the components $\mathbf{f}^{(0)}$, $\mathbf{f}^{(1)}$, and $\mathbf{f}^{(2)}$ from $\mathbf{f}$ exist,[1] as do simple circuits for producing the real and imaginary parts of $\mathbf{g}$ from $\mathbf{g}^{(0)}$, $\mathbf{g}^{(1)}$, and $\mathbf{g}^{(2)}$.

Experiments have been carried out to verify the ability to perform complex arithmetic. The source was an unfiltered, linear-filament, clear-envelope, incandescent bulb. The 30 × 30 matrix mask used to perform a 10-point DFT is shown in Fig. 2. This mask is designed so that the three entire vectors $\mathbf{f}^{(0)}$, $\mathbf{f}^{(1)}$, and $\mathbf{f}^{(2)}$ are entered side by side, whereas the three output components $g_k^{(0)}$, $g_k^{(1)}$, and $g_k^{(2)}$ for the kth Fourier coefficient appear side by side. Thus the output display shows each DFT component as a triplet of real and nonnegative components.

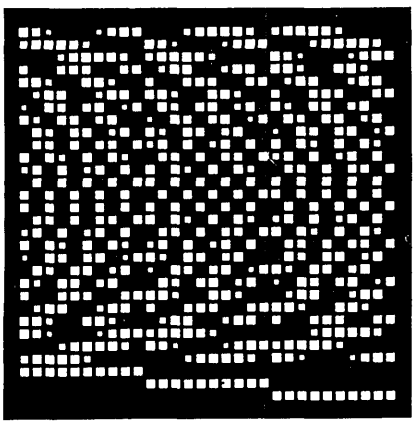

Fig. 2. Matrix mask for a 10-point DFT.

For this experiment the input functions were entered by hand as masks placed against the matrix mask, and output functions were detected on a 1024-element Reticon CCD detector array. Figure 3 shows both theoretical output distributions and experimentally obtained output distributions, the latter being photographed from an oscilloscope display. In parts (a) and (b), the function to be transformed consists of the sequence (1,0,0,0,0,0,0,0,0,0). The resulting DFT should be entirely real and of constant magnitude. As shown in these figures, the DFT components along the real axis are all nonzero and equal, whereas the components along 120° and 240° are all zero.

In parts (c) and (d), the input sequence was entirely real and constant. The DFT consists of a large, real zero-frequency component (on the far right), followed by triplets of equal strength for all other DFT components. Some thought shows that any DFT component with elements $g_k^{(0)}$, $g_k^{(1)}$, and $g_k^{(2)}$ exactly equal is equivalent to a zero result. Hence all DFT components, except the zero-frequency component, are zero.

Parts (e) and (f) show the results when the entire

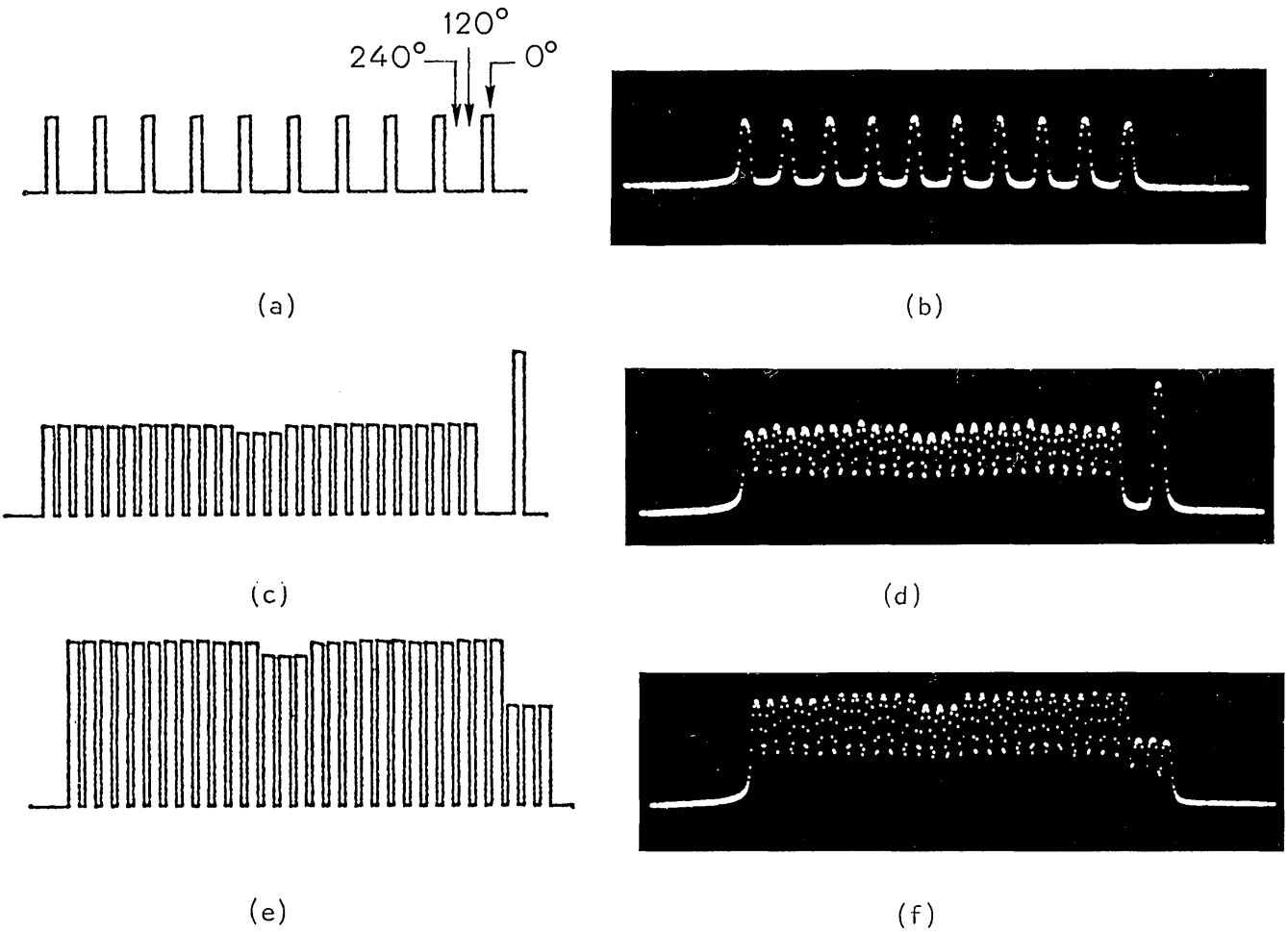

Fig. 3. Theoretical [(a), (c), (e)] and experimental [(b), (d), (f)] DFT results.

matrix mask is uniformly illuminated. In this case, some thought shows that the input is effectively a sequence containing all zeros. The output DFT shows triplets of equal strength, or a sequence of all zeros for the output.

A system composed of 96 high-speed LED's and 96 avalanche photodiodes would be capable of performing a 32-point DFT. Commercially available components have sufficient bandwidth, output power, and sensitivity to permit such a DFT to be performed every 10 nsec. The total throughput rate for such a processor is about 3×10^9 complex samples per second, whereas a corresponding number for special-purpose digital array processors is about 3×10^5 complex samples per second and a representative coherent optical processor[3] has a throughput of 3×10^7 real samples per second.

The chief significance of this processor is that the input data can be entered in parallel, and it is this fact that leads to its high throughput rate. Another system recently described[4,5] performs a similar matrix–vector product, but the data must be entered serially, and as a consequence the throughput rate is much lower. The processor described here is especially well suited for problems in which the elements of the input vector **f** are gathered by parallel sensors. Of course, matrices other than the DFT matrix can also be used if desired.

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References

1. J. W. Goodman and L. M. Woody, "Method for performing complex-valued linear operations on complex-valued data using incoherent light," Appl. Opt. **16,** 2611 (1977).
2. If one is sufficiently clever in eliminating unwanted terms at the output, real and imaginary components on biases can be used. However, the dynamic range of the system is reduced by such an approach.
3. We refer specifically to a system with an electron-beam addressed DKDP input light valve, which is capable of entering 10^6 data points 30 times per second. See D. Casasent, Proc. IEEE **65,** 143 (1977).
4. R. P. Bocker, Appl. Opt. **13,** 1670 (1974).
5. M. A. Monahan, K. Bromley, and R. P. Bocker, Proc. IEEE **65,** 121 (1977).